Number System

- Complex Numbers
  - Real
    - Rational
      - Integers $Z = \{-\infty \text{ to } +\infty\}$
        - –ve nos.
          - Even nos.
          - Odd nos.
        - Whole
          - Zero nos.
          - +ve nos. (Natural nos.) $N = \{1, 2, ...\infty\}$
            - Even nos.
              - Prime nos. $\{2\}$
              - Composite nos. $\{4, 6...\infty\}$
            - Odd nos.
              - Prime nos. $\{3, 5, 7...\infty\}$
              - Composite nos. $\{9, 15, ...\infty\}$
              - $\{1\}$
      - Fractions
        - Proper
        - Improper
        - Mixed
    - Irrational $\sqrt{2}, \frac{\sqrt{3}}{4}, \sqrt{7}$, etc
  - Imaginary $\sqrt{-3}, \sqrt{-2}$

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- $|a| = a$ (if a is positive)
- $|a| = -a$ (if a is negative)
- H.C.F. → Highest Common factor. (H.C.F. between 60 & 45 is 15)
- L.C.M. → Least Common Multiple (L.C.M. of 8 & 12 is 24).
- Remember the VBODMAS rule 2
  V → Vinculum (bar)
  B → Bracket
  O → of (multiplication)
  D → Division
  M → Multiplication
  A → Addition
  S → Subtraction

HCF

eg 1134 & 1215

$$1134)\,1215\,(1$$
$$-1134$$
$$81)\,1134\,(14$$
$$81$$
$$324$$
$$324$$
$$\times\times$$

81 = HCF

LCM

eg. 24, 36, 54

6	24	36	54
2	4	6	9
2	2	3	9
3	1	3	9
3	1	1	3
	1	1	1

$6 \{ 6$, $4 \{ 2 \times 2$, $9 \{ 3 \times 3$ = 216

$6 \times 4 \times 9 = 216$ LCM

* i.e. If a Natural no. 'N' can be expressed as $N = a^m \times b^n \times c^p$ where a, b, c are primes then total no.of factors of N including 1 & N is equal to $(m+1)(n+1)(p+1)$ 3

* $12 = 2^2 \times 3^1$

$(2+1)(1+1) = 6$ factors for 12

$100 = 5^2 \times 2^2$

$(2+1)(2+1) = 9$ factors

1752976 (1324
1
23 | 75
+3 | 69
262 | 629
+2 | 524
2644 | 10576
-10576

e.g. The factors of 12 are: – 12 can be expressed as $= 2^2 \times 3^1$

$\therefore 12 = (2^0 \times 3^1), (2^0 \times 3^0), (2^1 \times 3^1), (2^1 \times 3^0), (2^2 \times 3^0), (2^2 \times 3^1)$

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- Product of 2 nos. = (H.C.F) × (L.C.M)

e.g. If Product of 2 nos. = 600 and L.C.M = 120, $\therefore$ H.C.F $= \frac{600}{120} = 5$.

- **Fundamental laws of Indices:** –

1. $a^m \times a^n = a^{m+n}$
2. $a^m \times a^n \times a^p \times .... = a^{m+n+p+....}$
3. $(a^m)^n = a^{mn}$.
4. $a^m \div a^n = a^{m-n}$
5. $a^{p/q} = \sqrt[q]{a^p}$, where a & p are real nos. & $q \neq 0$.
6. $a^{1/n} = \sqrt[n]{a}$
7. $a^{-n} = 1/a^n$
8. $a^0 = 1$
9. $(a \times b)^m = a^m \times b^m$ & $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
10. If $a^m = a^n$ & $a \neq -1, 0, 1$, then m = n.

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2) To check 173 prime or not

$(14)^2 > 173$ upto 13 < 14

check 2, 3, 5, 7, 11, 13 are factors of 173

if not then prime.

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Test of divisibility

2 – even

3 – addition of digit % 3

4 – last 2 digits % 4

5 – 0 & 5 at U.P

6 – 2 & 3 condn

7 – take last digits, double subtract from rest no.

8 – last 3 digits

9 – additn % 9

11 – diff. of odd place additn – even place addn ÷ 3

12 – 3 & 4 condn.

- **Laws of surds:**

1. $(\sqrt[n]{a})^n = a$
2. $\sqrt[n]{a}.\sqrt[n]{b} = \sqrt[n]{ab}$, also $(\sqrt[n]{a}.\sqrt[n]{b})^n = a.b.$
3. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$, also $\left(\frac{\sqrt[n]{a}}{\sqrt[n]{b}}\right)^n = \frac{a}{b}$
4. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$
5. $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$

- Some properties of square nos.:

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1. A square cannot end with an odd no. of zeros.
2. A square no. cannot end with 2, 3, 7 or 8.
3. A square no. is a multiple of 3 or exceeds the multiple of 3 by unity.
4. Every square no. is multiple of 4 or exceeds the multiple of 4 by unity.

} ~~(× of 12) or (multiple +1~~ of 12)

5. If a square no. ends in 9, the preceding digit is even.

- **Remainder theorem:**

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e.g. Find the remainder when the product (1421 × 1423 × 1425) is divided by 12.

Ans. Divide each no. by 12. You get 5, 7 and 9 as remainders. Then, 5 × 7 × 9 when divided by 12 you get

$$\frac{35 \times 9}{12} = \frac{11 \times 9}{12} = 3 \text{ (Final Remainder).}$$

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Arithmetic Progression:

(10) Sum of the terms $(S_n) = \frac{n}{2}[2a + (n-1)d]$

$\therefore\ S_n = \frac{n}{2}[a + t_n]$

where, $t_n = a + (n-1)d$. (n^{th} term)

and $a \to 1^{st}$ term

$d \to$ common difference

$t_n \to n^{th}$ term

$n \to$ total no. of terms

Arithmetic Mean of 2 nos.:

- $A.M. = \frac{a+b}{2} = A$.

$\therefore$ a, A, b are in A.P and $2A = a + b$.

i.e. $2 \times$ (middle term) = (first term) + (last term)

(9)

$(1+2+3 \ldots + n) = \frac{1}{2}n(n+1)$

$(1^2+2^2+3^2 \ldots + n^2) = \frac{1}{6}n(n+1)(2n+1)$

$(1^3+2^3+3^3 \ldots + n^3) = \left[\frac{1}{2}n(n+1)\right]^2$

0 →	0	0	0	0
1 →	1	1	1	1
2 →	2	4	8	6
3 →	3	9	7	1
4 →	4	6	4	6
5 →	5	5	5	5
6 →	6	6	6	6
7 →	7	9	3	1
8 →	8	4	2	6
9 →	9	1	9	1

Geometric Progression:

(11) Sum upto n^{th} term: –

- For $r > 1$: – $S_n = \frac{a(r^n - 1)}{r - 1}$
- For $r < 1$: – $S_n = \frac{a(1 - r^n)}{1 - r}$

Where, $r \to$ common ratio

n^{th} term $= ar^{n-1}$

Sum upto infinite terms: –

- $S_\infty = \frac{a}{1-r}$ or $\frac{a}{r-1}$

and Sum upto n terms: – $S_n = S_\infty \times (1 - r^n)$

Geometric mean :

- $G.M. = \sqrt{ab} = G$

$\therefore$ a, G, b are in G.M and $G^2 = ab$

i.e. $(\text{Middle term})^2 = (\text{first term}) \times (\text{last term})$

1	121	441	961	1
4	144	484	1024	8
9	169	529	1089	27
16	196	576	1156	64
25	225	625	1225	125
36	256	676		216
49	289	729		343
64	324	784		512
81	361	841		729
100	400	900		1000
				1331
				1728
				2197
				2744
				3375

Note : G.M. is generally used to calculate the rate of growth.

