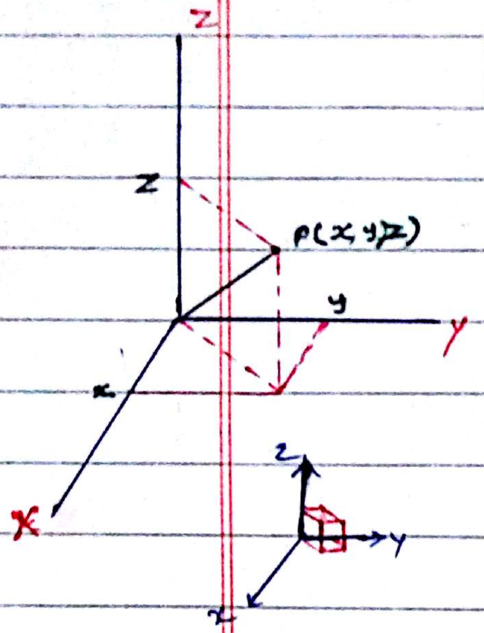
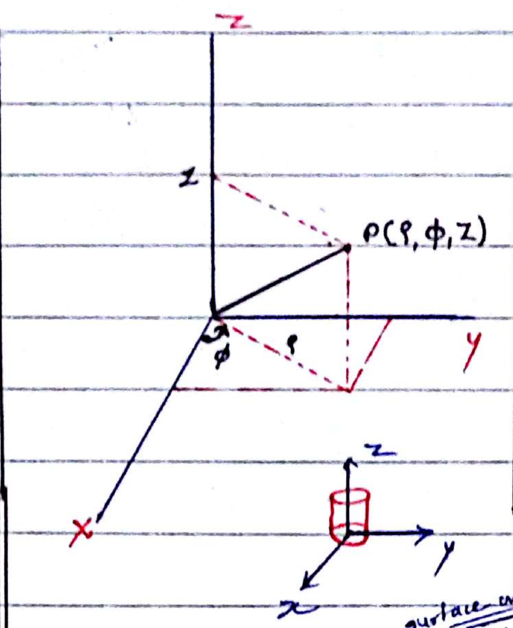


cartesian or Rectangular

$ds = dx\,dy$ or $dy\,dz$ or $dx\,dz$

$dv = dx \cdot dy \cdot dz$.



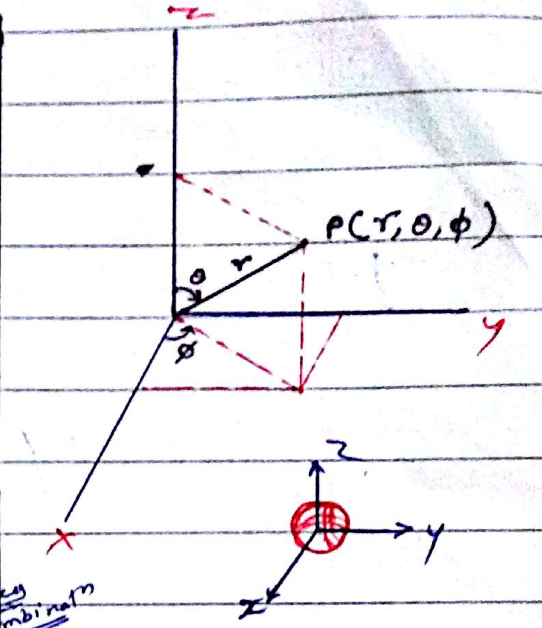
cylindrical

surface area combination

$\rho\, d\phi$, $d\rho$, dz

$ds = \rho\, d\rho\, d\phi$ or $\rho\, d\phi\, dz$

$dv = \rho\, d\rho\, d\phi\, dz$



spherical.

$ds = r^2 \sin\theta\, d\theta\, d\phi$

$dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$.

$x = \rho\cos\phi$ $\quad y = \rho\sin\phi$ $\quad \rho = r\sin\theta$ $\quad z = r\cos\theta$

$x = r\sin\theta\cos\phi$ $\quad y = r\sin\theta\sin\phi$ $\quad \rho = \sqrt{x^2+y^2}$ $\quad r = \sqrt{x^2+y^2+z^2}$

$\theta = \cos^{-1}\left(\frac{z}{r}\right)$ $\quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$

$\epsilon_0 \rightarrow 8.854 \times 10^{-12}$ F/m

$\rightarrow \frac{10^{-9}}{36\pi}$

$\alpha \rightarrow$ directly proportional
$\frac{1}{\alpha} \rightarrow$ inversely proportional

* **colomb's law :** $|\bar{F}| = \frac{1}{4\pi\epsilon} \frac{Q_1 Q_2}{|\bar{R}|^2} \cdot \frac{\bar{R}}{|\bar{R}|}$ Newton

force of attraction or repulsion bet^n Q_1 & Q_2
① α to $Q_1 \cdot Q_2$ & ② $\frac{1}{\alpha}$ to $\frac{1}{|\bar{R}|^2}$

* **Electric field :**

$$\frac{|\bar{F}|}{Q_0} = \bar{E} = \frac{Q}{4\pi\epsilon|\bar{R}|^2} \frac{\bar{R}}{|\bar{R}|}$$ } due to point charge Q.

$$\bar{E} = \int_l \frac{\rho_l \, dl}{4\pi\epsilon|\bar{R}|^2} \cdot \hat{a}_R$$ } ~~due to line charge~~ distribution

$\rho_l \rightarrow$ line charge density.

$$\bar{E} = \frac{\rho_l}{2\pi\epsilon|\bar{R}|} \cdot \frac{\bar{R}}{|\bar{R}|}$$. in genral for $\bar{E}$ due to infinite line with uniform charge density.

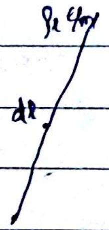
$\rho_s \rightarrow$ surface charge density.

$$\bar{E} = \int_S \frac{\rho_s \, ds}{4\pi\epsilon|\bar{R}|^2} \hat{a}_R$$ } due to sheet charge distribution

$$\bar{E} = \frac{\rho_s}{2\epsilon} \hat{a}_n$$ in general $\bar{E}$ due to infinite sheet charge density.

$\hat{a}_n \rightarrow$ normal vector normal to sheet.

* **Total charge distribution**

line charge	surface charge	Volume charge
 $dQ = \rho_l \, dl$ $Q = \int_l \rho_l \, dl$ $\hookrightarrow$ line integral single	 $dQ = \rho_s \, ds$ $Q = \int_S \rho_s \, ds$ $\hookrightarrow$ surface inte. Double inte	 $dQ = \rho_v \, dv$ $Q = \int_V \rho_v \, dv$ $\hookrightarrow$ volume inte triple integ

* **Electric flux :**

It is lines of forces originates, or ends in charge
i.e. $\psi = Q$ ∴ charge calculation are equal to flux calculation

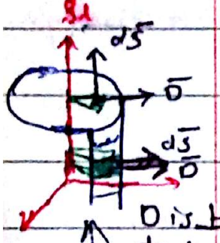
* Gaus law :

Net electric flux passing through any closed surface is equal to charge enclosed by that surface.

$$\psi_{net} = Q_{enclosed}.$$

* Electric flux density :

ψ. (Electric flux) per unit area.



ψ density $= \bar{D}$ for sphere $= \dfrac{Q}{4\pi r^2}\hat{a}_r$ → charge enclosed in sphere. ↳ radial vector Normal to surface

direction of $\bar{D}$ is purely depend upon charge configuration i.e. not alway ⊥ to surface

D is ⊥ to ds ⊥ to surface

$$\therefore \quad \bar{D} = \epsilon \bar{E}$$

$\therefore \bar{D} = \dfrac{Q}{4\pi r^2}\hat{a}_r$ — point charge

$\bar{D} = \dfrac{\rho_l}{2\pi\rho}\hat{a}_\rho$ — line charge

$\bar{D} = \dfrac{\rho_s}{2}\hat{a}_n$ — sheet charge.

