**Basic algebra and graphing for electric circuits**

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Resources and methods for learning about these subjects (list a few here, in preparation for your research):

**Questions**

Question 1

When evaluating (calculating) a mathematical expression, what order should you do the various expressions in? In other words, which comes first: multiplication, division, addition, subtraction, powers, roots, parentheses, etc.; and then what comes after that, and after that?

file 03052

Question 2

Follow proper order of operations to evaluate these expressions:

$$\frac{13+2}{3}+8= \qquad\qquad 25+(3+2)^2\times 2=$$

file 03320

Question 3

Follow proper order of operations to evaluate these expressions:

$$\frac{15-3}{3}+7= \qquad\qquad 20+(1+3)^2\times 3=$$

file 03321

Question 4

When evaluating an expression such as this, it is very important to follow proper order of operations. Otherwise, the correct result will be impossible to arrive at:

$$3\log 2^5+14$$

To show what the proper order of operations is for this expression, I show it being evaluated *step by step* here[†]:

$$3\log 2^5+14$$

$$3\log 32+14$$

$$3\times 1.5051+14$$

$$4.5154+14$$

$$18.5154$$

Do the same for each of the following expressions:

[†] By the way, this is a highly recommended practice for those struggling with mathematical principles: *document each and every step* by re-writing the expression. Although it takes more paper and more effort, it will save you from needless error and frustration!

- $10 - 25 \times 2 + 5$
- $-8 + 10^3 \times 51$
- $12^4 \times (3 + 11)$
- $21^{(7-4)} \times 40$
- $\log \sqrt{6 + 35^2}$
- $\sqrt{\left(\frac{220}{16} - 2.75\right) \times 2}$

file 03053

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Question 5

Observe the following equivalence:

$$4^3 \times 4^2 = (4 \times 4 \times 4) \times (4 \times 4)$$

Since all operations are the same (multiplication) and reversible, the parentheses are not needed. Therefore, we may write the expression like this:

$$4 \times 4 \times 4 \times 4 \times 4$$

Of course, the simplest way to write this is $4^5$, since there are five 4's multiplied together.

Expand each of these expressions so that there are no exponents either:

- $3^5 \times 3^2 =$
- $10^4 \times 10^3 =$
- $8^2 \times 8^3 =$
- $20^1 \times 20^2 =$

After expanding each of these expressions, re-write each one in simplest form: one number to a power, just like the final form of the example given ($4^5$). From these examples, what pattern do you see with exponents of products. In other words, what is the general solution to the following expression?

$$a^m \times a^n =$$

file 03054

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Question 6

Observe the following equivalence:

$$\frac{4^3}{4^2} = \frac{4 \times 4 \times 4}{4 \times 4}$$

It should be readily apparent that we may cancel out two quantities from both top and bottom of the fraction, so in the end we are left with this:

$$\frac{4}{1}$$

Re-writing this using exponents, we get $4^1$.

Expand each of these expressions so that there are no exponents either:

- $\frac{3^5}{3^2} =$
- $\frac{10^6}{10^4} =$
- $\frac{8^7}{8^3} =$
- $\frac{20^5}{20^4} =$

After expanding each of these expressions, re-write each one in simplest form: one number to a power, just like the final form of the example given ($4^1$). From these examples, what pattern do you see with exponents of products. In other words, what is the general solution to the following expression?

$$\frac{a^m}{a^n} =$$

file 03055

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Question 7

Observe the following equivalence:

$$\frac{4^2}{4^3} = \frac{4 \times 4}{4 \times 4 \times 4}$$

It should be readily apparent that we may cancel out two quantities from both top and bottom of the fraction, so in the end we are left with this:

$$\frac{1}{4}$$

Following the rule of $\frac{a^m}{a^n} = a^{m-n}$, the reduction of $\frac{4^2}{4^3}$ should be $4^{-1}$. Many students find this confusing, as the intuitive concept of exponents (how many times a number is to be multiplied by itself) fails here. How in the world do we multiply 4 by itself -1 times?!

Expand each of these expressions so that there are no exponents either:

- $\frac{3^2}{3^5} =$
- $\frac{10^4}{10^6} =$
- $\frac{8^3}{8^7} =$
- $\frac{20^4}{20^5} =$

After expanding each of these expressions, re-write each one in simplest form: one number to a power, just like the final form of the example given ($4^{-1}$), following the rule $\frac{a^m}{a^n} = a^{m-n}$. From these examples, what easy-to-understand definition can you think of to describe negative exponents?

