

R,L,C

- * Ohm's law in fields theory form - $\bar{J} = \sigma \bar{E}$
 - ↓ current density
 - ↑ conductivity
 - electric field
- * temp. rises from room temp. conductivity rises.

- * In presence of active sources $\rightarrow R,L,C$ absorbs energy.
current through them will be from +ve to -ve.

In the absence of Active sources \rightarrow stored energy in the memory element (L & C) will be delivered to memoryless Resistance. current through them -ve to +ve.

$$* J = \sigma E \quad J = \frac{i}{s}, \quad E = \frac{V}{l} \quad (\sigma = \frac{\text{out flow}}{\text{Area}} = n \mu_n q_n + p \mu_p q_p)$$

$$\therefore V = \frac{1}{\sigma s} i \Rightarrow V = \frac{sl}{s} i \Rightarrow V = Ri \quad \begin{array}{l} \text{Ohm's law} \\ \text{in circuit} \\ \text{theory.} \end{array}$$

$$R = \frac{sl}{s} = \frac{l}{s} = \frac{dQ}{dt} \quad i = \frac{dq}{dt}$$

$$S\text{-resistivity} = \frac{1}{\sigma} = \frac{RS}{l} \quad i = GV \quad \begin{array}{l} \text{3rd form of} \\ \text{Ohm's law.} \end{array}$$

instantaneous power.

$$P = VI \quad V = \frac{dq}{dt} \quad i = \frac{dq}{dt}$$

$$\therefore P = \frac{d\omega}{dt} \Rightarrow \omega = \int_P dt \quad \begin{array}{l} \text{Electrical energy.} \\ \boxed{P = i^2 R = \frac{V^2}{R}} \end{array}$$

- * Resistor is a linear, passive, bilateral and time invariant in V-I characteristic (if $I, S, \sigma \rightarrow \text{constant}$) i.e. R constant.

- * $\Psi = N\phi$ when conductor wound like spring - see coil flux. ↓ Number of turns. \downarrow Flux per turn, because coil will exhibit ideal inductance.

$$\Psi \propto i \quad \Psi = Li$$

$$V = \frac{d\Psi}{dt} \quad V = L \frac{di}{dt} \Rightarrow i = \frac{1}{L} \int_{-\infty}^t V dt$$

$$P = VI \quad \boxed{W = \frac{1}{2} Li^2} \quad \text{Instantaneous Energy}$$

* Inductor is a linear, passive, bilateral and time invariant in Ψ - i plane (L is constant)

* $i = \frac{dq}{dt}$ $q \propto v$ $q = Cv$

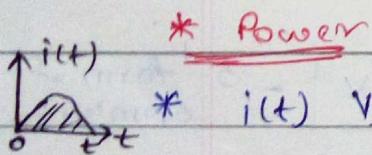
$$i = C \frac{dv}{dt} \quad v = \frac{1}{C} \int_{-\infty}^t i dt \quad P = vi$$

$$W = \frac{1}{2} Cv^2$$

instantaneous
energy.

* capacitor is linear, passive, bilateral and time-variant in q - v plane (c kept constant)

* $Z_L = j\omega L$ $Z_C = j\omega C$



* Power

* $i(t)$ vs t graph charge applied

$$q = \text{area under } i(t) = \int_0^t i(t) dt$$

* for a periodic signal.

$$P_{avg} = \frac{\text{The Energy Observed Over one period}}{\text{Time period.}}$$

$$\begin{aligned} P_{avg} &= \frac{\left[\int_0^T i(t)^2 R dt \right]}{T} = \frac{\left[\int_0^T V(t)^2 / R dt \right]}{T} \\ &= I_{rms}^2 \cdot R = \frac{V_{rms}^2}{R} \end{aligned}$$

* find f^n $i(t)$ or $v(t)$ by $y = mx + c$ line equation.

- * ① The average ~~power~~ true power or real power or Active power delivered by the source
 = absorbed by the load impedance (R)
 = $P_{avg} = I^2_{rms} \cdot R$ (watt)

- ② The average reactive power or imaginary power Delivered by the source
 = absorbed by the load impedance (X)
 = $Q_{avg} = I^2_{rms} \cdot X$ (VAR)

- ③ The average complex power or apparent power.
 = absorbed by the load impedance (Z) ($Z = R + jX$)
 = $S_{avg} = I^2_{rms} \cdot Z$ (VA)
 $|S_{avg}| = I^2_{rms} \cdot |Z|$

$$\textcircled{1} + \textcircled{2} = \textcircled{3}$$

v.i

* energy absorbed by resistance = $\int_0^t i^2(t) \cdot R dt$ ($V = IR$)

~~when $i(t)$ is given convert V into terms of i for $V(t)$~~

energy absorbed by inductor = $\int_0^t L \frac{di}{dt} dt$ ($V = L \frac{di}{dt}$)

energy absorbed by capacitor = $\int_0^t \frac{1}{C} \left(\int_{-\infty}^t i(t) dt \right) i dt$
 $V = \frac{1}{C} \int_{-\infty}^t i(t) dt$

* If current through an Ideal inductor is constant then energy absorbed is 0. $\because \frac{di(t)}{dt} = 0$ constant

My constant capacitor voltage energy absorbed is 0 ($\because \frac{dv}{dt} = 0$)

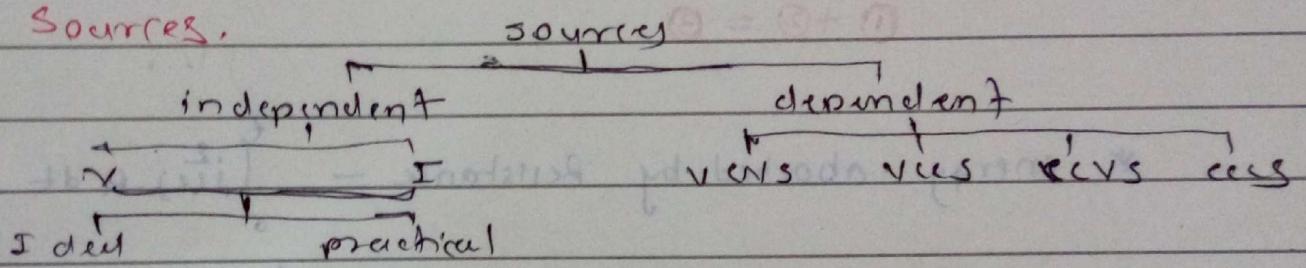
* Energy absorbed = energy stored + energy Dissipated.
 (by R, L, C) (by L, C) (by R)

* Types of Element.

- 1) Active & passive I or III quadrant - passive \rightarrow $P = VI$
II or IV quadrant Active $-V \cdot P = -VI$
- 2) Linear & Non linear (straight line through origin V & I
else non linear)
- 3) Bilateral & Unilateral Bilateral \rightarrow symmetry about origin
else unilateral
- 4) Distributed & lumped lumped - size is very small compared
to wavelength of applied signal.
Distributed - electrically inseparable
- 5) time invariant & time variant

* All linear elements are always bilateral, converse can't say

* Sources.



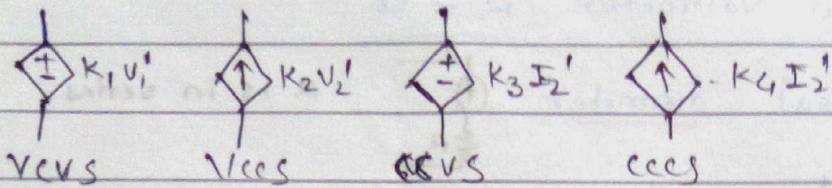
* in ideal voltage source, the load voltage is independent of the load current.

* All the ^{ideal} sources are non-linear, since the voltage & current relation are non-linear, they are active & unilateral

* Practical voltage source load voltage is a function of load current drawn.

Why for Ideal & practical current sources.

* w.r.t. to controlled variables the dependent source are said to be linear, active, bilateral



* KCL (Node) — $\sum \text{current} = 0$ OR incoming $I = \text{outgoing } I$

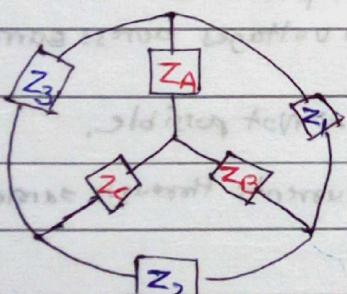
\rightarrow KCL + Ohm's law \rightarrow Nodal Analysis

* KVL (mesh) — Net voltage is 0.

KVL + Ohm's law \rightarrow Mesh analysis

* Voltage divider Rule — $V_1 = V \cdot \frac{Z_1}{Z_1 + Z_2}$

* Current division Rule — $I_1 = I \cdot \frac{Z_2}{Z_1 + Z_2}$



$$Z_A = \frac{Z_1 \cdot Z_3}{Z_1 + Z_2 + Z_3}$$

$$Z_B = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2 + Z_3}$$

$$Z_C = \frac{Z_2 \cdot Z_3}{Z_1 + Z_2 + Z_3}$$

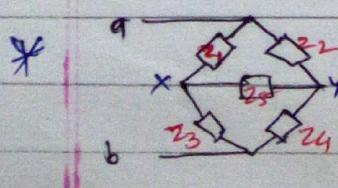
$$Z_1 = Z_A + Z_B + \frac{Z_A Z_B}{Z_C}$$

$$Z_2 = Z_B + Z_C + \frac{Z_B Z_C}{Z_A}$$

$$Z_3 = Z_A + Z_C + \frac{Z_C Z_A}{Z_B}$$

* KVL & KCL are independent to each other.

* Ohm's law applicable only for linear ^{passive} elements (or devices) not for active & Non-linear.

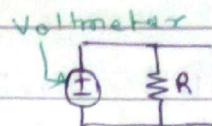


If $\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}$ or $Z_1 Z_4 = Z_2 Z_3$ Then bridge is balanced

No current flows through Z_5

? Z_5 branch is redundant & can be removed
 $\therefore Z_1 || Z_2$ & $Z_3 || Z_4$ Short key

* Practical voltmeter



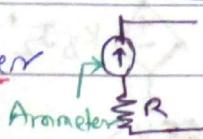
R is in parallel

↳ Internal Resistance of voltmeter

Ideal voltmeter

$$R = \infty$$

Practical ammeter

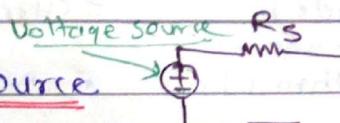


R is in series.

Ideal ammeter

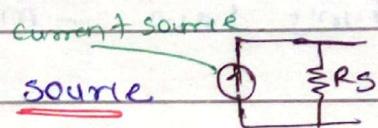
$$R = 0$$

* Practical voltage source



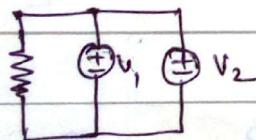
for Ideal voltage source $R_s = 0$

Practical current source



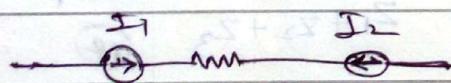
for Ideal current source $R_s = \infty$

* dont violate KVL & KCL Rule in circuit.



$V_1 \neq V_2 \Rightarrow$ Not possible

two voltages across same resistor X



$I_1 \neq I_2 \Rightarrow$ Not possible.

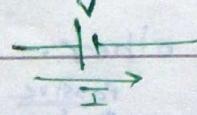
two currents through same resistor X

* Tellegen's theorem.

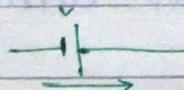
* power absorbed = power delivered.

* find branch current & voltages through each element then

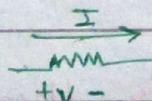
find power by $P = VI$



$VI \Rightarrow$ Power absorbed



$VI \Rightarrow$ Power delivered.



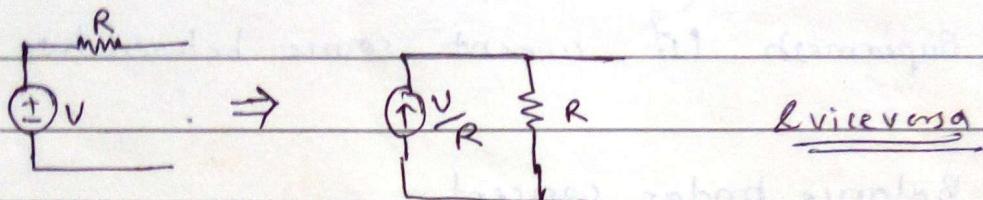
$VI \Rightarrow$ Power absorbed

* Nodal analysis

* Mesh analysis.