* Gaussian law in integral form

$$\psi_{net} = \oint_S \bar{D}\cdot d\bar{s} = Q_{enclosed}$$

↳ surface integral

$$\oint_S \bar{D}\cdot d\bar{s} = \int_v \rho_v\, dv \qquad \because Q_{enc} = \int_v \rho_v\, dv.$$

using divergence thm

↳ $\nabla\cdot\bar{D} = \rho_v$ — gauss law in pt form or gaus divergence thm.

$$\nabla\cdot\bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\nabla\cdot\bar{D} = \frac{1}{\rho}\frac{\partial(\rho D_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$$

$$\nabla\cdot\bar{D} = \frac{1}{r^2}\frac{\partial(r^2 D_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, D_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}$$

T.R. → ⓐ $\nabla\cdot\bar{D} = \dfrac{\partial D_1}{\partial 1} + \dfrac{\partial D_2}{\partial 2} + \dfrac{\partial D_3}{\partial 3}$

ⓑ $\nabla\cdot\bar{D} = \dfrac{1}{\rho}\,\rho + \dfrac{1}{\rho}$

ⓒ $\nabla\cdot\bar{D} = \dfrac{1}{r^2}\,r^2 + \dfrac{1}{r\sin\theta}\sin\theta + \dfrac{1}{r\sin\theta}$

1	2	3	
x	y	z	ⓐ
ρ	ϕ	z	ⓑ
r	θ	ϕ	ⓒ

* work (Joules)

$$w = Q \int_{initial}^{final} \bar{E} \cdot d\bar{l} \quad \text{Joule unit}$$

$$d\bar{l} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

PR3 → circle extra

$$d\bar{l} = d\rho\,\hat{a}_\rho + \rho\, d\phi\,\hat{a}_\phi + dz\,\hat{a}_z$$

$$d\bar{l} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\, d\phi\,\hat{a}_\phi$$

work done over a closed surface is 0

$$\therefore \oint_l \bar{E} \cdot d\bar{l} = 0 \quad \because Q \text{ is not } 0.$$

* Potential or (scalar electric potential)

$$V_{AB} = \frac{w}{Q} = -\int_B^A \bar{E} \cdot d\bar{l} \; (J/C \text{ or } V)$$

unit.

$$V_P = \frac{Q}{4\pi\epsilon |\bar{R}|} + C$$

* Potential function.

$$V = \frac{Q}{4\pi\epsilon\sqrt{x^2+y^2+z^2}}$$

* relatⁿ between Electric field & potential gradient

$$\bar{E} = -\nabla V$$

* electric field is negative gradient of scalar electric potential.

* Electric field project normal to an equipotential surface.

* $\bar{E}$ would be projecting from higher potential surface towards lower potential surface

* Potential varies its values Normal to equipotential surface

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* Potential due to dipole

$$V_p = \frac{Qd\cos\theta}{4\pi\epsilon r^2}$$

* poisson's equation :- $\nabla^2 V = \frac{\rho_v}{\epsilon}$

* Laplacian equation :- $\nabla^2 V = 0$ ($\rho_v = 0$, charge free region)

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

T.R.2

extra

in (b) TR1 $-\!\!-\!\!- + \frac{1}{\rho} + -\!\!-\!\!-$ in (c) TR1 $-\!\!-\!\!- + \frac{1}{r} + \frac{1}{r}$ & $D_1 = \frac{\partial D_1}{\partial 1}$, $D_2 = \frac{\partial D_2}{\partial 2}$, $D_3 = \frac{\partial D_3}{\partial 3}$ $= V$

* current & current density.

$I = \frac{dQ}{dt}$ $\quad \bar{J}_c = \rho_v \bar{v}$ $\quad \bar{J}_c = \sigma \bar{E}$

$\bar{J}_c$ – conduction current density (A/m²) | v – drift velocity. (m/s)

σ – conductivity. (℧/m) $\quad \rho_v$ - charge density.

$$I = \int_S \bar{J}_c \cdot d\bar{s} \quad \text{ampere.}$$

continuity of current equation.

$J_c = \sigma \bar{E}$

$\nabla \cdot J_c = \sigma \nabla \cdot \bar{E}$

$= \sigma \frac{\nabla \cdot \bar{D}}{\epsilon}$

$= \frac{\sigma}{\epsilon}\rho_v$

$$\oint_S \bar{J}_c \cdot d\bar{s} = -\frac{\partial Q}{\partial t} = -\frac{\partial}{\partial t}\int_v \rho_v \, dv$$

i.e. $\nabla \cdot \bar{J}_c = -\frac{\partial \rho_v}{\partial t} \Rightarrow \frac{\sigma}{\epsilon}\rho_v = -\frac{\partial \rho_v}{\partial t}$

$$\Rightarrow \frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\epsilon}\rho_v = 0$$

$$\rho_v = c_1 e^{-t/\tau} \qquad \tau = \frac{\epsilon}{\sigma} \Big\} \text{ Relaxation time.}$$

* electric field inside conductor is 0

* Boundry conditions

Case 1: conductor to dielectric Interface

$$\oint \bar{E}\cdot d\bar{l} = 0 \qquad \bar{E} = E_n \hat{a}_n + E_t \hat{a}_t$$

$$d\bar{l} = dl\, \hat{a}_t$$

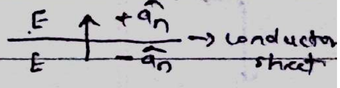
$$\therefore \oint \bar{E}\cdot d\bar{l} = \int E_t \, dl = 0 \quad \therefore \boxed{E_t = 0}$$

* the tangential component of E-field across a conductor to dielectric ~~[illegible]~~ interface vanishing. i.e. 0.
* if surface charge density of interface is ρ_s then Normal component of electric flux density is ρ_s.

$$D_n = \rho_s \Rightarrow \bar{D} = \rho_s \hat{a}_n \Rightarrow \bar{E} = \frac{\rho_s}{\epsilon}\hat{a}_n$$

$$\bar{E} = \frac{\rho_s}{\epsilon}\hat{a}_n \longrightarrow \text{for conductor}$$

$$\bar{E} = \frac{\rho_s}{2\epsilon}\hat{a}_n \text{ (one side)} \quad \text{or} \quad \bar{E} = \frac{\rho_s}{2\epsilon}(-\hat{a}_n) \text{ (other side)} \rightarrow \text{for conductor sheet.}$$



Case 2: Dielectric to Dielectric interface

$$\boxed{E_{t_1} = E_{t_2}}$$

* tangential components are continuous in Dielectric to Dielectric
* if ρ_s is there then $\boxed{D_{n_2} - D_{n_1} = \rho_s}$

D_{n_2} & D_{n_1} are normal component of electric flux density.

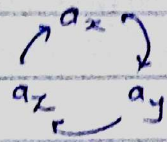
if $\rho_s = 0$ then $\boxed{D_{n_2} = D_{n_1}}$

Case 3: white sheet

$$\bar{E} = \frac{\rho_s}{2\epsilon}\hat{a}_n$$

* magnetic field

$$d\bar{H} = \frac{I\, d\bar{l} \times \hat{a}_r}{4\pi |R|^2} \qquad \bar{H} = \int_l \frac{I\, d\bar{l} \times \hat{a}_r}{4\pi |\bar{R}|^2}$$



↳ also called as Biot savart's law.

