INST 155 (Networks and Systems), section 1

Recommended schedule

Day 1

Topics: *Basic algebra review*
Questions: *1 through 10*
Lab Exercise: *Work on project (progress report: Question 61)*

Day 2

Topics: *Algebraic equation manipulation*
Questions: *11 through 20*
Lab Exercise: *Work on project (progress report: Question 62)*

Day 3

Topics: *Algebraic substitution*
Questions: *21 through 30*
Lab Exercise: *Work on project (progress report: Question 63)*

Day 4

Topics: *Solving simultaneous systems of equations*
Questions: *31 through 40*
Lab Exercise: *Work on project (progress report: Question 64)*

Day 5

Topics: *Resistor network analysis using simultaneous equations*
Questions: *41 through 50*
Lab Exercise: *Voltage divider with limited range (Question 65)*

Day 6

Topics: *Review*
Questions: *51 through 60*
Lab Exercise: *Work on project (progress report: Question 66)*

Impending deadlines

Project due at end of INST155, Section 2
Question 67: Sample project grading criteria

INST 155 (Networks and Systems), section 1

Skill standards addressed by this course section

This needs to be completed!

EIA *Raising the Standard; Electronics Technician Skills for Today and Tomorrow*, June 1994

B Basic and Practical Skills – Communicating on the Job

B.01 Use effective written and other communication skills. *Met by group discussion and completion of labwork.*

B.03 Employ appropriate skills for gathering and retaining information. *Met by research and preparation prior to group discussion.*

B.04 Interpret written, graphic, and oral instructions. *Met by completion of labwork.*

B.06 Use language appropriate to the situation. *Met by group discussion and in explaining completed labwork.*

B.07 Participate in meetings in a positive and constructive manner. *Met by group discussion.*

B.08 Use job-related terminology. *Met by group discussion and in explaining completed labwork.*

B.10 Document work projects, procedures, tests, and equipment failures. *Met by project construction and/or troubleshooting assessments.*

C Basic and Practical Skills – Solving Problems and Critical Thinking

C.01 Identify the problem. *Met by research and preparation prior to group discussion.*

C.03 Identify available solutions and their impact including evaluating credibility of information, and locating information. *Met by research and preparation prior to group discussion.*

C.07 Organize personal workloads. *Met by daily labwork, preparatory research, and project management.*

C.08 Participate in brainstorming sessions to generate new ideas and solve problems. *Met by group discussion.*

D Basic and Practical Skills – Reading

D.01 Read and apply various sources of technical information (e.g. manufacturer literature, codes, and regulations). *Met by research and preparation prior to group discussion.*

E Basic and Practical Skills – Proficiency in Mathematics

E.01 Determine if a solution is reasonable.

E.02 Demonstrate ability to use a simple electronic calculator.

E.06 Translate written and/or verbal statements into mathematical expressions.

E.07 Compare, compute, and solve problems involving binary, octal, decimal, and hexadecimal numbering systems.

E.12 Interpret and use tables, charts, maps, and/or graphs.

E.13 Identify patterns, note trends, and/or draw conclusions from tables, charts, maps, and/or graphs.

E.15 Simplify and solve algebraic expressions and formulas.

E.16 Select and use formulas appropriately.

E.18 Use properties of exponents and logarithms.

E.20 Graph functions.

E.29 Graph basic functions using polar and/or Cartesian coordinate systems.

Question 1

When evaluating (calculating) a mathematical expression, what order should you do the various expressions in? In other words, which comes first: multiplication, division, addition, subtraction, powers, roots, parentheses, etc.; and then what comes after that, and after that?

file 03052

Answer 1

Do what is inside parentheses first (the furthest "inside" parentheses if there are multiple layers of parentheses), powers and roots, functions (trig, log, etc.), multiplication/division, and finally addition/subtraction.

Notes 1

Order of operations is extremely important, as it becomes critical to recognize proper order of evaluation when "stripping" an expression down to isolate a particular variable. In essence, the normal order of operations is reversed when "undoing" an expression, so students must recognize what the proper order of operations is.

Question 2

When evaluating an expression such as this, it is very important to follow proper order of operations. Otherwise, the correct result will be impossible to arrive at:

$$3 \log 2^5 + 14$$

To show what the proper order of operations is for this expression, I show it being evaluated *step by step* here†:

$$3 \log 2^5 + 14$$

$$3 \log 32 + 14$$

$$3 \times 1.5051 + 14$$

$$4.5154 + 14$$

$$18.5154$$

Do the same for each of the following expressions:

- $10 - 25 \times 2 + 5$
- $-8 + 10^3 \times 51$
- $12^4 \times (3 + 11)$
- $21^{(7-4)} \times 40$
- $\log \sqrt{6 + 35^2}$
- $\sqrt{\left(\frac{220}{16} - 2.75\right) \times 2}$

file 03053

Answer 2

I'll let you determine and document the proper order of operations, but here are the results of each expression:

- $10 - 25 \times 2 + 5 = -35$
- $-8 + 10^3 \times 51 = 50992$
- $12^4 \times (3 + 11) = 290304$
- $21^{(7-4)} \times 40 = 370440$
- $\log \sqrt{6 + 35^2} = 1.5451$
- $\sqrt{\left(\frac{220}{16} - 2.75\right) \times 2} = 4.6904$

† By the way, this is a highly recommended practice for those struggling with mathematical principles: *document each and every step* by re-writing the expression. Although it takes more paper and more effort, it will save you from needless error and frustration!

Notes 2

Order of operations is extremely important, as it becomes critical to recognize proper order of evaluation when "stripping" an expression down to isolate a particular variable. In essence, the normal order of operations is reversed when "undoing" an expression, so students must recognize what the proper order of operations is.

Question 3

Observe the following equivalence:

$$4^3 \times 4^2 = (4 \times 4 \times 4) \times (4 \times 4)$$

Since all operations are the same (multiplication) and reversible, the parentheses are not needed. Therefore, we may write the expression like this:

$$4 \times 4 \times 4 \times 4 \times 4$$

Of course, the simplest way to write this is 4^5, since there are five 4's multiplied together.

Expand each of these expressions so that there are no exponents either:

- $3^5 \times 3^2 =$
- $10^4 \times 10^3 =$
- $8^2 \times 8^3 =$
- $20^1 \times 20^2 =$

After expanding each of these expressions, re-write each one in simplest form: one number to a power, just like the final form of the example given (4^5). From these examples, what pattern do you see with exponents of products. In other words, what is the general solution to the following expression?

$$a^m \times a^n =$$

file 03054

Answer 3

$$a^m \times a^n = a^{m+n}$$

Notes 3

I have found that students who cannot fathom the general rule ($a^m \times a^n = a^{m+n}$) often understand for the first time when they see concrete examples.

Question 4

Observe the following equivalence:

$$\frac{4^3}{4^2} = \frac{4 \times 4 \times 4}{4 \times 4}$$

It should be readily apparent that we may cancel out two quantities from both top and bottom of the fraction, so in the end we are left with this:

$$\frac{4}{1}$$

Re-writing this using exponents, we get 4^1.

Expand each of these expressions so that there are no exponents either:

- $\frac{3^5}{3^2} =$
- $\frac{10^6}{10^4} =$
- $\frac{8^7}{8^3} =$
- $\frac{20^5}{20^4} =$

After expanding each of these expressions, re-write each one in simplest form: one number to a power, just like the final form of the example given (4^1). From these examples, what pattern do you see with exponents of products. In other words, what is the general solution to the following expression?

$$\frac{a^m}{a^n} =$$

file 03055

Answer 4

$$\frac{a^m}{a^n} = a^{m-n}$$

Notes 4

I have found that students who cannot fathom the general rule ($\frac{a^m}{a^n} = a^{m-n}$) often understand for the first time when they see concrete examples.

Question 5

Observe the following equivalence:

$$\frac{4^2}{4^3} = \frac{4 \times 4}{4 \times 4 \times 4}$$

It should be readily apparent that we may cancel out two quantities from both top and bottom of the fraction, so in the end we are left with this:

$$\frac{1}{4}$$

Following the rule of $\frac{a^m}{a^n} = a^{m-n}$, the reduction of $\frac{4^2}{4^3}$ should be 4^{-1}. Many students find this confusing, as the intuitive concept of exponents (how many times a number is to be multiplied by itself) fails here. How in the world do we multiply 4 by itself -1 times?!

Expand each of these expressions so that there are no exponents either:

- $\frac{3^2}{3^5} =$
- $\frac{10^4}{10^6} =$
- $\frac{8^3}{8^7} =$
- $\frac{20^4}{20^5} =$

After expanding each of these expressions, re-write each one in simplest form: one number to a power, just like the final form of the example given (4^{-1}), following the rule $\frac{a^m}{a^n} = a^{m-n}$. From these examples, what easy-to-understand definition can you think of to describe negative exponents?

Also, expand the following expression so there are no exponents, then re-write the result in exponent form following the rule $\frac{a^m}{a^n} = a^{m-n}$:

$$\frac{5^3}{5^3}$$

What does this tell you about exponents of zero?

file 03056

Answer 5

A negative exponent is simply the reciprocal ($1/x$) of its positive counterpart. A zero exponent is always equal to 1.

Notes 5

I have found that students who cannot fathom the meaning of negative or zero exponents often understand immediately when they construct their own definition based on the general rule ($\frac{a^m}{a^n} = a^{m-n}$).

Question 6

The equation for calculating total resistance in a parallel circuit (for any number of parallel resistances) is sometimes written like this:

$$R_{total} = (R_1^{-1} + R_2^{-1} + \cdots R_n^{-1})^{-1}$$

Re-write this equation in such a way that it no longer contains any exponents.

file 00297

Answer 6

$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}}$$

Notes 6

This question is an exercise in basic algebra, specifically the meaning of negative exponents.

Question 7

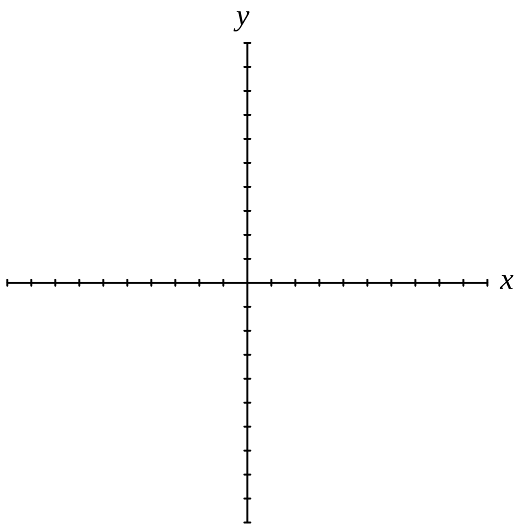
A *function* is a mathematical relationship with an input (usually x) and an output (usually y). Here is an example of a simple function:

$$y = 2x + 1$$

One way to show the pattern of any given function is with a table of numbers. Complete this table for the given values of x:

x	$2x + 1$
0	
1	
2	
3	
4	
5	

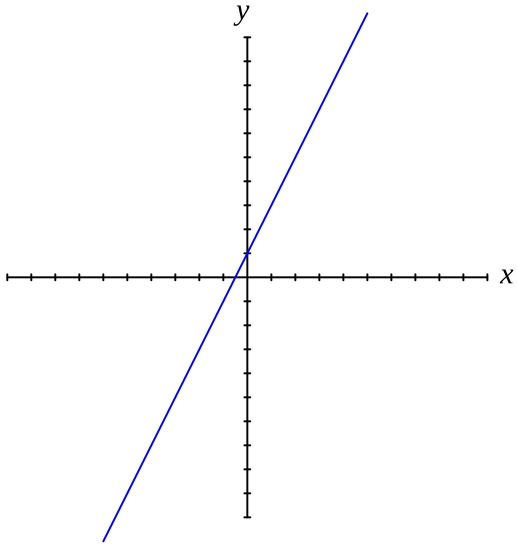
A more common (and intuitive) way to show the pattern of any given function is with a *graph*. Complete this graph for the same function $y = 2x + 1$. Consider each division on the axes to be 1 unit:



file 03057

Answer 7

x	$2x+1$
0	1
1	3
2	5
3	7
4	9
5	11



Notes 7

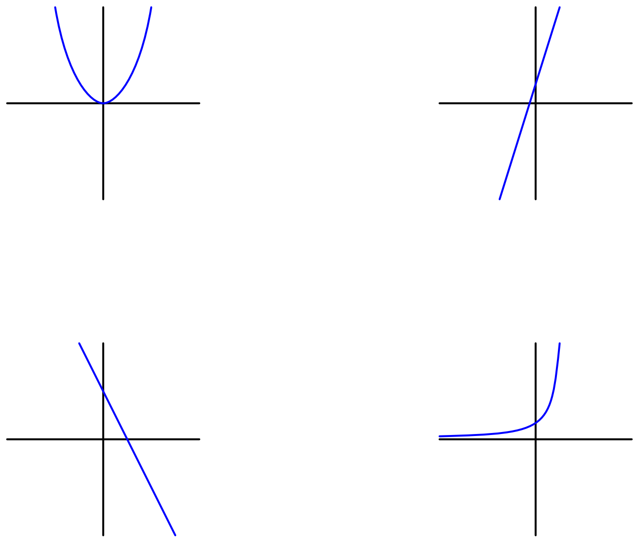
It is very important for your students to understand graphs, as they are very frequently used to illustrate the behavior of circuits and mathematical functions alike. Discuss with them how the line represents a continuous string of points and not just the integer values calculated in the table.