(12) **Harmonic mean:** – Harmonic mean is generally used to find the average speed when equal distances are covered at different speeds. *

$$H.M = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots\ldots} = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$$

IMP:

(13) 1. If x_1, x_2, x_3 ...... are in G.P. (& positive), then

$\log x_1, \log x_2, \log x_3$ .... is an A.P. & vice–versa.

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2. 3 terms of a G.P. should preferably be taken as a/r, a, ar and 4 terms should be taken as a/r^3, a/r, ar, ar^3.
Else, terms should be taken as a, ar, ar^2, etc....

3. 3 terms of an A.P. should preferably be taken as a–d, a, a + d and 4 terms as a–3d, a–d, a + d, a + 3d.

14 • **H.M. of 2 nos.:**

$$H.M. = \frac{2ab}{a+b}$$

16 1) No. betn 17 to 80 table by 6
No starts from 18 up to 78
∴ a = 18, tn = 78, d = 6 find n by AP

15 **Relation between A.M, G.M. & H.M.:**

$(GM)^2 = (AM) \times (HM)$ and AM > GM > HM

17 Reminder:-
0 → (BU) → Even ⇒ 6; odd ⇒ 1 (except 5)
1 → $(BU)' = BU$.
2 → $(BU)^2$
3 → $(BU)^3$

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Averages

$$\text{Average} = \frac{\text{Sum of all the items}}{\text{no.of items}}$$

18 Preceded by ← $\boxed{\text{No.}}$ → followed by
5 is followed by 3 & preceded by 7
⇒ 7 $\boxed{5}$ 3

- **Addition or removal of items and change in average:**

9) **Case I:** When one or more than one new items are added.

Let the average of N items = A.
Now 'n' new items are added and the average increases or decreases by x, then: –

$$\text{Average of New items added} = A \pm \left(1+\frac{N}{n}\right)x$$

No need to Remember formula

$\begin{pmatrix}\text{Use } (-\text{ve}) \text{ when avg. decreases} \\ (+\text{ve}) \text{ when avg. increases}\end{pmatrix}$

Case II: When one or more than one items are removed.

$$\text{Average of items removed} = A \pm \left(1-\frac{N}{n}\right)x$$

$\begin{pmatrix}\text{Use } (-\text{ve}) \text{ when avg. decreases} \\ (+\text{ve}) \text{ when avg. increases}\end{pmatrix}$

- **Replacement of some of the items:**
Sometimes, when a number of items of a group are removed & replaced with equal no. of different items, then the average of the group changes (increases or decreases) by x.

20 Let there be N items in the group, then: –

(Sum of new items added) – (Sum of removed items) = ± Nx

$\begin{pmatrix}\text{Use } (-\text{ve}) \text{ when avg. decreases} \\ (+\text{ve}) \text{ when avg. increases}\end{pmatrix}$

- **Median:** – In a set of 'n' nos. arranged in ascending or descending order, the median is the middle no. (if 'n' is odd) or the average of 2 middle nos. (if 'n' is even).

- **Mode:** – Mode is the no. that occurs most frequently in a given set of nos.

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Mixtures & Allegations

$$\frac{\text{Quantity of cheap}}{\text{Quantity of Dearer}} = \frac{(\text{Price of Dearer}) - (\text{Mean / Avg. Price})}{(\text{Mean / Avg. Price}) - (\text{Price of Cheap})}$$

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Price of Dearer — Price of Cheap

Avg / Mean Price

(Mean Price – Price of Cheap) — (Price of Dearer– Avg /Mean Price)

If the amount of Mixture = Q, then,

- Quantity of 1st ingredient in the mixture = $\frac{q_1}{q_1 + q_2} \times Q$
- Quantity of 2nd ingredient in the mixture = $\frac{q_2}{q_1 + q_2} \times Q$

  (Similarly for 3 quantities)

24 suppose a container contains x units of liquid from which y units are taken out and replaced by water. after n operations the quantity of pure liquid $= \left[x\left(1-\frac{y}{x}\right)^n\right]$ units

- If C. P. of 1st ingredient = C_1, C.P. of 2nd ingredient = C_2, C.P. of 3rd ingredient = C_3 then,
  - Mean price = C_m (Assuming $C_3 > C_2 > C_m > C_1$)
- If proportion of 1st ingredient = q_1, proportion of 2nd ingredient = q_2, proportion of 3rd ingredient = q_3 then,
  - $q_1 = (C_2 - C_m)(C_3 - C_m)$ (here cost of q_1, is absent)
  - $q_2 = (C_m - C1)(C_3 - C_m)$ (here cost of q_2, is absent)
  - $q_3 = (C_2 - C_m)(C_m - C_1)$ (here cost of q_3, is absent)
  - $\therefore q_1 : q_2 : q_3 = (C_2 - C_m)(C_3 - C_m) : (C_m - C_1)(C_3 - C_m) : (C_2 - C_m)(C_m - C_1)$

25 $q_1 : q_2 : q_3$

$c_1 \quad c_2 \quad c_3 \Rightarrow c_m$

c_m

$(C_3-C_m)(C_2-C_m) : (C_3-C_m)(C_m-C_1) : (C_2-C_m)(C_m-C_1)$

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Percentages

- $\frac{1}{2} = 50\%$, $\frac{2}{2} = 100\%$, $\frac{3}{2} = 150\%$....
- $\frac{1}{3} = 33.33\%$, $\frac{2}{3} = 66.67\%$, $\frac{3}{3} = 100\%$....
- $\frac{1}{4} = 25\%$, $\frac{2}{4} = 50\%$, $\frac{3}{4} = 75\%$, $\frac{4}{4} = 100\%$.....
- $\frac{1}{5} = 20\%$, $\frac{2}{5} = 40\%$, $\frac{3}{5} = 60\%$, $\frac{4}{5} = 80\%$, $\frac{5}{5} = 100\%$....

- $\frac{1}{6} = 16.67\%$, $\frac{2}{6} = 33.33\%$, $\frac{3}{6} = 50\%$, $\frac{4}{6} = 66.67\%$, $\frac{5}{6} = 83.33\%$, $\frac{6}{6} = 100\%$....
- $\frac{1}{7} = 14.28\%$, $\frac{2}{7} = 28.56\%$, $\frac{3}{7} = 42.84\%$, $\frac{4}{7} = 57.14\%$, $\frac{5}{7} = 71.42\%$, $\frac{6}{7} = 85.70\%$, $\frac{7}{7} = 100\%$....

(Note: – Similarly remember all the fractions in '%' form at least upto $\frac{1}{15}$)

- P% of Q = $\frac{P \times Q}{100}$

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- What % of P is Q = $\frac{Q \times 100}{P}$
- Percentage Change between two absolute values A & B is given by = $\frac{\text{Change} \times 100}{\text{Original Value}}$
- If A is M% more than B, then B is $\frac{100M}{(100+M)}$% less than A.

27 50% more $\frac{100}{150}\frac{B}{A}$ $\frac{50}{150} \times 100$

- If A is M% less than B, then B is $\frac{100M}{(100-M)}$% more than A.
- If a quantity R is increased initially by M% then decreased by N% and finally increased by P%, then the final value is = $\frac{R\,(100+M)(100-N)(100+P)}{100 \times 100 \times 100}$.

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- If initial population is 'P', rate of growth is M%, then population after 'n' years can be calculated as = $P\left[1 \pm \frac{M}{100}\right]^n$

minus for reduction 29 As compound interest

- If initial population is 'P', rate of growth is M%, then population 'n' years ago can be calculated as = $\frac{P}{\left[1+\frac{M}{100}\right]^n}$

e.g.