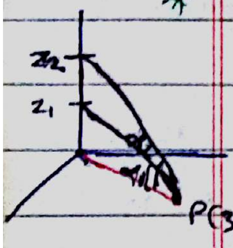
* $\bar{H}$ due to long infinite filamentary conductor

$$\bar{H} = \frac{I}{2\pi \rho}\, \hat{a}_\phi$$

Biot savart's law:- $\bar{H} = \frac{I}{2\pi |\bar{R}|}$ (unit vector of $d\bar{l} \times \hat{a}_R$)

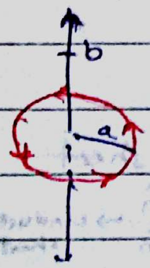
$$= \frac{I}{2\pi |\bar{R}|}\, \hat{a}_\phi$$

* $\bar{H}$ due to finite filament z_1 to z_2 carrying current I amp. at $p(\rho, \phi, 0)$ in terms of α_1 & α_2



$$\bar{H} = \frac{I}{4\pi \rho}\left[\sin\alpha_2 - \sin\alpha_1\right] \hat{a}_\phi \qquad \alpha_1 = \frac{z_1}{\sqrt{\rho^2 + z_1^2}} \qquad \alpha_2 = \frac{z_2}{\sqrt{\rho^2 + z_2^2}}$$

* $\bar{H}$ on the axis of circular current loop of radius a. at distace 'b' on axis.



$$\bar{H} = \frac{I a^2}{2(a^2 + b^2)^{3/2}}\, \hat{a}_z \ \ A/m$$

* Amperes law.

line integral of magnetic field intensity around a close path is equal to current enclosed by the path.

$$\oint_l \bar{H} \cdot d\bar{l} = I_{enclosed}$$

$$\oint_l \bar{H} \cdot d\bar{l} = \int_s \bar{J}_c \cdot d\bar{s}$$

using curl we get

$$\nabla \times \bar{H} = \bar{J}_c \longrightarrow \text{point form amperr law.}$$

$$\therefore \oint_l \bar{H} \cdot d\bar{l} = \int_s (\nabla \times \bar{H}) \cdot d\bar{s} \rightarrow \text{stokes theorem}$$

$\nabla \times \vec{H}$

Cartesian form	Polar form	Spherical form
$\begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix}$	$\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ H_\rho & \rho H_\phi & H_z \end{vmatrix}$	$\frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ H_r & rH_\theta & r\sin\theta\,H_\phi \end{vmatrix}$

* Magnetic flux density $(\bar{B})$ tesla or wb/m^2

magnetic flux $\rightarrow \Phi$ wb

$$B = \mu\bar{H} = \mu_0\mu_r\bar{H}$$

$$\Phi = \int_S \bar{B}\cdot d\bar{S}$$

gauss law for magnetic field.

$$\nabla\cdot\bar{B} = 0$$

* Boundary conditions

case 1 :- ~~dielectric~~ conductor to dielectric medium

$$H_n = 0$$

$$\hat{a}_n \times \bar{H} = \bar{J}_s$$

case 2 :- dielectric to dielectric medium

$$B_{n_1} = B_{n_2}$$

$$(H_1 - H_2)\times\hat{a}_n = \bar{J}_s$$

$$H_{t_1} = H_{t_2} \quad \text{if } \bar{J}_s = 0.$$

case 3 :- infinite sheet.

$$\bar{H} = \frac{1}{2}\bar{J}_s \times \hat{a}_n$$

* $\bar{E} = Re\{E_s e^{j\omega t}\}$

  └ phasor form [s suffix]

* $\bar{D} = \epsilon \bar{E}$, $\bar{B} = \mu \bar{H}$, $J_c = \sigma E$, $J_D = \frac{\partial \bar{D}}{\partial t} = \epsilon \frac{\partial \bar{E}}{\partial t}$

Maxwell's Equation's

1] Farady's Law

Integral form → $\oint \bar{E} \cdot d\bar{l} = -\frac{\partial}{\partial t} \int \bar{B} \cdot d\bar{s}$ $\because \frac{\partial \Phi}{\partial t} = \int \bar{B} \cdot d\bar{s}$

Point form → $\nabla \times \bar{E} = -\frac{\partial B}{\partial t} = -\mu \frac{\partial \bar{H}}{\partial t}$

phasor form → $\nabla \times \bar{E}_s = -j\omega \bar{B}_s = -j\omega \mu \bar{H}_s$

2] Modified Ampere's Law

Integral form → $\oint_l \bar{H} \cdot d\bar{l} = \int_s (\bar{J}_c + \bar{J}_D) \cdot d\bar{s}$

Point form → $\nabla \times \bar{H} = \bar{J}_c + \bar{J}_D = \sigma \bar{E} + \epsilon \frac{\partial \bar{E}}{\partial t}$

phasor form → $\nabla \times \bar{H}_s = \bar{J}_{cs} + \bar{J}_{Ds} = \sigma \bar{E}_s + j\omega \epsilon \bar{E}_s$

3] Gauss Law for electric field

Integral form → $\oint_s \bar{D} \cdot d\bar{s} = Q_{enclosed}$.

Point form → $\nabla \cdot \bar{D} = \rho_v$

phasor form → $\nabla \cdot \bar{D}_s = \rho_v$.

4) Gauss law for magnetic field.

Integral form → $\oint_S \bar{B} \cdot \bar{ds} = 0$

Point form → $\nabla \cdot \bar{B} = 0$

phasor form → $\nabla \cdot \bar{B}_s = 0$.

Modified amper's law

$$\nabla \times \bar{H} = \bar{J}_c + \bar{J}_D$$

* J_c lacks J_D by $90°$.

* magnitude of J_c & J_D are equal at $f = \frac{\sigma}{2\pi\epsilon}$

$$\frac{|J_{cs}|}{|J_{Ds}|} = 1 = \frac{\sigma E_s}{j\omega\epsilon E_s} \Rightarrow \frac{\sigma}{\omega\epsilon} = 1 \Rightarrow \omega\epsilon = \sigma$$

$$\therefore f = \frac{\sigma}{2\pi\epsilon} \text{ Hertz}$$

Medium

1) linear medium → $\bar{D}$ & $\bar{E}$ have samedirection $\bar{B}$ & $\bar{H}$ also.

2) Homogeneous medium → μ & ϵ are constant

3) Isotrophic medium → μ & ϵ are scalar constant
it is homogeneous but viceversa cant say

4) charge free medium → $\rho_v = 0$

5) Non-conductive medium → $\sigma = 0$

6) Unbounded medium → No boundries.

$$\frac{|J_{cs}|}{|J_{Ds}|} = \frac{|\sigma E_s|}{|j\omega \epsilon E_s|} = \frac{\sigma}{\omega \epsilon} = \tan\theta .$$

$\frac{\sigma}{\omega\epsilon} \gg 1 \Rightarrow$ good conductor $(\sigma \gg \omega\epsilon)$ $(\epsilon = \epsilon_0)$

$= \infty \Rightarrow$ Perfect conductor $(\epsilon \approx 0)$

$= 0 \Rightarrow$ Perfect dielectric $(\sigma = 0)$

$\ll 1 \Rightarrow$ good dielectric $(\sigma \ll \omega\epsilon)$ lossless

$\approx 1 \Rightarrow$ quasi-condonctors or semiconductors.

* EM wave motion through diff media.

Free space $(\sigma = 0, \varepsilon = \varepsilon_0, \mu = \mu_0)$ $\therefore \rho_v = 0$

lossless dielectrics $(\sigma = 0, \varepsilon = \varepsilon_r\varepsilon_0, \mu = \mu_r\mu_0)$

conducting medium.

lossy dielectrics $(\sigma \neq 0, \epsilon = \epsilon_r\epsilon_0, \mu = \mu_r\mu_0)$

Good conductors. $(\sigma \simeq \infty, \epsilon = \epsilon_0, \mu = \mu_r\mu_0)$

* EM wave through ~~space~~ free space $(\sigma = 0, \rho_v = 0)$

$\therefore$ Maxwells equations becomes

$$\nabla \times \bar{E} = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \times \bar{H} = \epsilon \frac{\partial \bar{E}}{\partial t} \quad (\because \sigma = 0)$$

$$\nabla \cdot \bar{D} = 0 \; ; \; \nabla \cdot \bar{E} = 0 \; (\because \rho_v = 0)$$

$$\nabla \cdot \bar{B} = 0 \; ; \; \nabla \cdot \bar{H} = 0 .$$

$$\left.\begin{aligned} \nabla^2 \bar{E} &= \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2} \quad (1) \\ \nabla^2 \bar{H} &= \mu\epsilon \frac{\partial^2 H}{\partial t^2} \end{aligned}\right\} \text{vector wave equat}^{n}.$$

$\nabla^2 \bar{E} = -\omega^2 \mu\epsilon E$ Helmholtz equation

(1) $$\frac{\partial^2 \bar{E}}{\partial x^2} + \frac{\partial^2 \bar{E}}{\partial y^2} + \frac{\partial^2 \bar{E}}{\partial z^2} = \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2} \rightarrow \text{Cartesian form}$$

2nd order PDE 4 dimention

lly for H.

if we assume wave is propagating along z direction since there is no boundries on x & y direction.

$\therefore \frac{\partial}{\partial y}() = 0$ & $\frac{\partial}{\partial x}() = 0$.

