Linear Algebra Done Right
Solutions To Chapter 1 Exercises

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1. $\frac{1}{a+bi} = c + di$
   $1 = (c+di)(a+bi)$
   $1 = ac + adi + bci + bd(i)^2$
   $1 = ac - bd + (ad+bc)i$

   $ac - bd = 1$
   $ad + bc = 0, \quad c = \frac{-ad}{b}$
   $\frac{a(-ad)}{b} - bd = 1$
   $\frac{-a^2 d}{b} - bd = 1$
   $(-a^2 - b^2)d = b$
   $d = \frac{b}{-a^2 - b^2} = \frac{-b}{a^2+b^2}$ thus, $\quad c = \frac{-ab}{(-a^2-b^2)b} = \frac{a}{a^2+b^2}$
   $d = \frac{-b}{a^2+b^2} \quad c = \frac{a}{a^2+b^2}$

2. To show $\frac{-1+3\sqrt{3}i}{2}$ is a cube root of 1 we can just take $\frac{-1+3\sqrt{3}i}{2}$ and cube it.
   $\left(\frac{-1+3\sqrt{3}i}{2}\right)^2 = \frac{1-2\sqrt{3}i-3}{4}$
   $\frac{(1-2\sqrt{3}i-3)}{4}\frac{(-1+3\sqrt{3}i)}{2} = \frac{(-1+3\sqrt{3}i+9-3\sqrt{3}i)}{8} = \frac{8}{8} = 1,$ which is the result we were looking for. Thus $\frac{-1+3\sqrt{3}i}{2}$ is a cubed root of 1.

3. Let v $\in V$ where V is a vector space, then $-$ v is the additive inverse of v.
   Consider $-(-\text{v})$. By the properties of of vector spaces $-(-\text{v}) = 0 + -(-\text{v})$
   $= \text{v} + -\text{v} + -(-\text{v}) = \text{v}$, as desired.