- **Results on Depreciation:**

  1. Value of Machine after 'n' years = $P\left[1-\frac{R}{100}\right]^n$
  2. Value of Machine 'n' years ago = $\frac{P}{\left[1-\frac{R}{100}\right]^n}$

- If the price of commodity increases by R%, then reduction in consumption, so as not to increase expenditure is = $\left[\frac{R}{(100+R)} \times 100\right]$%

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- If the price of commodity decreases by R%, then increase in consumption, so as not to decrease the expenditure is = $\left[\frac{R}{(100-R)} \times 100\right]$%

- Whenever there are two successive changes of a% and then of b%, then the net change is given by: $-\left[a+b+\frac{ab}{100}\right]\%$

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- If the **Selling Price** of two commodities is the same and one is sold at a Profit of P% and other at a Loss of P%, then in the whole transaction there is always a Loss of $\frac{P^2}{100}\%$. 32

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Profit and Loss

- Profit = Selling Price – Cost Price.
- Loss = Cost Price – Selling Price. 33
- Profit % $= \frac{\text{Profit} \times 100}{\text{Cost Price}}$.
- Loss % $= \frac{\text{Loss} \times 100}{\text{Cost Price}}$.
- Cost Price $= \frac{100}{(100 + \text{Profit }\%)} \times$ Selling Price.
- Cost Price $= \frac{100}{(100 - \text{Loss }\%)} \times$ Selling Price.
- Selling Price $= \frac{(100 + \text{Profit }\%)}{100} \times$ Cost Price 34
- Selling Price $= \frac{(100 - \text{Loss }\%)}{100} \times$ Cost Price
- Marked Price (1 – %Discount) = Selling Price

  or Customer Price = Cost Price (1 + %Gain)

if a trader profe to sell his goods at cost price but uses false weight then gain $= \frac{\text{Error}}{(\text{True Value} - \text{Error})} \times 100\%$

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Simple Interest and Compound Interest

- **Simple Interest (S.I.)** $= \frac{PNR}{100}$ 36

  where, P = Principal Sum,

  N = Number of Years,

  R = Rate of Interest.

- Amount on S.I. $= \frac{PNR}{100} + P$

- Change in S.I.

  $= \frac{(\text{Product of Fixed Parameters})}{100} \times \frac{(\text{Difference of Product of Variable Parameters})}{100}$

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- **Compound Interest (C.I.)** $= P\left(1+\frac{r}{100}\right)^n - P$ .... (for 'n' years).

- Amount on C.I. $= P\left(1+\frac{r}{100}\right)^n$ .... (for 'n' years).

present worth of Rs x due to n years is given by

Present worth $= \frac{x}{\left(1+\frac{R}{100}\right)^n}$

- $\frac{1}{2}$ yearly C.I. $= P\left(1+\frac{r}{200}\right)^{2n} - P$

- Quarterly C.I. $= P\left(1+\frac{r}{400}\right)^{4n} - P$

Case – I: – When rate is not the same, the basic formula can be written as: –

$$A = P\left(1+\frac{R_1}{100}\right)^{N_1} \times \cancel{P}\left(1+\frac{R_2}{100}\right)^{N_2} \ldots$$

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Case – II: – When Interest is Compounded Yearly, but Time is a Fraction, e.g.: – $5\frac{3}{4}$ years, then in this case the basic formula can be written as: –

$$A = P\left(1+\frac{R}{100}\right)^5 \times \cancel{P}\left(1+\frac{\frac{3}{4}\times R}{100}\right)$$

when rates are different years $R_1\%$, $R_2\%$, $R_3\%$ for 1st, 2nd & 3yr respect.

Amount $= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$

Difference between S.I. and C.I.:

$$C.I. - S.I. = P\left[\left(1+\frac{R}{100}\right)^N - 1 - \frac{RN}{100}\right]$$

- When N = 2 years, we can directly use the formula: –

$$C.I. - S.I. = P\left(\frac{R}{100}\right)^2$$

- When N = 3 years, we can directly use the formula: –

$$C.I. - S.I. = P\left[\left(\frac{R}{100}\right)^3 + 3\left(\frac{R}{100}\right)^2\right]$$

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Ratio, Proportion and Variation

If $\frac{a}{b} = \frac{c}{d}$ are in relation (proportion) then,

- $\frac{b}{a} = \frac{d}{c}$ (Invertendo)
- $\frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)
- $\frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)
- $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (Componendo – Dividendo)
- If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = ... = k$ then, $\frac{a+c+e+....}{b+d+f+....} = k$.
- If $\frac{a}{b} = \frac{b}{c}$, i.e.: $b^2 = ac$, then 'b' is called the mean (or geometric mean) (or third proportional) between 'a' and 'c'.

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$a:b$ ratio
$a^2:b^2$ duplicate ratio
$a^3:b^3$ triplicate ratio
$\sqrt{a}:\sqrt{b}$ subduplicate ratio.

if $a:b = c:d$ d is called fourth proportional.

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Partnership

When the periods of investment are the same:

- $\frac{\text{Investment of A}}{\text{Investment of B}} = \frac{\text{Profit of A}}{\text{Profit of B}}$ (Similarly for Loss as well).
- If there are three Partners, then,

  Investment of A : Investment of B : Investment of C = Profit of A : Profit of B : Profit C
  (Similarly for Loss as well)

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When the periods of investment are different:

- $\frac{(\text{Investment of A}) \times (\text{Period})}{(\text{Investment of B}) \times (\text{Period})} = \frac{\text{Profit of A}}{\text{Profit of B}}$ (Similarly for Loss as well).

i.e. $\frac{\text{A's Monthly Equivalent of Investment}}{\text{B's Monthly Equivalent of Investment}} = \frac{\text{Profit of A}}{\text{Profit of B}}$ (Similarly for Loss as well).

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A invest Rs x for P months & B invests Rs y for q months then
(A's share of profit) : (B's profit) = $xp : yq$

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Work, Pipe and Cisterns

1. If A can do a piece of work in n days, then A's 1 day's work = $\frac{1}{n}$.

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2. If A's 1 day's work = $\frac{1}{n}$, then A can finish the work in n days.

Similar to pipe & cisterns

3. If A is thrice as good a workman as B, then:
   Ratio of work done by A and B = 3 : 1.
   Ratio of times taken by A and B to finish a work = 1 : 3.

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4. Inlet : A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet.
   Outlet : A pipe connected with a tank or a cistern or a reservoir, emptying it, is known as an outlet.

5. (i) If a pipe can fill a tank in x hours, then :
   part filled in 1 hour = $\frac{1}{x}$.
   (ii) If a pipe can empty a full tank in y hours, then :
   part emptied in 1 hour = $\frac{1}{y}$
   (iii) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where y > x), then on opening both the pipes, the net part filled in 1 hour = $\left(\frac{1}{x}-\frac{1}{y}\right)$.
   (iv) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where x > y), then on opening both the pipes, the net part emptied in 1 hour = $\left(\frac{1}{y}-\frac{1}{x}\right)$.

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Time, Speed and Distance

If a person can do a piece of work in 'N' days, then in one day he can do $\left(\frac{1}{N}\right)$ units of work.

Basic Formulae:

If $Q_1 \propto Q_2$ then,

1. No. of persons ∝ Amount of work done
   i.e. (Man) ∝ (Quantity) – More persons, more work
2. No. of day ∝ Amount of work
   i.e. (Days) ∝ (Quantity) – More days, more work

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3. Working rate ∝ Amount of work
   i.e. (Hr/day) ∝ (Quantity) – More working rate, more work
4. Efficiency of Man ∝ Amount of work
   i.e. (Efficiency) ∝ (Quantity) – More efficiency of man, more work.

If $Q_1 \propto \frac{1}{Q_1}$ then,

5. No. of men $\propto \dfrac{1}{\text{No. of days}}$

i.e. more the men, less the no. of days required.