Also, expand the following expression so there are no exponents, then re-write the result in exponent form following the rule $\frac{a^m}{a^n} = a^{m-n}$:

$$\frac{5^3}{5^3}$$

What does this tell you about exponents of zero?

file 03056

Question 8

Perform the following calculations:

$$\frac{8^{12}}{8^{10}} = \qquad \frac{5^{3}}{5^{4}} = \qquad \left(2^{4}\right)\left(2^{-1}\right) =$$

file 03314

Question 9

Perform the following calculations:

$$\frac{10^{6}}{10^{3}} = \qquad \frac{3^{2}}{3^{3}} = \qquad \left(2^{6}\right)\left(2^{-4}\right) =$$

file 03315

Question 10

The equation for calculating total resistance in a parallel circuit (for any number of parallel resistances) is sometimes written like this:

$$R_{total} = (R_1^{-1} + R_2^{-1} + \cdots R_n^{-1})^{-1}$$

Re-write this equation in such a way that it no longer contains any exponents.

file 00297

Question 11

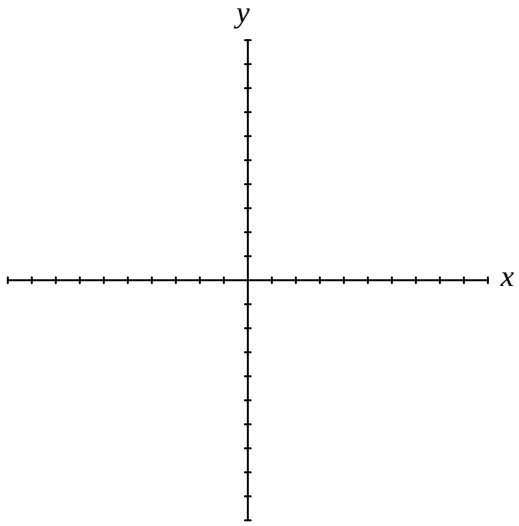
A *function* is a mathematical relationship with an input (usually $x$) and an output (usually $y$). Here is an example of a simple function:

$$y = 2x + 1$$

One way to show the pattern of any given function is with a table of numbers. Complete this table for the given values of $x$:

| $x$ | $2x + 1$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |

A more common (and intuitive) way to show the pattern of any given function is with a *graph*. Complete this graph for the same function $y = 2x + 1$. Consider each division on the axes to be 1 unit:

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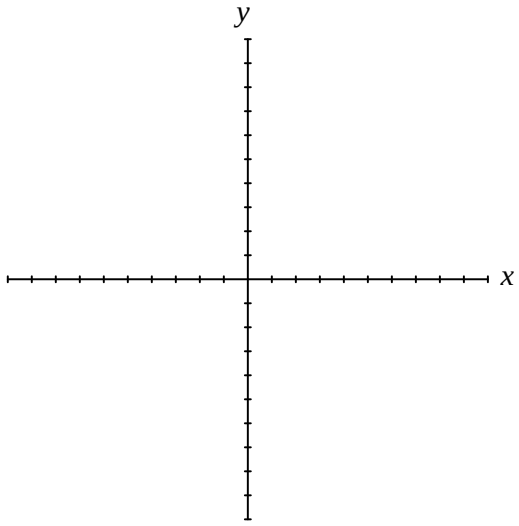
Question 12

A famous illustrative story for understanding exponents goes something like this:

*A pauper saves the life of a king. In return, the king offers the pauper anything he desires as a reward. The pauper, being a shrewd man, tells the king he does not want much, only a grain of rice today, then double that (two grains of rice) the next day, then double that (four grains of rice) the next day, and so on. The king asks how long he is to give the pauper rice, and the pauper responds by saying one day for every square on a chess board (64 days). This does not sound like much to the king, who never took a math course, and so he agrees.*

In just a short amount of time, though, the king finds himself bankrupted to the pauper because the quantity of rice is so enormously large. Such is the nature of exponential functions: they grow incredibly large with modest gains in $x$.

Graph the pauper's rice function ($y = 2^x$), with each division on the horizontal axis representing 1 unit and each division on the vertical axis representing 100 units.

