

## Digital codes

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Resources and methods for learning about these subjects (list a few here, in preparation for your research):

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### Question 1

Counting practice: count from zero to thirty-one in binary, octal, and hexadecimal:

	Binary	Octal	Hex
Zero			
One			
Two			
Three			
Four			
Five			
Six			
Seven			
Eight			
Nine			
Ten			
Eleven			
Twelve			
Thirteen			
Fourteen			
Fifteen			

	Binary	Octal	Hex
Sixteen			
Seventeen			
Eighteen			
Nineteen			
Twenty			
Twenty one			
Twenty two			
Twenty three			
Twenty four			
Twenty five			
Twenty six			
Twenty seven			
Twenty eight			
Twenty nine			
Thirty			
Thirty one			

[file 01221](#)

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### Answer 1

No answers given here – compare with your classmates!

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### Notes 1

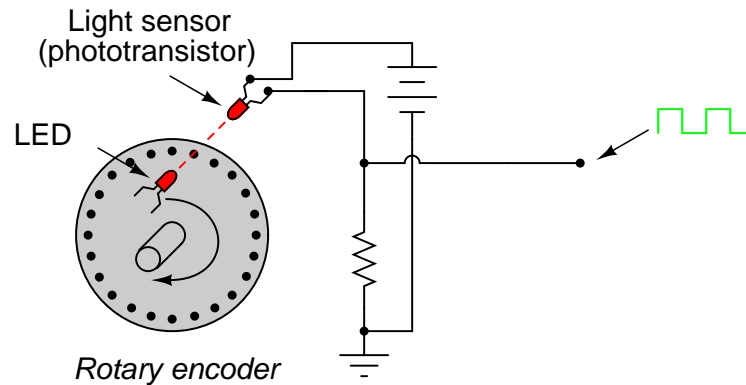
In order to familiarize students with these "strange" numeration systems, I like to begin each day of digital circuit instruction with counting practice. Students need to be *fluent* in these numeration systems by the time they are finished studying digital circuits!

One suggestion I give to students to help them see patterns in the count sequences is "pad" the numbers with leading zeroes so that all numbers have the same number of characters. For example, instead of writing "10" for the binary number two, write "00010". This way, the patterns of character cycling (especially binary, where each successively higher-valued bit has half the frequency of the one before it) become more evident to see.

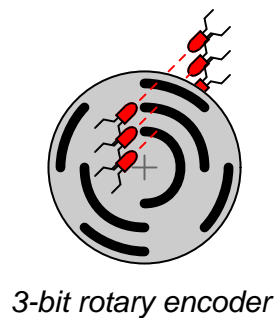
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### Question 2

*Rotary encoders* are electromechanical devices used to convert an angular position (shaft rotation) into a digital signal. The simplest form of rotary encoder uses a slotted wheel with a single LED/photodetector pair to generate pulses as the wheel turns:



Some rotary encoder designs have multiple-bit outputs, with each LED/photodetector pair reading a different "track" of slots in the disk:



In the illustration shown above, identify which LED/photodetector pairs represent the MSB (Most Significant Bit) and LSB (Least Significant Bit) of the binary output. Also, identify which direction the wheel must turn in order to produce an increasing count.

Note: assume that the darkest areas on the illustration represent slots cut through the disk, while the grey areas represent parts of the disk that are opaque.

file 01236

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### Answer 2

I'll let you figure out the MSB, LSB, and up-count direction on your own! It isn't difficult to do if you have mastered counting in binary.

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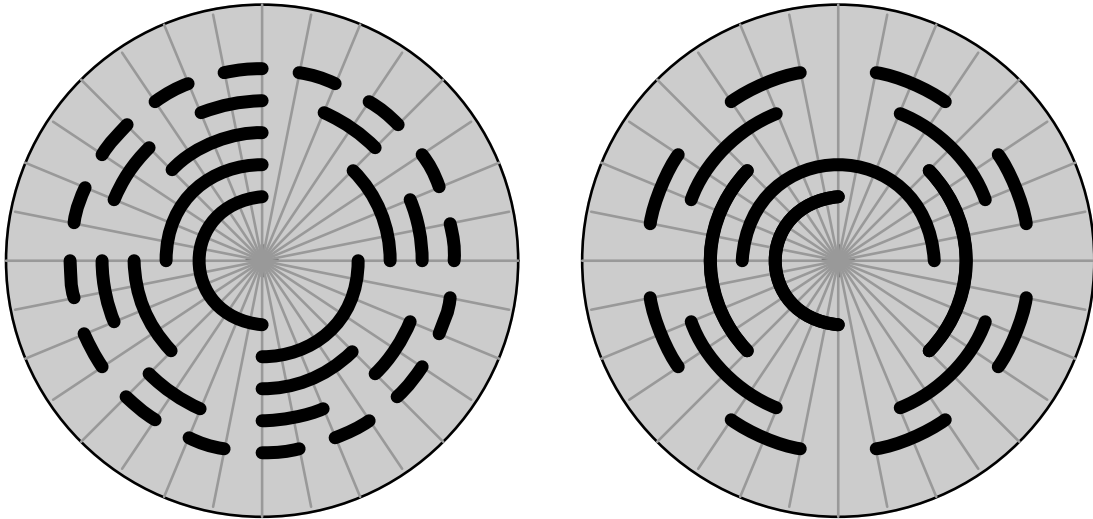
### Notes 2

Ask your students to brainstorm possible applications for rotary encoders. Where might we use such a device? Also, ask them to contrast the two encoder types (1 bit versus 3-bit) shown in the question. What applications might demand the 3-bit, versus only require a 1-bit encoder?