$\therefore$ equation ① becomes $\frac{\partial^2 E}{\partial z^2} = \mu\epsilon \frac{\partial^2 E}{\partial t^2}$ } 2nd order 2 dimen PDE

but in free space $\nabla \cdot \bar{E} = 0$.

$$\therefore \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \Rightarrow \frac{\partial E_z}{\partial z} = 0 \quad \therefore \underline{\underline{E_z = 0}}$$

$\therefore$ If we assume wave propagating z direction $\therefore E_z = H_z = 0$ lly for others & remaining E_x, E_y, H_x, H_y lies on transverse plane.

$\therefore$ in above case $\bar{E} = E_x \hat{a}_x + E_y \hat{a}_y$ only No E_z

$\therefore$ we get $\frac{\partial^2 E}{\partial z^2} = \mu\epsilon \frac{\partial \bar{E}}{\partial t^2} \Rightarrow \frac{\partial^2 \bar{E}_x}{\partial z^2} = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2}$ lly for y

solving by variable separable method

$$\frac{d^2 E_{xs}}{dz^2} + \omega^2 \mu\epsilon E_{xs} = 0$$

$\cdots(\omega t - \beta z)\cdots$ (β → +ve z direct^n)

$$\beta^2 = \omega^2 \mu\epsilon \Rightarrow \beta = \omega\sqrt{\mu\epsilon}$$

f = frequency

λ = wavelength.

$V_0 = \frac{1}{\sqrt{\mu\epsilon}}$ for free space μ_0 & ϵ_0

$V_0 = C = 3\times 10^8$ m/s.

$$V_0 = \frac{1}{\sqrt{\mu\epsilon}} = f\lambda \qquad \beta = \frac{\omega}{V_0} = \frac{2\pi f}{f\lambda} = \frac{2\pi}{\lambda}$$

η - intrinsic impedance

$\eta = \sqrt{\frac{\mu}{\epsilon}}$ for free space $\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi\ \Omega$.

if wave is propagating along

+Z axis

$\frac{\partial}{\partial x}() = 0; \frac{\partial}{\partial y} = 0$

→ +Z

$$\frac{E_x}{H_y} = \eta = -\frac{E_y}{H_x}$$

-ve Z axis

$\frac{\partial}{\partial x}() = 0, \frac{\partial}{\partial y} = 0$

→ -Z

$$\frac{E_x}{H_y} = -\eta = -\frac{E_y}{H_x}$$

lly for ~~...~~ x & y also

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* EM wave through lossy medium. ($\sigma \neq 0$)

charge free ($\rho_v = 0$)

maxwells eq^{n}s.

$$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}$$

$$\nabla \times \bar{H} = \sigma \bar{E} + \epsilon \frac{\partial \bar{E}}{\partial t}$$

$$\nabla \cdot \bar{D} = 0 \; ; \; \nabla \cdot \bar{E} = 0$$

$$\nabla \cdot \bar{B} = 0 \; ; \; \nabla \cdot \bar{H} = 0$$

$$\nabla^2 E = \mu \sigma \frac{\partial E}{\partial t} - \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$ = vector wave eq^n.

is phasor form

$$\nabla^2 E_s - j\omega\mu\sigma E_s - (j\omega)^2 \mu \epsilon E_s \Rightarrow \nabla^2 E_s - \gamma^2 E_s = 0$$

propagation constant $\rightarrow \gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$

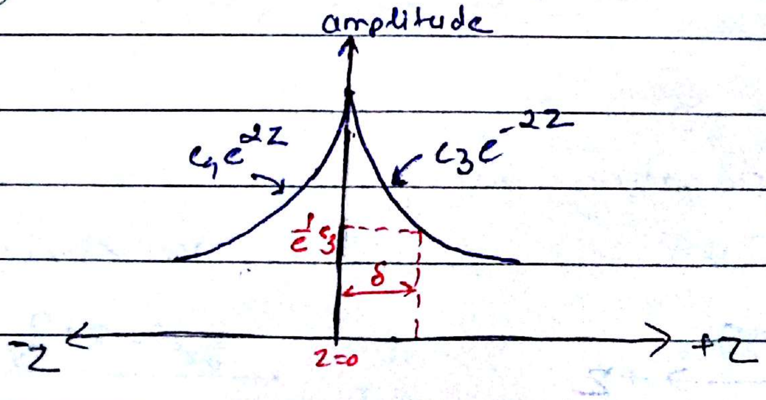
$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta.$$

attenuation constant $\rightarrow \alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$ Np/m

phase shift constant $\rightarrow \beta = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}$ rad/m

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = |\eta| \angle \theta_\eta$$ θ_η - phase angle

$\boxed{\tan\phi = \frac{\sigma}{\omega\epsilon} \quad \phi = 2\theta_n}$



δ = skin depth

It is z when

Amp is $\frac{1}{e} c_3$ or $0.36\, c_3$

$$\delta = \frac{1}{\alpha} = \alpha^{-1} \text{ m/Np.}$$

* good conducting medium. $\sigma >> \omega\epsilon$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

$\therefore$ In good conductor $\bar{E}$ leads by $\bar{H}$ by 45°

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$ valid for good conductor only.

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

* $\frac{\sigma}{\omega\epsilon}$ is called dessipation factor 'DF'/loss tangent

$$'D_F' = \frac{\sigma}{\omega\epsilon} \Rightarrow \tan\phi = \frac{\sigma}{\omega\epsilon} \Rightarrow \phi = \tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)$$

$$\phi = 2\theta_n \quad (\theta_n \rightarrow \text{phase angle of } \eta.)$$

↳ for good conductor.

* Polarisation.

- linear (E_x or E_y or both, $\phi_p = \pm n\pi$)
  - VP
  - HP
- circular ($E_{x_0} = E_{y_0}$, $\phi_p = n\pi/2$)
  - LHCP
  - RHCP
- Elliptical ($E_{x_0} \neq E_{y_0}$, $\phi_p \neq 0, 180^\circ$)
  - LHEP
  - RHEP

linear $\rightarrow$ at least one transverse field

If having both component phase difference

$$\phi_p = \pm n\pi \quad n = 0, 1, 2 \ldots$$

circular $\rightarrow$ two transverse component with equal magnitude

$$\phi_p = n\pi/2 \quad n = 1, 2, 3, \ldots$$

Elliptical $\rightarrow$ two transverse component must have unequal ~~ampl~~ magnitude

$$\phi_p \neq 0^\circ, 180^\circ, \ldots$$

* Reflection

* snell's law $\theta_i = \theta_r$

* Normal Incident ($\hat{a}_n$ should be $\perp$ to plane)

Incident	Reflected	Transmitted
$\bar{E}_{is} = E_{io} e^{-\gamma_1 z} \hat{a}_x$	$\bar{E}_{rs} = E_{ro} e^{+\gamma_1 z} \hat{a}_x$	$\bar{E}_{ts} = E_{to} e^{-\gamma_2 z} \hat{a}_x$
$\bar{H}_{is} = \frac{E_{io}}{\eta} e^{-\gamma_1 z} \hat{a}_y$	$\bar{H}_{rs} = \frac{E_{ro}}{\eta} e^{-\gamma_1 z} \hat{a}_y$	$\bar{H}_{ts} = \frac{E_{to}}{\eta} e^{-\gamma_2 z} \hat{a}_y$

$$E_{io} + E_{ro} = E_{to}$$

Reflection coeff. $\gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

Transmitted coeff. $T = \frac{2\eta_2}{\eta_2 + \eta_1}$

no impedance variation

→ if $\eta_1 \hat{=} \eta_2$ then $\gamma = 0$ & $T = 1$

→ if medium 2 is perfect conductor then

$\eta_2 = 0$ $T = 1$ & $\gamma = -1$

$|\gamma|^2$ → Power reflection coefficient

$|\gamma|^2 \times 100\%$ → % power reflected

$(1 - |\gamma|^2) 100\%$ → % power transmitted.