Question 8

Match each written function ($y = \cdots$) with the sketched graph it fits best:

$$y = 3x + 2 \qquad y = 5 - 2x$$

$$y = x^2 \qquad y = 2^x$$



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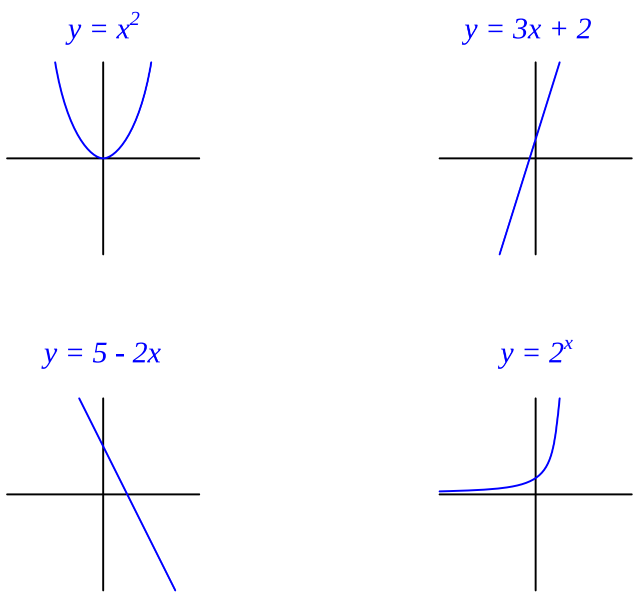
Answer 8

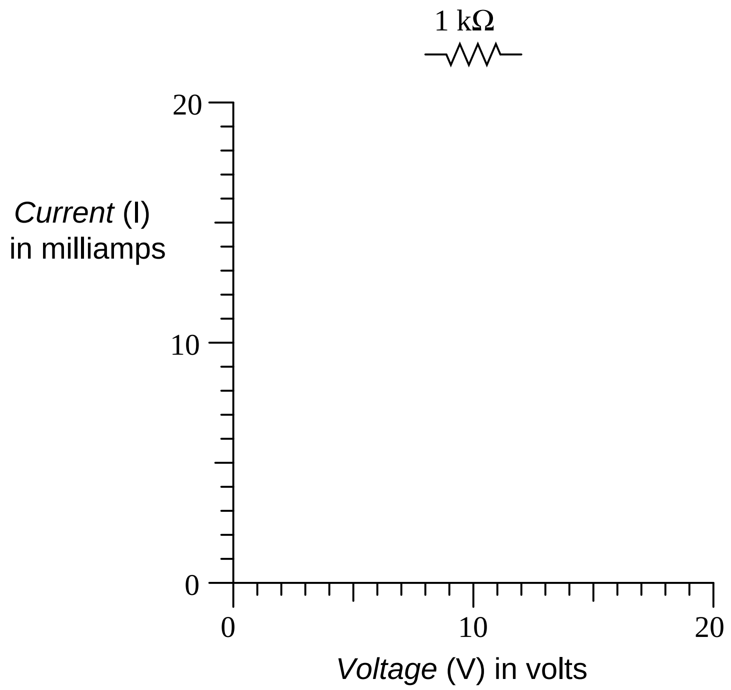


Notes 8

The primary purpose of this question is to have students figure out *how* to match each expression to a graph. Of course, one could take the time to plot each function one by one, but there exist much simpler ways to determine the "character" of a function without plotting the whole thing.

Question 9

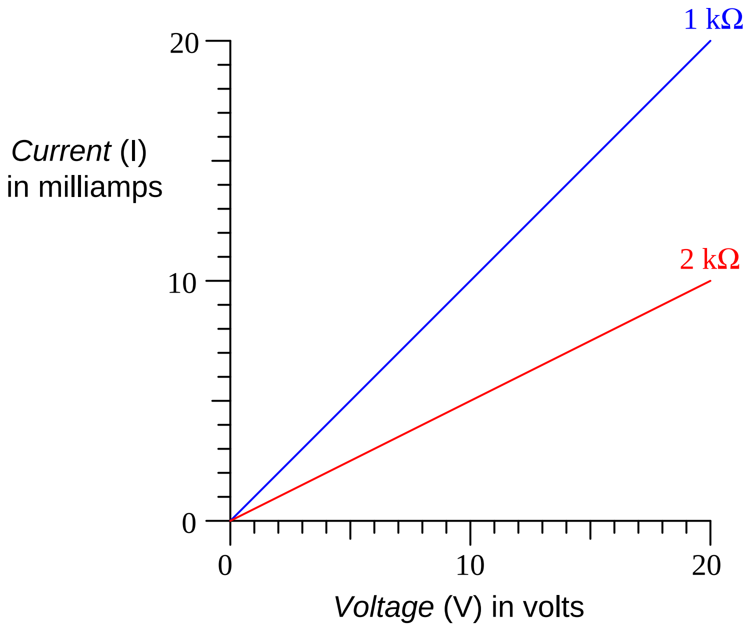
Many different equations used in the analysis of electric circuits may be graphed. Take for instance Ohm's Law for a 1 kΩ resistor:



Plot this graph, following Ohm's Law. Then, plot another graph representing the voltage/current relationship of a 2 kΩ resistor.

file 03059

Answer 9

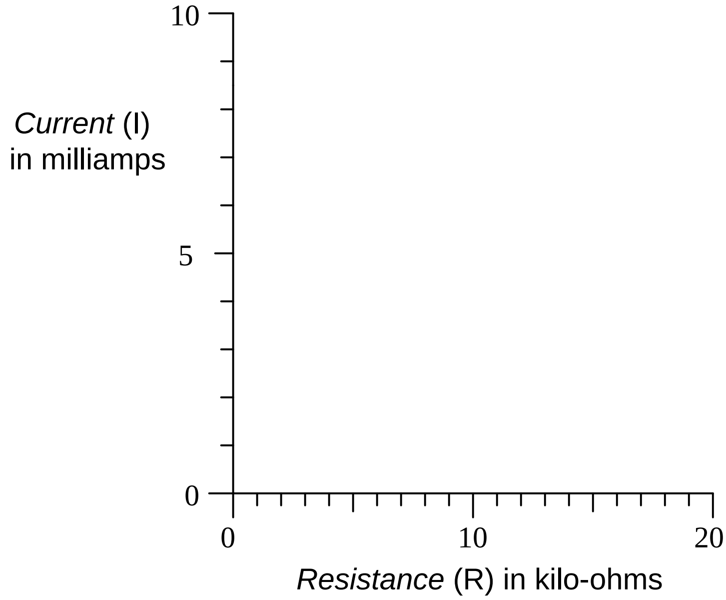


Notes 9

Ask your students to explain how they plotted the two functions. Did they make a table of values first? Did they draw dots on the paper and then connect those dots with a line? Did anyone plot dots for the endpoints and then draw a straight line in between because they knew this was a linear function?

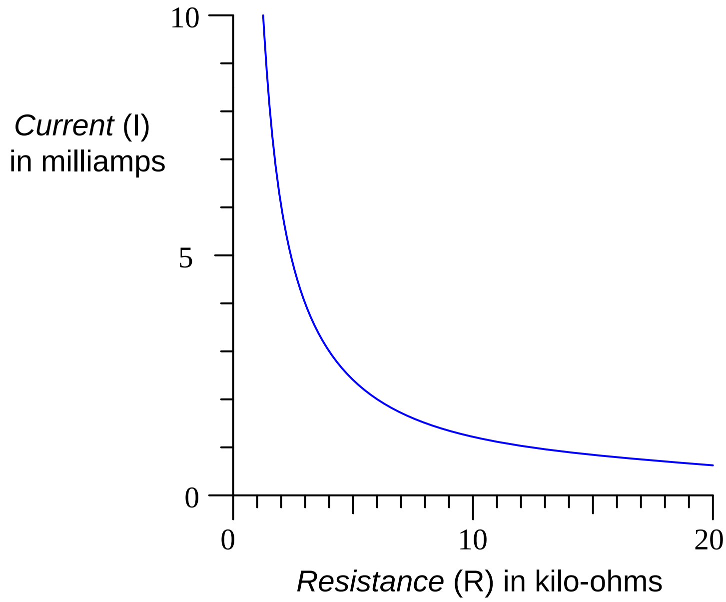
Question 10

Many different equations used in the analysis of electric circuits may be graphed. Take for instance Ohm's Law for a variable resistor connected to a 12 volt source:



Plot this graph, following Ohm's Law.
file 03060

Answer 10



Notes 10

Ask your students to explain how they plotted the two functions. Did they make a table of values first? Did they draw dots on the paper and then connect those dots with a line? Did anyone plot dots for the endpoints and then draw a straight line in between because they knew this was a linear function?

Many students are surprised that the plot is nonlinear, being that resistors are considered linear devices!

Question 11

Solve for n in the following equations:

Equation 1: $-56 = -14n$

Equation 2: $54 - n = 10$

Equation 3: $\frac{4}{n} = 12$

Equation 4: $28 = 2 - n$

file 03063

Answer 11

Equation 1: $n = 4$

Equation 2: $n = 44$

Equation 3: $n = 0.\overline{333}$

Equation 4: $n = -26$

Notes 11

Have your students come to the front of the class and show everyone else the techniques they used to solve for the value of a in each equation. Remind them to document each and every step in the process, so that nothing is left to guess or to chance.

Equations 2 through 4 require two steps to solve for n. Equation 1 only requires a single step, but the two negative numbers may be a bit confusing to some.

Question 12

The formula for calculating total resistance of three series-connected resistors is as follows:

$$R = R_1 + R_2 + R_3$$

Algebraically manipulate this equation to solve for one of the series resistances (R_1) in terms of the other two series resistances (R_2 and R_3) and the total resistance (R). In other words, write a formula that solves for R_1 in terms of all the other variables.

file 03066

Answer 12

$$R_1 = R - (R_2 + R_3) \qquad \text{or} \qquad R_1 = R - R_2 - R_3$$

Notes 12

This question is nothing more than practice algebraically manipulating equations. Ask your students to show you how they solved it, and how the two given answers are equivalent.

Question 13

Manipulate this equation to solve for resistor value R_1, given the values of R_2 and $R_{parallel}$:

$$R_{parallel} = \frac{R_1 R_2}{R_1 + R_2}$$

Then, give an example of a practical situation where you might use this new equation.
file 01988

Answer 13

$$R_1 = \frac{R_2 R_{parallel}}{R_2 - R_{parallel}}$$

I'll let you figure out a situation where this equation would be useful!

Notes 13

This question is really nothing more than an exercise in algebraic manipulation.

Question 14

The formula for calculating total resistance of three parallel-connected resistors is as follows:

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Algebraically manipulate this equation to solve for one of the parallel resistances (R_1) in terms of the other two parallel resistances (R_2 and R_3) and the total resistance (R). In other words, write a formula that solves for R_1 in terms of all the other variables.

file 03067

Answer 14

$$R_1 = \frac{1}{\frac{1}{R} - (\frac{1}{R_2} + \frac{1}{R_3})} \qquad \text{or} \qquad R_1 = \frac{1}{\frac{1}{R} - \frac{1}{R_2} - \frac{1}{R_3}}$$

Notes 14

This question is nothing more than practice algebraically manipulating equations. Ask your students to show you how they solved it, and how the two given answers are equivalent.

Question 15

The following equations solve for the output voltage of various switching converter circuits (unloaded), given the switch duty cycle D and the input voltage:

$$V_{out} = D\,V_{in} \qquad \text{(Buck converter circuit)}$$

$$V_{out} = \frac{V_{in}}{1-D} \qquad \text{(Boost converter circuit)}$$

$$V_{out} = \frac{D\,V_{in}}{1-D} \qquad \text{(Inverting or Cuk converter circuit)}$$

Manipulate each of these equations to solve for duty cycle (D) in terms of the input voltage (V_{in}) and desired output voltage (V_{out}). Remember that duty cycle is always a quantity between 0 and 1, inclusive.

file 02161

Answer 15

$$D = \frac{V_{out}}{V_{in}} \qquad \text{(Buck converter circuit)}$$

$$D = 1 - \left(\frac{V_{in}}{V_{out}}\right) \qquad \text{(Boost converter circuit)}$$

$$D = \frac{V_{out}}{V_{in} + V_{out}} \qquad \text{(Inverting or Cuk converter circuit)}$$

Notes 15

Given the equations for these converter circuit types solving for output voltage in terms of input voltage and duty cycle D, this question is nothing more than an exercise in algebraic manipulation.

Note to your students that all of these equations assume a condition of zero load on the converter circuit. When loads are present, of course, the output voltage will not be the same as what is predicted by these neat, simple formulae. Although these DC-DC power converter circuits are commonly referred to as "regulators," it is somewhat misleading to do so because it falsely implies a capacity for self-correction of output voltage. Only when coupled to a feedback control network are any of these converter circuits capable of actually *regulating* output voltage to a set value.

Question 16

The power dissipation of a transistor is given by the following equation:

$$P = I_C \left(V_{CE} + \frac{V_{BE}}{\beta} \right)$$

Manipulate this equation to solve for beta, given all the other variables.

file 00502

Answer 16

$$\beta = \frac{V_{BE}}{\frac{P}{I_C} - V_{CE}}$$

Notes 16

Although this question is essentially nothing more than an exercise in algebraic manipulation, it is also a good lead-in to a discussion on the importance of power dissipation as a semiconductor device rating.

High temperature is the bane of most semiconductors, and high temperature is caused by excessive power dissipation. A classic example of this, though a bit dated, is the temperature sensitivity of the original germanium transistors. These devices were extremely sensitive to heat, and would fail rather quickly if allowed to overheat. Solid state design engineers had to be very careful in the techniques they used for transistor circuits to ensure their sensitive germanium transistors would not suffer from "thermal runaway" and destroy themselves.

Silicon is much more forgiving then germanium, but heat is still a problem with these devices. At the time of this writing (2004), there is promising developmental work on silicon carbide and gallium nitride transistor technology, which is able to function under *much* higher temperatures than silicon.

Question 17

The equation for voltage gain (A_V) in a typical inverting, single-ended opamp circuit is as follows:

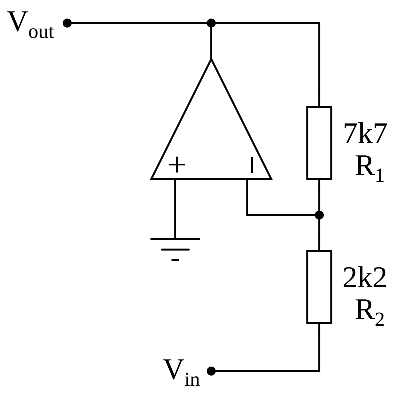
$$A_V = \frac{R_1}{R_2}$$

Where,

R_1 is the feedback resistor (connecting the output to the inverting input)

R_2 is the other resistor (connecting the inverting input to voltage signal input terminal)

Suppose we wished to change the voltage gain in the following circuit from 3.5 to 4.9, but only had the freedom to alter the resistance of R_2:



Algebraically manipulate the gain equation to solve for R_2, then determine the necessary value of R_2 in this circuit to give it a voltage gain of 4.9.

file 02708

Answer 17

$$R_2 = \frac{R_1}{A_V}$$

For the circuit shown, R_2 would have to be set equal to 1.571 kΩ.

