* Basics & intro to control.

* Any system which is not having sensor and not having provision to select the Reference i/p. is open loop system
Having sensor and provision to select Reference i/p is closeloop system.

* fan, tubelights, air-cooler, automatic washing machine, traffic lights & so on are open loop system. metadyne

* AC, Refregirator, human being, automatic iron box, gizer are clossed loop system. Fied control, D.c. motor, Automatic toaster

* Steady state error valid only for unity feedback system if non unity feedback is given to calculate steady state error we require to convert into unity feedback.

* $$T.F = \left.\frac{L(O/P)}{L(i/p)}\right|_{I_i=0} = \underline{L(IR)}$$ tranfer f^n analysis initial condition must be zero.

* impulse Responce also called as system responce, natural Responce or free forced responce.

* Basically system described in the form of openloop transfer function. $$\frac{C(s)}{R(s)} = G(s) = \frac{K(1+s\tau_1)(1+s\tau_2)\cdots}{s^n(1+s\tau_a)(1+s\tau_b)}$$ $\Rightarrow$ standard form.

$\tau \rightarrow$ time constant

Order $\Rightarrow$ maximum power in the denomenator i.e. total no. of poles in s-plane

type $\Rightarrow$ No. of poles at origin

* To find value of K convert the given f^n in standard form.

* Type and order of closeloop system defined for open loop transfer function of unity feedback system.
$\therefore$ To get type & order of close loop system required G(s) H(s) should equal to 1.

* ||

* denominator of T.F. equate to zero becomes Characteristic equation.
for close loop system characteristic eqⁿ is $1+G(s)H(s)=0$

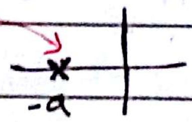
* Pole and zeros are nothing but -ve of inverse of system timeconstant $\left(-\frac{1}{T}\right)$ from standard eqⁿ.
pole $\left(-\frac{1}{T_a}\right)$ at which magnitude of T.F. is ∞
~~pole~~ zeros $\left(-\frac{1}{T_1}\right)$ at which magnitude of T.F. is 0.

* Time constant gives behaviour of system.
τ large $\Rightarrow$ slowest system Responce.
$\Rightarrow$ large time to take steady state.

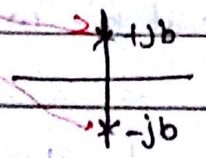
* Dominent pole :- pole which is close to imagenary axis.
Insignificant pole :- lies left most. $ISP(\tau) \leq \frac{DP(\tau)}{5}$
ISP $\leq$ 5 times D.P.

* ISP is best pole ∵ very quick Responce & more stable. because of DP system Responce becomes slow & less stable. if ISP neglected no much change in system Responce

$L[e^{-at}] = \frac{1}{s+a}$

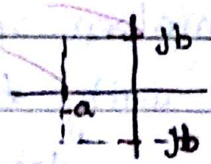


$L[\sin \text{ or } \cos(bt)] = \frac{b \text{ or } s}{s^2+b^2}$



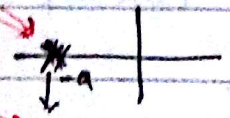
* exponential power gives real part of poles

$L[e^{-at}\sin bt]$



* sin & cosine terms give imaginary part of pole

$L[t^n e^{-at}]$



n+1 poles repeated

* t^n terms represent repeated nature of poles.

* To find equivalent system by neglecting ISP first do T.F. in standard form then value of T & K then neglect.

* finding system Responce, system stability, system time constant we consider only poles but not zeros.
  open loop zero not affect openloop stability.
  close loop zero not affect close lopp stability.
  but open loop zero affect close loop stability.

* → movement of pole in the s-plane is nothing but varying the system component values.
  → absolute stability ⇒ stable for all values of system component i.e. K from 0 to ∞.
  → conditional stable ⇒ stable for certain range of system component.
  → Addition pole or zero to TF. means adding RLC component.

* Real part of pole gives Time constant & imagnary part of pole gives frequency.

* Non Repeated poles on jω axis give constant amplitude & frequency Response undamped system. system is marginaly stable

* Pole lies on left side of splane decaying amplitude & oscillations damped system system is stable.

* poles lies on Right side of s-plane amplitude & oscillation exponentially rise to ∞ overdamped & unstable.

* Transfer f^n of opamp.

Inverting mode $\Rightarrow$ $\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$

Non-inverting mode $\Rightarrow$ $\frac{V_o(s)}{V_i(s)} = \left[1 + \frac{Z_2(s)}{Z_1(s)}\right]$

* T.F. to the diff. equations.

$$\frac{d^n y}{dt^n} = s^n Y(s)$$

T.F. is not defined for Non-linear System.