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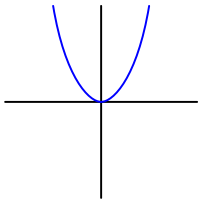
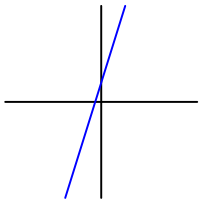
Question 13

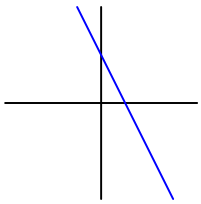
Match each written function ($y = \cdots$) with the sketched graph it fits best:

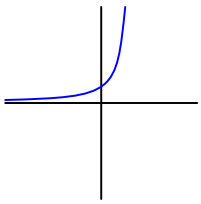
$y = 3x + 2$ $\qquad$ $y = 5 - 2x$

$y = x^2$ $\qquad$ $y = 2^x$









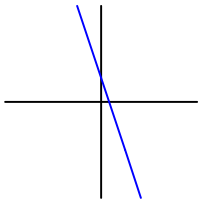
file 03061

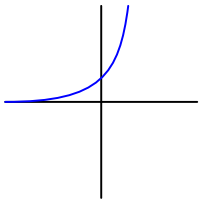
Question 14

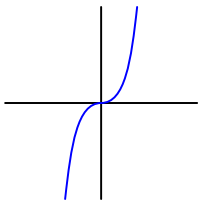
Match each written function ($y = \cdots$) with the sketched graph it fits best:

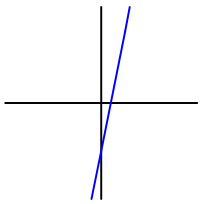
$y = 5x - 2$ $\qquad$ $y = 1 - 3x$

$y = x^3$ $\qquad$ $y = 3^x$









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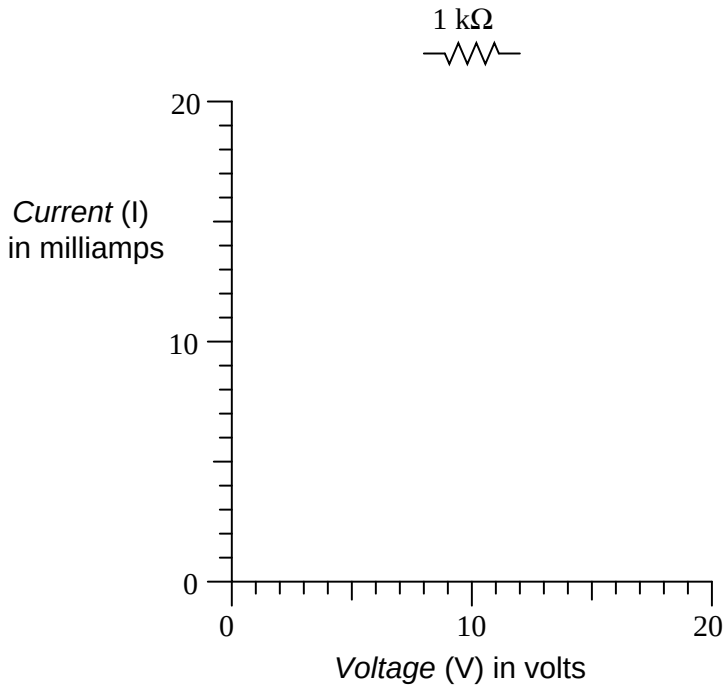
Question 15

Many different equations used in the analysis of electric circuits may be graphed. Take for instance Ohm's Law for a 1 kΩ resistor:

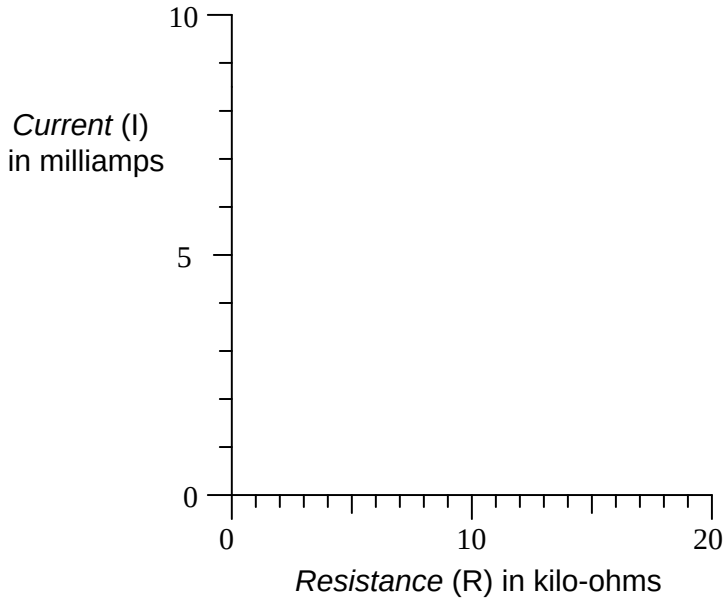



Plot this graph, following Ohm's Law. Then, plot another graph representing the voltage/current relationship of a 2 kΩ resistor.