Notes 17

Nothing more than a little algebra to obtain the answers for this question!

Question 18

The equation for voltage gain (A_V) in a typical noninverting, single-ended opamp circuit is as follows:

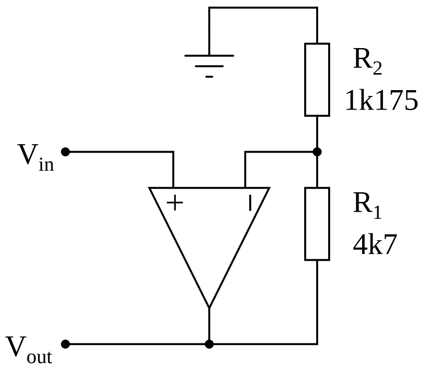
$$A_V = \frac{R_1}{R_2} + 1$$

Where,

R_1 is the feedback resistor (connecting the output to the inverting input)

R_2 is the other resistor (connecting the inverting input to ground)

Suppose we wished to change the voltage gain in the following circuit from 5 to 6.8, but only had the freedom to alter the resistance of R_2:



Algebraically manipulate the gain equation to solve for R_2, then determine the necessary value of R_2 in this circuit to give it a voltage gain of 6.8.

file 02707

Answer 18

$$R_2 = \frac{R_1}{A_V - 1}$$

For the circuit shown, R_2 would have to be set equal to 810.3 Ω.

Notes 18

Nothing more than a little algebra to obtain the answers for this question!

Question 19

The decay of a variable over time in an RC or LR circuit follows this mathematical expression:

$$e^{-\frac{t}{\tau}}$$

Where,

e = Euler's constant (≈ 2.718281828)
t = Time, in seconds
τ = Time constant of circuit, in seconds

For example, if we were to evaluate this expression and arrive at a value of 0.398, we would know the variable in question has decayed from 100% to 39.8% over the period of time specified.

However, calculating the amount of time it takes for a decaying variable to reach a specified percentage is more difficult. We would have to manipulate the equation to solve for t, which is part of an exponent.

Show how the following equation could be algebraically manipulated to solve for t, where x is the number between 0 and 1 (inclusive) representing the percentage of original value for the variable in question:

$$x = e^{-\frac{t}{\tau}}$$

Note: the "trick" here is how to isolate the exponent $\frac{-t}{\tau}$. You will have to use the natural logarithm function!

file 02001

Answer 19

Showing all the necessary steps:

$$x = e^{-\frac{t}{\tau}}$$

$$\ln x = \ln \left(e^{-\frac{t}{\tau}} \right)$$

$$\ln x = -\frac{t}{\tau}$$

$$t = -\tau \ln x$$

Notes 19

In my experience, most American high school graduates are extremely weak in logarithms. Apparently this is not taught very well at the high school level, which is a shame because logarithms are a powerful mathematical tool. You may find it necessary to explain to your students what a logarithm is, and exactly why it "un-does" the exponent.

When forced to give a quick presentation on logarithms, I usually start with a generic definition:

$$\text{Given: } b^a = c$$

$$\text{Logarithm defined: } \log_b c = a$$

Verbally defined, the logarithm function asks us to find the power (a) of the base (b) that will yield c.

Next, I introduce the common logarithm. This, of course, is a logarithm with a base of 10. A few quick calculator exercises help students grasp what the common logarithm function is all about:

$$\log 10 =$$

$$\log 100 =$$

$$\log 1000 =$$

$$\log 10000 =$$

$$\log 100000 =$$

$$\log \frac{1}{10} =$$

$$\log \frac{1}{100} =$$

$$\log \frac{1}{1000} =$$

After this, I introduce the *natural logarithm*: a logarithm with a base of e (Euler's constant):

$$\text{Natural logarithm defined: } \ln x = \log_e x$$

Have your students do this simple calculation on their calculators, and explain the result:

$$\ln 2.71828 =$$

Next comes an exercise to help them understand how logarithms can "undo" exponentiation. Have your students calculate the following values:

$$e^2 =$$

$$e^3 =$$

$$e^4 =$$

Now, have them take the natural logarithms of each of those answers. They will find that they arrive at the original exponent values (2, 3, and 4, respectively). Write this relationship on the board as such for your students to view:

$$\ln e^2 = 2$$

$$\ln e^3 = 3$$

$$\ln e^4 = 4$$

Ask your students to express this relationship in general form, using the variable x for the power instead of an actual number:

$$\ln e^x = x$$

It should now be apparent that the natural logarithm function has the ability to "undo" a power of e. Now it should be clear to your students why the given sequence of algebraic manipulations in the answer for this question is true.

Question 20

The electrical resistance of a conductor at any temperature may be calculated by the following equation:

$$R_T = R_r + R_r \alpha T - R_r \alpha T_r$$

Where,

R_T = Resistance of conductor at temperature T
R_r = Resistance of conductor at reference temperature T_r
α = Temperature coefficient of resistance at reference temperature T_r

Simplify this equation by means of factoring.

file 00509

Answer 20

$$R_T = R_r[1 + \alpha(T - T_r)]$$

Follow-up question: when plotted on a graph with temperature (T) as the independent variable and resistance (R_T) as the dependent variable (i.e. a two-axis graph with T on the horizontal and R on the vertical), is the resulting plot linear? Why or why not? How is it possible to tell just by looking at the equation, prior to actually plotting on a graph?

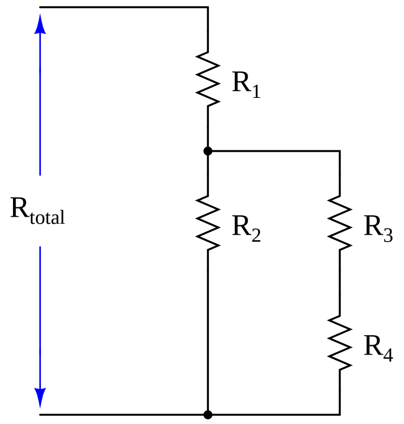
Notes 20

Just an exercise in algebra here!

Question 21

Substitution is a technique whereby we let a variable represent (stand in the place of) another variable or an expression made of other variables. One application where we might use substitution is when we must manipulate an algebraic expression containing a lot of similar-looking variables, as is often the case with science problems.

Take this series-parallel resistor circuit for example:



The equation expressing total resistance as a function of the four resistor values looks like this:

$$R_{total} = R_1 + \frac{R_2(R_3 + R_4)}{R_2 + R_3 + R_4}$$

Now imagine being asked to manipulate this equation to solve for R_3. When the only visual feature distinguishing each of the variables is the subscript (*total*, *1*, *2*, *3*, or *4*), it becomes very easy to lose track of where one is in the algebraic manipulation. A very common mistake is to exchange or needlessly repeat subscripts during the process, effectively mis-placing one or more variables. To help avoid such mistakes, you may *substitute* different letter variables for R_{total}, R_1, R_2, R_3, and R_4 like this:

Substitution table

Original variable	New variable
R_{total}	y
R_1	a
R_2	b
R_3	c
R_4	d

$$y = a + \frac{b(c + d)}{b + c + d}$$

After doing the algebraic manipulation to solve for c (R_3), the equation looks like this:

$$c = \frac{(y - a)(b + d) - bd}{a + b - y}$$

Back-substitute the original R variables in place of a, b, c, d, and y as you see them in the above equation to arrive at a form that directly relates to the schematic diagram.

file 03089

Answer 21

$$R_3 = \frac{(R_{total} - R_1)(R_2 + R_4) - R_2 R_4}{R_1 + R_2 - R_{total}}$$

Challenge question: show all the steps you would take to solve for R_3 in the original equation.

Notes 21

Here I show an application of substitution that is useful only because the human brain has difficulty distinguishing similar-looking symbols. More powerful uses of algebraic substitution exist, of course, but this is a start for students who have never seen the concept before.

Question 22

Substitution is a technique whereby we let a variable represent (stand in the place of) another variable or an expression made of other variables. One application where we might use substitution is when we must manipulate an algebraic expression containing multiple instances of the same sub-expression. For example, suppose we needed to manipulate this equation to solve for c:

$$1 = \frac{a + b(d^2 - f^2) + c}{d^2 - f^2}$$

The sub-expression $d^2 - f^2$ appears twice in this equation. Wouldn't it be nice if we had something simpler to put in its place during the time we were busy manipulating the equation, if for no other reason than to have less variables to write on our paper while showing all the steps to our work? Well, we can do this!

Substitute the variable x for the sub-expression $d^2 - f^2$, and then solve for c. When you are done manipulating the equation, back-substitute $d^2 - f^2$ in place of x.

file 03090

Answer 22

Original equation:

$$1 = \frac{a + b(d^2 - f^2) + c}{d^2 - f^2}$$

After substituting x:

$$1 = \frac{a + bx + c}{x}$$

After manipulating the equation to solve for c:

$$c = x(1 - b) - a$$

Back-substituting the original sub-expression in place of x:

$$c = (d^2 - f^2)(1 - b) - a$$

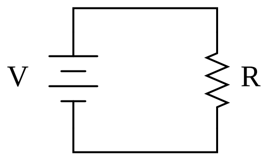
Notes 22

Here I show an application of substitution that is useful only because the human brain has an easier time dealing with a single symbol than with a collection of different symbols. More powerful uses of algebraic substitution exist, of course, but this is a start for students who are new to the concept.

Question 23

Substitution is the term we give to the mathematical equivalence of one variable to one or more other variables in an expression. It is a fundamental principle used to combine two or more equations into a single equation (among other things).

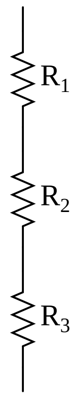
For example, we know that the formula for calculating current in a simple one-resistor circuit is as follows:



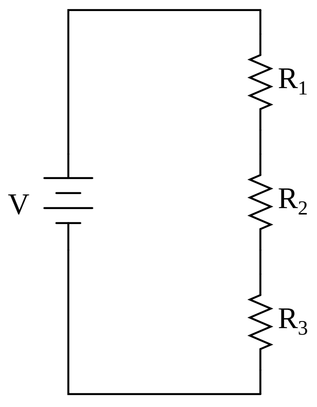
$$I = V / R$$

We also know that the total resistance (R) of a three-resistor series circuit is as follows:

$$R = R_1 + R_2 + R_3$$



Combine these two equations together using substitution so that we have a single equation for calculating current I in a three-resistor series circuit given the source voltage V and each resistance value R_1, R_2, and R_3:



$$I = f(V, R_1, R_2, R_3)$$

"I is some function of V, R_1, R_2, and R_3"

In other words, you need to have as your answer a single equation that begins with "$I =$" and has all the variables V, R_1, R_2, and R_3 on the other side of the "equal" sign.

file 03064

Answer 23

$$I = \frac{V}{R_1 + R_2 + R_3}$$

Notes 23

I like to speak of the process of substitution in terms of *definitions* for variables. In this particular case, $R_1 + R_2 + R_3$ is a *definition* for R that we put in R's place in the first equation ($I = \frac{V}{R}$).

The notation shown in the third schematic, $I = f(V, R_1, R_2, R_3)$, is known as *function notation.* It merely means that the value of I is determined by the values of all those variables within the parentheses, rather than just one.

Question 24

We know that the current in a series circuit may be calculated with this formula:

$$I = \frac{E_{total}}{R_{total}}$$

We also know that the voltage dropped across any single resistor in a series circuit may be calculated with this formula:

$$E_R = IR$$

Combine these two formulae into one, in such a way that the I variable is eliminated, leaving only E_R expressed in terms of E_{total}, R_{total}, and R.

file 00360

Answer 24

$$E_R = E_{total}\left(\frac{R}{R_{total}}\right)$$

Follow-up question: algebraically manipulate this equation to solve for E_{total} in terms of all the other variables. In other words, show how you could calculate for the amount of total voltage necessary to produce a specified voltage drop (E_R) across a specified resistor (R), given the total circuit resistance (R_{total}).

Notes 24

Though this "voltage divider formula" may be found in any number of electronics reference books, your students need to understand how to algebraically manipulate the given formulae to arrive at this one.

Question 25

We know that the voltage in a parallel circuit may be calculated with this formula:

$$E = I_{total}R_{total}$$

We also know that the current through any single resistor in a parallel circuit may be calculated with this formula:

$$I_R = \frac{E}{R}$$

Combine these two formulae into one, in such a way that the E variable is eliminated, leaving only I_R expressed in terms of I_{total}, R_{total}, and R.

file 00368

Answer 25

$$I_R = I_{total}\left(\frac{R_{total}}{R}\right)$$

How is this formula similar, and how is it different, from the "voltage divider" formula?

Notes 25

Though this "current divider formula" may be found in any number of electronics reference books, your students need to understand how to algebraically manipulate the given formulae to arrive at this one.

At first it may seem as though the two divider formulae (voltage versus current) are easy to confuse. Is it $\frac{R}{R_{total}}$ or $\frac{R_{total}}{R}$? However, there is a very simple way to remember which fraction belongs with which formula, based on the numerical value of that fraction. Mention this to your students and at least one of them will be sure to recognize the pattern.

Question 26

There are two basic Ohm's Law equations: one relating voltage, current, and resistance; and the other relating voltage, current, and power (the latter equation is sometimes known as *Joule's Law* rather than Ohm's Law):

$$E = IR$$

$$P = IE$$

In electronics textbooks and reference books, you will find twelve different variations of these two equations, one solving for each variable in terms of a unique pair of two other variables. However, you need not memorize all twelve equations if you have the ability to algebraically manipulate the two simple equations shown above.

Demonstrate how algebra is used to derive the ten "other" forms of the two Ohm's Law / Joule's Law equations shown here.

file 00088

Answer 26

I won't show you how to do the algebraic manipulations, but I will show you the ten other equations. First, those equations that may be derived strictly from $E = IR$:

$$I = \frac{E}{R}$$

$$R = \frac{E}{I}$$

Next, those equations that may be derived strictly from $P = IE$:

$$I = \frac{P}{E}$$

$$E = \frac{P}{I}$$

Next, those equations that may be derived by using algebraic *substitution* between the original two equations given in the question:

$$P = I^2 R$$

$$P = \frac{E^2}{R}$$

And finally, those equations which may be derived from manipulating the last two power equations:

$$R = \frac{P}{I^2}$$

$$I = \sqrt{\frac{P}{R}}$$

$$E = \sqrt{PR}$$

$$R = \frac{E^2}{P}$$

Notes 26

Algebra is an extremely important tool in many technical fields. One nice thing about the study of electronics is that it provides a relatively simple context in which fundamental algebraic principles may be learned (or at least illuminated).