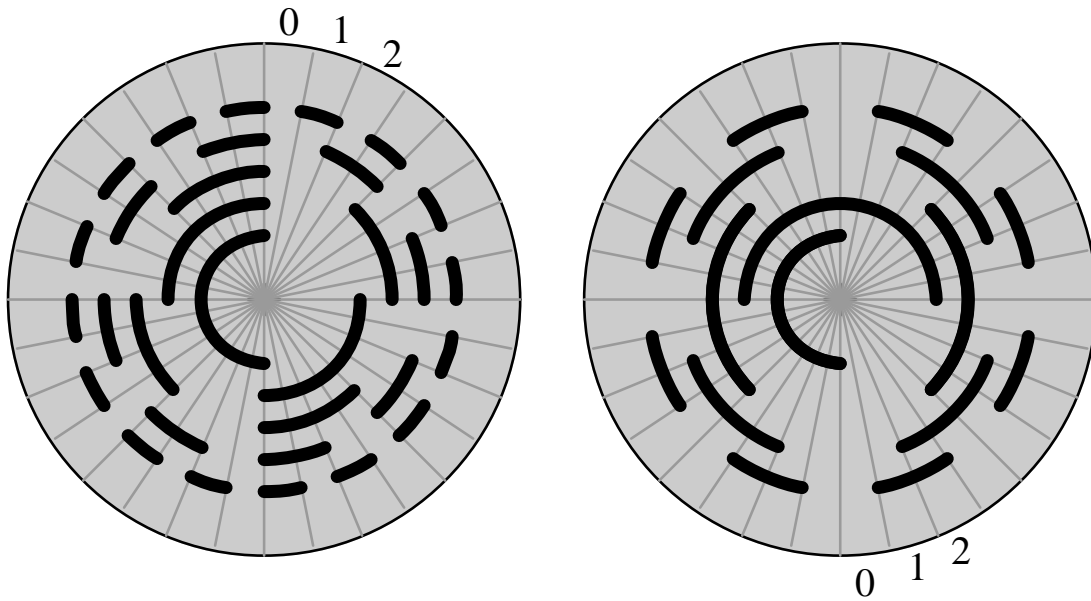
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### Question 3

Absolute rotary encoders often use a code known as *Gray code* rather than binary, to represent angular position. This code was patented by Frank Gray of Bell Labs in 1953, as a means of reducing errors in rotary encoder output. Examine each of these encoder disks, and determine which one is binary and which one is Gray code:



Assuming that the darkest areas on the illustration represent slots cut through the disk, and the grey areas represent parts of the disk that are opaque, mark the "zero," "one," and "two" sectors on each disk.  
file 01237



I won't directly tell you which disk is which, but I will provide a comparison of 5-bit binary versus Gray code, to help you in your analysis:

Binary	Gray
00000	00000
00001	00001
00010	00011
00011	00010
00100	00110
00101	00111
00110	00101
00111	00100
01000	01100
01001	01101
01010	01111
01011	01110
01100	01010
01101	01011
01110	01001
01111	01000
10000	11000
10001	11001
10010	11011
10011	11010
10100	11110
10101	11111
10110	11101
10111	11100
11000	10100

11001	10101
11010	10111
11011	10110
11100	10010
11101	10011
11110	10001
11111	10000

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### Notes 3

Ask your students what patterns they notice in the Gray code sequence, as compared to the binary count. What difference do they see between binary and Gray code, analyzing the bit transitions from one number to the next?

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#### Question 4

Explain why Gray code is often used in rotary encoders rather than binary coding. What difference does it make what type of code we use to mark the sectors of an encoder disk, so long as each sector possesses a unique number?

file 01238

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#### Answer 4

Gray code markings are more tolerant of sensor misalignment than binary markings, because there is no need for perfect synchronization of multiple bit transitions between sectors.

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#### Notes 4

This is perhaps the most important reason for using Gray code in encoder marking, but it is not necessarily obvious *why* to the new student. I found that making a physical mock-up of a binary-coded wheel versus a Gray-coded wheel helped me better present this concept to students. Those students with better visualization/spatial relations skills will grasp this concept faster than the others, so you might want to solicit their help in explaining it to the rest of the class.