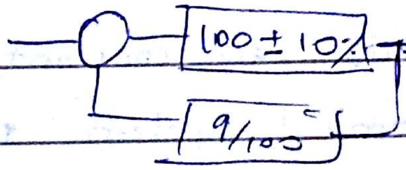
Sensitivity:- relative variation in o/p to the change in parameters ① $G(s)$ ② $H(s)$

$$S^T_G = \frac{\%\text{ of change in T.F.}}{\%\text{ of change in } G(s)} = \frac{\partial T/T}{\partial G/G}$$

||y $S^T_H = \frac{\partial T/T}{\partial H/H}$.

for open loop system $T = G$ $\quad S^T_G = 1$ highly sensitive

for close loop system $T = \frac{G}{1+GH}$ $\quad S^T_G = \frac{1}{1+GH}$ less sensitive than OLS.



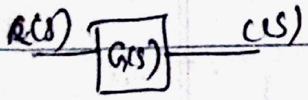
Go by $S^T_G = \frac{\%\text{change in } T}{\%\text{ changeing } G} = \frac{1}{1+GH}$ calculate from given GH

required to find.

10% given

* block diagram.

* Open loop :- * $T.F = \frac{C(s)}{R(s)} = G(s)$ O.L.T.F.

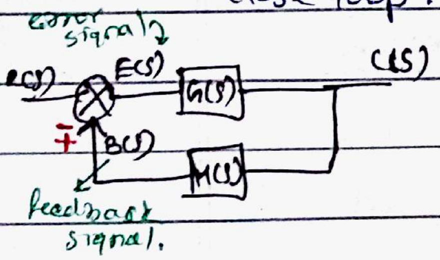


* gain is same not affected.
* more stable
* less accurate
* more sensitive w.r.t. noise & disturbance.
* B.W. is constant ∵ Gain x Bandwidth = constant. $BW(\uparrow) \propto speed(\uparrow)$ (↑)$BW \propto \frac{1}{T_r(s)}$ (rise time)
* more reliable.
* Not necessary to measure o/p errors. are not generated.
* sensors are not essential
* design easy.

* close loop :- * Forward path gain $G(s) = \frac{C(s)}{E(s)}$

Backword path gain $H(s) = \frac{B(s)}{C(s)}$

$\boxed{G(s)\,H(s) = \text{loop gain}}$



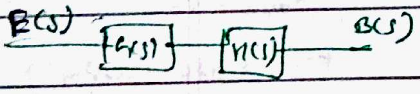
+ve feed back 0 or ±360° phase shift

-ve feed back ±180° phase shift.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

* gain is decrease by factor $1+GH$
* stability depends on loop gain.

Loop gain = 0 ⇒ OL stability = CL stability.



$G(s)H(s)$ OLTF of N.U.F. system

$H(s) = 1$ OLTF. of UF system.

* Accuracy depends on feed back N/W.

Highly accurate than OL. System.

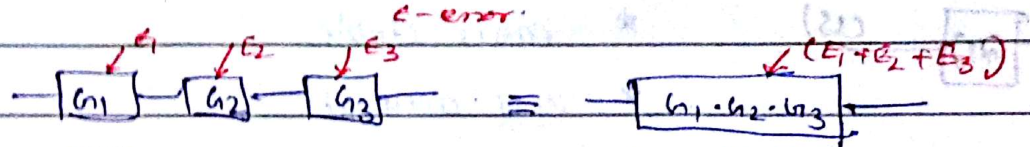
* less sensitive w.r.t. noise & disturbance.
* gain is reduced by $1+GH$ so B.W. is increased by $1+GH$. factor.
* less reliable
* o/p must be measured errors are generated.
* sensors are essential ~~design is complex~~
* design is complex

* Block diagram Reduction techniques.

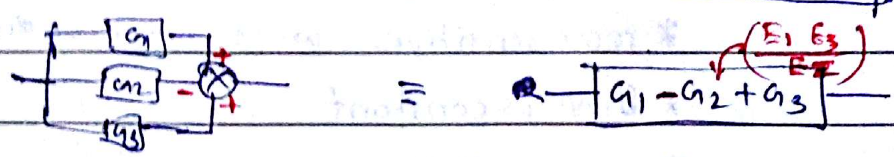
* imp. rule is gain should remain same before shifting & after shifting. maintain o/p same at each termination.

c - error.