The same may be said for calculus concepts as well: basic principles of derivative and integral (with respect to time) may be easily applied to capacitor and inductor circuits, providing students with an accessible context in which these otherwise abstract concepts may be grasped. But calculus is a topic for later worksheet questions . . .

Question 27

Suppose we only knew the emitter and base currents for an operating transistor and wished to calculate β from that information. We would need a definition of beta cast in terms of I_E and I_B instead of I_C and I_B.

Apply algebraic substitution to the formula $\beta = \frac{I_C}{I_B}$ so that beta (β) is defined in terms of I_E and I_B. You may find the following equation helpful in your work:

$$I_E = I_C + I_B$$

file 02387

Answer 27

$$\beta = \frac{I_E}{I_B} - 1$$

Notes 27

This question is nothing more than an exercise in algebraic manipulation.

Question 28

The resistance of a piece of copper wire at temperature T (in degrees Celsius) is given by the following formula:

$$R_T = R_o \left[1 + 0.004041(T - 20)\right]$$

Suppose you wished to alter this formula so it could accept values for T in units of degrees Fahrenheit instead of degrees Celsius. Suppose also that the only formula you are able to find for converting between Fahrenheit (T_F) and Celsius (T_C) is this one:

$$T_F = T_C \left(\frac{9}{5}\right) + 32$$

Combined these two formulae into one solving for the resistance of a copper wire sample (R_T) at a specific temperature in degrees Celsius (T_C), given the specimen's "reference" resistance (R_o) at 20^o Celsius (room temperature).

file 03068

Answer 28

$$R_T = R_o \left[1 + 0.004041 \left(\frac{5}{9} T_F - 37.\overline{77}\right)\right]$$

Notes 28

Solving this algebraic problem requires both manipulation of the temperature equation *and* substitution of variables. One important detail I incorporated into this question is the lack of a subscript for T in the original resistance formula. In the first sentence I identify that temperature as being in degrees Celsius, but since there is no other T variables in the equation, I did not have to include a "C" subscript. When students look to the Celsius-Fahrenheit conversion formula to substitute into the resistance formula, they must decide which T in the conversion formula to use, T_F or T_C. Here, I purposely wrote the conversion formula in terms of T_F to see how many students would blindly substitute T_F for T in the resistance formula instead of properly identifying T_C as the variable to substitute and doing the work of manipulation.

Far from being a "trick" question, this scenario is very realistic. Formulae found in reference manuals do not necessarily use standardized variables, but rather cast their variables according to context. Multiple formulae will most likely not be written with *identical* subscripted variables just waiting to be substituted. It is the domain of the intelligent technician, engineer, or scientist to figure out what variables are appropriate to substitute based on context!

Question 29

A bipolar junction transistor parameter similar to β is "alpha," symbolized by the Greek letter α. It is defined as the ratio between collector current and emitter current:

$$\alpha = \frac{I_C}{I_E}$$

Apply algebraic substitution to this formula so that alpha is defined as a *function* of beta: $\alpha = f(\beta)$. In other words, substitute and manipulate this equation until you have alpha by itself on one side and no variable except beta on the other.

You may find the following equations helpful in your work:

$$\beta = \frac{I_C}{I_B} \qquad\qquad I_E = I_C + I_B$$

file 02064

Answer 29

$$\alpha = \frac{\beta}{\beta + 1}$$

Follow-up question: what range of values might you expect for α, with a typical transistor?

Notes 29

This question is nothing more than an exercise in algebraic manipulation.

Question 30

The Q factor of a series inductive circuit is given by the following equation:

$$Q = \frac{X_L}{R_{series}}$$

Likewise, we know that inductive reactance may be found by the following equation:

$$X_L = 2\pi f L$$

We also know that the resonant frequency of a series LC circuit is given by this equation:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Through algebraic substitution, write an equation that gives the Q factor of a series resonant LC circuit exclusively in terms of L, C, and R, without reference to reactance (X) or frequency (f).

file 01683

Answer 30

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Notes 30

This is merely an exercise in algebra. However, knowing how these three component values affects the Q factor of a resonant circuit is a valuable and practical insight!

Question 31

Solve this equation for the value of x:

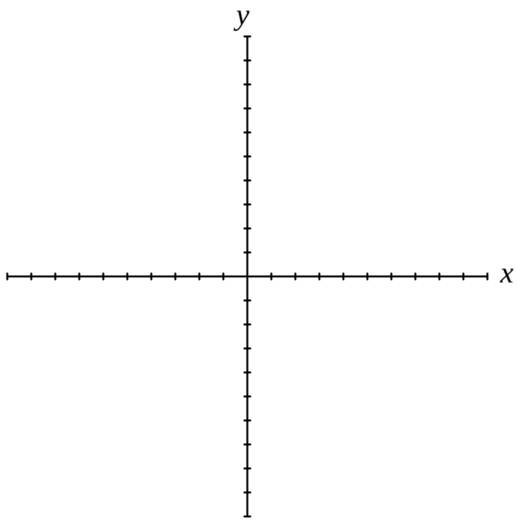
$$x + 5 = 8$$

How many exact solutions does the above equation have?

Now, determine a few *different* solutions for the following equation:

$$x + y = 8$$

How many exact solutions does this equation have? Plot this equation's solutions on the following graph:

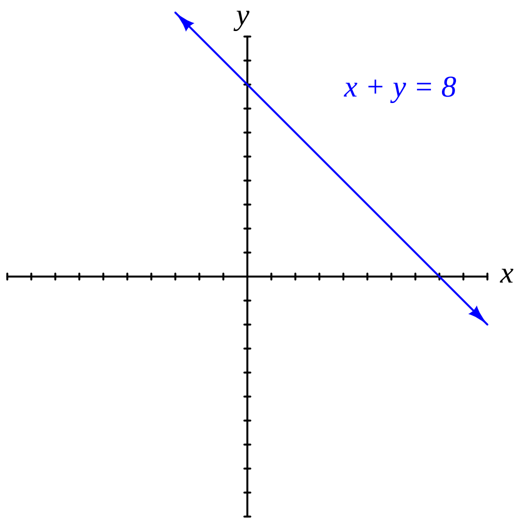


file 03091

Answer 31

If $x + 5 = 8$, then $x = 3$ (exactly one solution)

If $x + y = 8$, then there are an infinite number of solutions:



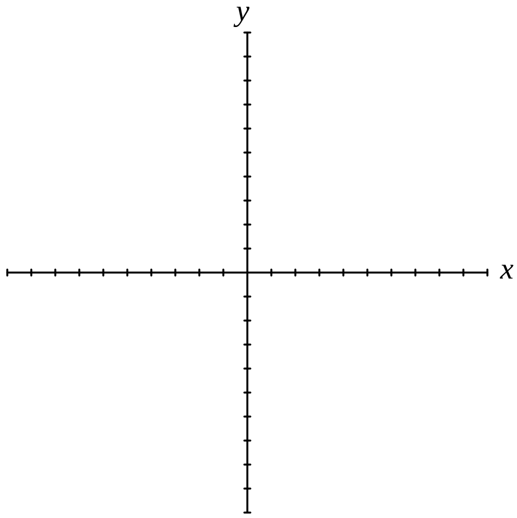
Follow-up question: find the solution to $x + 5 = 8$ on this same graph.

Notes 31

This question begins with an extremely simple equation having one solution and moves on to another simple equation having an infinite number of solutions. While an infinitude of correct answers may seem impossible to rationally deal with, a graph handles it quite nicely, the multitude of correct answer pairs represented as a line on a graph with infinite length.

Question 32

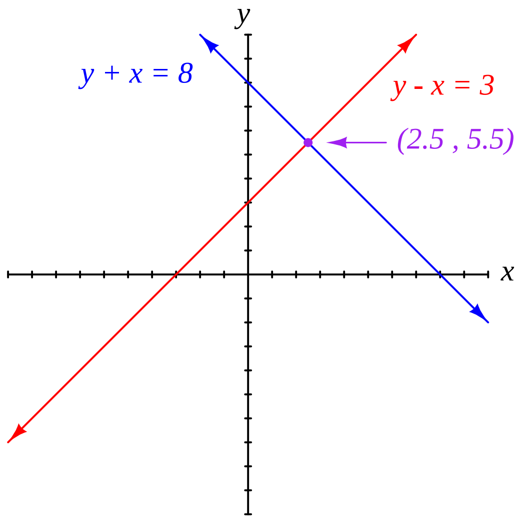
Plot the solutions to the equation $y + x = 8$ on a graph:



On the same graph, plot the solutions to the equation $y - x = 3$. What is the significance of the point where the two lines cross?

file 03092

Answer 32



The point of intersection between the two lines represents the one solution set that satisfies *both* equations (where $x = 2.5$ and $y = 5.5$).

Notes 32

The purpose of this question is to gently introduce students to the concept of simultaneous systems of equations, where a set of solutions satisfies more than one equation at a time. It is important for students to understand the basic concepts of graphing before they try to answer this question, though.

Question 33

What does it actually mean to obtain a solution for a "simultaneous" system of equations? For example, if $2x + y = 7$ and $x - y = -1$, what do the solution values ($x = 2$; $y = 3$) represent?

If we were to graph both these linear equations on a Cartesian (x, y) coordinate system, where would the solution (2,3) be located on the graph?

file 01040

Answer 33

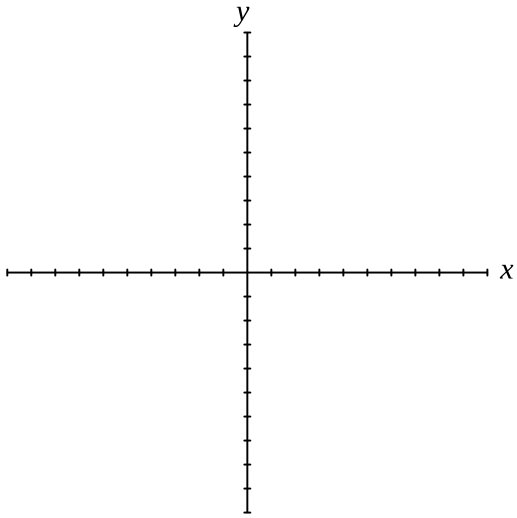
The solutions for a system of equations represent a unique combination of values that satisfy all equations in that system. For a two variable system, the solution is the intersection of two lines.

Notes 33

Many students have difficulty grasping the significance of systems of equations. Discuss the meaning of equations, and systems of equations, with your students, making sure that the concept of simultaneity (solutions satisfying all equations at once) is made clear.

Question 34

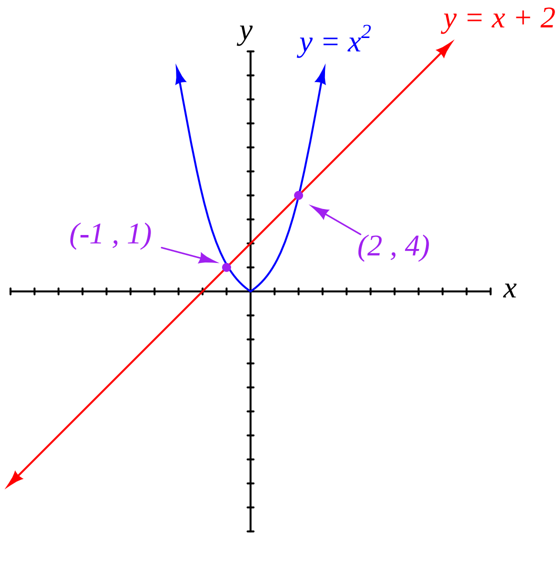
Plot the equation $y = x^2$ on the following graph:



On the same graph, plot the equation $y = x + 2$. What is the significance of the point where the two plots cross?

file 03101

Answer 34



Here there are *two* points of intersection between the parabola (curve) and the straight line, representing two different solution sets that satisfy *both* equations.

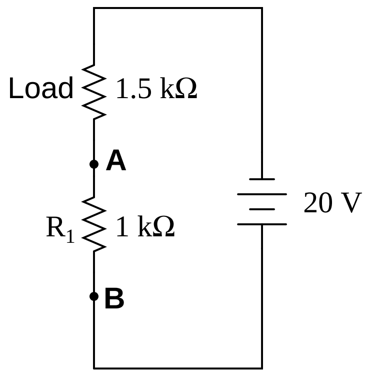
Challenge question: solve this simultaneous system of equations without graphing, but by symbolically manipulating the equations!

Notes 34

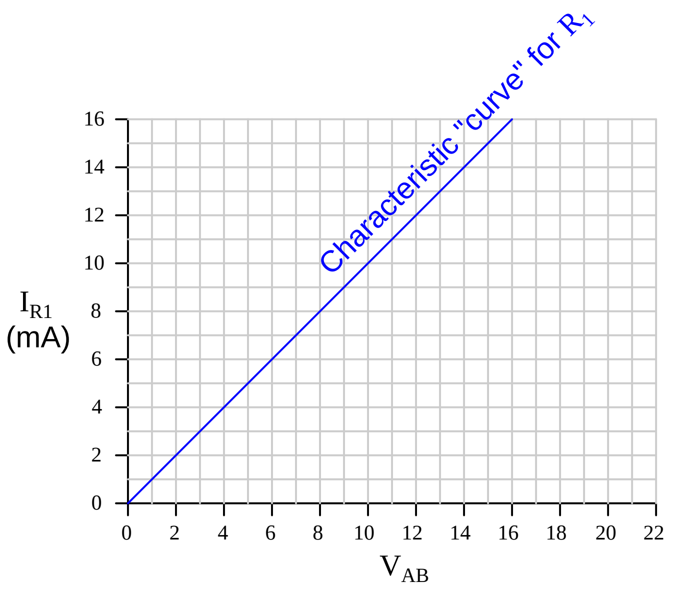
Here, solution by graphing may be a bit easier than the symbolic solution. In principle we may determine solutions for *any* pair of equations by graphing, with about equal difficulty. The only real problem is precision: how closely we may interpret to points of intersection. A practical example of non-linear simultaneous function solution is *load line* analysis in semiconductor circuitry.