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Important Formulae:

1. $$\frac{\text{Man}_1 \times \text{Days}_1 \times \text{Work rate}_1}{(\text{Amount of Work Done})_2} = \frac{\text{Man}_2 \times \text{Days}_2 \times \text{Work rate}_2}{(\text{Amount of Work Done})_2}$$

2. $$\frac{(\text{No. of Engine})_1 \times (\text{Hrs.})_1 \times (\text{Consumption Rate})}{(\text{Amount of coal burnt})_1} = \frac{(\text{No. of Engine})_1 \times (\text{Hrs.})_1 \times (\text{Consumption Rate})}{(\text{Amount of coal burnt})_1}$$

3. $$\frac{(\text{No. of Examiner})_1 \times (\text{Days})_1 \times (\text{Work Rate})_1}{(\text{No. of Answer books checked})_1} = \frac{(\text{No. of Examiner})_2 \times (\text{Days})_2 \times (\text{Work Rate})_2}{(\text{No. of Answer books checked})_2}$$

From 1, 2 & 3 we get: –

$$\frac{(\text{No. of persons})_1 \times (\text{Days})_1 \times (\text{Work Rate})_1 \times (\text{Efficiency})_1}{(\text{Quantity of Work})_1} = \frac{(\text{No. of persons})_2 \times (\text{Days})_2 \times (\text{Work Rate})_2 \times (\text{Efficiency})_2}{(\text{Quantity of Work})_2}$$

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* in 60 min the min hand gains 55 min on the hr hand
* 5 min = 30° ⇒ 1 min = 6°
* in 1 hr hr hand moves 30°
* in 12 min hr hand moves 1 min
* in 1 min hr hand move $\frac{1}{12}$ min
* min hand crosses hr hand once in hr

* leap year divisible by 4
* leap year feb = 29 days
* Normal year → 365 days → 1 odd day
Leap year → 366 days → 2 odd days
* one one day diff on same date of consicutive years
~~20~~ 1 Jan 2013 → friday
1 Jan 2014 → sat → for Normal
~~leap~~ sunday – for leap.
* to estimate day on certain date
$\frac{\text{count days}}{7}$ reminder ~~(mode)~~ shift day ahead

We have: – Speed = $\dfrac{\text{Distance}}{\text{Time}}$

Basic Conversions:

1 km/hr = 5/18 m/sec
1 m/sec = 18/5 km/hr

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Special Cases:

- If a person 'P' is 'n' times as good a workman as Q, then lone time for P = $\dfrac{\text{alone time for Q}}{n}$ & after same time $\dfrac{\text{P's work}}{\text{Q's work}} = \dfrac{n}{1}$
- If more than one person are engaged on payment basis for doing a work, then the total wages distributed to each person are
  i) in proportion to the work done by each person or
  ii) in proportion to the 1 days work of each person or
  iii) in inverse proportion to the alone time of each person.

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alone time means time require to complete work by alone person.

- Two vehicles A & B start at the same time from points P & Q towards each other, they take M & N hrs to reach their respective ends, then,

  (A's speed) : (B's speed) = $\left(\dfrac{n}{m}\right)^{1/2}$

  (⇒Similar concept of work is applicable in Pipes & Cisterns as well)

- When a train passes a pole (or any stationary object) it covers a distance equal to its own length.
- When a train passes a platform it covers a distance equal to the sum of the length of the platform & its own length.
- When a train A passes a moving train B, it covers a distance equal to the sum of the length of both the trains A &B.
- When a train passes a person sitting on the window seat in another moving train, the train covers a distance equivalent to its own length.

Stoppage time per hour for a train:

For the same distance of travel, if a train runs: –

→ at an average speed V_1 km/hr (without stopping)
→ at an average speed V_2 km/hr (with stoppage), then,

$$\text{Stopping time/hr} = \left(\frac{V_1 - V_2}{V_1}\right)\text{hr}$$

$$= \left(\frac{\text{difference in speed}}{\text{faster speed}}\right)\text{hr}$$

Time taken with 2 different modes of transport:

(Time taken by any one transport both ways) – (Time taken by mixed transport) = Time gained or lost

Time & distance between 2 moving bodies:

2 persons A & B: –
Speed of A = V_1 km/hr
Speed of B = V_2 km/hr

$s = \frac{dist}{time}$

If they walk in same direction:

- A & B will be $(V_1 - V_2)$ km apart in 1 hr.
- Conversely A & B will be 1 km apart in $\left(\frac{1}{V_1 - V_2}\right)$ hrs.
- A & B will be x km apart in $\left(\frac{x}{V_1 - V_2}\right)$ hrs.

49 similar concepts as that of trains

Parallely if they walk in opposite directions:

- A & B will be $(V_1 + V_2)$ km apart in 1 hr.
- Conversely A & B will be 1 km apart in $\left(\frac{1}{V_1 + V_2}\right)$ hrs.
- A & B will be x kms apart in $\left(\frac{x}{V_1 + V_2}\right)$ hrs

Time to cross an object moving in the direction of train = $\dfrac{(\text{length of train})+(\text{length of object})}{(\text{Speed of train})-(\text{Speed of object})}$

Types of objects	Time to cross
Pole, standing man, etc	$t=\dfrac{\text{length of train}}{\text{speed of train}}$
Platform, train at rest	$t=\dfrac{\text{length of }(\text{train}+\text{object})}{\text{speed of train}}$
Object is moving & is of negligible length e.g. man – running, car-moving, etc	$t=\dfrac{\text{length of train}}{\text{speed of }(\text{train}-\text{object})}$
Object is moving & has some length e.g. another train running	$t=\dfrac{\text{length of }(\text{train}+\text{object})}{\text{speed of }(\text{train}-\text{object})}$

Two trains crossing each other in both directions: –

$L_1 \rightarrow$ length of one train

$L_2 \rightarrow$ length of 2^{nd} train

They cross each other in opposite direction in t_1 seconds.
They cross each other in same direction in t_2 seconds.

Speed of Faster Train = $\dfrac{L_1+L_2}{2}\left[\dfrac{1}{t_1}+\dfrac{1}{t_2}\right]$

Speed of slower Train = $\dfrac{L_1+L_2}{2}\left[\dfrac{1}{t_1}-\dfrac{1}{t_2}\right]$

Two trains lengths a & b moving opposite direction at u & v m/s
time taken by train to cross each other = $\left(\dfrac{a+b}{u+v}\right)$ sec
moving in same direction = $\left(\dfrac{a+b}{u-v}\right)$ sec

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Boats & Streams:

Important Formulae:

Man's rate of rowing in still water = x

Speed of stream = y

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speed of boat in still water = u km/hr
speed of steam = v km/hr
(a) Speed down stream = u+v
(b) Speed up stream = u−v
$u=\dfrac{a+b}{2}$ $v=\dfrac{a-b}{2}$

1. Down stream rate = $x+y=\dfrac{\text{Down stream distance}}{\text{time to cover it}}=\dfrac{d_{down}}{t_{down}}$

2. Upstream rate = $x-y=\dfrac{\text{up stream dist.}}{\text{time to coer it}}=\dfrac{d_{up}}{t_{up}}$

3. Man's rate in still water or speed of Boat,

$$x=\frac{1}{2}\left[\frac{d_{down}}{t_{down}}+\frac{d_{up}}{t_{up}}\right]=\frac{1}{2}[(x+y)+(x-y)]$$

4. Speed of stream,

$$y=\frac{1}{2}\left[\frac{d_{down}}{t_{down}}-\frac{d_{up}}{t_{up}}\right]=\frac{1}{2}[(x+y)-(x-y)]$$

5. When downstream dist. = upstream distance, then

a. $\dfrac{\text{Man's in still water}}{\text{speed of stream}} = \dfrac{t_{up} + t_{down}}{t_{up} - t_{down}}$

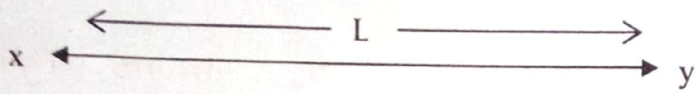
b. Also, Average speed for total journey (up + down) = $\dfrac{\text{Upstream rate} \times \text{Downstream rate}}{\text{Man's rate in still water}}$

c. Total journey time ($t_{up} + t_{down}$) = $\dfrac{(\text{Man's rate in still water} \times \text{Total distance})}{\text{Upstream rate} \times \text{Downstream rate}}$

d. Total Distance = downstream distance + upstream distance = 2 × any one side dist. (as both are equal)

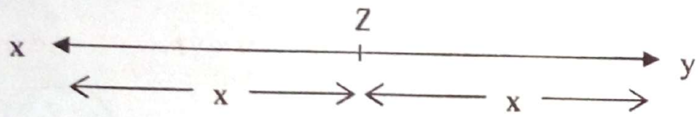
Races:

1. A beats B by X metres: –



i.e. A & B start together at X.