file 03059

Question 16

Many different equations used in the analysis of electric circuits may be graphed. Take for instance Ohm's Law for a variable resistor connected to a 12 volt source:




Plot this graph, following Ohm's Law.
file 03060

**Answers**

Answer 1

Do what is inside parentheses first (the furthest "inside" parentheses if there are multiple layers of parentheses), powers and roots, functions (trig, log, etc.), multiplication/division, and finally addition/subtraction.

Answer 2

$$\frac{13+2}{3}+8=13 \qquad 25+(3+2)^2\times 2=75$$

Answer 3

$$\frac{15-3}{3}+7=11 \qquad 20+(1+3)^2\times 3=68$$

Answer 4

I'll let you determine and document the proper order of operations, but here are the results of each expression:

- $10-25\times 2+5=-35$
- $-8+10^3\times 51=50992$
- $12^4\times(3+11)=290304$
- $21^{(7-4)}\times 40=370440$
- $\log\sqrt{6+35^2}=1.5451$
- $\sqrt{\left(\frac{220}{16}-2.75\right)\times 2}=4.6904$

Answer 5

$$a^m\times a^n=a^{m+n}$$

Answer 6

$$\frac{a^m}{a^n}=a^{m-n}$$

Answer 7

A negative exponent is simply the reciprocal $(1/x)$ of its positive counterpart. A zero exponent is always equal to 1.

Answer 8

$$\frac{8^{12}}{8^{10}}=64 \qquad \frac{5^3}{5^4}=\frac{1}{5}=0.2 \qquad \left(2^4\right)\left(2^{-1}\right)=8$$

Answer 9

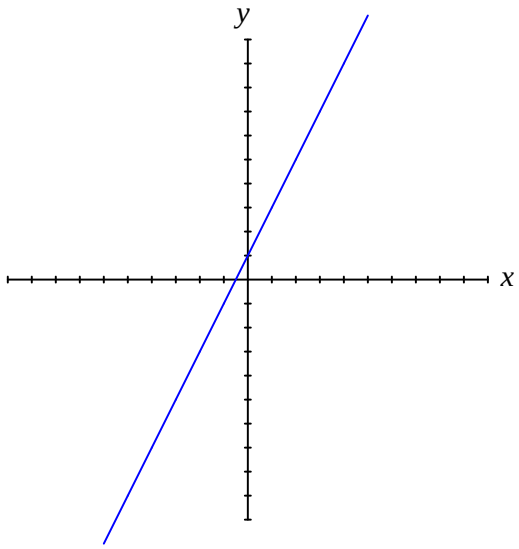
$$\frac{10^6}{10^3} = 1000 \qquad \frac{3^2}{3^3} = \frac{1}{3} \approx 0.333 \qquad \left(2^6\right)\left(2^{-4}\right) = 4$$

Answer 10

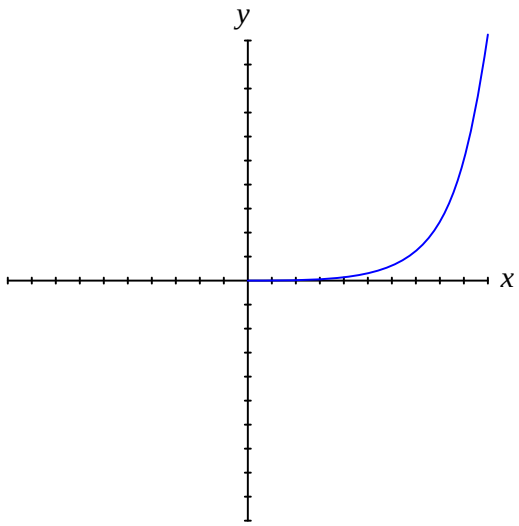
$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}}$$