$i_R = \frac{V_2 - V_1}{R}$	$i_C = C \frac{d(V_2 - V_1)}{dt}$	$i_L = \frac{1}{L} \int_{-\infty}^t (V_2 - V_1) dt$
$V_R = (I_2 - I_1)R$	$V_C = \frac{1}{C} \int_{-\infty}^t (I_2 - I_1) dt$	$V_L = L \frac{d(I_2 - I_1)}{dt}$

* Source Transformation.



Voltage series with $R \rightarrow$ current parallel with R .

* OP-Amp.

$$V_{id} = V_1 - V_2 \quad V_o = A_v \cdot V_{id}$$

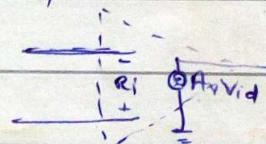
* for ideal OP-amp $A_v = \infty \quad V_{id} = 0 \quad i.e. V_1 = V_2$

* So for Ideal opamp Voltage at Non-inverting is equal to voltage at Inverting terminal.

* for ideal OPAMP $R_i = \infty, R_o = 0$.

Input Resistance $R_i = \infty$

\therefore possible to write nodal eqn at i/p side



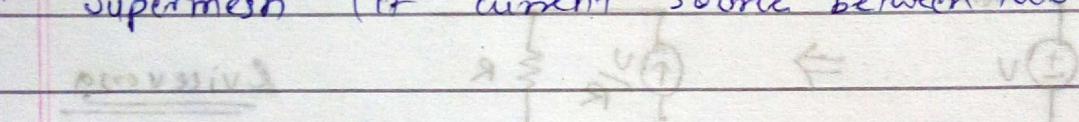
O/p Resistance $R_o = 0$

exhibit voltage source
not possible to
write Nodal at o/p.

* for practical OPAMP:-

* in star delta for a problem like cube, if network is symmetrical and if connection also symmetrical use KVL for the ans.
* see problems for detail

* Supernode (if voltage source between two node.)
* Supermesh (if current source between two loop).



* Balance bridge concept.

23, 31, 33, 60, 138, 141, 150, 152*, 210, 238

976-90 *

$v = n \cdot k$ $\theta = \pi v$ $\theta = \pi \cdot 1$ $\theta = \pi$ Instinct
latter is preferred due to simpler analysis Instinct not so
latter preferred to analyze it
 $\theta = \pi$ $\theta = \pi$ $\theta = \pi$ Instinct
 $\theta = \pi$ $\theta = \pi$ $\theta = \pi$ Instinct

* Superposition theorem:- (SPT)

- Properties
 - applicable only for linear Networks,
 - ideal voltage source short & current source open.
 - SPT applicable for both active & passive Networks.

Homogeneity principle:- If excitation is multiplied by 'K'

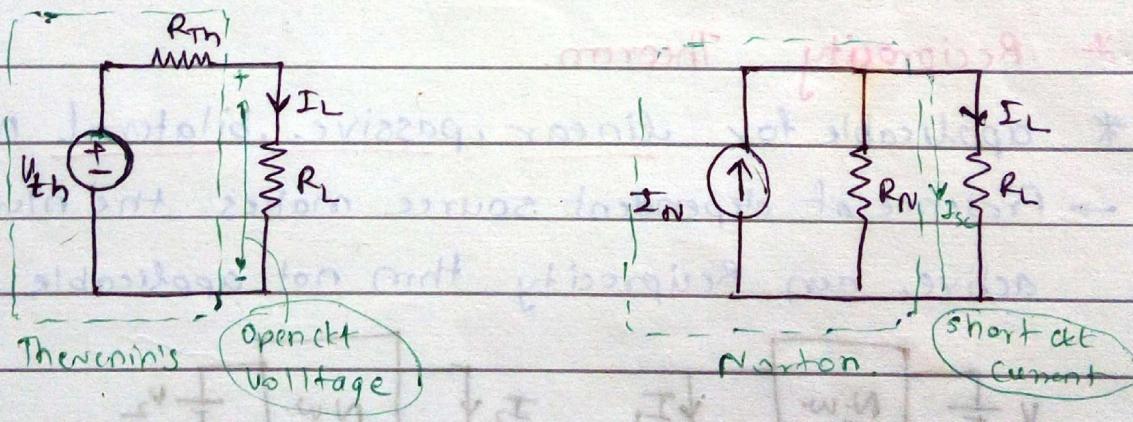
then response of all other Network are also multiplied by K. (homogeneity obeyed by all the linear networks)

→ When multiple sources are present then superposition theorem apply first then homogeneity

Before homogeneity always ensure one independent source in the Network.

- * maximum or minimum power dissipated when two sources with resistor R is $(\sqrt{P_1} + \sqrt{P_2})^2 / R$ W.

* Thevenin's & Norton theorem.



- * if ideal voltage & current source connected in parallel \rightarrow Norton equivalent Not possible
- connected in series \rightarrow Thevenin's equivalent Not possible.

- * properties similar to SPT.

* Maximum Power Transfer Thm.

* This theorem applicable only when load is variable otherwise choose minimum internal impedance so that maximum current flows through load & more power dissipates.

max. power transfer when $R_L = R_S$.

$$P_{\text{max}} = \frac{V_S^2}{4R_L} \quad \text{at } R_L = R_S,$$

$$\text{Total power} = \frac{V_S^2}{2R_L}$$

$$\% \text{ efficiency} = \frac{P_{\text{max}}}{\text{Total power}} \times 100 = \frac{\frac{V_S^2}{4R_L}}{\frac{V_S^2}{2R_L}} \times 100 = 50\%$$

∴ max. efficiency is 50%

* Properties similar to SPT.

* If load is variable then theorem

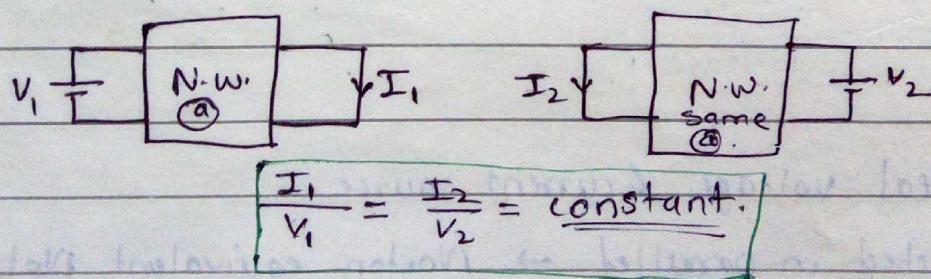
calculate V_{TH} , R_{TH} $\frac{V_{TH}^2}{4R_{TH}}$ = max power.

If load is R_{LC} try to get max. current through load

* Reciprocity Theorem.

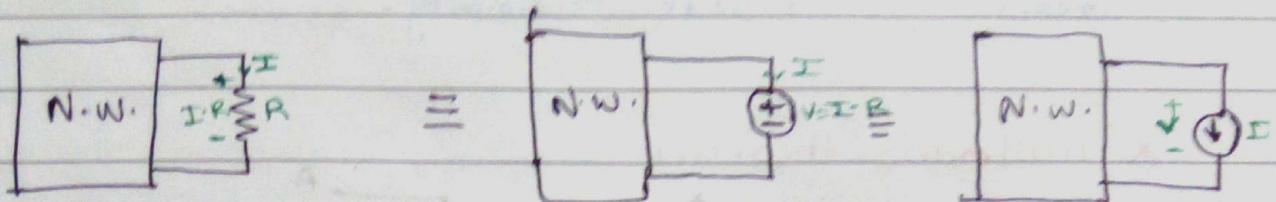
* Applicable for linear, passive, bilateral N/w.

→ Presence of dependent source makes the n/w an active one. Hence Reciprocity Thm not applicable.



* For complex n/w. use superposition Thm first in that after use Reciprocity &/or homogeneity.

* The Substitution theorem, or compensation thm



$$P_R = P_V = P_I = V \cdot I = R \cdot I^2 = I^2 R \text{ (abs)}.$$

- * whenever two different frequencies are operating on the network simultaneously always the SPT is used to evaluate the response, since reactive elements are frequency sensitive $Z_L = j\omega L$, $Z_C = \frac{1}{j\omega C}$.
- * properties same as SPT

* Duality Principal.

* The N.W & its dual are only equal with respect to performance but the connection & elements point of view 'they are not equal'.

* No. of mesh eqns = No. of nodal eqns. (n eqns)
(N loops) (N nodes) $\quad (N+1)$ principal nodes

$$i(t) \leftrightarrow v(t) \quad I \leftrightarrow V \quad R \leftrightarrow G \quad L \leftrightarrow C$$

$$Z \leftrightarrow Y \quad \text{OC} \leftrightarrow \text{SC} \quad \text{series} \leftrightarrow \text{parallel}$$

$$\text{star} \leftrightarrow \Delta \quad R \cdot i(t) \leftrightarrow G \cdot V(t) \quad KCL \leftrightarrow KVL$$

$$L \frac{di(t)}{dt} \leftrightarrow C \frac{dV(t)}{dt} \quad \frac{1}{C} \int i(t) dt \leftrightarrow \frac{1}{2} \int V(t) dt$$

Nodal \leftrightarrow Mesh

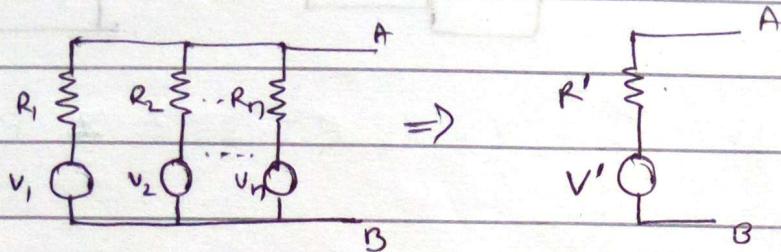
Thevenins \leftrightarrow Norton

Twig \leftrightarrow link

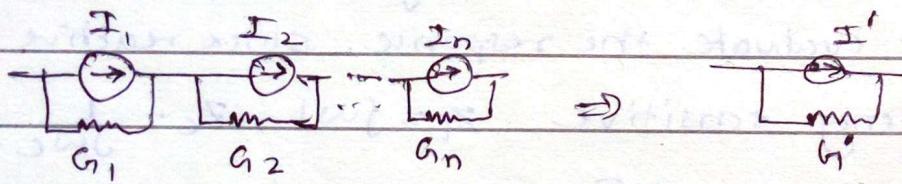
loop \leftrightarrow f-cutset.

* whenever network is complex ~~we can~~ name nodes separately
redraw the network to simplify.