* Poynting Vector (Poyting thm)

to calculate power & power densities

$$\bar{P} = \bar{E} \times \bar{H}$$

average power $\bar{P}_{avg} = \frac{1}{2} \bar{E}_s \times \bar{H}_s^* \; w/m^2$ complex conjugate

average power across surface $W_{avg} = \int_s \bar{P}_{avg} \cdot d\bar{s}$ watts.

$$P_{avg} = \frac{1}{2} \frac{|E|^2}{\eta}$$

$$* \quad \frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}}$$

* oblique incident:

parallel polarisation. | $\perp$ lar polarisation

$$\Gamma_{\parallel} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$\Gamma_{\perp} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$T_{\parallel} = 1 + \Gamma_{\parallel}$$

$$T_{\perp} = 1 - \Gamma_{\perp}$$

Brestens angle

$$\sin\theta_{B_{\parallel}} = \sqrt{\frac{1 - \frac{\mu_2\epsilon_1}{\mu_1\epsilon_2}}{1-(\epsilon_1/\epsilon_2)^2}}$$

for a non magnetic media

$\mu_1 = \mu_2 = 0$

$$\therefore \tan\theta_{B_{\parallel}} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\therefore \theta_{B_{\parallel}} = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

Breston's angle

$$\sin\theta_{B_{\perp}} = \sqrt{\frac{1 - \frac{\mu_1\epsilon_2}{\mu_2\epsilon_1}}{1-(\mu_1/\mu_2)^2}}$$

for a non magnetic media

$\mu_1 = \mu_2 = 0.$

$\sin\theta_{B_{\perp}} = \infty$ not possible

$\therefore$ Breston's angle Not exist

in general EM wave can represent

$$\bar{E} = Re[\bar{E}_{s0}\, e^{j(\omega t - \bar{K}\cdot\bar{r})}]$$

$$|\bar{K}|^2 = \beta^2 = \left(\frac{2\pi}{\lambda}\right)^2$$

$$\bar{K} = K_x\hat{a}_x + K_y\hat{a}_y + K_z\hat{a}_z$$

$$\bar{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$$

$$* \quad VSWR(S) = \frac{E_{max}}{E_{min}} \qquad \Gamma = \frac{S-1}{S+1} \qquad S = \frac{1+|\Gamma|}{1-|\Gamma|}$$

* Refractive index $= \sqrt{\epsilon_r}$

* waveguides

→ conductor boundries, linear-homogeneous - isotrophic - charge free - non conducting field medium inside.

→ Rectangular, circular, elliptical → Rectangular prefered

→ tangential component of electric field & normal component of magnetic field is 0 at boundry.

$\therefore$ at $x = 0, x = a \longrightarrow E_y = E_z = 0 \; ; H_x = 0$

$\therefore$ at $y = 0, y = b \longrightarrow E_x = E_z = 0 \; ; H_y = 0$

→ Maxwell equations

$$\nabla \times \bar{E} = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \times \bar{H} = \epsilon \frac{\partial E}{\partial t} \qquad \because \sigma = 0 \quad \text{Non conducting medium}$$

$$\nabla \cdot \bar{D} = 0 \quad \nabla \cdot \bar{E} = 0 \qquad \rho_v = 0, \text{ charge free medium.}$$

$$\nabla \cdot \bar{B} = 0 \quad \nabla \cdot \bar{H} = 0.$$

$$\nabla^2 E = \mu\epsilon \frac{\partial^2 E}{\partial t^2} \qquad \text{vector wave eq}^n.$$

$$\Rightarrow \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = \mu\epsilon \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial ()}{\partial x} \neq 0 \;\&\; \frac{\partial}{\partial y}() \neq 0 \qquad \because \text{it is bounded medium.}$$

$$\therefore \frac{\partial^2 E_s}{\partial x^2} + \frac{\partial^2 E_s}{\partial y^2} + \frac{\partial^2 E_s}{\partial z^2} = -\omega^2 \mu\epsilon \bar{E}_s$$

we assume wave is propagating along z-direction

$$\therefore E_s = E_{s_0}(x,y,z)\, e^{-\bar{\gamma} z} \Rightarrow \frac{\partial^2 E_s}{\partial z^2} = +\bar{\gamma}^2 E_s$$

$$\therefore \frac{\partial^2 E_s}{\partial x^2} + \frac{\partial^2 E_s}{\partial y^2} + \bar{\gamma}^2 E_s + \omega^2 \mu\epsilon \bar{E}_s = 0$$

$$\Rightarrow \frac{\partial^2 E_s}{\partial x^2} + \frac{\partial E_s}{\partial y^2} + h^2 E_s = 0.$$

TEM wave not possible in waveguide.

* Waves in waveguide.

TM$_{mn}$ waves	TE$_{mn}$ Waves.
→ transverse magnetic	→ Transverse electric
→ Magnetic field lies in transverse plane $H_z = 0$	→ Electric field lies on transverse plane $E_z = 0$
→ also called E waves	→ also called H-waves.

wave eq^n → $f_1(x)$	$f_2(y)$	$f_3(z)$	$f_4(t)$
$\cos\left(\frac{m\pi}{a}x\right)$ or $\sin\left(\frac{m\pi}{a}x\right)$	$\cos\left(\frac{n\pi}{b}y\right)$ or $\sin\left(\frac{n\pi}{b}y\right)$	$e^{-\gamma z}$	$e^{j\omega t}$

TM$_{mn}$ Waves →

$$\xrightarrow{\qquad\qquad} +z$$

$$\frac{E_x}{H_y} = \eta_{TM_{mn}} = -\frac{E_y}{H_x}$$

$$E_{xs} = E_{x_0}\cos\left(\frac{m\pi}{a}\right)x\ \sin\left(\frac{n\pi}{b}\right)y\ e^{-\gamma z} \qquad H_{xs} = -\frac{E_{ys}}{\eta_{TM_{mn}}}$$

$$E_{ys} = E_{y_0}\sin \text{ —''— } \cos \text{ —''— } \qquad H_{ys} = \frac{E_{xs}}{\eta_{TM_{mn}}}$$

$$E_{zs} = E_{z_0}\sin \text{ —''— } \sin \text{ —''— } \qquad \underline{\underline{H_{zs} = 0}}$$

TE$_{mn}$ wave →

$$\xrightarrow{\qquad\qquad} +z$$

$$\frac{E_x}{H_y} = \eta_{TE_{mn}} = -\frac{E_y}{H_x}.$$

$$E_{xs} = E_{x_0}\cos\left(\frac{m\pi}{a}\right)x\ \sin\left(\frac{n\pi}{b}\right)y\ e^{-\gamma z} \qquad H_{xs} = \frac{-E_{ys}}{\eta_{TE_{mn}}},$$

$$E_{ys} = E_{y_0}\sin \qquad \cos \qquad H_{ys} = \frac{E_{xs}}{\eta_{TE_{mn}}}.$$

$$\underline{\underline{E_{zs} = 0}} \qquad H_{zs} = H_{z_0}\cos\left(\frac{m\pi}{a}\right)x\ \cos\left(\frac{n\pi}{b}\right)y\ e^{-\gamma z}$$

$$\gamma = \sqrt{\underbrace{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}_{P} - \underbrace{\omega^2\mu\epsilon}_{Q}} = \alpha + i\beta$$

$P > Q$ → γ real, $\beta = 0$ ⇒ wave propogation not possible

$P < Q$ → γ imag, $\alpha = 0, \beta \neq 0$ ⇒ wave propogat^n possible.

$P = Q$ → at cut off frequency.