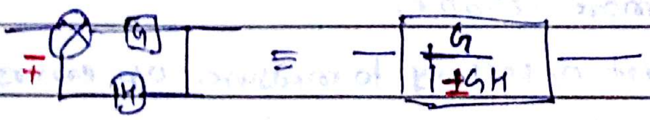
cascade * blocks



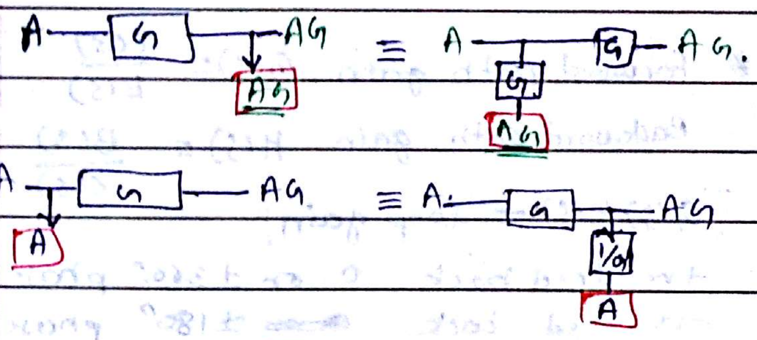
ll el *



Loop *

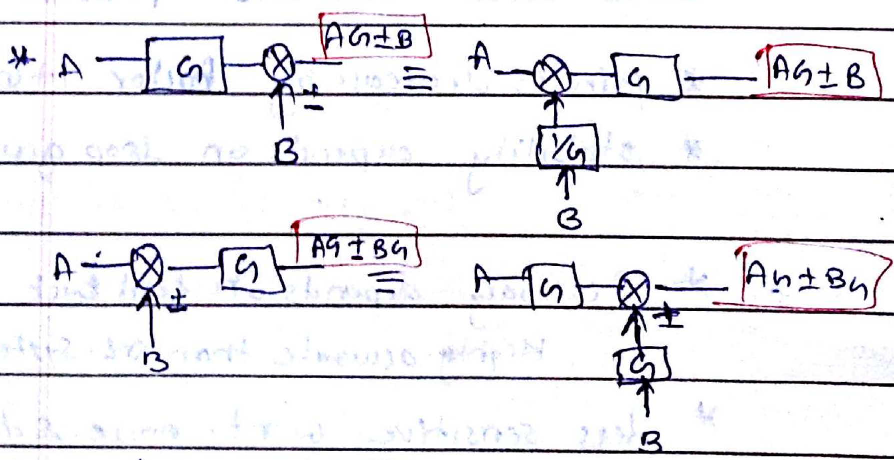


Summing pt * No change but blog should not be there in betn two s.p.



*anything loose multiply

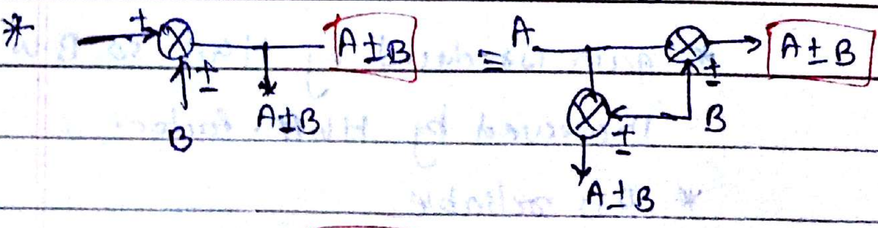
*anything extra devide

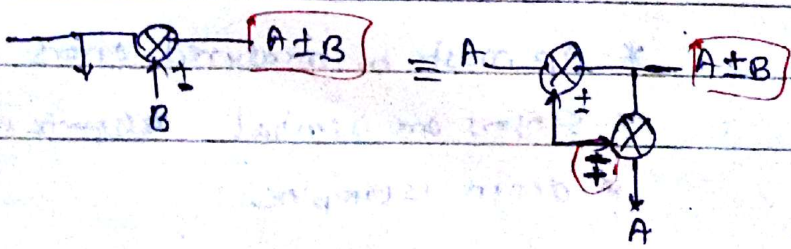


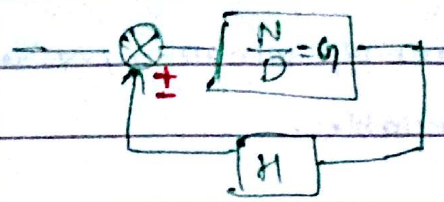
Condn to be follow

(□ → before shifting equals to

▭ → after shifting)





*  $= \frac{G}{1 \mp GH} = \boxed{\frac{N}{D \mp N \cdot H}}$

* while shifting take care of forward path gain before shifting should ~~not~~ be chang.
If forward path is charging. eleminate that factor by multiplyng ~~or~~ deviding that block in additional path.