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#### Question 5

Convert the following binary numbers into Gray code:

- $100110_2 =$
- $110010_2 =$
- $101001_2 =$
- $1010100110_2 =$

file 01376

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#### Answer 5

- $100110_2 = 110101_{\text{Gray}}$
  - $110010_2 = 101011_{\text{Gray}}$
  - $101001_2 = 111101_{\text{Gray}}$
  - $1010100110_2 = 1111110101_{\text{Gray}}$
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#### Notes 5

There are many textbook references for the conversion process between binary and Gray code. Let your students research how the conversions are done!



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#### Question 6

Convert the following Gray code numbers into binary:

- $111110_{\text{Gray}} =$
- $100001_{\text{Gray}} =$
- $101110_{\text{Gray}} =$
- $1110001111_{\text{Gray}} =$

file 01377

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#### Answer 6

- $111110_{\text{Gray}} = 101011_2$
- $100001_{\text{Gray}} = 111110_2$
- $101110_{\text{Gray}} = 110100_2$
- $1110001111_{\text{Gray}} = 1011110101_2$

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#### Notes 6

There are many textbook references for the conversion process between binary and Gray code. Let your students research how the conversions are done!

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#### Question 7

A nearly universal standard for representing text data in digital form is the *ASCII* code. What does the acronym "ASCII" stand for, and what is the format of this code?

file 01233

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#### Answer 7

"ASCII" = American Standard Code for Information Interchange. Basic ASCII is a seven-bit binary code capable of representing all alphabetical characters used in the English language (upper-case as well as lower), as well as Arabic numerals, English punctuation marks, and some miscellaneous control codes for teletype machines.

Challenge question: although ASCII technically requires only 7 bits, a full 8 bits (1 byte) is usually reserved for each ASCII character in computer systems. Explain why.

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#### Notes 7

ASCII is arguably the *lingua franca* of the digital world. Despite its humble beginnings and Anglo-centric format, it is used worldwide in digital computer and telecommunication systems. Let your students know that every plain-text computer file is nothing more than a collection of ASCII codes, one code for each text character (including spaces).

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Question 8

Explain how the *Morse Code* resembles ASCII in structure and purpose.  
file 01241

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Answer 8

Morse Code is digital, being composed of only two types of characters, just like ASCII. Also, its purpose is to convey alphanumeric information, just like ASCII.

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Notes 8

An interesting point to bring up to students about Morse Code is that it is *self-compressing*. Note how different Morse characters possess different "bit" lengths, whereas ASCII characters are all 7 bits each (or 8 bits for Extended ASCII). This makes Morse a more efficient code than ASCII, from the perspective of bit economy!

Ask your students what ramifications this "self-compressing" aspect of Morse Code would have if we were to choose it over ASCII for sending alphanumeric characters over digital communications lines, or store alphanumeric characters in some form of digital memory media.

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Question 9

Decode this set of ASCII characters, to reveal a secret message (all codes given in hexadecimal format):

49 20 4C 6F 76 65 20 45 6C 65 63 74 72 6F 6E 69 63 73 21

file 01234

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Answer 9

I'll let you decode this message on your own!

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Notes 9

This question provides students with practice using an ASCII reference table.

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Question 10

Standard ASCII codes are seven bits in length, but communications equipment usually sends data in eight-bit (byte) groups. The extra bit is often used as a *parity bit*. What is the purpose of a "parity bit," and how is it used?

file 01235

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Answer 10

"Parity bits" are used as a primitive form of error detection. Communications equipment making use of parity for error detection may either be configured for "even parity" or for "odd parity". I'll let you research and explain how parity bits are used in the transmission of ASCII data.

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Notes 10

The concept of parity is not very complex. It should be well within the reach of students to research on their own and report their findings to the class as a whole.

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Question 11

Explain why Binary Coded Decimal (BCD) is sometimes referred to as the "8421" code. Why is this code used at all?

file 01239

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Answer 11

BCD uses groups of four binary bits to represent each digit of a decimal number. The LSD place weights are 8-4-2-1, while the next significant digit's place weightings are 80-40-20-10, and so on.

Follow-up question: the four bits used for each BCD character could be called half of a byte (8 bits). There is a special word for a four-bit grouping. What is that word?

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Notes 11

Discuss with your students the purpose of using BCD to represent decimal quantities. While not an efficient usage of bits, BCD certainly is convenient for representing decimal figures with discrete (0 or 1) logic states.

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#### Question 12

Convert the following decimal numbers into BCD form:

- $739_{10}$
- $25_{10}$
- $92241_{10}$

Convert the following BCD numbers into decimal form:

- 1000 1001
- 0100 0111 0110
- 0011 1000 0101 0001

file 01240

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#### Answer 12

- $739_{10} = 0111\ 0011\ 1001$
- $25_{10} = 0010\ 0101$
- $92241_{10} = 1001\ 0010\ 0010\ 0100\ 0001$
  
- $1000\ 1001 = 89_{10}$
- $0100\ 0111\ 0110 = 476_{10}$
- $0011\ 1000\ 0101\ 0001 = 3851_{10}$

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#### Notes 12

Nothing but straightforward conversions here!