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2} \qquad \underline{\omega = 2\pi f}$$

$$\therefore \boxed{\gamma = i\beta}$$

$$f_c\lambda_c = \frac{1}{\sqrt{\mu\epsilon}} = v_0 \quad \Rightarrow \quad \lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

$\therefore f > f_c$ or $\lambda < \lambda_c \Rightarrow$ Propagation is allowed

$\therefore f < f_c$ or $\lambda > \lambda_c \rightarrow$ Propagation is not allowed.

phase shift constant $\rightarrow$ $$\beta = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

phase velocity $\rightarrow$ $$v = \frac{\omega}{\beta}$$

free space wavelength $$\bar{\lambda} = \frac{2\pi}{\beta}$$

λ – guide wavelength inside the waveguide.

$$\frac{1}{\lambda^2} = \frac{1}{\bar{\lambda}^2} + \frac{1}{\lambda_c^2}$$

* Intrinsic impedance in waveguide

$$\eta_{TE} = \frac{\eta}{\sqrt{1-(f_c/f)^2}} = \frac{\eta}{\sqrt{1-(\lambda/\lambda_c)^2}}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}}\ \Omega$$

$$\eta_{TM} = \eta\sqrt{1-(f_c/f)^2} = \eta\sqrt{1-(\lambda/\lambda_c)^2}$$

$$\eta_{TE} \cdot \eta_{TM} = \eta^2$$

at $f = f_c$

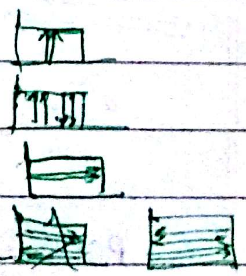
$\beta \rightarrow 0$, $v \rightarrow \infty$, $\bar{\lambda} \rightarrow \infty$, $\eta_{TE} \Rightarrow \infty$, $\eta_{TM} \Rightarrow 0$

$TE_{10} \rightarrow$ no. of field along x 1 $\rightarrow$

$TE_{20} \rightarrow$ no. of fields along x 2 $\rightarrow$

$TE_{01} \rightarrow$ no. of field along y 1 $\rightarrow$

$TE_{02} \rightarrow$ $\rightarrow$



$\bar{v}_p \cdot v_g = v_c^2$ (group velocity; phase velocity)

Dominant Mode :-

lowest possible propagating mode having lowest cutoff frequency & highest cut off wavelength.

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short form

Transmission Lines. (TX^n l)

* It is symmetrical Network. i.e. i/p → o/p o/p → i/p

characteristic imped.

* when a symmetrical N/W. is terminated by Z_0 then the impedance seen at the input of the N/W is also Z_0 ($\because$ through out N/W $Z_0 = Z_0$. same)

* Two wire TX lines have

due to voltage drop across line

due to current in line

series impedance $Z = (R + j\omega L)\ \Omega m$.

shunt admittance $Y = (G + j\omega C)\ \mho m$.

due to charges build on the line

due to leakage conductance

Propagation constant

* $$\frac{d^2V}{dx^2} - \gamma^2 V = 0$$
$$\frac{d^2I}{dx^2} - \gamma^2 I = 0$$
Homogeneous eqn.

$-\frac{dV}{dx} = IZ$

$-\frac{dI}{dx} = VY$

attenuation constant

Phaseshift constant

$$\gamma = \sqrt{ZY} = \sqrt{(R+j\omega L)(G+j\omega C)} = \alpha + j\beta$$

$$V = C_1 e^{-\gamma x} + C_2 e^{+\gamma x} \qquad I = C_3 e^{-\gamma x} + C_4 e^{\gamma x} \qquad Z_0 = \sqrt{\frac{Z}{Y}}$$

$$V = V_s \cosh \gamma x - I_s Z_0 \sinh \gamma x$$
$$I = I_s \cosh \gamma x - \frac{V_s}{Z_0} \sinh \gamma x.$$
general eq^{n}s.

for $Z_r = Z_0$ (No-reflection) ------ only $-\gamma$

$$V = V_s \cosh \gamma x - V_s \sinh \gamma x \Rightarrow V = V_s e^{-\gamma x}$$
$$I = I_s \cosh \gamma x - I_s \sinh \gamma x \Rightarrow I = I_s e^{-\gamma x}$$

* Distortion

① frequency → R, L, C, G are constant in γ but $\omega \Rightarrow 2\pi f$

$Re(\gamma) \Rightarrow \alpha$ is function of f $\therefore$ frequency destortion.

② Delay → $v_p = \frac{\omega}{\beta}$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$
$$= \sqrt{L\left(\frac{R}{L} + j\omega\right) C\left(\frac{G}{C} + j\omega\right)}$$

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Distortion less condition $\frac{R}{L} = \frac{G}{C}$

$\gamma = \alpha + j\beta$ $\therefore$ $\gamma = \sqrt{LC}\left(\frac{R}{L} + j\omega\right)$ or $\sqrt{LC}\left(\frac{G}{C} + j\omega\right)$

Now.

$\alpha = R\sqrt{\frac{C}{L}}$ or $\alpha = G\sqrt{\frac{L}{C}}$ → not a f^n of frequency

i.e. no distortion

$\beta = \omega\sqrt{LC}$

$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$; Delay $= \frac{1}{v_p} = \sqrt{LC}$

$Z_0 = \sqrt{\frac{L}{C}}$

$\therefore$ $R = \alpha Z_0$ $\quad C = \frac{1}{Z_0 v_p}$

$L = \frac{Z_0}{v_p}$ $\quad G = \frac{\alpha}{Z_0}$

* TX l. & ABCD parameters

from general eqns. }

considering
$x = y = l \Rightarrow$ i/p end
i.e. $V = V_S$ & $I = I_S$
$V_S = V_R$ & $I_S = I_R$

$$\begin{array}{c} V_1 \\ I_1 \end{array}\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{array}{cc} A & B \\ \left[\begin{matrix} \cosh\gamma l & Z_0 \sinh\gamma l \\ \frac{1}{Z_0}\sinh\gamma l & \cosh\gamma l \end{matrix}\right] & \\ C & D \end{array} \begin{array}{c} V_2 \\ \begin{bmatrix} V_R \\ I_R \end{bmatrix} \\ I_2 \end{array}$$

$A = D$; $AD - BC$

* For loss less TX line $\alpha = 0$.

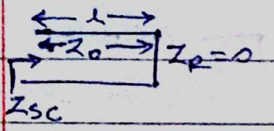
l - TX$^{\text{ion}}$ length
βl — Electrical length

$\therefore$ $\gamma = j\beta$. $\Rightarrow \cosh\gamma l = \cos\beta l$
$\sinh\gamma l = j\sin\beta l$.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta l & jZ_0\sin\beta l \\ \frac{j}{Z_0}\sin\beta l & \cos\beta l \end{bmatrix}$$

* SC lines & OC lines

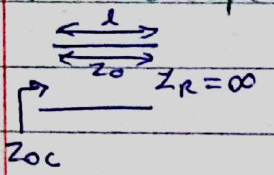
for SC (short circuit) $Z_R = 0$



$Z_{sc} = Z_0 \tanh \gamma l$ (for lossy)

$Z_{sc} = jZ_0 \tan \beta l$ (for lossless)

for OC (open circuit) $Z_R = \infty$



$Z_{oc} = Z_0 \coth \gamma l$ (for lossy)

$Z_{oc} = jZ_0 \cot \beta l$ (for lossless)

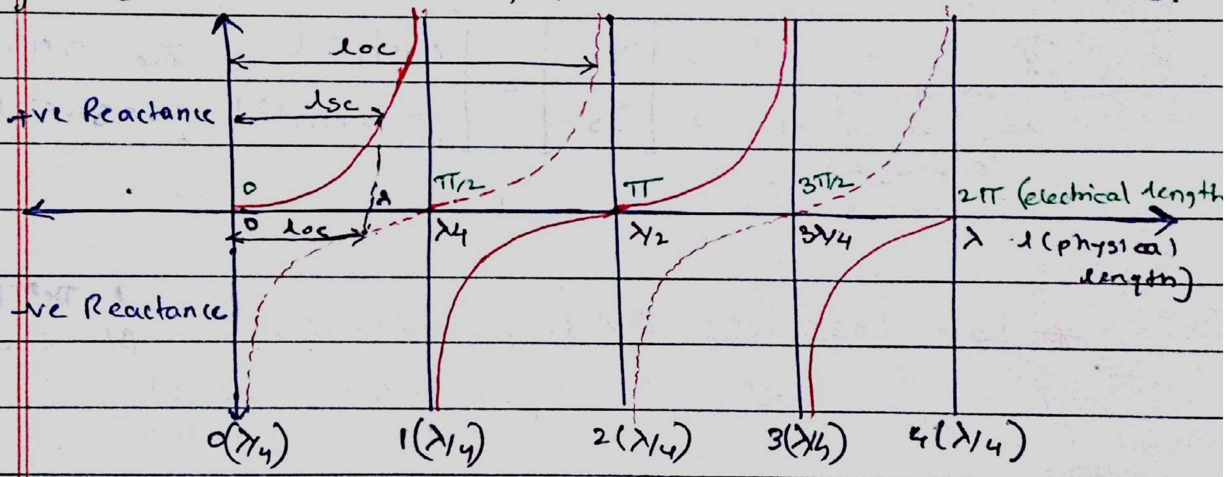
$$Z_{oc} \times Z_{sc} = Z_0^2$$

$Z_{sc}, Z_{oc}, Y_{sc}, Y_{oc} \rightarrow$ stubs. $\left(Z = \frac{1}{Y}\right)$

	Impedance	Admittance
inductor	$\rightarrow j\omega L = jX_L$	$\frac{1}{j\omega L} = -jB_L$
capacitor	$\rightarrow \frac{1}{j\omega C} = -jX_C$	$j\omega C = jB_C$

Reactances — Susceptances.

lossy lines considered then graph of $\tan \beta l$ & $\cot \beta l$ for Z_{sc} & Z_{oc}.