2. A gives B a start of X metres: –



A starts at X but B starts at Z i.e. x metres ahead of A at the same moment.

3. A beats B by t seconds: –

A & B start together at X but A finishes at Y t secs before B finishes. 52

4. A gives B a start of t secs: – Parallely as (2)

5. $\dfrac{\text{Winner's time}}{\text{loser's distance}} = \dfrac{\text{Loser's time}}{\text{Loser's distance}} = \dfrac{(\text{beat time}) + (\text{start time})}{(\text{beat dist}) + (\text{start dist})}$

□□□□□□

Geometry & Trigonometry

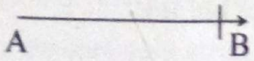
Geometry :

1. (i) A line contains infinitely many points.
   (ii) Through a given point, there pass an infinite number of lines.
   (iii) Given two distinct points A and B, there is one and only one line that contains both the points.
   (iv) Three or more than three points are said to be collinear if there is a line which contains them all.

(v) Two lines having a common point are known as intersecting lines. Two lines can intersect at the most at one point.

(vi) Two lines which are both parallel to the same line are parallel to each other.

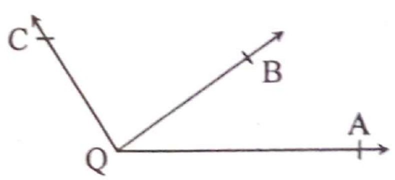
(vii)The part of a line with two end points A and B, is called a line segment $\overrightarrow{AB}$.



(viii) A part of a line which has only one end point is called a ray.

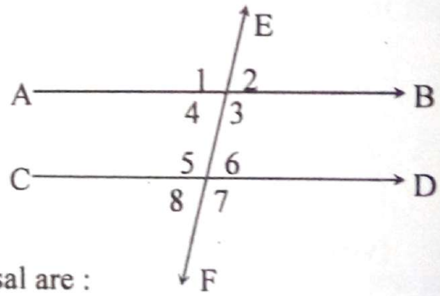
2. (i) inclination between two rays having a common end point is called an angle.
(ii) An angle θ is said to be :
I. Acute, if $\theta < 90°$ II. Obutse, if $\theta > 90°$
III. Straight, if $\theta = 180°$ IV. Right Angle, if $\theta = 90°$
V. Reflex, if $180° < \theta < 360°$.
(iii) Complement of θ is $(90° - \theta)$. Supplement of θ is $(180° - \theta)$.

3. (i) **Adjacent Angles :** Two angles are said to be adjacent, if they have a common vertex and a common arm, and the other arm of one angle is on one side of the common arm and that of the other is one the opposite side.
(ii) Linear Pair : Two adjacent angles are said to form a linear pair of angles, if their non–common arms are two opposite rays.
(iii) The sum of a linear pair of angles is 180°.
(iv) The sum of all the angles around a point is 360°.



4. Vertically opposite Angles : If two lines intersect, then vertically opposite angles are equal.

5. Suppose AB || CD and a transversal EF cuts them.
Then :
I. Pairs of corresponding $\angle$ s are :
$(\angle 1 \ \& \ \angle 5)$, $(\angle 2 \ \& \ \angle 6)$.
$(\angle 4 \ \& \ \angle 8)$, $(\angle 3 \ \& \ \angle 7)$.
II. Pairs of Alternate angles are :
$(\angle 3 \ \& \ \angle 5)$, $(\angle 4 \ \& \ \angle 6)$.
III. Pairs of interior angles on the same side of the transversal are :
$(3 \ \& \ \angle 6)$, $(\angle 4 \ \& \ \angle 5)$.



6. If two parallel lines are intersected by a transversal, then :
(i) each pair of corresponding angles are equal ;
(ii) each pair of alternate angles are equal ;
(iii) interior angles on the same side of the transversal are supplementary.

7. (i) Sum of all the angles of a triangle is 180°.
(ii) If one side of a triangle is produced, them the exterior angle so formed is equal to sum of the interior opposite angles.

Polygons

(i) It is a closed plane figure bounded by some straight lines. A polygon is called a Triangle, Quadrilateral, Pentagon, Hexagon, Heptagon, Octagon, Nonagon and Decagon according as it contains 3, 4, 5, 6, 7, 8, 9, 10 sides respectively.

(ii) Convex & Concave Polygon : A polygon in which none of its interior angles is more than 180°, is known as a convex polygon.
On the other hand, if at least one angle of a polygon is more than 180°, then it is said to be concave.

(iii) Regular Polygon: A polygon having all sides equal and all angles equal is called a regular polygon.

I. Each exterior angle of a regular polygon $= \left(\dfrac{360}{\text{Number of sides}}\right)^{0}$.

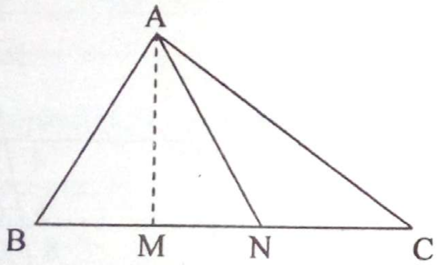
II. Each interior angle = 180° – (exterior angle).

(iv) In a convex polygon of n sides, we have :
I. Sum of all interior angles = (2n – 4) right angles :
II. Sum of all exterior angles = 4 right angles.

(v) Number of diagonals of a polygon of n sides $= \dfrac{n(n-1)}{2} - n$

Useful Facts For solving Problems On Triangles :

1. (i) The sum of all the angles of a triangle is 180°.
(ii) If one side of a triangle is produced, then the exterior angle so formed is equal to the sum of two interior opposite angles.
(iii) If the three sides of a triangle be produced in order, then the sum of all the exterior angles so formed is 360°.
(iv) In a ΔABC, if the bisectors of $\angle B$ and $\angle C$ meet at O, then $\angle BOC = 90° + \dfrac{1}{2}\angle A$.
(v) In ΔABC, if sides AB and AC are produced to P and Q respectively and the bisectors of $\angle PBC$ and $\angle QCB$ intersect at Q then $\angle BOC = 90° - \dfrac{1}{2}\angle A$.
(vi) In ΔABC, if AM is the bisector of $\angle BAC$ and $AN \perp BC$, then $\angle MAN = \dfrac{1}{2}(\angle B - \angle C)$.



(vii) In ΔABC, if BC is produced to D and AL is the bisector of $\angle A$, then : $\angle ABC + \angle ACD = 2\angle ALC$.
(viii) In a ΔABC, if side BC is produced to D and bisectors of $\angle ABC$ and $\angle ACD$ meet at E, then $\angle BEC = \dfrac{1}{2}\angle A$.

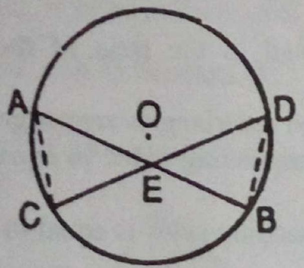
2. (i) In a triangle :
(a) Orthocentre is the point of intersection of the altitudes.
(b) Circumcentre is the point of intersection of the perpendicular bisectors of the sides.
(c) Incentre is the point of intersection of the angular bisectors.
(d) Centroid is the point of intersection of the medians.
(ii) The circumfcentre of a triangle is equivalent from its vertices.
(iii) The incentre of a triangle is equidistant from its sies.
(iv) The centroid divides a median in the ratio 2 : 1.
(v) The orthocenter of a right angled triangle lies at the vertex containing the right angle.