* Millman's theorem,



$$V' = \frac{v_1 G_1 + v_2 G_2 + \dots + v_n G_n}{G_1 + G_2 + \dots + G_n} \quad R' = \frac{1}{G_1 + G_2 + \dots + G_n}$$



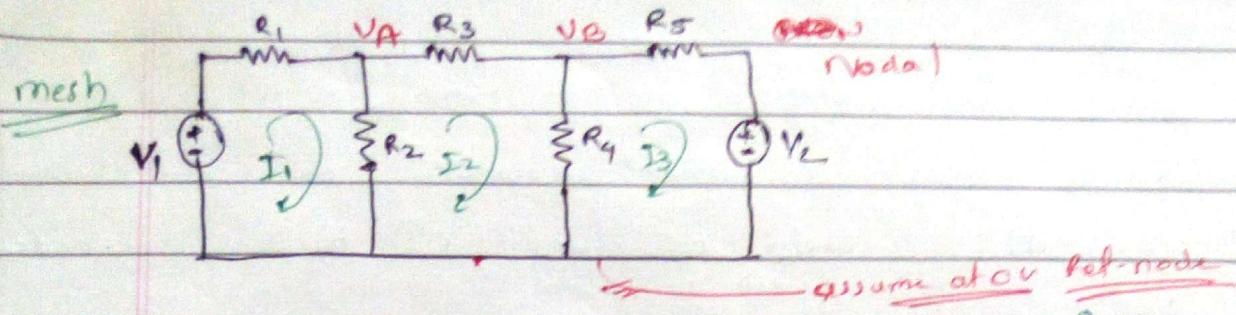
$$I' = \frac{I_1 R_1 + I_2 R_2 + \dots + I_n R_n}{R_1 + R_2 + \dots + R_n} \quad G' = \frac{1}{R_1 + R_2 + \dots + R_n}$$

* Properties same as SPT

* for a complex impedances maximum power transfer condition is ~~Z~~ $Z_L = Z_s^*$ (i.e. load impedance should be complex conjugate of source Impedance)

* for complex impedance ($Z_s = x + iy$) max power transfer by Z_L which is purely Resistive. then

$$Z_L = \sqrt{x^2 + y^2}$$



$$\text{mesh } ① (R_1 + R_2)I_1 - R_2 I_2 = V_1$$

$$② \cancel{(R_2 + R_3 + R_4)I_2} + (R_2 + R_3 + R_4)I_2 - R_4 I_3 = 0$$

$$③ \cancel{- R_4 I_2} + (R_4 + R_5)I_3 = -V_2$$

solve by eqⁿ 3 Unknown

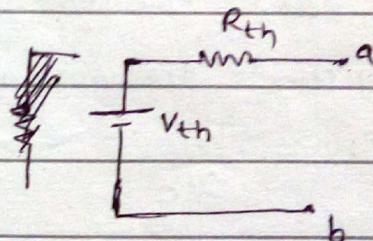
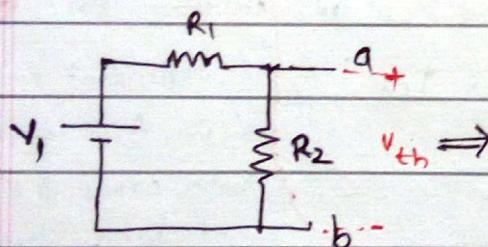
Nodal

$$① V_A \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) - V_B \left(\frac{1}{R_3} \right) = V_1 \left(\frac{1}{R_1} \right)$$

$$-V_A \left(\frac{1}{R_3} \right) + V_B \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \right) = V_2 \left(\frac{1}{R_5} \right)$$

give
polarity
accordingly
here V₁, R₁, V₂ are
+ve

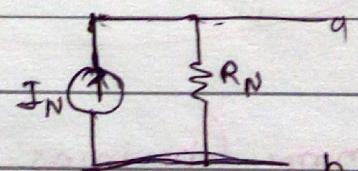
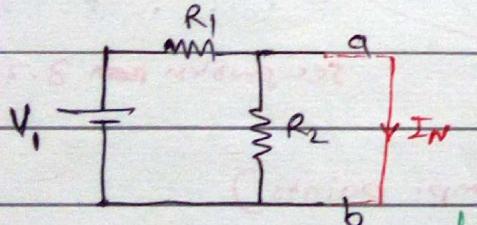
Thevenin



$$V_{th} = \frac{V_1 R_2}{R_1 + R_2}$$

$$R_{th} = R_1 || R_2$$

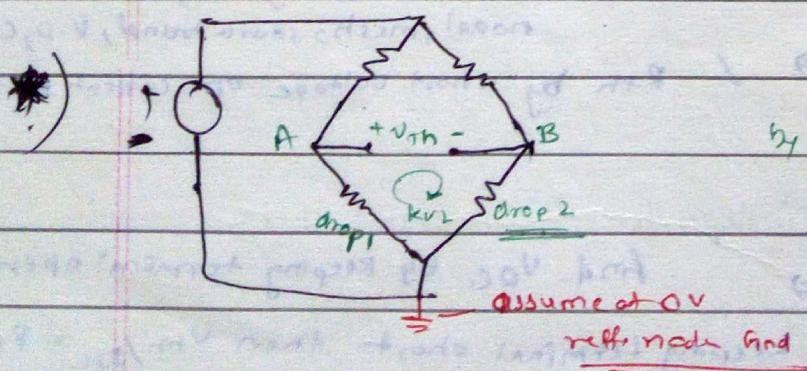
Norton



$$I_N = \frac{V_1}{R_1}$$

$$R_N = R_1 || R_2$$

for R_{th} or R_N short current
voltage source open current source
& calculate R_{th}

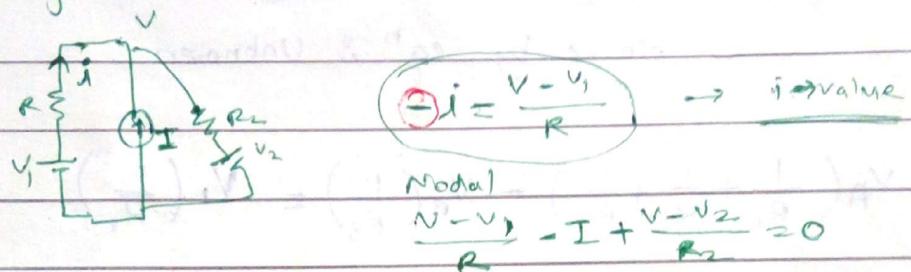


$$\text{by KVL } V_{th} = V_A - V_B$$

▷ source transformation

- * → Voltage division or current division., series & parallel connection
- * 3) Star-delta for resist. or simplify NW by separating nodes
- 5) Nodal or mesh
- 7) Thvenins or norton.
- 6) SPT.

* during nodal be careful about current signs.



to find Thvenins equivalent.

- ① first open the terminal find V_{th}
 - ② then short the terminal find I_{sc}
 - ③ $R_{th} = \frac{V_{th}}{I_{sc}}$
- in both case ① & ② if dependent source is there then find its value in both cases ① & ② don't take dependent source value in ① into ②.

see problem 3.23 pg 181

Summary (Thvenins) (imp. points)

case ① All are independent source: calculate V_{th} by any method
nodal, mesh, source trans, V-D, C-D etc.
Normal method: \Rightarrow R_{th} by short voltage open current source.

case ② if both dependent & independent sources are there.

\Rightarrow case 2 method \Rightarrow find V_{oc} by keeping terminal open

then find I_{sc} by keeping terminal short then $V_{th}/I_{sc} = R_{th}$.

*** [or connect $\frac{V_o}{I_o}$ at open terminal & find $\frac{V_o}{I_o} = R_{th}$] $\begin{bmatrix} V_{source SC} \\ I_{source OC} \end{bmatrix}$ Independent

case ③ only dependent source no independent source

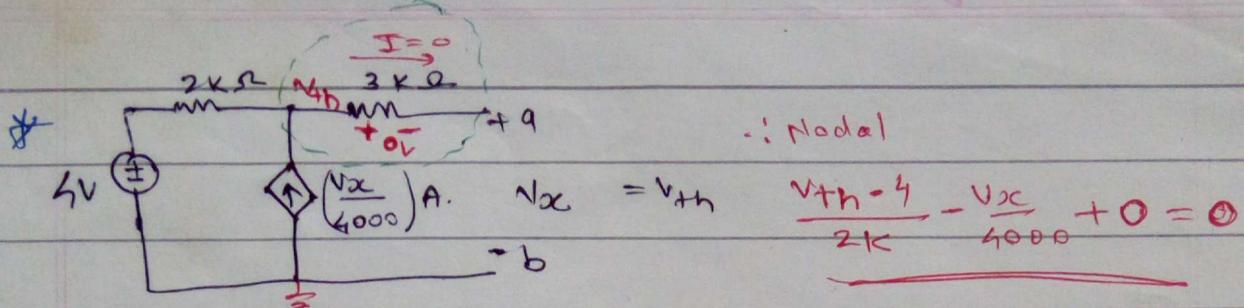
In this case $V_{th}=0$, $I_{sc}=0$ but $R_{th} \neq 0$

find R_{th} in this case Thvenins = Norton = Resistor behaviour

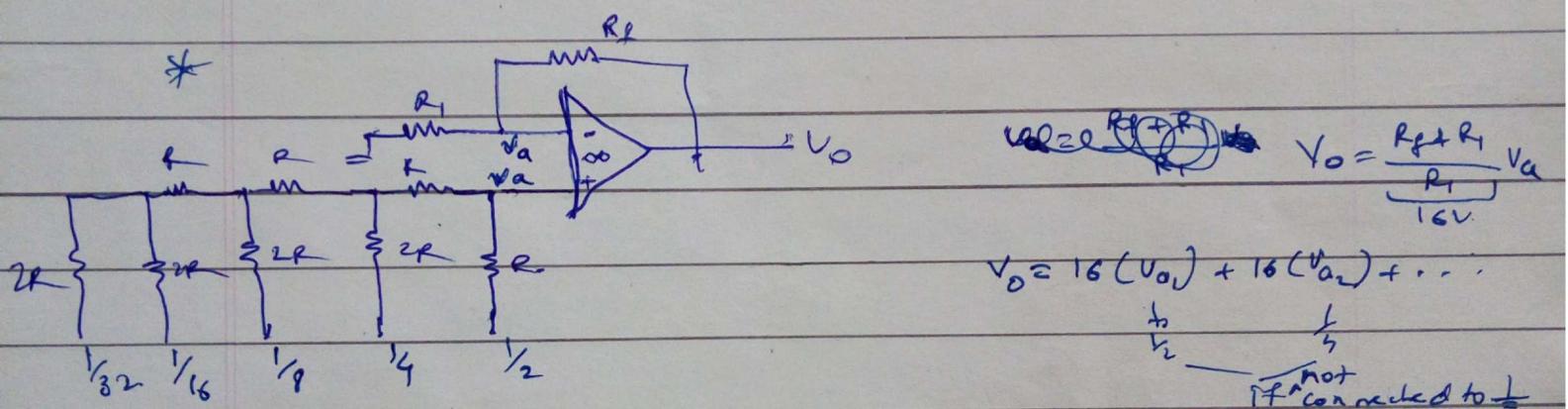
*** (In this case consider or assume voltage or current at terminal)

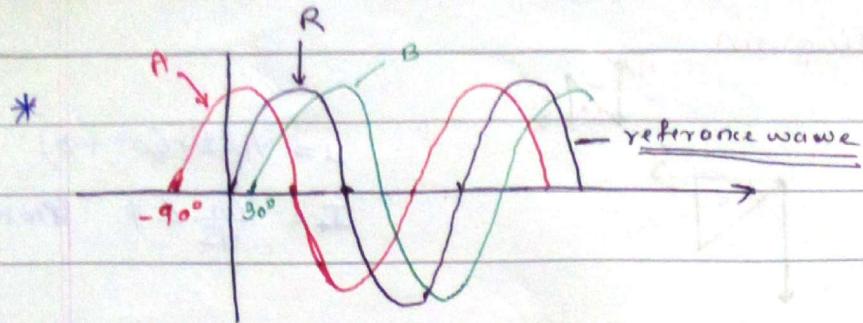
$$R_{th} = \frac{a}{b+c} R_L$$

(8)



current in O.C. is 0





wave A → leads the reference wave by 90° .

wave B → lags the reference wave by 30° .

phase difference between A & B is 120° .