Question 35

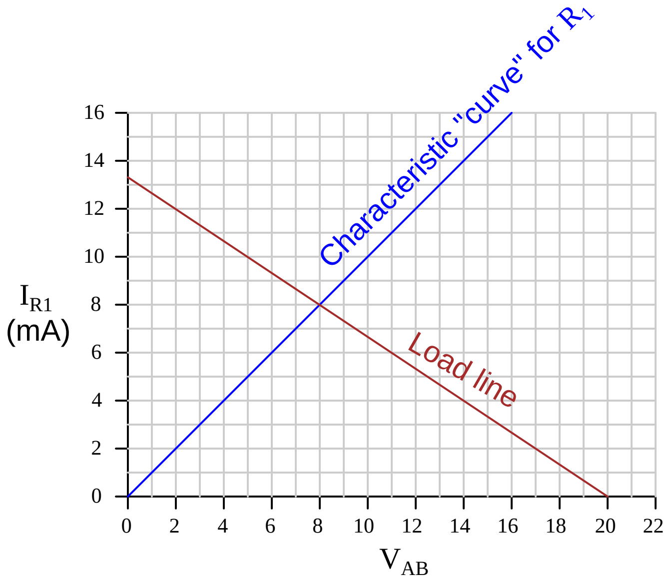
Load lines are useful tools for analyzing transistor amplifier circuits, but they may be hard to understand at first. To help you understand what "load lines" are useful for and how they are determined, I will apply one to this simple two-resistor circuit:



We will have to plot a load line for this simple two-resistor circuit along with the "characteristic curve" for resistor R_1 in order to see the benefit of a load line. Load lines really only have meaning when superimposed with other plots. First, the characteristic curve for R_1, defined as the voltage/current relationship between terminals **A** and **B**:



Next, I will plot the load line as defined by the 1.5 kΩ load resistor. This "load line" expresses the voltage available between the same two terminals (V_{AB}) as a function of the load current, to account for voltage dropped across the load:



At what value of current (I_{R1}) do the two lines intersect? Explain what is significant about this value of current.

file 00953

Answer 35

$I_R = 8$ mA is the same value of current you would calculate if you had analyzed this circuit as a simple series resistor network.

Follow-up question: you might be wondering, "what is the point of plotting a 'characteristic curve' and a 'load line' in such a simple circuit, if all we had to do to solve for current was add the two resistances and divide that total resistance value into the total voltage?" Well, to be honest, there is no point in analyzing such a simple circuit in this manner, except to illustrate *how* load lines work. My follow-up question to you is this: where would plotting a load line actually be helpful in analyzing circuit behavior? Can you think of any modifications to this two-resistor circuit that would require load line analysis in order to solve for current?

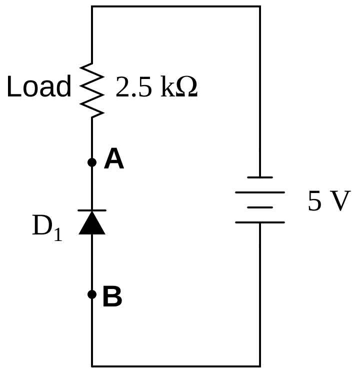
Notes 35

While this approach to circuit analysis may seem silly – using load lines to calculate the current in a two-resistor circuit – it demonstrates the principle of load lines in a context that should be obvious to students at this point in their study. Discuss with your students how the two lines are obtained (one for resistor R_1 and the other plotting the voltage available to R_1 based on the total source voltage and the load resistor's value).

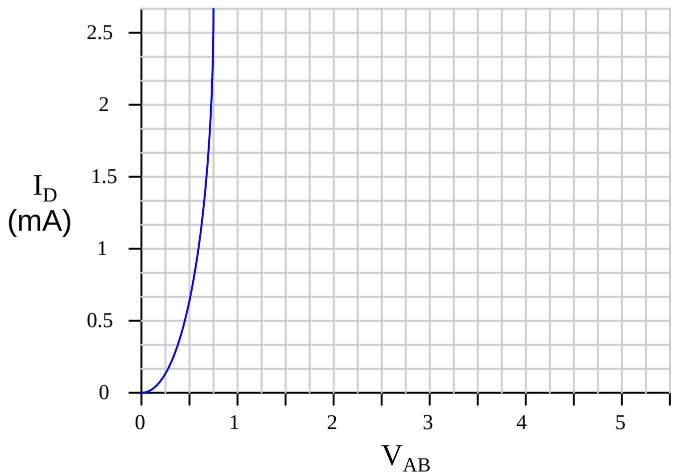
Also, discuss the significance of the two line intersecting. Mathematically, what does the intersection of two graphs mean? What do the coordinate values of the intersection point represent in a system of simultaneous functions? How does this principle relate to an electronic circuit?

Question 36

Load lines are useful tools for analyzing transistor amplifier circuits, but they may be applied to other types of circuits as well. Take for instance this diode-resistor circuit:



The diode's characteristic curve is already plotted on the following graph. Your task is to plot the load line for the circuit on the same graph, and note where the two lines intersect:

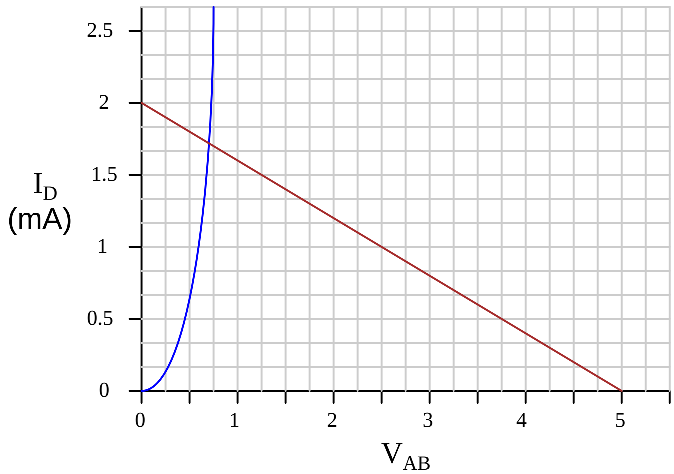


What is the practical significance of these two plots' intersection?

file 00954

Answer 36

The two lines intersect at a current of approximately 1.72 mA:



Follow-up question: explain why the use of a load line greatly simplifies the determination of circuit current in such a diode-resistor circuit.

Challenge question: suppose the resistor value were increased from 2.5 kΩ to 10 kΩ. What difference would this make in the load line plot, and in the intersection point between the two plots?

Notes 36

While this approach to circuit analysis may seem silly – using load lines to calculate the current in a diode-resistor circuit – it demonstrates the principle of load lines in a context that should be obvious to students at this point in their study. Discuss with your students how the load line is obtained for this circuit, and why it is straight while the diode's characteristic curve is not.

Also, discuss the significance of the two line intersecting. Mathematically, what does the intersection of two graphs mean? What do the coordinate values of the intersection point represent in a system of simultaneous functions? How does this principle relate to an electronic circuit?

Question 37

Suppose you were given the following two equations and asked to find solutions for x and y that will satisfy *both* at the same time:

$$y + x = 8$$

$$y - x = 3$$

If we manipulate the second equation so as to solve for y, we will have a definition of y in terms of x that we may use for substitution in the first equation:

$$y = x + 3$$

Show the process of substitution into the first equation, and how this leads to a single solution for x. Then, use that value of x to solve for y, resulting in a solution set valid for *both* equations.

file 03094

Answer 37

If $y + x = 8$ and $y = x + 3$, then $(x + 3) + x = 8$. Therefore,

$$x = 2.5 \text{ and } y = 5.5$$

Notes 37

This question demonstrates one of the (many) practical uses of algebraic substitution: solving simultaneous systems of equations.

Question 38

Suppose you were given the following two equations and asked to find solutions for x and y that will satisfy *both* at the same time:

$$y + x = 8$$

$$y - x = 3$$

Now, you know that we may do anything we want to either equation as long as we do the same thing to both sides (on either side of the "equal" sign). This is the basic rule we follow when manipulating an equation to solve for a particular variable. For example, we may take the equation $y + x = 8$ and subtract x from both sides to yield an equation expressed in terms of y:

$$\begin{array}{c} y + x = 8 \\ \underline{\quad -x \qquad -x \quad} \\ y = 8 - x \end{array}$$

Following the same principle, we may take two equations and combine them either by adding or subtracting both sides. For example, we may take the equation $y - x = 3$ and add both sides of it to the respective sides of the first equation $y + x = 8$:

$$\begin{array}{c} y + x = 8 \\ \underline{\quad y - x = 3 \quad} \\ 2y = 11 \end{array}$$

What beneficial result comes of this action? In other words, how can I use this new equation $2y = 11$ to solve for values of x and y that satisfy both of the original equations?

file 03093

Answer 38

We may use the result ($2y = 11$) to solve for a value of y, which when substituted into either of the original equations may be used to solve for a value of x to satisfy *both* equations at the same time.

Notes 38

While not intuitively obvious to most people, the technique of adding two entire equations to each other for the purpose of eliminating a variable is not only possible to do, but very powerful when looking for solutions to satisfy both original equations. Discuss with your students why it is allowable for us to add $y - x$ to $y + x$ and to add 3 to 8. to yield the equation $2y = 11$.

Question 39

Solve for values of x and y that will satisfy both of the following equations at the same time:

$$x + 2y = 9$$

$$4x - y = -18$$

file 03100

Answer 39

$x = -3$ $\qquad$ $y = 6$

Follow-up question: solve this system of simultaneous equations using both substitution (solving for one variable in one of the equations and substituting that into the other equation) and addition (adding the two equations together to produce a third equation with only one unknown).

Notes 39

Nothing special here – just practice solving for a two-variable system of equations.

Question 40

Many circuit analysis techniques require the solution of "systems of linear equations," sometimes called "simultaneous equations." This question is really a series of practice problems for solving simultaneous linear equations, the purpose being to give you lots of practice using various solution techniques (including the solution facilities of your calculator).

Systems of two variables:

$x + y = 5$
$x - y = 1$

$x - y = -6$
$2x - y = 4$

$2x + y = 7$
$x - y = 2$

$3x - 2y = -1$
$5x + y = -6$

$-10x + 2y = 0$
$-3x - 5y = -28$

$3x - 5y = -13$
$-x + 2y = 5$

$1000x - 500y = 0$
$550x + 2500y = 5550$

$-15000x + 2200y = -66200$
$7900x - 2800y = 28300$

$9100x - 5000y = 24$
$-5200x - 2700y = -6.5$

Systems of three variables:

$x - y + z = 1$
$-x - y + z = -1$
$x + y + z = 3$

$3x + 2y - 5z = -21$
$x - 3y + z = 8$
$-x - y - z = -12$

$x + y + z = 0$
$2x - y - 4z = -9$
$-2x + 2y - z = 12$

$x + y - 2z = -12$
$3x - 2y + z = 19$
$-4x + 3y - 5z = -45$

$-4x - 3y + 2z = -32$
$x - 2y + 3z = -1$
$-2x + 7y - z = 3$

$19x - 6y + 20z = -33$
$4x + 5y - 3z = -17$
$-7x + 2y - 8z = 9$

$890x - 1000y + 2500z = -1500$
$3300x + 7200y - 5100z = 21500$
$-x + y - z = 0$

$2750x - 6200y + 4500z = 17500$
$-10000x + 5300y - 1000z = 8100$
$6x - 2y - 3z = 5$

file 01039

Answer 40

Systems of two variables:

$x + y = 5$
$x - y = 1$
$\mathbf{x = 3 \; ; \; y = 2}$

$x - y = -6$
$2x - y = 4$
$\mathbf{x = 10 \; ; \; y = 16}$

$2x + y = 7$
$x - y = 2$
$\mathbf{x = 3 \; ; \; y = 1}$

$3x - 2y = -1$
$5x + y = -6$
$\mathbf{x = -1 \; ; \; y = -1}$

$-10x + 2y = 0$
$-3x - 5y = -28$
$\mathbf{x = 1 \; ; \; y = 5}$

$3x - 5y = -13$
$-x + 2y = 5$
$\mathbf{x = -1 \; ; \; y = 2}$

$1000x - 500y = 0$
$550x + 2500y = 5550$
$\mathbf{x = 1 \; ; \; y = 2}$

$-15000x + 2200y = -66200$
$7900x - 2800y = 28300$
$\mathbf{x = 5 \; ; \; y = 4}$

$9100x - 5000y = 24$
$-5200x - 2700y = -6.5$
$\mathbf{x = 0.001924 \; ; \; y = -0.001298}$

Systems of three variables:

$x - y + z = 1$
$-x - y + z = -1$
$x + y + z = 3$
$\mathbf{x = 1 \; ; \; y = 1 \; ; \; z = 1}$

$3x + 2y - 5z = -21$
$x - 3y + z = 8$
$-x - y - z = -12$
$\mathbf{x = 4 \; ; \; y = 1 \; ; \; z = 7}$

$x + y + z = 0$
$2x - y - 4z = -9$
$-2x + 2y - z = 12$
$\mathbf{x = -3 \; ; \; y = 3 \; ; \; z = 0}$

$x + y - 2z = -12$
$3x - 2y + z = 19$
$-4x + 3y - 5z = -45$
$\mathbf{x = 2 \; ; \; y = -4 \; ; \; z = 5}$

$-4x - 3y + 2z = -32$
$x - 2y + 3z = -1$
$-2x + 7y - z = 3$
$\mathbf{x = 6 \; ; \; y = 2 \; ; \; z = -1}$

$19x - 6y + 20z = -33$
$4x + 5y - 3z = -17$
$-7x + 2y - 8z = 9$
$\mathbf{x = -5 \; ; \; y = 3 \; ; \; z = 4}$

$890x - 1000y + 2500z = -1500$
$3300x + 7200y - 5100z = 21500$
$-x + y - z = 0$
$\mathbf{x = 2.215 \; ; \; y = 1.378 \; ; \; z = -0.8376}$

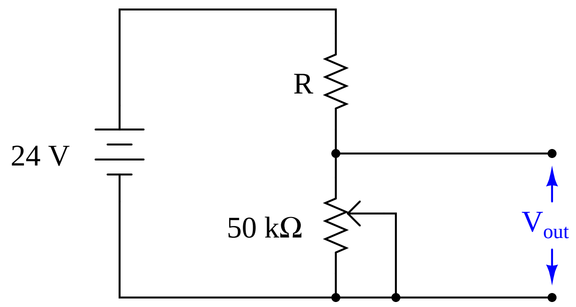
$2750x - 6200y + 4500z = 17500$
$-10000x + 5300y - 1000z = 8100$
$6x - 2y - 3z = 5$
$\mathbf{x = -5.171 \; ; \; y = -9.322 \; ; \; z = -5.794}$

Notes 40

I suggest you let your students discover how to use the equation-solving facilities of their scientific calculators on their own. My experience has been that students both young and old take to this challenge readily, because they realize learning how to use their calculators will save them a tremendous amount of hand calculations!