Answer 11

| $x$ | $2x+1$ |
|---|---|
| 0 | 1 |
| 1 | 3 |
| 2 | 5 |
| 3 | 7 |
| 4 | 9 |
| 5 | 11 |

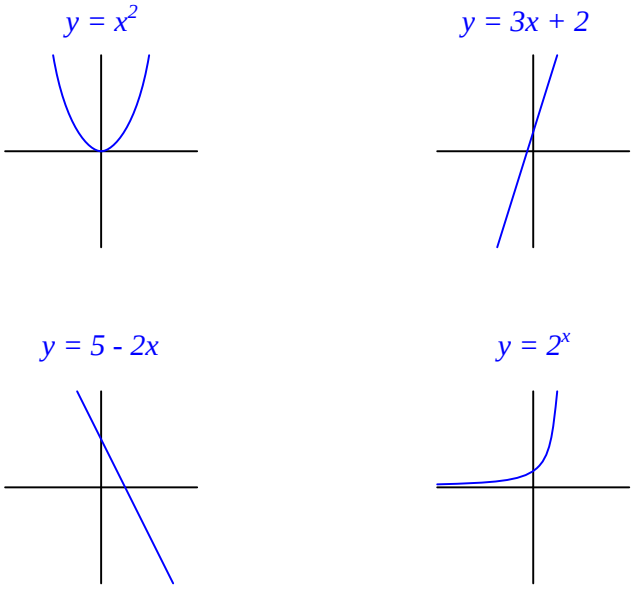


Answer 12

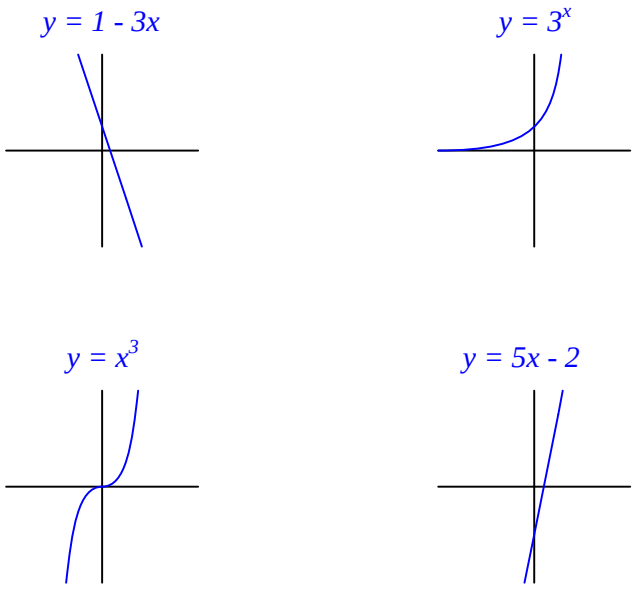



Follow-up question: what do you think this graph will look like for negative values of $x$?