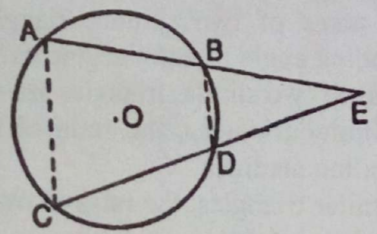
3. (i) If two angles of a triangle are unequal, the greater angle has the greater side opposite to it.
(ii) In a ΔABC, $\angle B$ is a right angle, an obtuse angle or an acute angle according as : $AC^2 = AB^2 + BC^2$, $AC^2 > AB^2 + BC^2$ or $AC^2 < AB^2 + BC^2$ respectively.

4. (i) In ΔABC, if $\angle B$ is obtuse, then $AC^2 = AB^2 + BC^2 + 2\,BC.AD$.
(ii) In ΔABC, if $\angle C$ is acute, then $AB^2 = BC^2 + AC^2 - 2BC.\,CD$, where $AD \perp BC$.

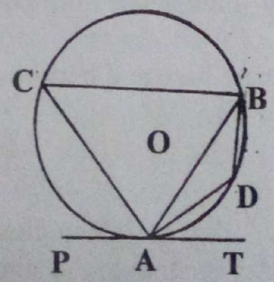
Circles :

1. The perpendicular from the centre of a circle to a chord bisects the chord.
2. The line joining the centre of a circle to the mid point of a chord is perpendicular to the chord.
3. Perpendicular bisector of a chord passes through the centre.
4. Equal chords of a circle (or of congruent circles) are equidistant from the centre.
5. Chords which are equidistant from the centre are equal.
6. There is one and only one circle passing through three non–collinear points.
7. The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.
8. Angles in the same segment of a circle are equal.
9. The angle in a semi circle is a right angle.
10. The sum of a pair of opposite angles of a cyclic quadrilateral is 180°.
11. If a side of a cyclic quadrilateral is produced, then the exterior angle is equal to the interior opposite angle.
12. Equal chords are made by equal arcs.
13. Equal chords (or equal arcs) subtend equal angles at the centre.
14. The quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.
15. If two chords AB and CD of a circle intersect inside or outside the circle when produced at a point E, then AE × EB = DE × EC.





16. Suppose a circle of radius r is inscribed in a triangle of area A. If the semiperimeter of the triangle be s, then $r = \frac{A}{s}$.
17. A tangent at any point of a circle is perpendicular to the radius through the point of contact.
18. The lengths of two tangents drawn from an external point to a circle are equal.
19. From an external point, if two tangents are drawn to a circle,
    (a) they subtend equal angles at the centre.
    (b) they are equally inclined to the line segment joining the centre to that point.
20. If A is an external point from which the tangents to the circle with centre O touch it at B and C, then OA is the perpendicular bisector of BC.
21. If PAB is a secant intersecting the circle at A and B, and PT is a tangent, then $PA \times PB = PT^2$.
22. If all the sides of a parallelogram touch a circle, the parallelogram is a rhombus.
23. If from the point of contact of tangent, a chord is drawn then the angles which the chord makes with the tangent line are equal respectively to the angles formed in the corresponding alternate segments.
    ∴ In the adjoining Fig.
    $\angle BAT = \angle ACB$ and $\angle BAP = \angle ADB$



Trigonometry :

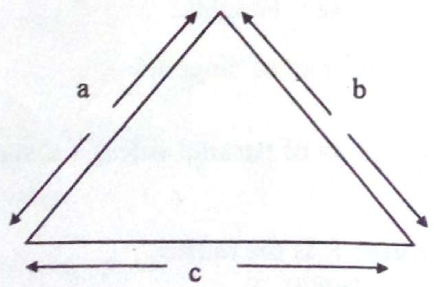
- $\tan\theta = \dfrac{1}{\cot\theta} = \dfrac{\text{Opp. side}}{\text{adjacent side}} = \dfrac{\sin\theta}{\cos\theta}$
- $\sin\theta = \dfrac{1}{\operatorname{cosec}\theta} = \dfrac{\text{Opp. side}}{\text{hypotenuse}}$
- $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$
- $\tan\theta = \dfrac{\sec\theta}{\operatorname{cosec}\theta}$
- $\cot\theta = \dfrac{\operatorname{cosec}\theta}{\sec\theta}$
- $\cos^2\theta + \sin^2\theta = 1$
- $\sec^2\theta - \tan^2\theta = 1$
- $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

Angles / Trig. Ratio	$90^0-\theta$	$90^0+\theta$	$180^0-\theta$	$180^0+\theta$	$270^0-\theta$	$270^0+\theta$	$360^0-\theta$ or $(-\theta)$
sin	$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$
cos	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$	$\cos\theta$
tan	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$
cot	$\tan\theta$	$-\tan\theta$	$-\cot\theta$	$\cot\theta$	$\tan\theta$	$-\tan\theta$	$-\cot\theta$
sec	$\operatorname{cosec}\theta$	$-\operatorname{cosec}\theta$	$-\sec\theta$	$-\sec\theta$	$-\operatorname{cosec}\theta$	$\operatorname{cosec}\theta$	$\operatorname{Sec}\theta$
cosec	$\sec\theta$	$\sec\theta$	$\operatorname{cosec}\theta$	$-\operatorname{cosec}\theta$	$-\sec\theta$	$-\sec\theta$	$-\operatorname{cosec}\theta$

	0^0	30^0	45^0	60^0	90^0
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞

Sine rule (for any triangle):

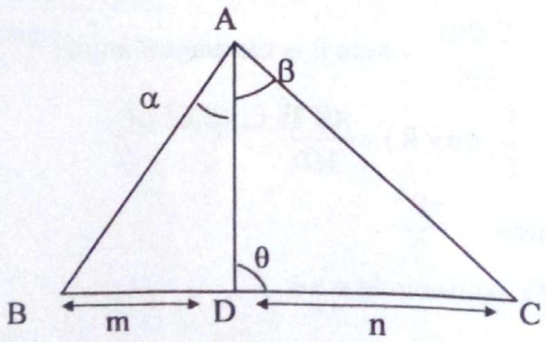
$\Rightarrow \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin c}$



$\Rightarrow$ In any Δ ABC

If $\dfrac{BD}{DC} = \dfrac{m}{n}$

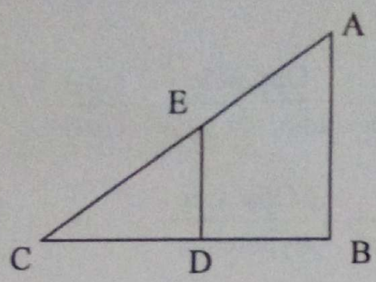
$\& \angle BAD = \alpha$
$\angle CAD = \beta$
$\angle ADC = \theta$



Then, $(m + n)\cot\theta = m.\cot\alpha - n.\cot\beta$

$\Rightarrow$ In a right – angled triangle ABC,

If DE || AB, then $\dfrac{AB}{DE} = \dfrac{BC}{DC}$



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Mensuration

I. 1. Area of a rectangle = (Length × Breadth).

$\therefore$ Length = $\left(\dfrac{\text{Area}}{\text{Breadth}}\right)$ and Breadth = $\left(\dfrac{\text{Area}}{\text{Length}}\right)$.