Question 41

Suppose you needed to choose a fixed resistor value (R) to make a voltage divider circuit, given a known potentiometer resistance value, the source voltage value, and the desired range of adjustment:



Desired range for V_{out} = *0 to 17 volts*

Solve for R, and show the equation you set up in order to do it.

Hint: remember the series resistor voltage divider formula . . .

$$V_R = V_{total} \left(\frac{R}{R_{total}} \right)$$

file 03102

Answer 41

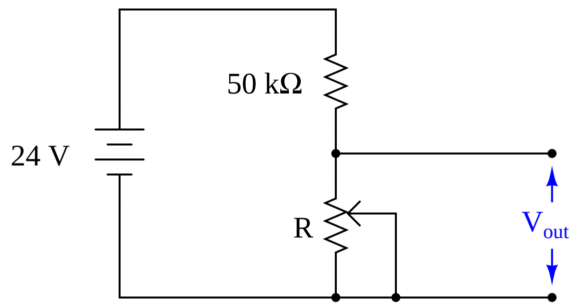
$R = 20.588$ kΩ

Notes 41

Be sure to have your students set up their equations in front of the class so everyone can see how they did it. Some students may opt to apply Ohm's Law to the solution of R, which is good, but for the purpose of developing equations to fit problems it might not be the best solution. Challenge your students to come up with a *single equation* that solves for R, with all known quantities on the other side of the "equal" sign.

Question 42

Suppose you needed to choose a potentiometer value (R) to make a voltage divider circuit, given a known fixed resistor value, the source voltage value, and the desired range of adjustment:



Desired range for V_{out} = *0 to 17 volts*

Solve for R, and show the equation you set up in order to do it.

Hint: remember the series resistor voltage divider formula . . .

$$V_R = V_{total} \left(\frac{R}{R_{total}} \right)$$

file 03103

Answer 42

$R = 121.43$ kΩ

Follow-up question: you will not be able to find a potentiometer with a full-range resistance value of exactly 121.43 kΩ. Describe how you could take a standard-value potentiometer and connect it to one or more fixed-value resistors to give it this desired full-scale range.

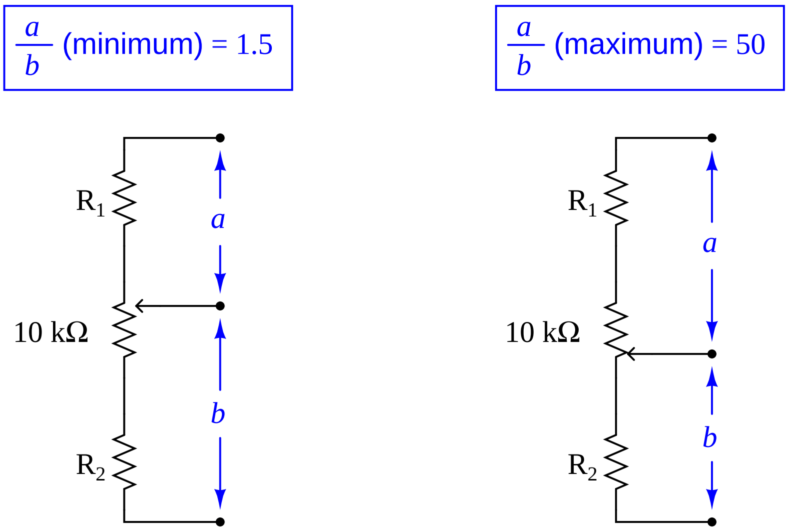
Notes 42

Be sure to have your students set up their equations in front of the class so everyone can see how they did it. Some students may opt to apply Ohm's Law to the solution of R, which is good, but for the purpose of developing equations to fit problems it might not be the best solution. Challenge your students to come up with a *single equation* that solves for R, with all known quantities on the other side of the "equal" sign.

The follow-up question is very practical, as it is impossible to find potentiometers ready-made to arbitrary values of full-scale resistance. Instead, you must work with what you can find, which is usually nominal values such as 10 kΩ, 100 kΩ, 1 MΩ, etc.

Question 43

An engineer needs to calculate the values of two resistors to set the minimum and maximum resistance ratios for the following potentiometer circuit:



First, write an equation for each circuit, showing how resistances R_1, R_2, and the 10 kΩ of the potentiometer combine to form the ratio $\frac{a}{b}$. Then, use techniques for solving simultaneous equations to calculate actual resistance values for R_1 and R_2.

file 02769

Answer 43

$$\frac{a}{b}\text{ (minimum)} = \frac{R_1}{R_2 + 10000} \qquad \frac{a}{b}\text{ (maximum)} = \frac{R_1 + 10000}{R_2}$$

$R_1 = 15.77$ kΩ

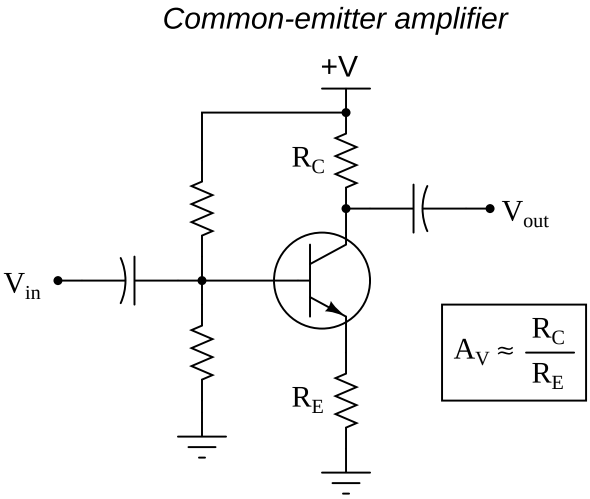
$R_2 = 515.5\ \Omega$

Notes 43

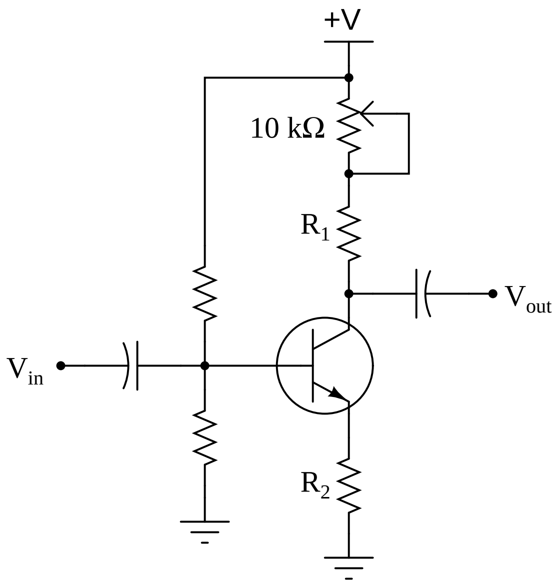
This very practical application of simultaneous equations was actually used by one of my students in establishing the lower and upper bounds for the voltage gain adjustment of an inverting opamp circuit!

Question 44

The voltage gain of a common-emitter transistor amplifier is approximately equal to the collector resistance divided by the emitter resistance:



Knowing this, calculate the necessary resistance values for the following fixed-value resistors (R_1 and R_2) to give this common-emitter amplifier an adjustable voltage gain range of 4 to 7:



file 03107

Answer 44

$R_1 = 13.33$ kΩ

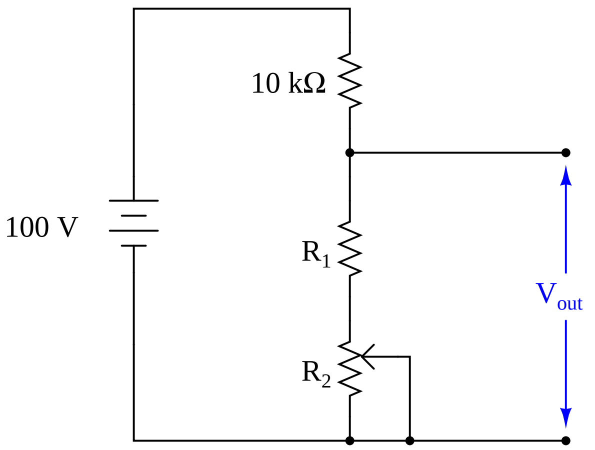
$R_2 = 3.333$ kΩ

Notes 44

Have your students show how they set up the system of equations to solve for the two resistor values. This is a good exercise to do in front of the class, so everyone can see (possibly) different methods of solution.

Question 45

Suppose you needed to choose two resistance values to make a voltage divider with a limited adjustment range. One of these resistors will be fixed in value (R_1), while the other will be variable (a potentiometer connected as a rheostat– R_2):



Desired range for V_{out} = *30 volts to 70 volts*

Set up a system of simultaneous equations to solve for both R_1 and R_2, and show how you arrived at the solutions for each.

Hint: remember the series resistor voltage divider formula . . .

$$V_R = V_{total} \left(\frac{R}{R_{total}} \right)$$

file 03104

Answer 45

R_1 (fixed) = 4.286 kΩ

R_2 (pot) = 19.048 kΩ

Follow-up question: you will not be able to find a potentiometer with a full-range resistance value of exactly 19.048 kΩ. Describe how you could take a standard-value potentiometer and connect it to one or more fixed-value resistors to give it this desired full-scale range.

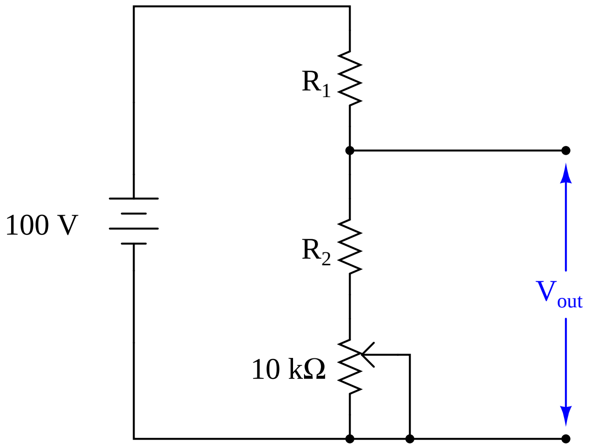
Notes 45

Be sure to have your students set up their equations in front of the class so everyone can see how they did it. Some students may opt to apply Ohm's Law to the solution of both resistors, which is good, but for the purpose of developing equations to fit problems it might not be the best solution. Challenge your students to come up with a *set of equations* that solve for R_1 and R_2, then use techniques for solution of simultaneous equations to arrive at solutions for each.

The follow-up question is very practical, as it is impossible to find potentiometers ready-made to arbitrary values of full-scale resistance. Instead, you must work with what you can find, which is usually nominal values such as 10 kΩ, 50 kΩ, 100 kΩ, etc.

Question 46

Suppose you needed to choose two resistance values to make a voltage divider with a limited adjustment range:



Desired range for V_{out} = *30 volts to 70 volts*

Set up a system of simultaneous equations to solve for both R_1 and R_2, and show how you arrived at the solutions for each.

Hint: remember the series resistor voltage divider formula . . .

$$V_R = V_{total}\left(\frac{R}{R_{total}}\right)$$

file 03105

Answer 46

$R_1 = 5.25$ kΩ

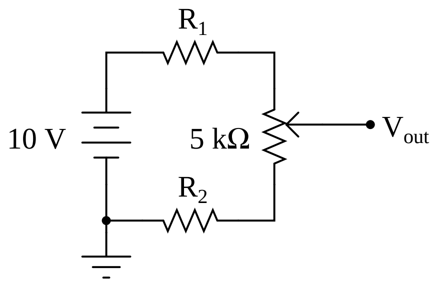
$R_2 = 2.25$ kΩ

Notes 46

Be sure to have your students set up their equations in front of the class so everyone can see how they did it. Some students may opt to apply Ohm's Law to the solution of both resistors, which is good, but for the purpose of developing equations to fit problems it might not be the best solution. Challenge your students to come up with a *set of equations* that solve for R_1 and R_2, then use techniques for solution of simultaneous equations to arrive at solutions for each.

Question 47

Use simultaneous equations to calculate the values of R_1 and R_2 necessary to give this voltage divider the range of adjustment specified:



V_{out} (minimum) = 3 volts V_{out} (maximum) = 8 volts

$R_1 =$ $R_2 =$

file 03110

Answer 47

$R_1 = 2$ kΩ

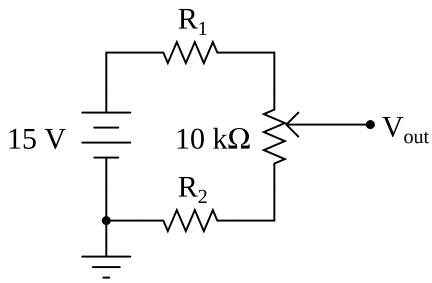
$R_2 = 3$ kΩ

Notes 47

Have your students show their methods of solution in class, so you may observe their problem-solving ability and they may see multiple methods of solution.

Question 48

Use simultaneous equations to calculate the values of R_1 and R_2 necessary to give this voltage divider the range of adjustment specified:



V_{out} (minimum) = 5 volts $\quad\quad$ V_{out} (maximum) = 12 volts

$R_1 =$ $\quad\quad\quad\quad$ $R_2 =$

file 03109

Answer 48

$R_1 = 4.2857$ kΩ

$R_2 = 7.1429$ kΩ

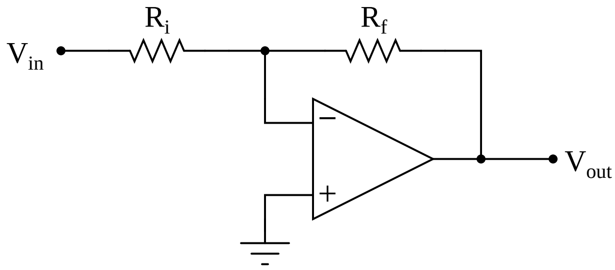
Notes 48

Have your students show their methods of solution in class, so you may observe their problem-solving ability and they may see multiple methods of solution.