R is Reference wave we can say that it is in phase ~~if ref is in wave~~

A & B both are out of phase ~~: ref is sine wave~~

$$V(t) = V_m \sin \omega t. \quad V_m \rightarrow \text{Peak Voltage.}$$

$$v(t) = V_m \sin(\omega t + 90^\circ) \quad \text{— A wave.}$$

$$v(t) = V_m \sin(\omega t - 30^\circ) \quad \text{— B wave}$$

V_m = peak voltage

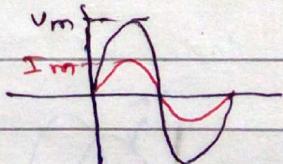
$$V_{avg} = \frac{1}{T} \int_0^T v(t) dt.$$

$$\text{peak factor} = \frac{V_m}{V_{rms}} = 1.414 \Rightarrow \frac{V_m}{\frac{V_m}{\sqrt{2}}} = \frac{\sqrt{2}}{1}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707 V_m. \\ = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt}$$

$$\text{form factor} = \frac{V_{rms}}{V_{avg}}$$

$$\textcircled{1} \quad V(t) = R i(t) \Rightarrow V_m = I_m R.$$



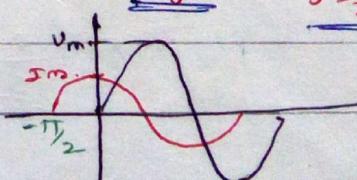
$$\textcircled{2} \quad V(t) = L \frac{di}{dt} \Rightarrow V_m = \omega L I_m = X_L I_m$$

$$\boxed{X_L = \omega L = 2\pi f L} \quad Z = j\omega L = jX_L \quad \text{in pure inductor } i(t) \text{ lags } v(t) \text{ by } \frac{\pi}{2}$$

$$\textcircled{3} \quad V(t) = \frac{1}{C} \int i(t) dt \Rightarrow V_m = \frac{I_m}{\omega C} = \frac{I_m}{j\omega C} = -jX_C$$

$$\boxed{X_C = \frac{1}{2\pi f C}}$$

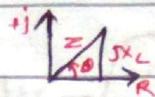
$$Z = -\frac{j}{\omega C} = -jX_C$$



in pure capacitor: $i(t)$ leads $v(t)$ by $\frac{\pi}{2}$

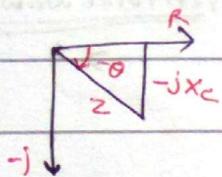
* impedance diagram

RL series -



$$i = V_m \sin(\omega t + \phi)$$

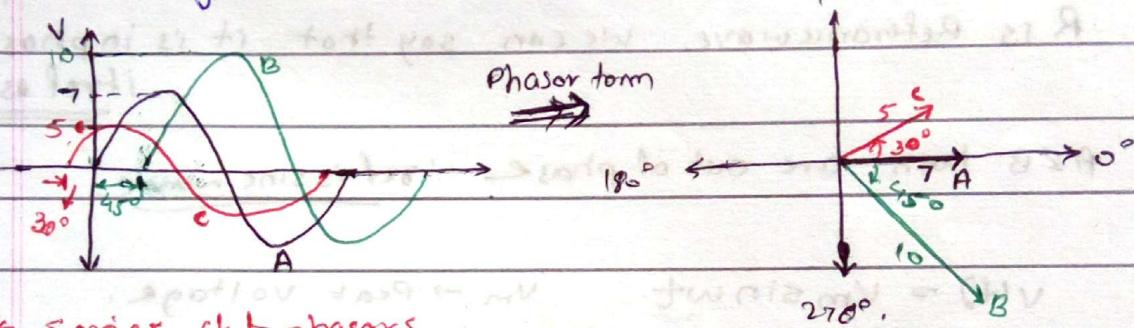
RC Series -



$$I = \frac{V_m}{\sqrt{Z}} \angle \phi \quad (\text{Phasor form})$$

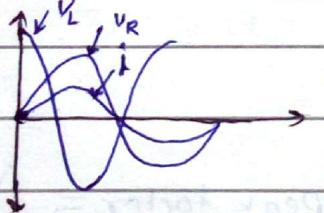
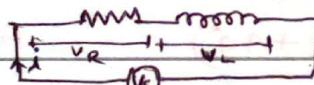
* phasor diagram.

it is used to represent sine wave in terms of magnitude and angular position.



* series ckt phasors

* Series R



Phasor form

$$(\text{pf}) \cos \theta = \left(\frac{V_R}{V_s} \right) \text{ lags}$$

$$V_R = IR$$

$$V_L = IX_L$$

$$V_s = IR + jIX_L$$

$$V_s = \sqrt{V_R^2 + V_L^2}$$

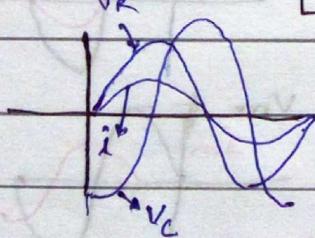
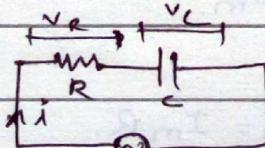
$$X_L = \omega L = 2\pi f L$$

$$i = \frac{V_s}{Z}$$

$$\theta = \tan^{-1} \left(\frac{X_L}{R} \right)$$

$$\theta = \tan^{-1} \left(\frac{V_L}{V_R} \right)$$

* Series RC



Phasor form

$$(\text{pf}) \cos \theta = \left(\frac{V_R}{V_s} \right) \text{ lags}$$

$$V_R = IR$$

$$V_C = IX_C$$

$$V_s = IR - jIX_C$$

$$V_s = \sqrt{V_R^2 + V_C^2}$$

$$X_C = \frac{1}{2\pi f C}$$

$$Z = \sqrt{R^2 + X_C^2}$$

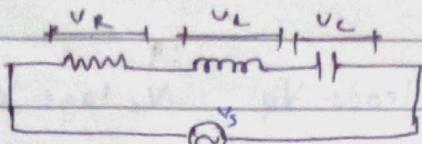
$$i = \frac{V_s}{Z}$$

$$\theta = \tan^{-1} \left(\frac{-X_C}{R} \right)$$

$$\theta = \tan^{-1} \left(\frac{-V_C}{V_R} \right)$$

for pure **R** or **C** $\theta = \pm 90^\circ$

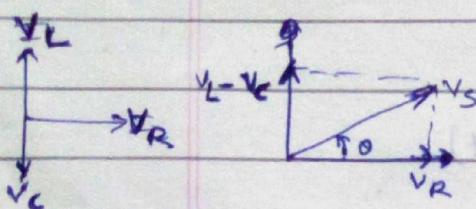
Series RLC



$$\textcircled{1} \quad V_L > V_C$$

$$\textcircled{2} \quad V_C > V_L$$

$$\textcircled{3} \quad V_L = V_C.$$



$$V_R = IR \quad V_S = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V_S = \sqrt{V_R^2 + (V_C - V_L)^2}$$

$$V_S = V_R$$

$$V_C = I X_C \angle 90^\circ \quad \theta = \tan^{-1} \left(\frac{V_L - V_C}{V_R} \right)$$

$$\theta = \tan^{-1} \left(\frac{V_C - V_L}{V_R} \right)$$

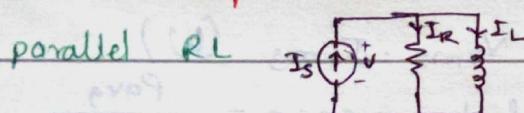
$$\theta = 0$$

$$V_C = I X_C \angle 90^\circ \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$Z = R$$

* Use dict phasors.



$$\text{parallel RL} \quad I_S \uparrow \quad I_R \downarrow \quad I_L \rightarrow \quad I_S = \sqrt{I_R^2 + I_L^2} \quad \theta = \tan^{-1} \left[\frac{I_L}{I_R} \right]$$

$$(pt) \cos \theta = \frac{I_R}{I_S} \quad \underline{\text{lag}}$$

$$I_R = \frac{V}{R} \quad I_L = \frac{V}{Z_L} \quad I_L = \frac{V}{X_L} \angle -90^\circ \quad Z = \frac{V}{I_S}$$

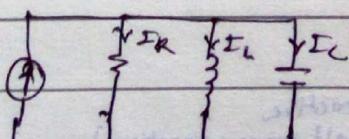
$$\text{parallel RC} \quad I_S \uparrow \quad I_R \downarrow \quad I_C \rightarrow \quad I_R = \frac{V}{R} \quad I_C = \frac{V}{Z_C} \quad I_C = \frac{V}{X_C} \angle 90^\circ$$

$$I_C \rightarrow \quad I_S \uparrow \quad I_R \downarrow \quad I_C \rightarrow \quad I_S = \sqrt{I_R^2 + I_C^2} \quad \theta = \tan^{-1} \left(\frac{I_C}{I_R} \right)$$

$$(pt) \cos \theta = \frac{I_R}{I_S} \quad \underline{\text{lead}}$$

$$Z = \frac{V}{I_S}$$

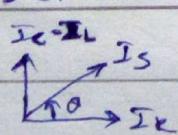
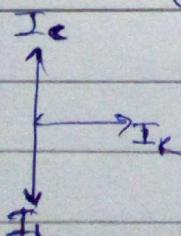
Parallel RLC



$$\textcircled{1} \quad I_C > I_L$$

$$\textcircled{2} \quad I_L > I_C$$

$$\textcircled{3} \quad I_L = I_C$$



$$I_S = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$I_S = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$I_S = I_R$$

$$\theta = \tan^{-1} \left(\frac{I_C - I_L}{I_R} \right)$$

$$\cos \theta = \frac{I_R}{I_S}$$

$$\theta = \tan^{-1} \left(\frac{I_L - I_C}{I_R} \right)$$

$$\cos \theta = \frac{I_R}{I_S}$$

$$\theta = 0$$

UPF

* In series V_L leads V_R V_C lags V_R I constant as $\frac{V}{Z}$
 in parallel $\frac{1}{L}$ leads $\frac{1}{R}$ $\frac{1}{C}$ lags $\frac{1}{R}$ V constant as $\frac{I}{Z}$

Instantaneous power:- $P_{inst}(t) = V(t) i(t)$

Average power :- $P_{avg} = \frac{1}{T} \int_0^T P_{inst}(t) dt = \frac{1}{T} \int_0^T V(t) i(t) dt$

$$P_{avg} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\theta \quad (\text{W})$$

$$= V_{rms} \cdot I_{rms} \cos\theta$$

$\cos\theta = 1$
when $V(t)$ & $i(t)$
are in phase

Apparent power:- $= V_{rms} \cdot I_{rms} \quad (\text{W})$

$$\text{Power factor} = \cos\theta = \frac{P_{avg}}{V_{rms} \cdot I_{rms}}$$

Reactive power:- $= V_{rms} \cdot I_{rms} \sin\theta \quad (\text{VAR})$

$$= I_{rms}^2 Z \sin\theta$$

$$= I_{rms}^2 X_L$$

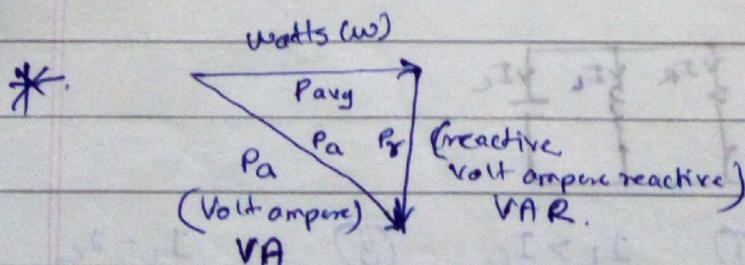
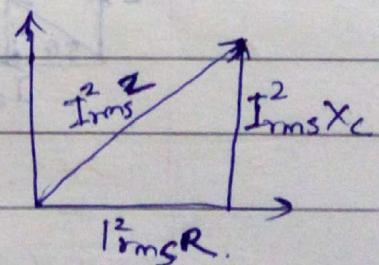
($X_L = Z \sin\theta$ from impedance
triangle)

$$P_a = I_{rms}^2 Z \Rightarrow P_{true} = P_a \cos\theta. \quad (\text{Apparent})$$

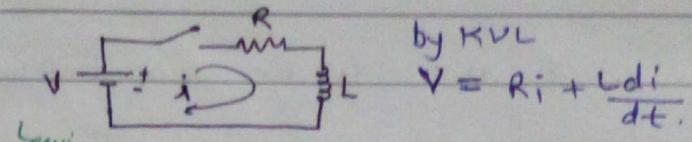
Apparent

$$P_{avg} = P_a \cos\theta.$$

$$P_{reactive} = P_a \sin\theta.$$



* DC Response for RL circuit



by KVL

$$V = Ri + \frac{L di}{dt}$$

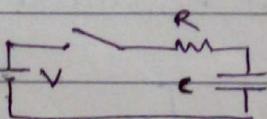
$$i = \frac{V}{R} - \frac{V}{R} e^{-\frac{t}{\tau}} \quad (\tau = \frac{L}{R} \text{ sec}) \quad (\because \text{by solving diff. eqns})$$

$$\text{V}_R = V [1 - e^{-\frac{t}{\tau}}] \quad \text{V}_L = V e^{-\frac{t}{\tau}}$$

$$P_R = V_R \cdot i \quad P_L = V_L \cdot i$$

* after 5τ transient part reaches more than 99% of its final value.