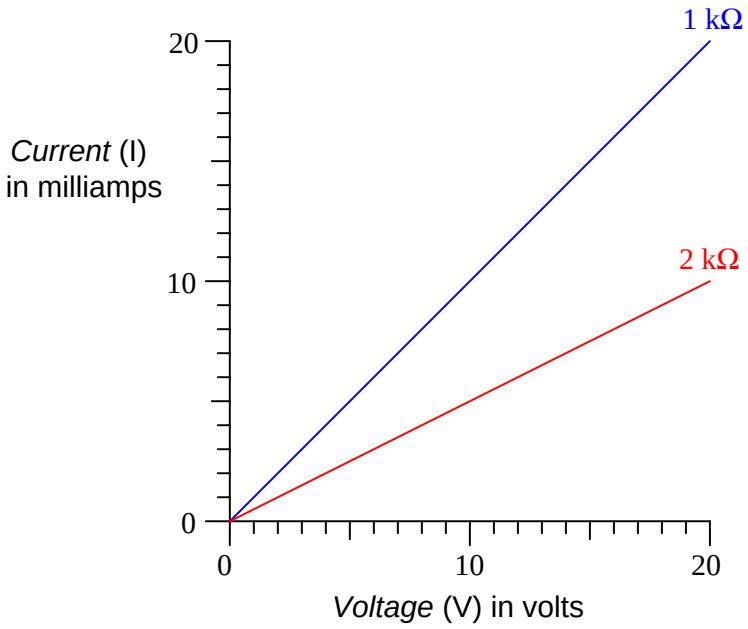
Answer 13



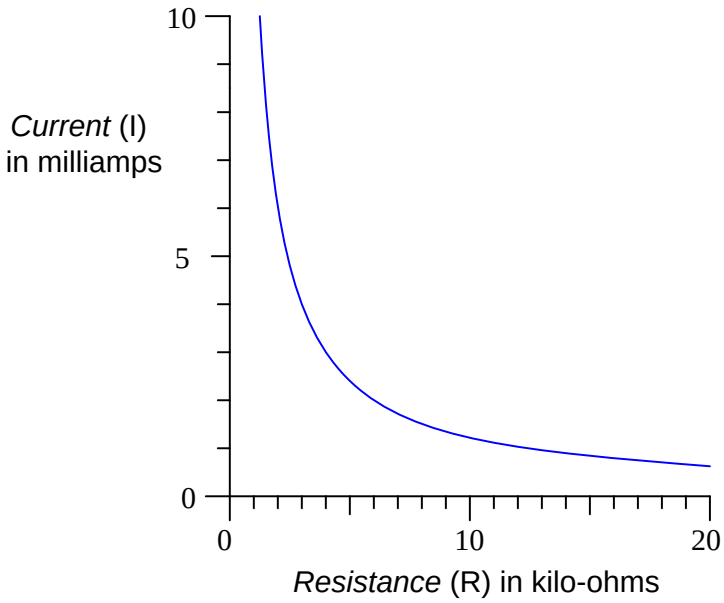
Answer 14




Answer 15



Answer 16



**Notes**

Notes 1

Order of operations is extremely important, as it becomes critical to recognize proper order of evaluation when "stripping" an expression down to isolate a particular variable. In essence, the normal order of operations is reversed when "undoing" an expression, so students must recognize what the proper order of operations is.

Notes 2

Nothing special here – just simple arithmetic problems that cannot be solved correctly unless proper order of operations is followed.

Notes 3

Nothing special here – just simple arithmetic problems that cannot be solved correctly unless proper order of operations is followed.

Notes 4

Order of operations is extremely important, as it becomes critical to recognize proper order of evaluation when "stripping" an expression down to isolate a particular variable. In essence, the normal order of operations is reversed when "undoing" an expression, so students must recognize what the proper order of operations is.

Notes 5

I have found that students who cannot fathom the general rule ($a^m \times a^n = a^{m+n}$) often understand for the first time when they see concrete examples.

Notes 6

I have found that students who cannot fathom the general rule ($\frac{a^m}{a^n} = a^{m-n}$) often understand for the first time when they see concrete examples.

Notes 7

I have found that students who cannot fathom the meaning of negative or zero exponents often understand immediately when they construct their own definition based on the general rule ($\frac{a^m}{a^n} = a^{m-n}$).

Notes 8

Nothing special here, just practice with exponents.

Notes 9

Nothing special here, just practice with exponents.

Notes 10

This question is an exercise in basic algebra, specifically the meaning of negative exponents.

Notes 11

It is very important for your students to understand graphs, as they are very frequently used to illustrate the behavior of circuits and mathematical functions alike. Discuss with them how the line represents a continuous string of points and not just the integer values calculated in the table.

Notes 12

From the graph shown, it may appear that the function approaches 0 as $x$ approaches zero. This is not the case, as a simple calculation ($y = 2^0$) shows. In order for students to adequately see what is going on near the origin, they will have to re-scale the graph.

Notes 13

The primary purpose of this question is to have students figure out *how* to match each expression to a graph. Of course, one could take the time to plot each function one by one, but there exist much simpler ways to determine the "character" of a function without plotting the whole thing.

Notes 14

The primary purpose of this question is to have students figure out *how* to match each expression to a graph. Of course, one could take the time to plot each function one by one, but there exist much simpler ways to determine the "character" of a function without plotting the whole thing.

Notes 15

Ask your students to explain how they plotted the two functions. Did they make a table of values first? Did they draw dots on the paper and then connect those dots with a line? Did anyone plot dots for the endpoints and then draw a straight line in between because they knew this was a linear function?

Notes 16

Ask your students to explain how they plotted the two functions. Did they make a table of values first? Did they draw dots on the paper and then connect those dots with a line? Did anyone plot dots for the endpoints and then draw a straight line in between because they knew this was a linear function?

Many students are surprised that the plot is nonlinear, being that resistors are considered linear devices!