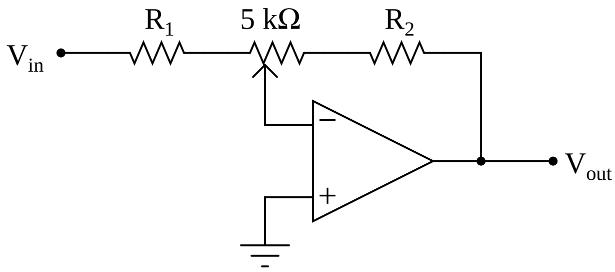
Question 49

The voltage gain of an inverting operational amplifier circuit is defined by the ratio of feedback to input resistance:

$$A_V = \frac{R_f}{R_i}$$



Calculate the necessary values of R_1 and R_2 to limit the minimum and maximum voltage gain of this opamp circuit to 5 and 30, respectively, given a potentiometer in the middle with a full-span resistance of 5 kΩ:



file 02770

Answer 49

$R_1 = 1.2$ kΩ

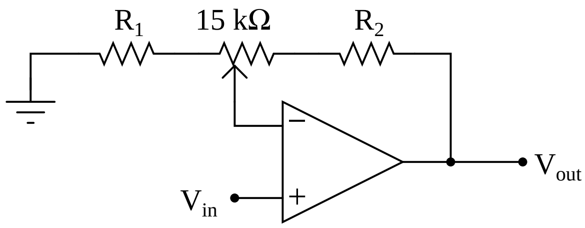
$R_2 = 31$ kΩ

Notes 49

This is a very practical example of using simultaneous equations in analog circuit design.

Question 50

Calculate the necessary values of R_1 and R_2 to limit the minimum and maximum voltage gain of this opamp circuit to 10 and 85, respectively:



file 02771

Answer 50

$R_1 = 2$ kΩ

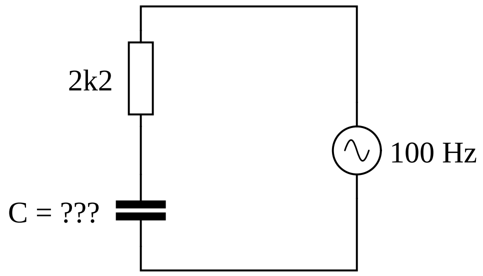
$R_2 = 153$ kΩ

Notes 50

This is a very practical example of using simultaneous equations in analog circuit design. A common mistake students make while setting up the equations is forgetting that a noninverting amplifier's gain is the ratio of the feedback and grounding resistors, *plus one!*

Question 51

Calculate the necessary size of the capacitor to give this circuit a total impedance (Z_{total}) of 4 kΩ, at a power supply frequency of 100 Hz:



file 04042

Answer 51

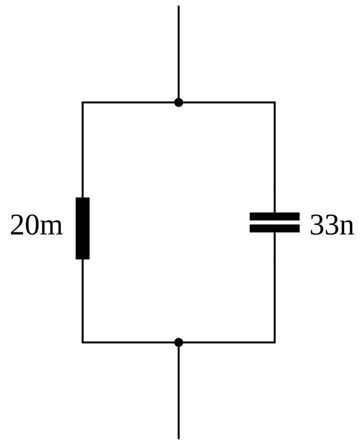
$C = 0.476\ \mu$F

Notes 51

Nothing special to note here, just practice with the impedance triangle (and the capacitive reactance formula).

Question 52

Calculate the resonant frequency of this parallel LC circuit, and qualitatively describe its total impedance (Z_{total}) when operating at resonance:



file 04043

Answer 52

$f_r = 6.195$ kHz

Z_{total} @ $f_r = \infty$

Notes 52

Nothing special to note here, just an application of the resonance formula and a review of parallel LC resonance.

Question 53

Suppose you were given a component and told it was either a resistor, an inductor, or a capacitor. The component is unmarked, and impossible to visually identify. Explain what steps you would take to electrically identify what type of component it was, and what its value was, without the use of any test equipment except a signal generator, a multimeter (capable of measuring nothing but voltage, current, and resistance), and some miscellaneous passive components (resistors, capacitors, inductors, switches, etc.). Demonstrate your technique if possible.

file 02120

Answer 53

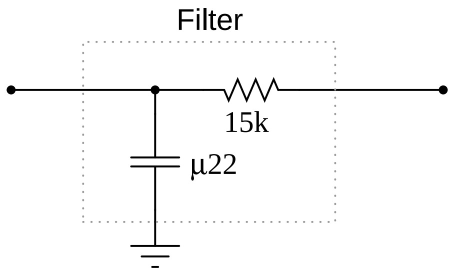
Did you really think I would give you the answer to this?

Notes 53

This is an excellent opportunity to brainstorm as a group and experiment on real components. There is obviously more than one way to make the determinations of identity and value! Use the class time to engage your students in lively discussion and debate over how to approach this practical problem.

Question 54

Identify what type of filter this circuit is, calculate its cutoff frequency, and distinguish the input terminal from the output terminal:



file 04044

Answer 54

This is a *low-pass* filter.

$f_{cutoff} = 48.23$ Hz

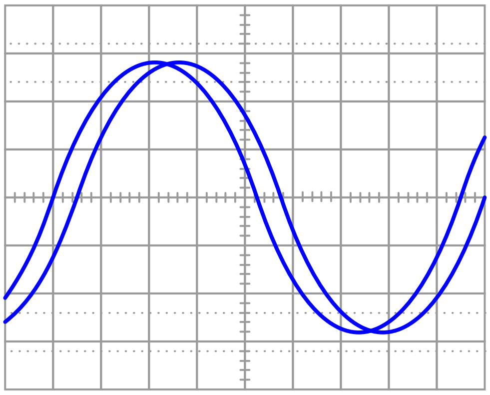
The input terminal is on the right, while the output terminal is on the left.

Notes 54

Be sure to ask students where they found the cutoff frequency formula for this filter circuit. Also, ask them how they were able to distinguish the input and output terminals. What would happen if these terminals were reversed (i.e. if the input signal were applied to the output terminal)?

Question 55

A dual-trace oscilloscope is used to measure the phase shift between voltage and current for an inductive AC load:



Calculate the following, given a load voltage of 110 volts, a load current of 3.2 amps, and a frequency of 60 Hz:

- Apparent power (S) =
- True power (P) =
- Reactive power (Q) =
- Θ =
- Power factor =
- Necessary parallel C size to correct power factor to unity =

file 02192

Answer 55

- Apparent power (S) = 352 VA
- True power (P) = 328.2 W
- Reactive power (Q) = 127.2 VAR
- $\Theta = 21.2^o$
- Power factor = 0.932
- Necessary parallel C size to correct power factor to unity = 27.9 μF

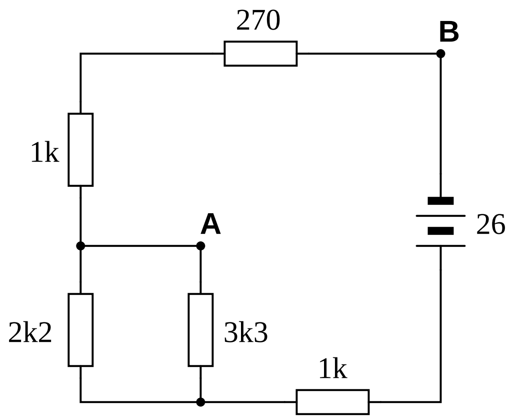
Follow-up question: identify which waveform represents voltage and which waveform represents current on the oscilloscope display.

Notes 55

There are multiple methods of solution for this problem, so be sure to have your students present their thoughts and strategies during discussion!

Question 56

Calculate the amount of voltage between points **A** and **B** in this circuit. Be sure to identify polarity as well as magnitude:



file 04045

Answer 56

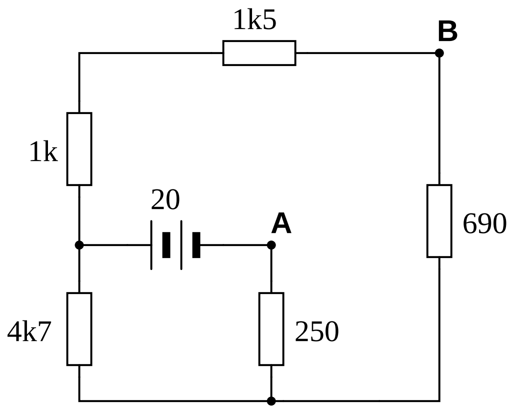
$V_{\mathbf{AB}}$ = 9.198 volts, **A** positive and **B** negative.

Notes 56

Nothing here but series-parallel calculation practice, combined with KVL. Ask your students to explain *how* they calculated this voltage, because there is definitely more than one way to do it!

Question 57

Calculate the amount of voltage between points **A** and **B** in this circuit. Be sure to identify polarity as well as magnitude:



file 04046

Answer 57

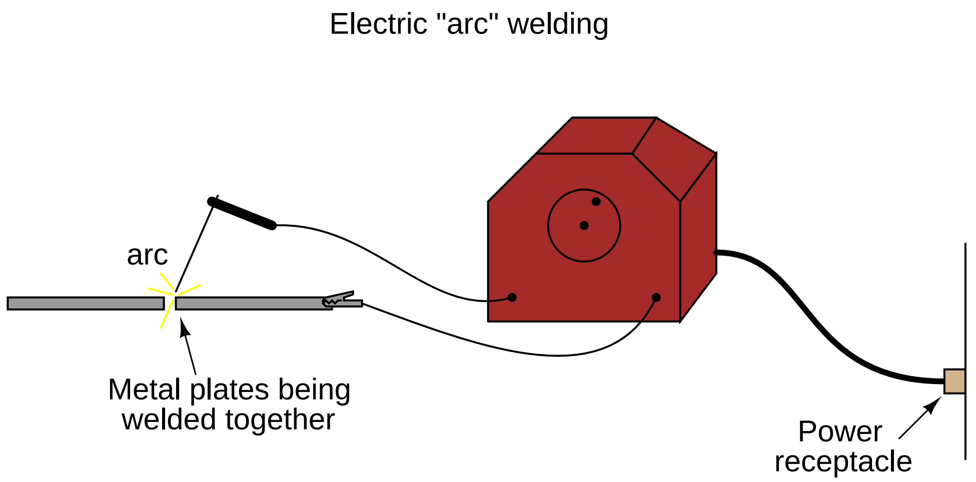
$V_{\mathbf{AB}} = 6.148$ volts, **A** negative and **B** positive.

Notes 57

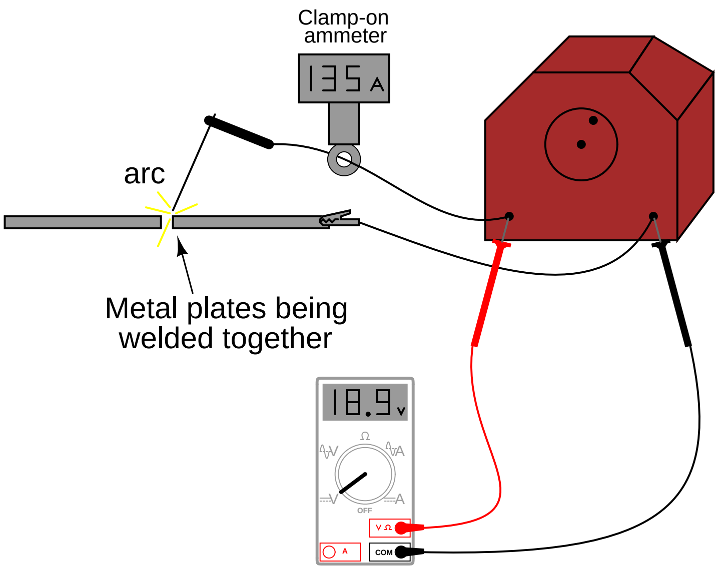
Nothing here but series-parallel calculation practice, combined with KVL. Ask your students to explain *how* they calculated this voltage, because there is definitely more than one way to do it!

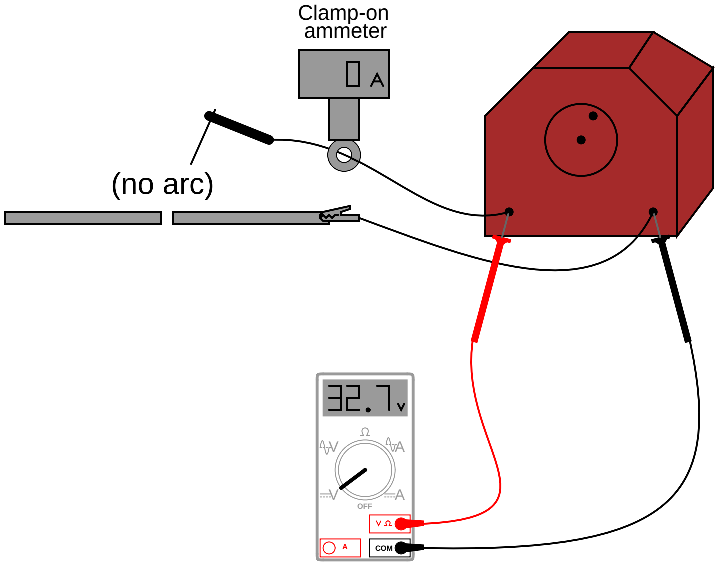
Question 58

An *electric arc welder* is a low-voltage, high-current power source designed to supply enough electric current to sustain an arc capable of welding metal with its high temperature:



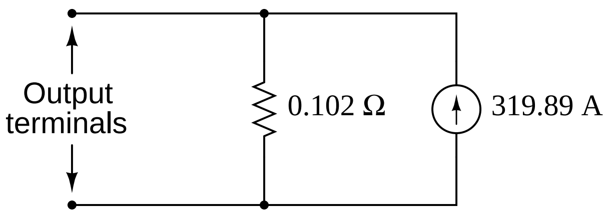
It is possible to derive a Norton equivalent circuit for an arc welder based on empirical measurements of voltage and current. Take for example these measurements, under *loaded* and *no-load* conditions:





Based on these measurements, draw a Norton equivalent circuit for the arc welder.
file 03292

Answer 58



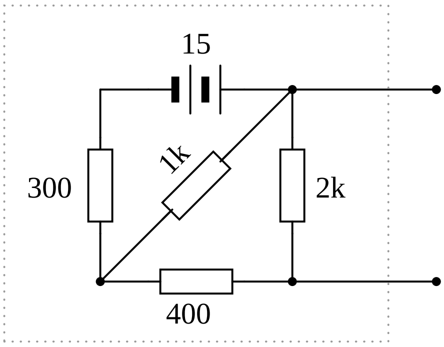
Notes 58

This practical scenario shows how Norton's theorem may be used to "model" a complex device as two simple components (current source and resistor). Of course, we must make certain assumptions when modeling in this fashion: we assume, for instance, that the arc welder is a linear device, which may or may not be true.