2. Perimeter of a rectangle = 2 (Length + Breadth).

II. Area of a square = $(\text{side})^2 = \dfrac{1}{2}(\text{diagonal})^2$.

III. Area of 4 walls of a room = 2 (Length + Breadth).

IV. 1. Area of a triangle = $\dfrac{1}{2} \times$ Base × Height.

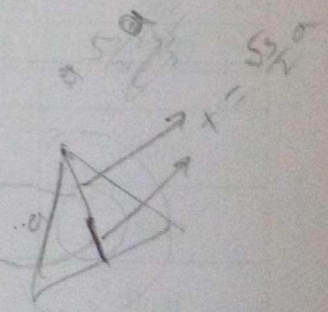
2. Area of a triangle = $\sqrt{s(s-a(s-b)(s-c)}$, where a, b, c are the sides of the triangle and $s = \dfrac{1}{2}(a+b+c)$.

3. Area of an equilateral triangle = $\dfrac{\sqrt{3}}{4} \times (\text{side})^2$.

4. Radius of incircle of an equilateral triangle of side a = $\dfrac{a}{2\sqrt{3}}$.

5. Radius of circumcircle of an equilateral triangle of side a = $\dfrac{a}{\sqrt{3}}$.

6. Radius of incircle of a triangle of area Δ and semi–perimeter s = $\dfrac{\Delta}{s}$.



V. 1. Area of a parallelogram = (Base × Height).

2. Area of a rhombus = $\dfrac{1}{2} \times$ (Product of diagonals).

3. Area of a trapezium = $\dfrac{1}{2} \times$ (sum of parallel sides) × distance between them.

VI. 1. Area of a circle = πR^2, where R is the radius.

2. Circumference of a circle = $2\pi R$.

3. Length of an arc = $\dfrac{2\pi R\theta}{360}$, where θ is the central angle.

4. Area of a sector = $\dfrac{1}{2}(\text{arc} \times R) = \dfrac{\pi R^2\theta}{360}$.

VII. 1. Area of a semi–circle = $\dfrac{\pi R^2}{2}$.

2. Circumference of a semi–circle = πR.

I. Cuboid

Let length = ℓ, breadth = b and height = h units. Then,

1. Volume = $(\ell \times b \times h)$ cubic units.
2. Surface area = $2(\ell b + bh + \ell h)$ sq. units.
3. Diagonal = $\sqrt{\ell^2 + b^2 + h^2}$ units.

II. Cube

Let each edge of a cube be of length a. Then,

1. Volume = a^3 cubic units.
2. Surface area = $6a^2$ sq. units.
3. Diagonal = $\sqrt{3}$ a units.

III. Cylinder

Let radius of base= r and Height (or length) = h. Then,

1. Volume = $(\pi r^2 h)$ cubic units.
2. Curved surface area = $(2\pi rh)$ sq. units.
3. Total surface area = $(2\pi rh + 2\pi r^2)$ sq. units. = $2\pi r\,(h + r)$ sq. units.

IV. Cone

Let radius of base = r and Height = h. then,

1. Slant height, $\ell = \sqrt{h^2 + r^2}$ units.
2. Volume = $\left(\frac{1}{3}\pi r^2 h\right)$ cubic units.
3. Curved surface area = $(\pi r\ell)$ sq. units.
4. Total surface area = $(\pi r\ell + \pi r^2)$ sq. units.

V. Sphere

Let the radius of the sphere be r. Then,

1. Volume = $\left(\frac{4}{3}\pi r^3\right)$ cubic units.
2. Surface area = $(4\pi r^2)$ sq. units.

VI. Hemisphere

Let the radius of a hemisphere be r. Then,

1. Volume = $\left(\frac{2}{3}\pi r^3\right)$ cubic units.
2. Curved surface area = $(2\pi r^2)$ sq. units.
3. Total surface area = $(3\pi r^2)$ sq. units.

   Remember: 1 litre = 1000 cm^3.

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Algebraic Equations

Linear Equations in Two Variables: An equation of the form $ax + by = c$, where a, b, c are real numbers is called a linear equation in two variables x and y.
The graph of a linear equation $ax + by = c$ is a straight line.

System of Linear Equations:

Consistent System: A system consisting of two simultaneous linear equations is said to be consistent, if it has at least one solution.

Inconsistent System: A system of two simultaneous linear equations is said to be inconsistent, if it has no solution at all.

Important Rules:

I. The system $a_1x + b_1y = c_1$, $a_2x + b_2y = c_2$ has:

(i) a unique solution, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$;;

(ii) an infinite number of solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(iii) no solution, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

II. **The homogeneous system $a_1x + b_2y = c_2$** has a non–zero solution only when $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ and in this case, the system has an infinite number of solutions.

III. The graphs of $a_1x + b_1y = c_1$, $a_2x + b_2y = c_2$ will be
(i) parallel, if the system has no solution;
(ii) coincident, if the system has an infinite number of solutions;
(iii) intersecting, if the system has a unique solution.

IV. The equations of the type $ax + by = c$ and $k\,ax + k\,by = kc$ are known as dependent equations.

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Quadratic Equations

- Monomial → $3xyz$, x^3, y^2, z etc
- Binomial → $x^3 + y^3$, $x + y$, $xyz + z^3t$ etc/
- Polynomials of higher order → $x + y + zx + ...$ etc.

Degree of a polynomial:
Polynomial of degree 1 → x, $x + y$, 2 etc.
Polynomial of degree 2 → xy, x^2, $x^2 + y^2$ etc. & so on.

- An equation is a statement of equality which involves one or more unknown quantities called the variables.

e.g (of Linear equation)
a. $2x + 3 = 1$

b. $\frac{3}{2}x+5=2x+7$

c. $\frac{5}{3}y-16=3y+4$

Method to solve simultaneous equation "

I) Eliminate one variable & solve for other

II) Consider 2 linear simultaneous equation

$a_1 \times (\text{variable})_1 + b_1 \times (\text{variable})_2 + c_1 = 0$

$a_2 \times (\text{variable})_1 + b_1 \times (\text{variable})_2 + c_2 = 0$

Step a : Recognise variables 1 & 2

Step b : By method of cross multiplication it is found that

$(\text{Variable})_1 = \frac{b_1c_2 - b_2c_1}{D}$

Where, $D = a_1b_2 - a_2b_1$

Step c : Put the value of $(\text{variable})_1$ calculated in step (b) in any one of the given linear equations & evaluate the $(\text{Variable})_2$ or use the formula : $(\text{Variable})_2 = \frac{c_1a_2 - c_2a_1}{D}$

- Test for consistency of system of linear equations

Consider 2 linear equations:

$q_1 \times (\text{variable})_1 + b_1 \times (\text{variable})_2 + c_1 = 0$

$q_2 \times (\text{variable})_2 + b_2 \times (\text{variable})_2 + c_2 = 0$

Test	Conclusion
$\frac{a_1}{a_2} + \frac{b_1}{b_2} = \text{or} \neq \frac{c_1}{c_2}$	Consistent & unique solution
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Consistent & Infinite solution
$\frac{a_1}{a_2} = \frac{b_1}{b_2} + \frac{c_1}{c_2}$	Inconsistent & No Solution

An equation of the form $ax^2 + bx + c = 0$ where $a \neq 0$, b & C are numbers and 'x' is unknown, quadratic equation in x.

A quadratic equation $ax^2 + bx + c = 0$ has 2 and only 2 roots given by

$$x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$X_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Where $(b^2 - 4ac) = D$ is called the Discriminate & x_1 & x_2 both satisfy the given quadratic equation

- Irresective of $D > 0, = 0, < 0$ the

Sum of roots $= -\frac{b}{a}$

Product of roots $= \frac{c}{a}$

Using the above 2 relations the quadratic equation can be formed as follows :
x^2 – [Sum of roots] x + [Product of roots] = 0
(where x is variable quantity unknown)

- If $D = (b^2 - 4\,ac) > 0$

Case I: – $(b^2 - 4\,ac)$ is a perfect square, then, Roots are real, unequal and rational.
Value of roots

$$X_1 = \left[\frac{-b+K}{2a}\right] \quad \text{where } K = \sqrt{D} = \text{whole no.}$$

$$X_2 = \left[\frac{-b-k}{2a}\right]$$

Case II: – $(b^2 - 4ac)$ is not a perfect square.
Then, Roots are real, Unequal and Irrational.