* DC Response for RC circuit



by KVL

$$V = Ri + \frac{1}{C} \int i dt$$

differentiating,

$$\frac{di}{dt} + \frac{1}{RC} i = 0$$

solving diff. eq. to get i

$$i = \frac{V}{R} e^{-\frac{t}{RC}}$$

Time constant.
 $\tau = RC$

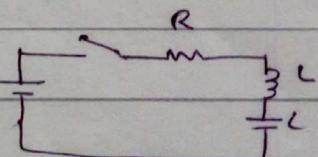
$$V_R = V e^{-\frac{t}{\tau}}$$

$$V_C = V(1 - e^{-\frac{t}{\tau}})$$

$$P_R = V_R \cdot i$$

$$P_C = V_C i$$

* DC Response for RLC series circuit



$$\text{By KVL} \quad V = Ri + \frac{L di}{dt} + \frac{1}{C} \int i dt$$

$$\text{to get } i \text{ different. w.r.t. } i \Rightarrow \frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = 0$$

$$\Rightarrow D^2 + \frac{R}{L} D + \frac{1}{LC} = 0 \quad \text{solving diff. eqn & get roots.}$$

$$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

① if k_2 is +ve

$$\left(\frac{R^2}{4L}\right) > \frac{1}{LC}$$

② if k_2 is -ve

$$\left(\frac{R^2}{4L}\right) < \frac{1}{LC}$$

③ if k_2 is zero.

$$\frac{R^2}{4L} = LC$$

$$[D - (k_1 + k_2)][D - (k_1 - k_2)]i = 0 \quad [D - (k_1 + jk_2)][D - (k_1 - jk_2)]i = 0 \quad (D - k_1)(D - k_2)i = 0$$

$$i = c_1 e^{(k_1 + k_2)t} + c_2 e^{(k_1 - k_2)t}$$

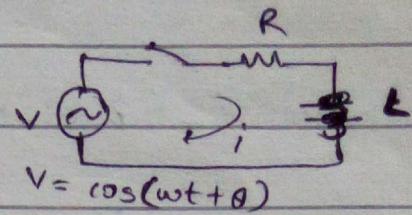
$$i = e^{k_1 t} [c_1 \cos k_2 t + c_2 \sin k_2 t]$$

$$i = e^{k_1 t} (c_1 + c_2 t)$$

e.g. 997

Q. Define
Response

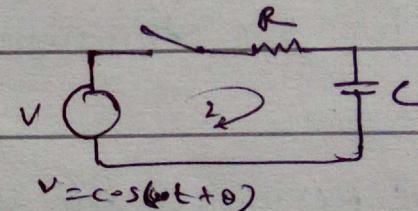
* Sinusoidal Response of RL ckt



$$i = e^{-t/\tau} \left[\frac{-V}{\sqrt{R^2 + (\omega L)^2}} \cos \left(\theta - \tan^{-1} \frac{\omega L}{R} \right) \right]$$

$$+ \frac{V}{\sqrt{R^2 + (\omega L)^2}} \cos \left(\omega t + \theta - \tan^{-1} \frac{\omega L}{R} \right)$$

* Sinusoidal Response of RC ckt



$$i = e^{-t/\tau} \left[\frac{V}{R} \cos \theta - \frac{V}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \cos \left(\theta + \tan^{-1} \frac{1}{\omega C R} \right) \right]$$

$$+ \frac{V}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \cos \left(\omega t + \theta + \tan^{-1} \frac{1}{\omega C R} \right)$$

* Sinusoidal Response of RLC ckt

* Behaviour of L & C elements at $t=0^+$ and $t \rightarrow \infty$ instants.

$$\text{we know } Z_L = sL \quad Z_C = \frac{1}{sC}$$

$$t=0^+ \Rightarrow s=\infty \Rightarrow Z_L = \infty \Rightarrow L \rightarrow 0\Omega$$

$$Z_C = 0 \Rightarrow C \rightarrow \infty \Omega.$$

$$t \rightarrow \infty \Rightarrow s \rightarrow 0 \Rightarrow Z_L = 0 \Rightarrow L \rightarrow \infty \Omega$$

$$Z_C = \infty \Rightarrow C \rightarrow 0\Omega.$$

* In steady state energy stored in the memory element is maximum and constant.

$$\frac{1}{2} L i_L^2 \Rightarrow \text{max & const.} \Rightarrow i_L = \text{max & const.}$$

My $\therefore V_L - i \frac{di}{dt} \Rightarrow i_L \rightarrow 0 \Rightarrow L \rightarrow \infty \Omega$:- Inductor act as constant current source in steady state.

$$\frac{1}{2} C V_C^2 \Rightarrow \text{max & const.} \Rightarrow V_C = \text{max & const.}$$

\therefore capacitor act as a constant voltage source in steady state

$$\therefore i_C = C \frac{dV_C}{dt} \Rightarrow i_C \rightarrow 0 \Rightarrow C \rightarrow 0\Omega.$$

* Inductor current cannot change instantaneously ($V_L(t) = \text{practical I.P.S.}$)

$$\text{i.e. } i_L(0^+) = i_L(0^-) \quad \text{My } E_L(0^+) = E_L(0^-)$$

(but) $V_L(t) = \delta(t)$

$$i_L(0^+) = i_L(0^-) + \frac{1}{L} \quad \text{My } E_L(0^+) > E_L(0^-)$$

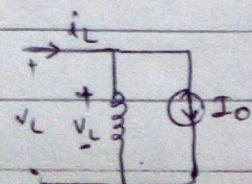
$$i_L(0^-) = 0 \quad \therefore i_L(0^+) = \frac{1}{L} \Rightarrow E_L(0^+) = \frac{1}{2L} (J)$$

* My $V_C(0^+) = V_C(0^-)$ For practical I.P.S.

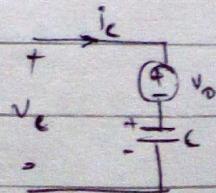
(but) For $i_C = \delta(t)$

$$V_C(0^+) = \frac{1}{C} \quad E_C(0^+) = \frac{1}{2C} (J)$$

$$i_L = I_0 + \frac{1}{L} \int_0^t V_L dt$$



$$V_C = V_0 + \frac{1}{C} \int_0^t i_C dt \quad (\text{Page 86 Ace Notes})$$



* DC Transient (According to Ace Notes)

① Source free circuits (i.e. without independent sources)

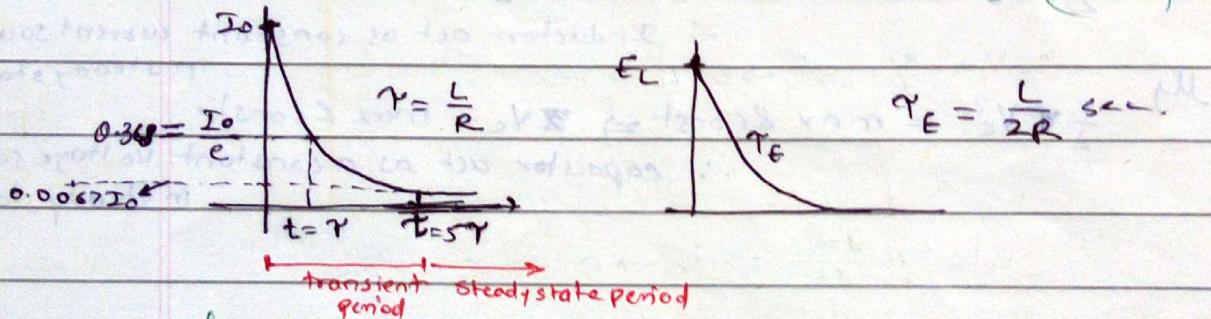
→ in all source free ckt. stored energy in memory elements at $t=0$ is maximum.

→ As a fn of time this stored energy is given to the memory less Resistance in an exponential decaying manner

→ Hence Energy ~~was~~ present in the network at steady state (i.e. $t \rightarrow \infty$) is zero

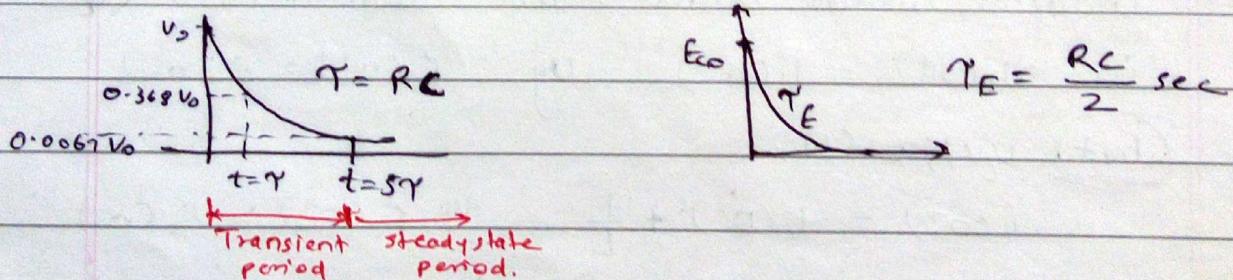
* $i(t) \rightarrow$ for source free RL ckt.

$$i(t) = I_0 e^{-\frac{t}{\tau_E}} \quad I_0 = i(0^+) \text{ (initially stored energy.)}$$



* for source free RC ckt.

$$V(t) = V_0 e^{-\frac{t}{\tau}}$$



Comparing τ we get

$$\boxed{\frac{L}{R} = RC}$$

* Resonance.

Ckt is said to be in resonance if current is in phase with the applied voltage.

* for series RLC ckt:-

for resonance $X_L = X_C$ is must conditn.

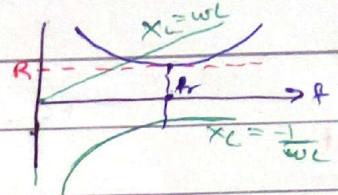
$$\therefore f_r = \frac{1}{2\pi\sqrt{LC}}$$

f_r - frequency at resonance.

Current leads source voltage $\Leftarrow f < f_r$ $X_L < X_C$ (at $f=0$ circuit is capacitive)

In phase $\Leftarrow f = f_r$ $X_L = X_C \Rightarrow Z = R$

Current lags behind applied voltage $\Leftarrow f > f_r$ $X_C > X_L$



(∴ impedance varies as frequency varies)

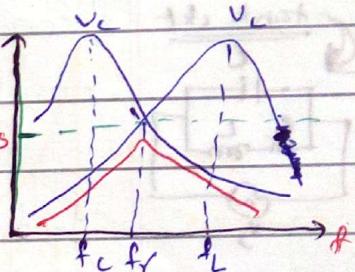
drop across R is maximum at $f=f_r$

$V_L + V_C = 0$ maximum voltage across capacitor at $f=f_C$

Like short combination max voltage across inductor is at $f=f_L$

$$f_L = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{R^2 C}{2L}}$$

$$f_C = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{1}{R^2} - \frac{2L}{C}}$$



Bandwidth of system is the range of frequency for which the current or o/p voltage is equal to 70.7% of its value at the resonant frequency. ($BW = f_L - f_C$)

$$\rightarrow \omega_r^2 = \omega_L \omega_C \quad ; \quad \omega_r^2 = \frac{1}{LC} \quad \omega_L - \omega_C = \frac{R}{L}$$

$$f_r^2 = f_L \cdot f_C$$

$$f_L + f_C = BW = \frac{R}{2\pi L} \Rightarrow f_C = f_r - \frac{R}{4\pi L} \quad f_L = f_r + \frac{R}{2\pi L}$$

Q factor, high at minity i.e. quality factor = ~~$\frac{1}{R}\sqrt{LC}$~~

$$\frac{f_L - f_C}{f_r} = \frac{1}{Q} \Rightarrow Q = \frac{f_r}{B.W.}$$

$$Q = \frac{\omega_r}{R} = \frac{1}{\omega_r C R}$$

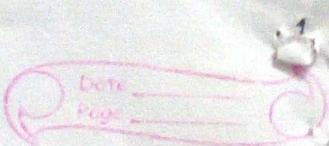
$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

→ At resonance frequency power is $P_{max} = I_{max}^2 R$.