Incidentally, there is such a thing as a DC-measuring clamp-on ammeter as shown in the illustrations, in case any one of your students ask. AC clamp-on meters are simpler, cheaper, and thus more popularly known, but devices using the Hall effect are capable of inferring DC current by the strength of an unchanging magnetic field, and these Hall-effect devices are available at modest expense.

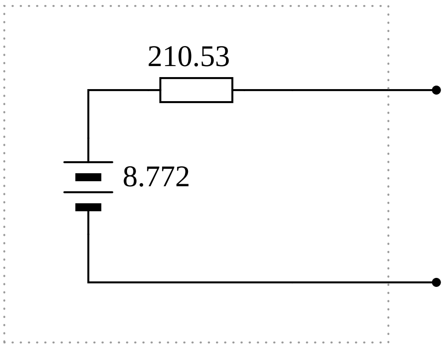
Question 59

Thévenize this resistive network:



file 04047

Answer 59

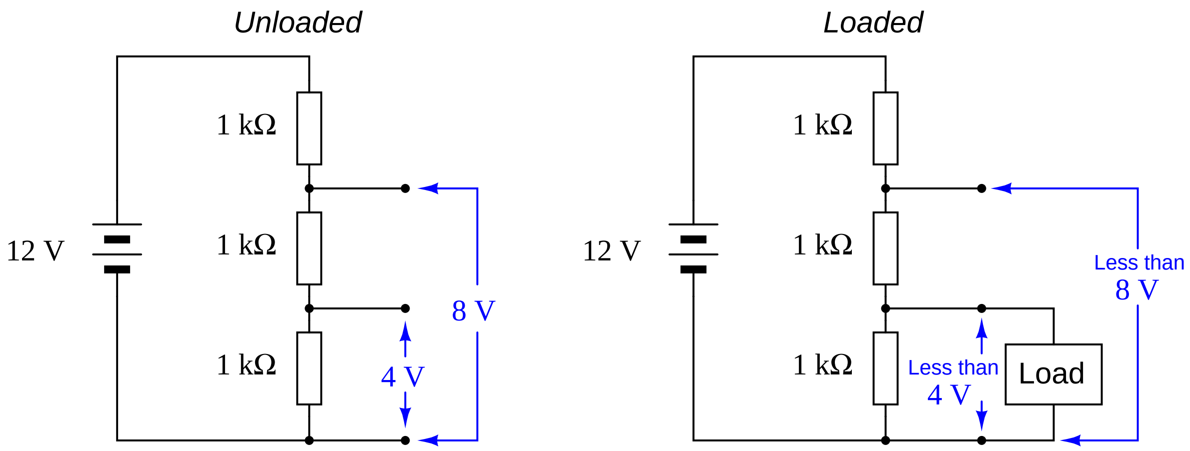


Notes 59

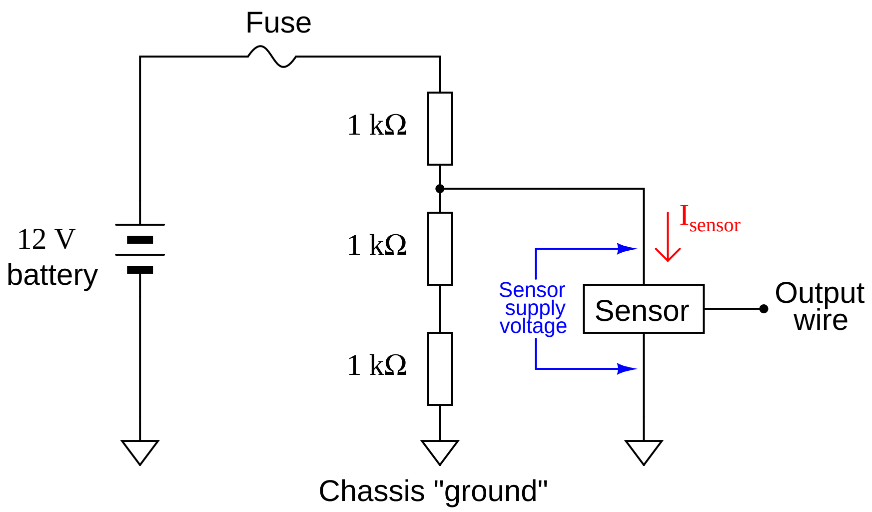
Nothing but practice here. Have your students demonstrate how they did the Thévenin conversion, step-by-step.

Question 60

Resistive voltage dividers are very useful and popular circuits. However, it should be realized that their output voltages "sag" under load:



Just how much a voltage divider's output will sag under a given load may be a very important question in some applications. Take for instance the following application where we are using a resistive voltage divider to supply an engine sensor with reduced voltage (8 volts) from the 12 volt battery potential in the automobile:



If the sensor draws no current (I_{sensor} = 0 mA), then the voltage across the sensor supply terminals will be 8 volts. However, if we were asked to predict the voltage across the sensor supply terminals for a variety of different sensor current conditions, we would be faced with a much more complex problem:

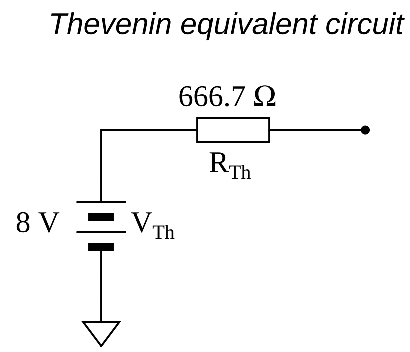
Sensor current (I_{sensor})	Sensor supply voltage
0 mA	8 volts
1 mA	
2 mA	
3 mA	
4 mA	
5 mA	

One technique we could use to simplify this problem is to reduce the voltage divider resistor network into a Thévenin equivalent circuit. With the three-resistor divider reduced to a single resistor in series with an equivalent voltage source, the calculations for sensor supply voltage become much simpler.

Show how this could be done, then complete the table of sensor supply voltages shown above.

file 03239

Answer 60



Sensor current (I_{sensor})	Sensor supply voltage
0 mA	8 volts
1 mA	7.333 volts
2 mA	6.667 volts
3 mA	6 volts
4 mA	5.333 volts
5 mA	4.667 volts

Follow-up question: if we cannot allow the sensor supply voltage to fall below 6.5 volts, what is the maximum amount of current it may draw from this voltage divider circuit?

Challenge question: figure out how to solve for these same voltage figures without reducing the voltage divider circuit to a Thévenin equivalent.

Notes 60

Students are known to ask, "When are we ever going to use Thévenin's Theorem?" as this concept is introduced in their electronics coursework. This is a valid question, and should be answered with immediate, practical examples. This question does exactly that: demonstrate how to predict voltage "sag" for a loaded voltage divider in such a way that is much easier than using Ohm's Law and Kirchhoff's Laws directly.

Note the usage of European schematic symbols in this question. Nothing significant about this choice – just an opportunity for students to see other ways of drawing schematics.

Note also how this question makes use of ground symbols, but in a way where the concept is introduced gently: the first (example) schematics do not use ground symbols, whereas the practical (automotive) circuit does.

Project progress report (1 day)
Date:
Description of progress made on this day

file 03995

Answer 61

Be sure to note *everything* accomplished for each day, so your instructor has a complete record of your progress.

Notes 61

The purpose of this report form is to familiarize students with the concept of time management as it relates to project completion. Too many students have a tendency to do little or nothing until just before their project is due. By assigning a grade value for progress made each day, you help them learn time management skills and also help them complete their projects sooner (and better!).

Question 62

Project progress report (1 day)
Date:
Description of progress made on this day

file 03995

Answer 62

Be sure to note *everything* accomplished for each day, so your instructor has a complete record of your progress.

Notes 62

The purpose of this report form is to familiarize students with the concept of time management as it relates to project completion. Too many students have a tendency to do little or nothing until just before their project is due. By assigning a grade value for progress made each day, you help them learn time management skills and also help them complete their projects sooner (and better!).

Question 63

Project progress report (1 day)
Date:
Description of progress made on this day

file 03995

Answer 63

Be sure to note *everything* accomplished for each day, so your instructor has a complete record of your progress.

Notes 63

The purpose of this report form is to familiarize students with the concept of time management as it relates to project completion. Too many students have a tendency to do little or nothing until just before their project is due. By assigning a grade value for progress made each day, you help them learn time management skills and also help them complete their projects sooner (and better!).

Project progress report (1 day)
Date:
Description of progress made on this day

file 03995

Answer 64

Be sure to note *everything* accomplished for each day, so your instructor has a complete record of your progress.

Notes 64

The purpose of this report form is to familiarize students with the concept of time management as it relates to project completion. Too many students have a tendency to do little or nothing until just before their project is due. By assigning a grade value for progress made each day, you help them learn time management skills and also help them complete their projects sooner (and better!).

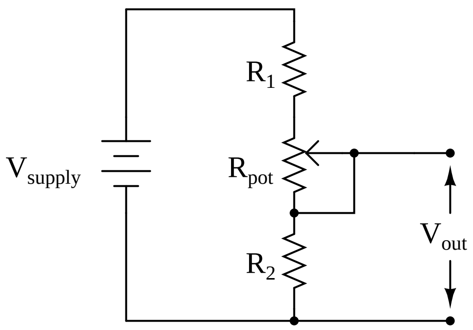
Question 65

Competency: **Voltage divider with limited range** Version:

Description

Use simultaneous equations to calculate values for resistors R_1 and R_2 that will limit the range of voltage adjustment to the specified minimum and maximum values.

Schematic



Given conditions

V_{supply} =

R_{pot} =

V_{out} (max) =

V_{out} (min) =

Parameters

	Calculated		Measured
R_1		V_{out} (min)	
R_2		V_{out} (max)	

Equations

file 03108

Answer 65

Use circuit simulation software to verify your predicted and measured parameter values.

Notes 65

The purpose of this exercise is for students to use simultaneous equations to arrive at values for fixed resistors R_1 and R_2 that will limit output voltage adjustment to the limits specified.

An extension of this exercise is to incorporate troubleshooting questions. Whether using this exercise as a performance assessment or simply as a concept-building lab, you might want to follow up your students' results by asking them to predict the consequences of certain circuit faults.

Question 66

Project progress report (1 day)
Date:
Description of progress made on this day

file 03995

Answer 66

Be sure to note *everything* accomplished for each day, so your instructor has a complete record of your progress.

Notes 66

The purpose of this report form is to familiarize students with the concept of time management as it relates to project completion. Too many students have a tendency to do little or nothing until just before their project is due. By assigning a grade value for progress made each day, you help them learn time management skills and also help them complete their projects sooner (and better!).

Question 67

NAME: _______________ **Project Grading Criteria** PROJECT: _______________

You will receive the highest score for which *all* criteria are met.

100 % *(Must meet or exceed all criteria listed)*
A. Impeccable craftsmanship, comparable to that of a professional assembly
B. No spelling or grammatical errors anywhere in *any* document, upon first submission to instructor

95 % *(Must meet or exceed these criteria in addition to all criteria for 90% and below)*
A. Technical explanation sufficiently detailed to teach from, inclusive of every component (supersedes 75.B)
B. Itemized parts list complete with part numbers, manufacturers, and (equivalent) prices for *all* components, including recycled components and parts kit components (supersedes 90.A)

90 % *(Must meet or exceed these criteria in addition to all criteria for 85% and below)*
A. Itemized parts list complete with prices of components purchased for the project, plus total price
B. No spelling or grammatical errors anywhere in *any* document upon final submission

85 % *(Must meet or exceed these criteria in addition to all criteria for 80% and below)*
A. "User's guide" to project function (in addition to 75.B)
B. Troubleshooting log describing all obstacles overcome during development and construction

80 % *(Must meet or exceed these criteria in addition to all criteria for 75% and below)*
A. All controls (switches, knobs, etc.) clearly and neatly labeled
B. All documentation created on computer, not hand-written (including the schematic diagram)

75 % *(Must meet or exceed these criteria in addition to all criteria for 70% and below)*
A. Stranded wire used wherever wires are subject to vibration or bending
B. Basic technical explanation of all major circuit sections
C. Deadline met for working prototype of circuit (Date/Time = __________ / __________)

70 % *(Must meet or exceed these criteria in addition to all criteria for 65%)*
A. All wire connections sound (solder joints, wire-wrap, terminal strips, and lugs are all connected properly)
B. No use of glue where a fastener would be more appropriate
C. Deadline met for submission of fully-functional project (Date/Time = __________ / __________) – supersedes 75.C if final project submitted by that (earlier) deadline

65 % *(Must meet or exceed these criteria in addition to all criteria for 60%)*
A. Project fully functional
B. All components securely fastened so nothing is "loose" inside the enclosure
C. Schematic diagram of circuit

60 % *(Must meet or exceed these criteria in addition to being safe and legal)*
A. Project minimally functional, with all components located inside an enclosure (if applicable)
B. Passes final safety inspection (proper case grounding, line power fusing, power cords strain-relieved)

0 % *(If any of the following conditions are true)*
A. Fails final safety inspection (improper grounding, fusing, and/or power cord strain relieving)
B. Intended project function poses a safety hazard
C. Project function violates any law, ordinance, or school policy

file 03173

Answer 67

Be sure you meet with your instructor if you have any questions about what is expected for your project!

Notes 67

The purpose of this assessment rubric is to act as a sort of "contract" between you (the instructor) and your student. This way, the expectations are all clearly known in advance, which goes a long way toward disarming problems later when it is time to grade.