Value of roots :

$$X_1 = \left[\frac{-b+\sqrt{D}}{2a}\right]$$

$$X_2 = \left[\frac{-b-\sqrt{D}}{2a}\right]$$

- If $D = (b^2 - 4ac) = 0$
  Then, Roots are Real, equal & Rational.

- If $D = (b^2 - 4ac) < 0$

then, Roots are Imaginary and Complex Conjugates.
eg. : $-1 + i\sqrt{3}$ & $-1 - i\sqrt{3}$

Common Root in 2 quadratic Equations: –

Let the 2 quadratic equation be
$a_1x^2 + b_1x + c_1 = 0$
$a_2x^2 + b_2x + c_2 = 0$

Then the two, equations have one root (say α) in common only if following condition is satisfied :

$$(a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1) = (c_1a_2 - c_2a_1)^2$$

$$\text{Common root } (\alpha) = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

If $(a_1b_2 - a_2b_1) = 0$, then (α) doesnot exist i.e. the 2 equation have no common root at all.

Relation between two Roots	Corresponding Formula
One root exceeds the other by K	$a^2k^2 = b^2 - 4ac$
Two roots are in the ratio of m:n	$\frac{b^2}{ac} = \frac{(m+n)^2}{mn}$
One root is p^{th} power of other	$\left(\frac{c}{a}\right)^{\frac{1}{P+1}} + \left(\frac{c}{a}\right)^{\frac{P}{P+1}} = \frac{-b}{a}$

Permutations & Combinations

Product rule: – [and → X]

- If event (I) can happen in 'm' ways and event (II) can happen in 'n' ways (Both mutually independent sub-events of the same event), then the event (I) & (II) together can happen in 'm × n' ways.

Addition rule: – [either, or → +]

- If event (I) can happen in 'm' ways & event (II) can happen in 'n' ways (Both mutually exclusive events), then the event (I) or (II) can occur in 'm + n' ways.

- **Combination → Selection of things**
- **Permutation → Arrangement of things**

Permutation includes combination. Always there is first combination & then permutation.

$\underline{|n} = n!$ denotation of factorial.

- Combination: – $^nC_r = \dfrac{n!}{(n-r)!r!}$
- Permutation: – $^nP_r = \dfrac{n!}{(n-r)!}$

Relation between Permutation & Combination:

- $^nC_r = \dfrac{^nP_r}{r!}$
- $^nP_r = {^nC_r} \times r!$
- Arranging 'n' things in a line or in a row = $n! = {^nP_n}$ (as → $O! = 1$)
- Arranging 'n' things in a circle = $(n-1)!$

Permutation of alike things:

e.g. LOLLYPOP = $\dfrac{8!}{3!\,2!\,2!}$

e.g. In how many ways 6 boys can be arranged in a row so that 2 particular boys are always together?
Ans. A B C D E F = $5! \times 2!$

e.g. In how many ways 6 boys can be arranged in a row so that 2 particular boys are never together?
Ans. $6! - 5! \times 2!$

e.g. In how many ways 6 boys can be arranged in a circle so that 2 particular boys are always together?
Ans. $(5-1)! \times 2! = 4! \times 2!$

e.g. In how many ways 6 boys can be arranged in a circle so that 2 particular boys are never together in a circle?
Ans. $(6-1)! - 2! \times 4! = 5! - 4! \times 2!$

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Probability

Probability of any event 'A' [P(A)]

$$= \frac{F}{T} = \frac{\text{No. of arrangements}}{\text{Combinations}} = \frac{\text{No. of arrangements / combination with restriction imposed}}{\text{No. of arrangements / combination without restriction}}$$

- $P(A) + P(\overline{A}) = 1$
- $0 \leq P(A)\ 1$
- If $P(A) = 0$, then event A is impossible
- If $P(A) = 1$, then event A is certain.
- P(no event occurs) = P(none) = 1 – P(atleast one event occurs)
- $P(A + B)$, $P(A \cup B) \rightarrow$ Probability of occurrence of alteast one of the events A & B (i.e. either A or B, or both)
- $P(AB)$, $P(A \cap B) \rightarrow$ Probability of occurrence of A as well as B or Probability of the simultaneous occurrence of A & B.
- $P(A\overline{B}) \rightarrow$ Probability of occurrence of only A but not of B.
- $P(A/B) \rightarrow$ Probability of occurrence of A assuming that B has already occurred. (Dependent or conditional Probability).
- $P(A/BC) \rightarrow$ Probability of occurrence of A assuming that B & C have already occurred.

Probability Theorem:

- $P(A + B) = P(A) + P(B) - P(AB)$
- $P(A + B + C) = P(A) + P(B) + P(C) - P(AB) - P(BC) - P(CA) + P(ABC)$
- $P(A/B) = \dfrac{P(A).P\left(B/A\right)}{P(B)}$
- Parallely: – $P\left(B/A\right) = \dfrac{P(B).P\left(A/B\right)}{P(A)}$
- Probability of repeated trials = $^{n}C_{r} \,.\, P^{r} .\, q^{n-r}$

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Other Important Formulae

- If 'n' is odd, then $n(n^2 - 1)$ is divisible by 24.
- If 'n' is odd, then $(2^n + 1)$ is divisible by 3.
- If 'n' is even, then $(2^n - 1)$ is divisible by 3.
- If 'n' is odd, then $(2^{2n} + 1)$ is divisible by 5.
- If 'n' is even, then $(2^{2n} - 1)$ is divisible by 5.
- If 'n' is even, then $(2^{2n} - 1)$ is divisible by 15.
- If 'n' is odd, then $(5^{2n} + 1)$ is divisible by 13.
- If 'n' is even, then $(5^{2n} - 1)$ is divisible by 13.
- If 'n' is any natural number, then $(5^{2n} - 1)$ is divisible by 24.
- If 'n' is co – prime to 5, then $n(n^4 - 1)$ is divisible by 30.
- * $(x^n + y^n)$ is divisible by $(x + y)$, when 'n' is an odd number.
- * $(x^n - y^n)$ is divisible by $(x + y)$, when 'n' is an ~~odd~~ Even number.
- * $(x^n - y^n)$ is divisible by $(x - y)$, when 'n' is an odd or even number.
- Sum of first 'n' Natural Numbers = $[n(n + 1) / 2]$.
- * Sum of first 'n' Odd Numbers = n^2.
- * Sum of first 'n' Even Numbers = $n(n + 1)$.

n ≠ nth

- Sum of the squares of the first 'n' Natural Numbers = $[n (n + 1) (2n + 1) / 6]$.
- Sum of the cubes of the first 'n' Natural Numbers = $[n(n + 1) / 2]^2$.

n = nth

- * L.C.M. of Fraction = $\dfrac{\text{L.C.M. of Numerators}}{\text{H.C.F of Denominators}}$ & H.C.F. of Fraction = $\dfrac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$
- $(a + b)^2 = a^2 + b^2 + 2ab = (a - b)^2 + 4ab$.
- $(a - b)^2 = a^2 + b^2 - 2ab = (a + b)^2 - 4ab$.
- $(a^2 - b^2) = (a + b)(a - b)$.
- $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$.
- $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$.
- $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.
- $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.
- $(x + y + z)^2 = [x^2 + y^2 + z^2 + 2(xy + yz + zx)]$
- $(x^3 + y^3 + z^3 - 3xyz) = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
- If $x + y + z = 0 \Rightarrow x^3 + y^3 + z^3 = 3xyz$.
- To find out the unit's digit in the multiplication of two or more than two numbers, multiply only the last digit of every number.

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