——— variation in Z_{sc} ($\tan \beta l$ graph)

- - - - - Variation in Z_{oc} ($\cot \beta l$ graph)

for Z_{sc} :→ l is $n(\lambda/4)$ $n = 0, 2, 4, 6, 8 \ldots$ (even)

then impedance or Reactance $= 0$ i.e. series resonance.

l is $n(\lambda/4)$ $n = 1, 3, 5, \ldots$ (odd)

then impedance or reactance $= \infty$ i.e. ||ld resonant.

for $Z_{oc} \rightarrow$ verse versa of above.

$$Z_{in} = Z_0 \frac{Z_R + jZ_0 \tan \beta l}{Z_0 + jZ_R \tan \beta l} \qquad \beta l = \frac{2\pi}{\lambda} \cdot l.$$

we draw next 3 conclusions →

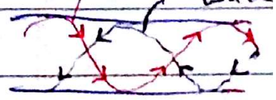
① $\frac{n\lambda}{2}$ line $(n=1,2,3\ldots)$ $\qquad Z_{in} = Z_R$

* High to low or low to high transformation.

② $n\frac{\lambda}{4}$ line $(n=1,3,5\ldots)$ $\qquad Z_{in} = \frac{Z_o^2}{Z_R}$ $\quad Z_R = 0 \Rightarrow Z_{in} = \infty$, $\quad Z_R = \infty \Rightarrow Z_{in} = 0$

③ $\frac{\lambda}{8}$ line $\qquad |Z_{in}| = |Z_o|$ $\quad$ * magnitude only.

Left to Right incident wave. R to L Reflected wave



*

if $Z_R \neq Z_o$ then impedance is discontinuous

$\therefore$ Part of the energy deliverd part of the energy Reflected.

we know $\quad V = \underbrace{C_1 e^{-\gamma x}}_{\text{incident}} + \underbrace{C_2 e^{+\gamma x}}_{\text{Reflected}}$ $\Big\}$ standing wave = incident + Reflected.

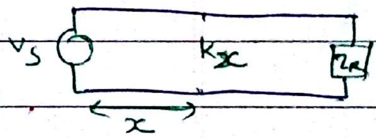
We can also prove by taking time factor. (see notes)

at $Z_R = Z_o$ only incident term exist No reflection $V = C_1 e^{-\gamma x}$

* Voltage Reflection Coefficient. $\qquad K = \frac{V_R}{V_i}$

$$K_x = \frac{C_2 e^{+\gamma x}}{C_1 e^{-\gamma x}} \Rightarrow \text{for lossy}$$

$$K_x = \frac{C_2}{C_1} e^{2j\beta x} \quad \text{for Lossyless.}$$



* Voltage standing waves.

* $Z_R \neq Z_o$ fluctuation of voltage betn V_{max} & V_{min} throughout length of xn line

* $Z_R = Z_o$ $V_{max} = V_{min}$ throughout same voltage.

* $Z_R = \infty$ i.e. (O.C)

* $Z_R = 0$ i.e. (S.C).

$Z_R \neq Z_o$ $\quad V = C_1 e^{-j\beta x} + C_2 e^{j\beta x} \Rightarrow C_1 \angle -\beta x + C_2 \angle \beta x.$

$V_{max} = |V_i| + |V_r|$ $\qquad V_{min} = |V_i| - |V_r|$

$$S = \frac{V_{max}}{V_{min}} = \frac{|V_i| + |V_R|}{|V_i| - |V_R|} = \frac{1+|K|}{1-|K|} \quad \because \frac{V_R}{V_i} = K$$

$$K = \frac{Z_R - Z_o}{Z_R + Z_o} \qquad K = |K| \angle \phi \qquad |K| = \frac{S-1}{S+1}$$

when $Z_R = Z_o$ $\quad K = 0$ $\quad |K| = 0$ $\quad S = 1$ $\quad |K_{max}| = 1$ $\quad |K_{min}| = 0$

$Z_R = 0$ $\quad K = -1 = 1\angle 180°$ $\quad |K| = 1$ $\quad S = \infty$ $\quad |S_{max}| = \infty$ $\quad |S_{min}| = 1$

$Z_R = \infty$ $\quad K = 1 = 1\angle 0°$ $\quad |K| = 1$ $\quad S = \infty$

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* $|K|^2$ – Power Reflection coeff.

% power Reflected $= 100\,|K|^2$ %

% Power transmitted $= 100\,(1-|K|^2)$ %

Return loss $= 20 \log_{10} |K|$

$Z_{max} = S Z_0$ $\quad Z_{min} = \frac{Z_0}{S}$

* Location of V_{max} & V_{min}.

$2\beta y_{max} - \phi = 2n\pi$

$2\beta y_{min} - \phi = (2n+1)\pi$

$n = 0, 1, 2, 3 \ldots$

$n=0$ – y_{max1} or y_{min1}

$n=1$ – y_{max2} or y_{min2}.

* Antenna (Radiator)

① Isotrophic Antenna

Radiating in all directions, impractical, used for reference equivalent to point source radiates all directions.

② Directional Antenna

Radiating or Recieving EM waves through some perticular directⁿ.
All practical antennas

③ Omni Directional Antenna

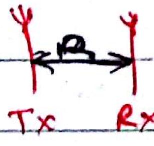
uniform radiation in azhimuthal plan $\theta = 90^\circ$
non-uniform radiation in elevation plane ϕ = constant.

* Radiation properties

① Radiation patterns ② Average Radiation densities
③ Average Radiation powers ④ Average radiation Intensity
⑤ Directive gain ⑥ Directivity ⑦ power gain of antenna
⑧ efficiency ⑨ Affective oparture area ⑩ Antena polarisatⁿ

① Radiation Patterns.

Locus of recieved field strength or power at fixed distance



$R > \frac{2D^2}{\lambda}$ fraunhoofer far field zone

$R < \frac{2D^2}{\lambda}$ —— " —— near field zone

$R \rightarrow$ distance betⁿ Tx & Rx
$D \rightarrow$ Max dimentⁿ of antenna
$\lambda \rightarrow$ operating wavelength

→ Radiation pattern in Isotrophic Antenna

Radiate in all direction ∴ Radiation pattern is sphere

① In Azhimuthal plane ($\theta = 90^\circ$) radiation is Uniform
② In elevation plane ($\phi = 0^\circ$ & $\phi = 180^\circ$) radiation is uniform

② Average radiation Density

average power radiated per unit area given by poynting vector.

$$\therefore\ P_{rad} = \frac{1}{2} E_s \times H_s^* \ \ W/m^2$$

↳ Radiated

③ Average Radiated power.

$$\therefore\ W_{rad} = \oint_S \overline{P_{rad}} \cdot \overline{ds} \qquad \overline{ds} \rightarrow r^2 \sin\theta\, d\theta\, d\phi\, \hat{a}_r$$

If $\overline{P_{rad}}$ is magnitude is independent on ϕ in Azhimuttal plane then it Uniformly radiate in Azhimuttal plane & if independent on θ in elevation plane the Uniformly radiate in evelvation plane.

Steradian → solid angle substended by sphere.
for all spheare its 4π like in circle rad $= 2\pi$

④ Average Radiation Intensity.

