Assignment 13 - Probability distribution

1 Problem 1

You are in charge of managing overbooking for an airline company. Since 30% of your booked customers do not show up when an airplane is leaving, your policy is to accept 28 bookings for a plane with 24 seats.

- What is the expected number of passengers on a plane?
- What is the expected number of empty seats?
- What is the probability that at least one booked passenger is not able to leave?
- If the penalty for each booked passenger that is not able to leave is 1000, what is the expected penalty per flight?

2 Solution

We can model the problem with a binomial random variable with $p = 0.7$, $1 - p = 0.3$ and $n = 28$. The expected number of passengers is:

$$E[X] = np = 28 \times 0.7 = 19.6 \simeq 19 \tag{1}$$

The expected number of empty seats is easily $24 - 19 = 5$. The probability that at least one booked passenger is not able to leave is:

$$P(i) = \binom{n}{i} p^i (1-p)^{n-i} \tag{2}$$

$$P(25) = \binom{28}{25} \times 0.7^{25} \times (0.3)^3 = 0.012 = 1.2\% \tag{3}$$

There are no expected passengers that can't leave, so the expected penalty per flight is 0.

3 Problem 2

Customers arrive at a certain store according to a Poisson process of rate $\lambda = 4$ per hour. Given that the store opens at 9:00 AM, what is the probability that exactly one customer has arrived by 9:30 AM and a total of five has arrived by 11:30 AM ?

4 Solution

We can use the Poisson probability:

$$P(k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!} \tag{4}$$

with $k = 1$, $\lambda = 4$ and $t = 0.5$:

$$P(1) = \frac{(4 \times 0.5)^1 e^{-4\times 0.5}}{1!} = 0.27 = 27\% \tag{5}$$

For the second request we have:

$$P(5) = \frac{(4 \times 2.5)^5 e^{-4\times 2.5}}{5!} = 0.03 = 3\% \tag{6}$$

5 Problem 3

Cars arrive at a gas station randomly every 2 minutes, on the average. Determine the probability that the interarrival times of cars does not exceed 1 minute

6 Solution

From the property of Poisson distribution we have that the interarrival time are independent and exponentially distributed with parameter λ. We have $\lambda = 1/2 = 0.5$. So we have:

$$P(X \leq x) = 1 - e^{-\lambda x} \Rightarrow P(X \leq 1) = 1 - e^{-0.5\times 1} = 0.39 = 39\% \tag{7}$$