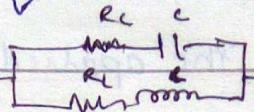
→ power at f_C & f_L (lower & higher wrt freq) $P_L = P_C = \frac{I_{max}^2}{2}$

Q factor is ratio of reactive power of inductor or capacitor to the true power $Q = 2\pi \times \frac{\text{max energy stored}}{\text{energy dissipated per cycle}}$

$$Q = \frac{\omega_L}{R} = \frac{1}{\omega_C R} = \frac{f_r}{BW} \quad \text{here } \omega_r = 2\pi f_r *$$



* Magnification :- $\alpha = \frac{V_L}{V}$ or $\frac{V_C}{V}$.
For series resonance



* Parallel Resonance

→ in parallel resonance also $X_L = X_C$ must condition.

→ current in each branch is same in magnitude, but 180° phase shift so overall current & voltage are in phase at resonance.

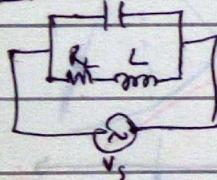
$$I_C + I_L = 0$$

LC like open circuit

$$\omega_r = \frac{1}{\sqrt{L C}} \sqrt{\frac{R_L^2 - (X_C)^2}{R_C^2 - (X_L)^2}} \quad \therefore \text{for } R_L = R_C, f_r = \frac{1}{2\pi\sqrt{LC}}$$

tank ckt Resonant frequency for tank ckt (parallel resonant ckt),

tank ckt \Rightarrow the stored energy transferred back & forth between capacitor & inductor.



$$f_r \text{ for tank ckt} = \frac{1}{2\pi\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}}$$

$$\text{Q factor for parallel Resonance} = Q_r = \frac{\omega_r R C}{R} = \frac{R}{\omega_r L} = R \sqrt{\frac{C}{L}}$$

* magnification for II^{th} resonance $\alpha_r = \frac{I_L}{I}$ or $\frac{I_C}{I}$

$$I_L = \alpha_r I L - 90^\circ = -j \alpha_r I$$

$$I_C = \alpha_r I C L - 90^\circ = j \alpha_r I$$

$$I_R = I \quad I_L + I_C = 0$$

* minimum R for physical resistance. (called critical resistance) in series RLC when ξ (damping factor) = 1

$$\xi = \frac{1}{2\sqrt{LC}} = 1 \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

$$R_{\text{critical}} (\text{for series RLC}) = 2\sqrt{LC}$$

* If for parallel Resonance R_{max} called critical Resistance

$$R_{\text{critical}} (\text{at resonance}) = \frac{1}{2} \sqrt{\frac{L}{C}}$$

* * for a Resonant frequency find impedance Z.

& equate j term with zero at Resonance. Total impedance is pure Resistive.

* In parallel resonance it will Resonate for all frequencies

if $R^2 = \frac{L}{C}$ out of infinite resonate frequency we select $\omega_0 = \frac{1}{\sqrt{LC}}$

* Average power in series resonance = $\frac{V_{\text{rms}}^2}{R} + \frac{I_{\text{rms}}^2}{R}$
 $R = \frac{V_{\text{rms}}^2}{P_{\text{avg}}}$

$$\text{Average power in parallel resonance} = I_L^2 \cdot \frac{V_{\text{rms}}^2}{R} + I_C^2 \cdot \frac{V_{\text{rms}}^2}{R} \\ = I_{\text{rms}}^2 \cdot R_{\text{eq}}$$

* for parallel resonance. $i(t) = \frac{v_i(t)}{Z} = i_R$.

$$V = V_C = V_L = V_R \Rightarrow V_R = i_R \cdot R$$

$$i_C = \frac{dV_C}{dt} = C \frac{dV_R}{dt}$$

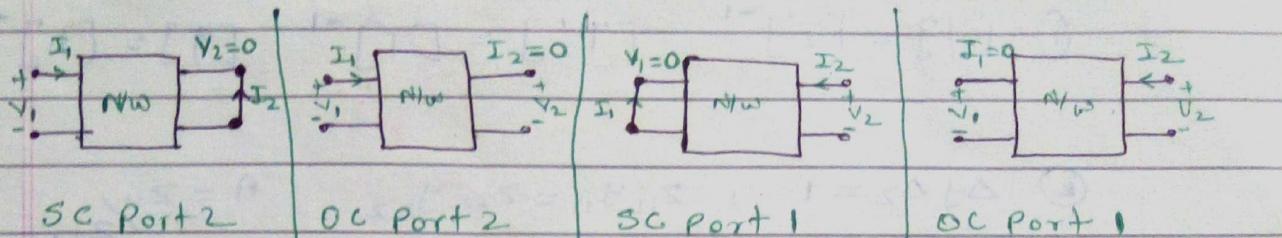
$$i_L = \frac{1}{L} \int v_L dt = \frac{1}{L} \int V_R dt$$

* for half wave rectifier average value = $\frac{\text{Peak Value}}{\pi}$

* Locus Diagram.

- * phasor diagram expanded to develop a curve known as locus.
- * locus diagram useful in determining the behaviour or response of an RLC circuit when one of its parameter is varied while frequency and voltage kept constant.
- * circle diagrams identifies locus plots that are either circular or semicircular.
- * based on leading of current in capacitor & lagging the current by the inductor concept. ~~modifying~~ modify the phasor diagram by varying the component and get locus of current
this is the locus diagram. For clarity see problems pg 323, 325
319, 320 etc.

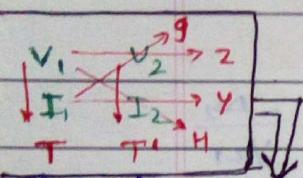
* Two port Network.



calculating $\mathbf{Y}_{11}, \mathbf{Y}_{21}$

$\mathbf{h}_{11}, \mathbf{h}_{21}$

\mathbf{B}, \mathbf{D}



$\mathbf{Z}_{11}, \mathbf{Z}_{21}$

$\mathbf{g}_{11}, \mathbf{g}_{21}$

\mathbf{A}, \mathbf{C}

$\mathbf{Y}_{12}, \mathbf{Y}_{22}$

$\mathbf{g}_{12}, \mathbf{g}_{22}$

\mathbf{B}', \mathbf{D}'

$\mathbf{Z}_{12}, \mathbf{Z}_{22}$

$\mathbf{h}_{12}, \mathbf{h}_{22}$

\mathbf{A}', \mathbf{C}'

Name of Parameters	Express	In terms of	equations	matrix Associated	Condition of reciprocity, symmetry.
--------------------	---------	-------------	-----------	-------------------	-------------------------------------

Open ckt $\mathbf{V}_1, \mathbf{I}_1, \mathbf{I}_2 \& \mathbf{I}_2$ $\mathbf{V}_1 = \mathbf{Z}_{11}\mathbf{I}_1 + \mathbf{Z}_{12}\mathbf{I}_2$ $[\mathbf{z}] = \begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} \end{bmatrix}$ $\mathbf{Z}_{12} = \mathbf{Z}_{21}$

Impedance (\mathbf{Z}) $\mathbf{V}_2, \mathbf{I}_1, \mathbf{I}_2 \& \mathbf{I}_2$ $\mathbf{V}_2 = \mathbf{Z}_{21}\mathbf{I}_1 + \mathbf{Z}_{22}\mathbf{I}_2$ $\mathbf{Z}_{11} = \mathbf{Z}_{22}$

Short ckt Admittance (\mathbf{Y}) $\mathbf{I}_1, \mathbf{V}_1 \& \mathbf{V}_2$ $\mathbf{I}_1 = \mathbf{Y}_{11}\mathbf{V}_1 + \mathbf{Y}_{12}\mathbf{V}_2$ $[\mathbf{y}] = \begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} \end{bmatrix}$ $\mathbf{Y}_{12} = \mathbf{Y}_{21}$

(\mathbf{T}) $\mathbf{I}_2, \mathbf{V}_1 \& \mathbf{V}_2$ $\mathbf{I}_2 = \mathbf{Y}_{21}\mathbf{V}_1 + \mathbf{Y}_{22}\mathbf{V}_2$ $\mathbf{Y}_{11} = \mathbf{Y}_{22}$

Transmission (\mathbf{T}) $\mathbf{V}_1, \mathbf{V}_2 \& \mathbf{I}_2$ $\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$ $[\mathbf{T}] = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{0} \end{bmatrix}$ $\mathbf{AD} - \mathbf{BC} = 1$

(\mathbf{ABCD}) $\mathbf{I}_1, \mathbf{V}_2 \& \mathbf{I}_2$ $\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$ $\mathbf{A} = \mathbf{D}$.

Inverse Transmission (\mathbf{T}') $\mathbf{V}_2, \mathbf{V}_1 \& \mathbf{I}_1$ $\mathbf{V}_2 = \mathbf{A}'\mathbf{V}_1 - \mathbf{B}'\mathbf{I}_1$ $[\mathbf{T}'] = \begin{bmatrix} \mathbf{A}' & \mathbf{B}' \\ \mathbf{C}' & \mathbf{0}' \end{bmatrix}$ $\mathbf{AD}' - \mathbf{BC}' = 1$

($\mathbf{A}'\mathbf{B}'\mathbf{C}'\mathbf{D}'$) $\mathbf{I}_2, \mathbf{V}_1 \& \mathbf{I}_1$ $\mathbf{I}_2 = \mathbf{C}'\mathbf{V}_1 - \mathbf{D}'\mathbf{I}_1$ $\mathbf{A}' = \mathbf{D}'$

Hybrid (\mathbf{H}) $\mathbf{V}_1, \mathbf{I}_1 \& \mathbf{V}_2$ $\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$ $[\mathbf{h}] = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix}$ $\mathbf{h}_{12} = -\mathbf{h}_{21}$

$\mathbf{I}_2, \mathbf{I}_1 \& \mathbf{V}_2$ $\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$ $\Delta \mathbf{h} = 1$

$\mathbf{I}_1 = \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2$ $[\mathbf{g}] = \begin{bmatrix} \mathbf{g}_{11} & \mathbf{g}_{12} \\ \mathbf{g}_{21} & \mathbf{g}_{22} \end{bmatrix}$ $\mathbf{g}_{12} = -\mathbf{g}_{21}$

$\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2$ $\Delta \mathbf{g} = 1$

* interconnection betⁿ parameters.

$$\textcircled{1} \quad \underline{\underline{[Y]}} = \underline{\underline{[Z]}^{-1}} \quad [T'] = [T]^{-1} \quad [g] = [h]^{-1}$$

$$\textcircled{2} \quad \Delta Y \Delta Z = 1, \quad z_{11}y_{11} = z_{22}y_{22}, \quad A = z_{11}/z_{22}$$

$$h_{11} = B_{1D} = 1/y_{11}, \quad \text{etc.}$$

$\Delta \rightarrow$ determinant of matrix

we obtained

$$\underline{\underline{Y \rightarrow Z}} \quad z_{11} = \frac{y_{22}}{\Delta Y}, \quad z_{12} = \frac{-y_{12}}{\Delta Y}, \quad z_{21} = \frac{-y_{21}}{\Delta Y}, \quad z_{22} = \frac{y_{11}}{\Delta Y}$$

"Y for rest T' + 0 T & g to h

$$A = \frac{z_{11}}{z_{22}} = \frac{-y_{22}}{y_{21}}, \quad B = \frac{\Delta Z}{z_{22}} = -\frac{1}{y_{21}}, \quad C = \frac{1}{z_{22}} = -\frac{\Delta Y}{y_{21}}$$

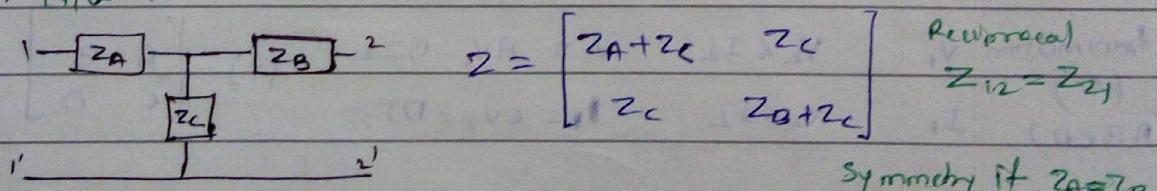
$$D = \frac{z_{22}}{z_{21}} = \frac{-y_{11}}{y_{21}}$$

* to get relation between parameter write equations.

of both parameter take conditions & apply to both equations.
& obtain relation. (pg 752)

* Typical Networks & parameters

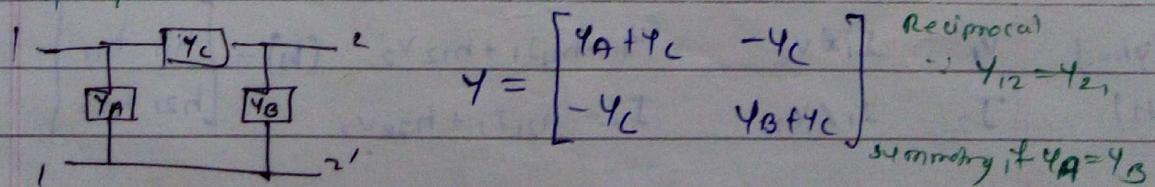
* TN/ ω :-



Symmetry if $z_A = z_B$

if $z_A = z_B = 0$, $z_C = z$ T becomes null element
(∴) ⇒ ⇒ Y not defined for N/w det [Z] = 0

* TTN/ ω .