Average power radiated per unit solid angle

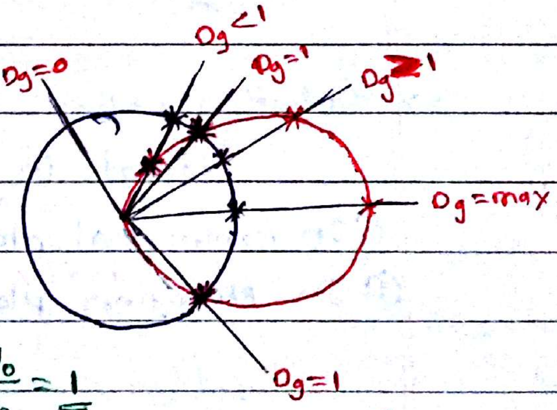
$$\overline{U} = r^2\, \overline{P_{rad}} \quad W/st.$$

$W_{rad}(Pract) = W_{rad}(Iso)$

W_{rad} must equal

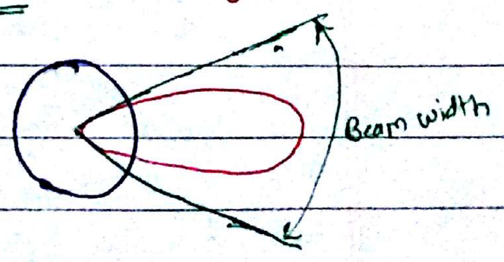
⑤ Directive Gain 'Dg'

$$D_g = \frac{U \rightarrow \text{Practical}}{U_0 \rightarrow \text{isotrophic}}$$



⑥ D_g max is called Directivity.

Directivity of isotrophic Antenna $= \frac{U_0}{U_0} = 1$



Communication
- Broadcasting → Beam width should high.
- Point to point → beam width should less

beam width ↑ Directive gain ↓

$D_g = \frac{U}{U_0}$ $\bar{U}_{iso} = U_0 \hat{a}_r$ $\bar{P}_{rad(iso)} = \frac{U_0 \hat{a}_r}{r^2}$

$W_{rad}(iso) = U_0 4\pi$

$W_{rad}(iso) = W_{rad}(\text{practical}) = W_{rad}$

$$\therefore \boxed{U_0 = \frac{W_{rad}}{4\pi}}$$

$$D_g = \frac{U}{U_0} = 4\pi \frac{U}{W_{rad}}$$ $P_{rad} = \frac{U}{r^2}$

$$D_0 = 4\pi \frac{U_{max}}{W_{rad}}$$ $P_{rad} = \frac{(E_{rms})^2}{\eta}$

(7) Power Gain (G_p)

$$G_p = 4\pi \frac{U}{W_{in}}$$ $W_{in} \rightarrow W_{rad} + W_{loss}$

max power gain $G_0 = 4\pi \frac{U_{max}}{W_{in}}$ $W = R I_{eff}^2$.

$W_{rad} = R_{rad} I_{eff}^2$.

for dipole $\rightarrow R_{rad} = 80\pi^2 \left(\frac{l}{\lambda}\right)^2 \Omega$

(8) Total effeciency (~~eff~~) (e_t)

$$e_t = \frac{W_{rad}}{W_{in}} = \frac{G_0}{D_0}$$ if $G_0 = D_0$ antenna is 100% efficient

$$e_t = \frac{R_{rad}}{R_{rad} + R_{loss}} \times 100\%$$ → for percentage

(9) effective aparture Area (A_e) m^2

it is physical size of antenna. large aparture area → large Antenna

Ratio of average power radiated to average power density.

$$A_e = \frac{\lambda^2}{4\pi} D_g$$

↑ D_0	↓ Beam width (smaller)	↑ A_e
↓ D_0	↑ Beam width (wider)	↓ A_e

$$A_{e\,max} = \frac{\lambda^2}{4\pi} D_0$$

By defination → $A_e = \frac{W_{rad}}{P_{rad}}$ (if antenna 100% efficient then

* $A_e = \frac{W_{recieved}}{P_{radiated}}$

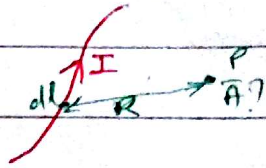
(10) Antenna Polarisation

Antenna polarisation is same as that of the wave polarisation.

eg. → for circular polarisation Antenna must radiate two transverse field with equal amplitude & the phase diff. $\pm n\frac{\pi}{2}$

* Magnetic Vector Potential ($\bar{A}$)

$$\bar{B} = \nabla \times \bar{A}$$



$$d\bar{A} = \frac{I\,d\bar{l}}{4\pi|\bar{R}|}$$

→ Horizian Dipole : $(l << \lambda)$ uniform current distribution

$$H_{\phi s} = j\,\frac{I_0 \beta l}{4\pi r}\,\sin\theta\; e^{-j\beta r}$$

$$W_{rad} = \underbrace{80\pi^2 \left(\frac{l}{\lambda}\right)^2}_{R_{rad}\,\Omega} I_{eff}^2 \text{ watts.}$$

$$D_g = \underbrace{1.5}_{D_0}\sin^2\theta$$

Half wave Dipole : $(l = \lambda/2)$ sinusoidal current Distributn

$$H_{\phi s} = j\,\frac{I_0}{2\pi r}\left(\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right) e^{-j\beta r}$$

Rrad Ω → Radiation Resistance
Self impedance.

$$W_{rad} = \underbrace{73}_{R_{rad}\,\Omega}\, I_{eff}^2$$

$$D_g = \underbrace{1.64}_{D_0}\,\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

Quarter wave monopole : $(l = \lambda/4)$ sinusoidal distr.

$$H_{\phi s} = j\,\frac{I_0}{2\pi r}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\, e^{j\beta r}$$

Rrad Ω → radiation Resistance.

$$W_{rad} = \underbrace{36.5}_{R_{rad}\,\Omega}\, I_{eff}^2$$

$$D_g = \underbrace{3.28}_{D_0}\,\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

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μ – Permiability.

is the measure of the ability of a material to support the formation of a magnetic field

It is the degree of magnetization that a material obtains in response to an applied magnetic field

unit – Henry/meter or Newton/Amp^2
H/m N/A^2

mag. flux density, magnetic field

$$B = \mu H$$

B – unit tesla or weber/meter^2

μ – H/m or N/A^2

H – units Ampere/meter

$$\text{tesla} = \frac{\text{Newton} \cdot \text{second}}{\text{coulomb} \cdot \text{metre}}$$

$$= \frac{\text{Newton} \cdot \cancel{\text{second}}}{\text{Amp} \cdot \cancel{\text{second}} \cdot \text{meter}} = \frac{\text{Newton}}{\text{amp} \cdot \text{meter}}$$

$$\text{Henry} = \frac{\text{tesla} \cdot \text{meter}}{\text{Amp}}$$

$$\mu = \mu_0 \mu_r$$

$$\mu_0 \rightarrow 4\pi \times 10^{-7} \ N/A^2$$

conductivity σ siemens/meter. $\mho/m$.
it is the materials ability to conduct electricity.

$$J = \sigma E$$

current density A/m^2

~~meters~~

volts/meter

$$\frac{1}{\Omega\,\text{meter}} = \frac{\mho}{\text{meter}}$$

ρ resistivity $\Rightarrow \frac{1}{\rho} = \sigma$ conductivity.

$\epsilon \rightarrow$ absolute permitivity (Farad/meter)
is the measure of resistance that is encountered when forming an electric field.
i.e ~~keeus~~ measure of how electric field affects & is affected by a dielectric medium.
The permitivity of medium describes how much electric field (more correctly, flux) is 'generated' per unit charge in that medium.
Thus, permitivity relates to a material ability to transmit (or permit) an electric field.

$\varepsilon = \varepsilon_0 \varepsilon_r \rightarrow$ Relative permitivity of material

$\varepsilon_0 \rightarrow$ Permitivity in freespace or vacuum 8.8541×10^{-12} F/m

$\varepsilon = (1+\chi)\varepsilon_0$

$\chi \rightarrow$ electric susceptibility

susceptibility is proportionality constant indicates degree of polarisation of a dielectric material in responce to an applied electric field.

$$D = \epsilon E$$

$$\frac{\text{coulomb}}{m^2} = \frac{\text{farad}}{m} \times \frac{\text{volts}}{me}$$

coulomb = farad × volts.

coulomb ↓ unit of electric charge

farad ↓ unit of capacitance

volts ↓ unit of electric potential or electromotive force.

1C = 1F × 1V

1 coulomb = 1Amp × 1sec