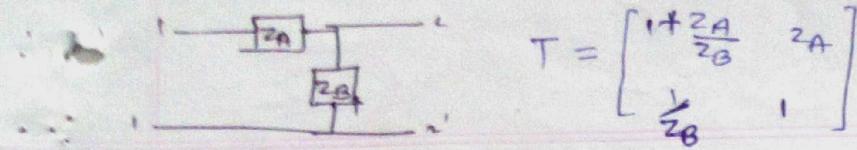


Symmetry if $y_A = y_B$

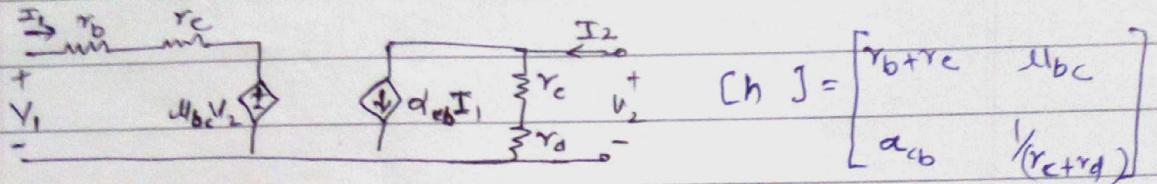
If $y_A = y_B = 0$ & $y_C \neq 0$ TT becomes series element

⇒ ⇒ Z not defined for such N/w det [Y] = 0,

* T Networks

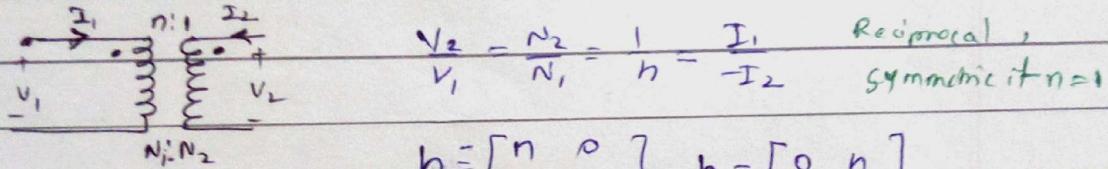


* Model of CE connector transistor.



Neither reciprocal nor symmetrical.

* Ideal Transformer



$$h = \begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix} \quad h = \begin{bmatrix} 0 & n \\ -n & 0 \end{bmatrix}$$

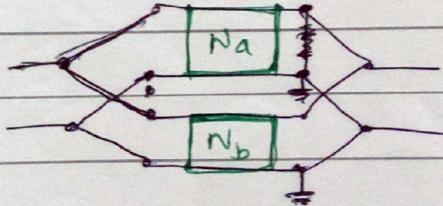
(Neither z nor y parameter defined)

* Interconnection of 2-port N/Ws.

* Parallel connections

Individual y parameters get added to get ~~z~~ y parameters of overall N/W.

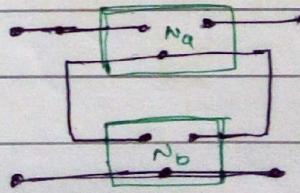
$$[y] = [y_a] + [y_b]$$



* Series connection

Individual z parameters get added to get z parameters of overall N/W

$$[z] = [z_a] + [z_b]$$



* Cascade connection

Here individual T-matrices are multiplied to get overall T parameter of N/W

$$[T] = [T_a] \times [T_b]$$

matrix multiplication?

$$A = A_a A_b + B_a B_b$$

$$\begin{array}{c} \xleftarrow{\hspace{1cm}} \begin{bmatrix} A_a & B_a \\ B_a & T_a \end{bmatrix} \xleftarrow{\hspace{1cm}} \begin{bmatrix} A_b & B_b \\ B_b & T_b \end{bmatrix} \xleftarrow{\hspace{1cm}} \end{array}$$

* Coupled circuits.

* if the current enters the dots or leaves the dot in both the coils simultaneously, then the mutual flux will aid to self flux else oppose the self flux.

$$\text{Diagram: } \begin{array}{c} \rightarrow \\ \text{dot} \\ \left\{ \begin{array}{l} \text{coil 1} \\ \text{coil 2} \end{array} \right. \\ \leftarrow \end{array} \Rightarrow L_1 \frac{di}{dt} + M \frac{di}{dt}$$

$$\text{Diagram: } \begin{array}{c} \rightarrow \\ \text{dot} \\ \left\{ \begin{array}{l} \text{coil 1} \\ \text{coil 2} \end{array} \right. \\ \rightarrow \end{array} \Rightarrow L_1 \frac{di}{dt} - M \frac{di}{dt}$$

→ coeff of coupling

$$K = M / \sqrt{L_1 L_2}$$

L_1 → self inductance
 M → mutual inductance

maximum possible inductance

$$M = \sqrt{L_1 L_2} \quad \underline{\underline{K=1}}$$

$$V = L \frac{di}{dt} \quad & V = N \frac{d\phi}{dt} \Rightarrow L = N \frac{d\phi}{dt}$$

$$L \propto N^2 \Rightarrow \frac{L_1}{L_2} = \frac{N_1^2}{N_2^2} = \frac{I_2^2}{I_1^2}$$

$$\frac{N_2^2}{N_1^2} = \frac{L_2}{L_1} = a^2 \quad \downarrow \text{turn ratio}$$

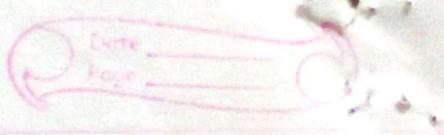
* Coupled inductors in series :-

$$\text{Diagram: } \begin{array}{c} \rightarrow \\ \text{dot} \\ \left\{ \begin{array}{l} \text{coil 1} \\ \text{coil 2} \end{array} \right. \\ \rightarrow \end{array} \Rightarrow L_{eq} = L_1 + L_2 + 2M$$

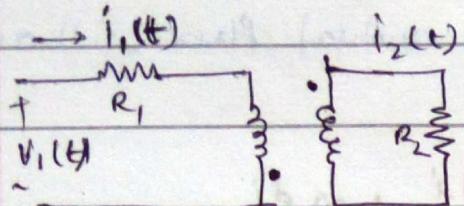
$$\text{Diagram: } \begin{array}{c} \rightarrow \\ \text{dot} \\ \left\{ \begin{array}{l} \text{coil 1} \\ \text{coil 2} \end{array} \right. \\ \leftarrow \end{array} \Rightarrow L_{eq} = L_1 + L_2 - 2M$$

$$\text{Diagram: } \begin{array}{c} \rightarrow \\ \text{dot} \\ \left\{ \begin{array}{l} \text{coil 1} \\ \text{coil 2} \end{array} \right. \\ \rightarrow \end{array} \Rightarrow L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

$$\text{Diagram: } \begin{array}{c} \rightarrow \\ \text{dot} \\ \left\{ \begin{array}{l} \text{coil 1} \\ \text{coil 2} \end{array} \right. \\ \leftarrow \end{array} \Rightarrow L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$



* Transform coupling.

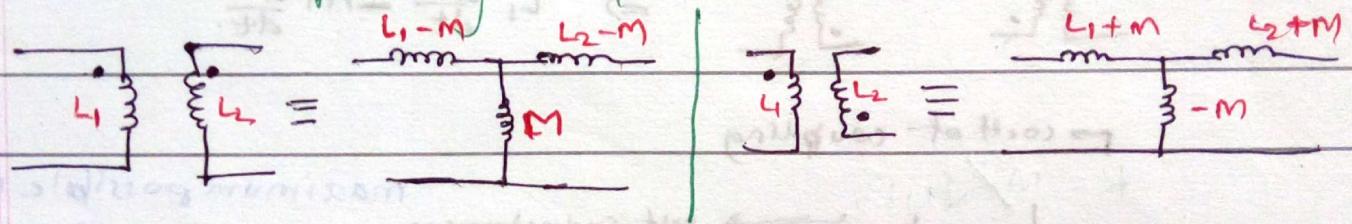


KVL:

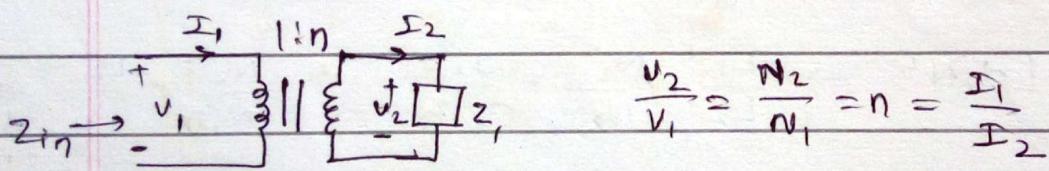
$$V_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} + R_1 i_1(t)$$

$$0 = L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt} + R_2 i_2(t)$$

* Transformer replaced by T-equivalent



* Transformer as a impedance matching properties.



$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n = \frac{I_1}{I_2}$$

$$Z_{in} = \frac{Z_L}{n^2}$$

$$\frac{Z_L}{Z_{in}} = n^2 \left(\frac{N_2}{N_1} \right)^2$$

for ideal transformer

$$\begin{cases} k=1, M=\sqrt{L_1 L_2} \\ L_1 \rightarrow \infty, L_2 \rightarrow \infty, M \rightarrow \infty \end{cases}$$

$$L_1 : L_2 : M = N_1^2 : N_2^2 : N_1 N_2$$

* Graph theory

* No. of Nodes in graph = No. of Nodes in tree.

→ if tree has n node $\Rightarrow (n-1)$ branches

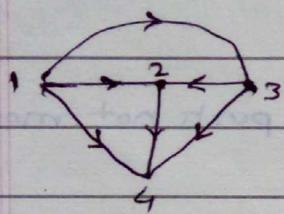
→ branches of tree are called twigs. twigs $(n-1) \Rightarrow$ tree value or rank of the tree

→ complementary set of branches are co-tree

→ branches of co-tree are links or chords.

→ N/W with b branches & n nodes have, $(n-1)$ twigs and $(b-n+1)$ links. i.e. $b-(n-1)$ links.

→ incident matrix. $(A)_{n \times b}$



	a	b	c	d	e	f
1	1	0	1	0	0	1
2	-1	-1	0	1	0	0
3	0	1	0	0	1	-1
4	0	0	-1	-1	-1	0

* Properties

→ incident matrix is unique having order $n \times b$.

→ rank of incident matrix is equal to rank of graph = $(n-1)$

→ det. of complete incident matrix of closed loop is always 0.

(or branch)

→ * The number of edges of complete graph with n nodes is

$$= \frac{n(n-1)}{2}$$

→ Number of possible trees of complete graph with n nodes

$$= n^{n-2}$$

→ Tree + co-tree = original graph.

* fundamental loop matrix or f-cut matrix or tie-set matrix.

→ f-loops or f-cuts or tie sets are minimum number of loop or mesh equations required to solve the corresponding N/W of a given graph.

adding 1 link at a time get f-link equations.

* Properties →

- Rank of Tie set matrix = $(b - n + 1)$ i.e., $b - (n - 1)$
- No. of f-loops always equal to no. of links = $\underline{(b - n + 1)}$
- every fundamental loop consists of only one link in its representation.
- One loop matrix for one tree & for one graph n^{n-2} trees possible. ∴ n^{n-2} loop matrices are possible.
- cut set is a minimal set of branches of a graph, removal of which divides the graph into two parts i.e. the cutset always comprise minimum number of removed branches from the original graph
- * → cut set must divide graph in two parts not more than two.
- * f-cutset matrix
 - gives minimum no. of nodal eq's required to solve corresponding
 - ① select tree ② removing one tree branch at a time result one cutset
- Properties
 - Rank of f-cutset matrix is $n - 1$
 - no. of f-cutset are always equal to the number of tree branches i.e. twigs = $\underline{(n - 1)}$
 - Every f-cut set consist of only one tree branch in its representation.
 - every tree will result one-f-cut set matrix ∴ for any graph no. of cutset matrices = no. of trees = $\underline{n^{n-2}}$

* Filters

$$D = 10 \log_{10} \frac{P_1}{P_2}$$

$\frac{P_1}{P_2} \Rightarrow \text{i/p power/o/p power}$

$$D = 20 \log_{10} \frac{V_1}{V_2}$$

$\frac{V_1}{V_2} \Rightarrow \text{i/p voltage/o/p voltage}$

$$1D = 0.115N$$

N - Neper D - Decibal.

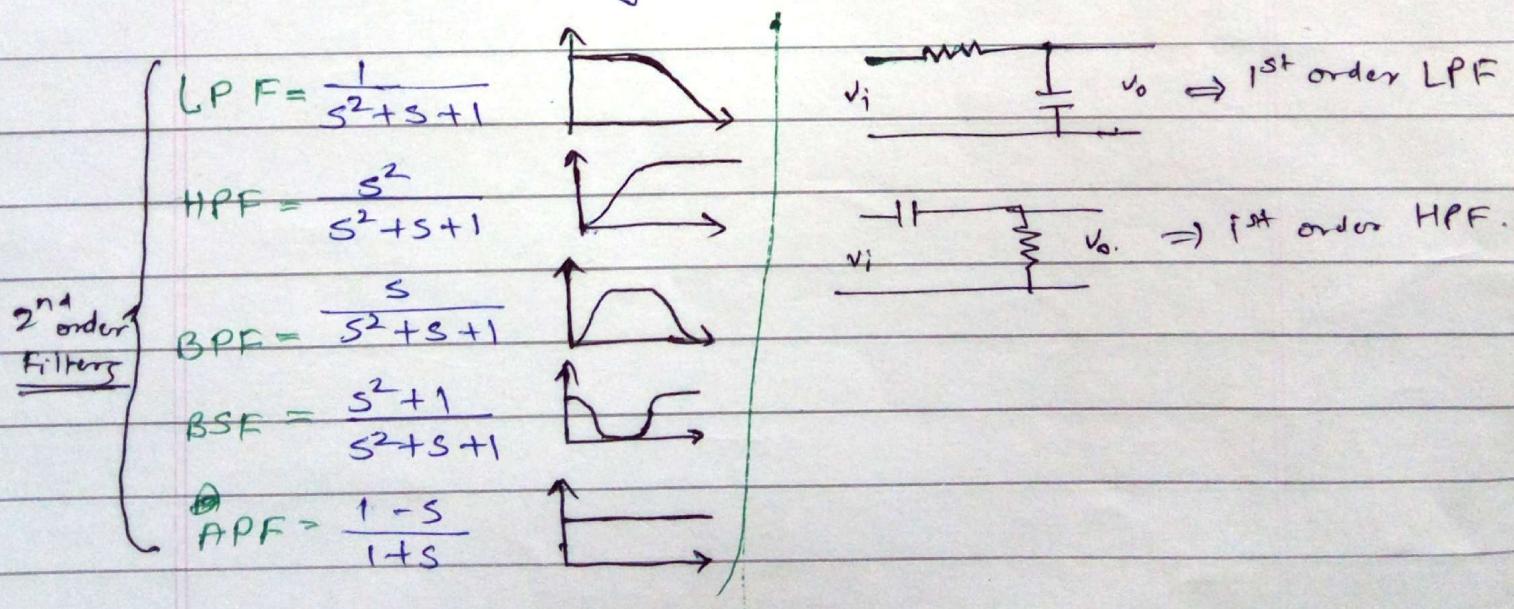
$$Z_R = R \Omega \quad Z_L = j\omega L \Omega \quad Z_C = \frac{1}{j\omega C} \Omega$$

* $\omega = 0 \Rightarrow Z_L = 0 \Rightarrow L \rightarrow SC \quad \& \quad Z_C = \infty \Rightarrow C \rightarrow OC$

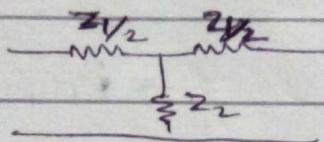
* $\omega = \infty \Rightarrow Z_L = \infty \Rightarrow L \rightarrow OC \quad \& \quad Z_C = 0 \Rightarrow C \rightarrow SC$

** To identify the filter put $\omega = 0$ & $\omega = \infty$ & find V_o at 0 & V_o at ∞ & conclude about filter.

* All pass Filter add min 0 & max ± 180 phase shift.
 \therefore APF is basically a phase shifter ckt.



* for T N/w.

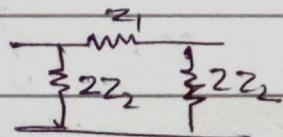


$$Z_{OT} = \sqrt{\frac{z_1^2}{4} + z_1 z_2} \quad Z_{OT} = \sqrt{Z_{OC} Z_{SC}}$$

$$\tanh \gamma^2 = \sqrt{\frac{Z_{SC}}{Z_{OC}}} \quad \cosh \gamma^2 = 1 + \left(\frac{z_1}{z_2}\right) = \sqrt{\frac{z_1}{z_2}}$$

Propagation constant for T N/w.

* for T N/w

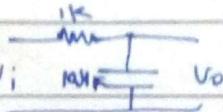


$$Z_{OT} = \sqrt{\frac{z_1 z_2}{1 + \frac{z_1}{2 z_2}}} = \frac{z_1 z_2}{Z_{OT}}$$

$$Z_{OT} = \sqrt{Z_{OC} Z_{SC}}$$

$$\cosh \gamma^2 = 1 + \frac{z_1}{2 z_2}$$

Propagation constant for T N/w.

* $V_i = 3 + 4 \sin 100t$ V . 

$\omega = 0$

$\omega = 100$

$$H(s) = \frac{1}{1+sCR} \Rightarrow H(j\omega) = \frac{1}{1+j\omega CR}$$

And $H(j\omega)$ for $\omega = 0$
 $\omega = 100$
 Then $\lim_{s \rightarrow 0} H(s)$

$$V_o(s) = H(s) \cdot V_i(s)$$

$$\frac{I(s)}{V(s)} = H(s) = Y(s) = \frac{1}{Z(s)}$$

- * if asked steady state current or voltage go by Laplace transform of ckt & find T.F. & ans. e.g. as above.

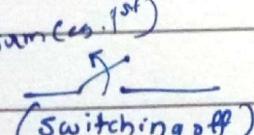
- * C is SC in DC & Depends on freq. in AC. (mostly OC)

C is SC at $t=0^+$ & OC at $t \rightarrow \infty$ (ss.)

L is OC at $t=0^+$ & SC at $t \rightarrow \infty$ (ss.)
 (T.S.)

* $\frac{di}{dt} + \frac{R}{L} i = 0 \Rightarrow i(t) = I_0 e^{-\frac{R_L t}{L}} = I_0 e^{-t/\tau} \quad \tau = \frac{L}{R}$

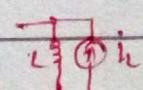
My $v(t) = V_0 e^{-t/\tau_{RC}} = V_0 e^{-t/\tau} \quad \tau = RC$

* for particular source (1st)
 \Rightarrow 3 states

① S is ON $\rightarrow t \rightarrow \infty$
 this is steady state $L = SC, C = O.C$

find i_L for L & or V_C for C.
 i through L as L is SC $\Rightarrow 0^-$
 Now S is just off 0^+

Now this is transient state or $V_C^{(0^+)} = V_C^{(\infty)}$
 $L = OC, C = SC$, & $i_L(0^+) = i_L(0^-)$
 but L have II d current source of i_L
 & C have V_C source in series.

③ Now $t \geq 0$ L is replaced by 

by CTS 

* If $i_L(0^+) = i_L(0^-)$ for L
 then $V_L(0^+) \neq V_L(0^-)$

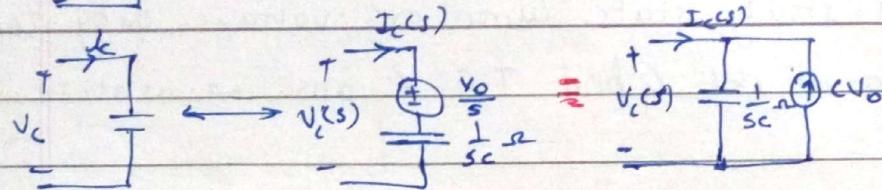
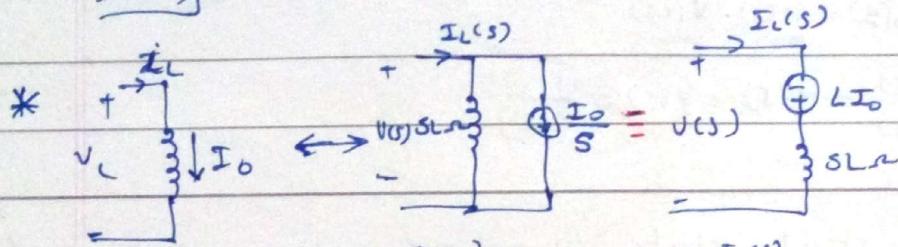
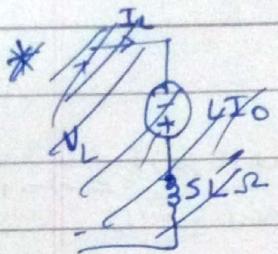
find $V_L(0^+)$

My for V_C also)

in this state
 ckt becomes
 Resistive.

* find τ for L or C. Now
 to have $i(t) R V(t)$ eqn.

* for all source free ckt of transient current will flow from
 one to the terminal. energy is decaying \therefore memory elements
 are discharging.



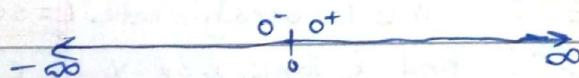
$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)] e^{-t/\tau}$$

My for source free $i_L = i_L(0) e^{-t/\tau} \therefore i_L(\infty) = 0$

$$V_C(t) = V_C(\infty) + [V_C(0) - V_C(\infty)] e^{-t/\tau}$$

for source free $V_C(t) = V_C(0) e^{-t/\tau} \therefore V_C(\infty) = 0$

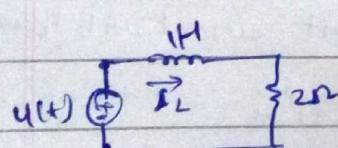
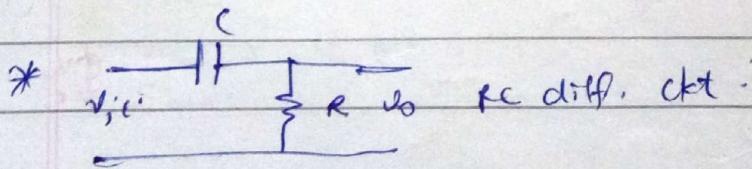
* Main concept



① for s.s. cap. is o.c. & Inductor is s.c. find V_C & I_L

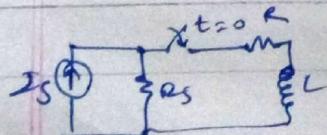
② at transient cap is sc. Inductor is oc with replacement of appropriate V_C & I_L sources.

③ at $t \rightarrow \infty$ again cap. o.c. & Inductor s.c.



method
① Laplace method $\frac{V(s)}{I(s)} = \frac{1}{s} \quad \frac{V(s)}{I(s)} = ? \quad \underline{i_L(t)}$

② find $i_L(0)$, $i_L(\infty)$ & then $\underline{i_L(t)}$



$$\frac{di_L(t)}{dt} = ?$$

$$i_L(0) = 0$$

$$\text{Ans} = \frac{I_0 C (R + sL) + I_s R s}{L}$$

$$\text{at } t \geq 0 \quad I_0 C s = i_L(t) + L \frac{di_L}{dt}$$