* if no FB is mention consider open loop response.

Masons gain formula:-

$$M = \frac{Y_{out}}{Y_{in}} = \sum_{k=1}^{N} \frac{M_k \Delta_k}{\Delta}$$

M = gain betn Y_{in} & Y_{out}

Y_{out} = O/p node variable.

Y_{in} = input node variable.

N = total numbe of forward path betn Y_{in} & Y_{out}.

M_k = gain of the k^{th} forword path betn Y_{in} & Y_{out}.

Δ = 1 − (sum of all individual loop gains) + (sum of gain of all possible combination of two non. touching loops)
(minus) $\mp$ (sum of the gain products of all possible comb. of three non. touching loops + ...

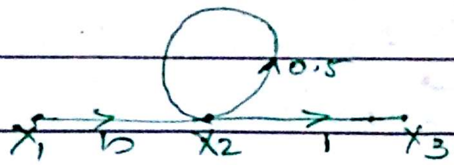
Δ_k is obtain from Δ by removing the loops touching the k^{th} fw. path.

* signal flow Graph (SFG) applied only for linear system.

* SFG may be defined as a graphical means of portraying the i/p − o/p relationship between the variables of a set of linear algebriaic equations.

* Block diagrams contains system o/p variables, system i/p variables & functional relation of the variables.

* in SFG. the nodes represent system variables.
Branch of SFG Represent functional relation of the variables.

* 

$\frac{X_2}{X_1} = \frac{1.0}{0.05} = 20$

$\frac{X_3}{X_2} = 1$

$\frac{X_3}{X_1} = 20 \times 1 = 20$

* O/p of F/B control system must be a f^n of reference i/p & error signal.

* AC control system has the advantage of smaller frame size of A.C. component

* T.F. of control system depends on system parameters only.

* Potentiometers are used in control system as error sensing transducers.

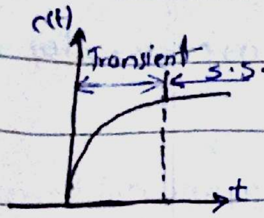
* T.F. of Tachometer is Ks.

* if BD is is complicated then go by mason's gain formula.

* if transportation lag $T_d = a$ sec

then L [transport lag T_d] $= e^{-as}$

* Time domain Analysis.

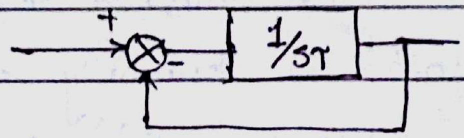


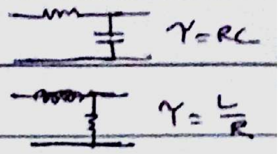
$T_{R} \cdot c(t) = C_{t} r(t) + C_{ss}(t)$

* exponential decay terms gives transient response.
  i.e. poles in the left side gives transient Response.
* Poles in the imagnary axis gives steady state respon

$\therefore$ e^{-at} terms are transient & constant or sine/cosine are SS.

* 1^{st} order system.





* $G(s) = \frac{1}{\tau s}$ $\quad H(s) = 1$ $\quad CLTF = \frac{1}{(1+s\tau)}$ | Type = 1 order = 1 (OL)

Type & order for close.L not defined

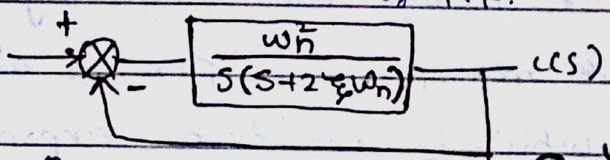
* Error is deviation of o/p from i/p $\quad e(t) = r(t) - c(t)$

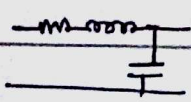
? Steady state error $= \lim_{t\to\infty} e(t)\,[r(t) - c(t)] = e_{ss}$

* to find various Response (eg. Impulse response, step response etc)
  take $R(s)$ (eg. for impulse $R(s)=1$, for step $R(s) = \frac{1}{s}$) multiply
  to T.F. & get $C(t)$ by ILT.
* for sinusoidal Response i/p. sinusoidal o/p also sinusoidal.

$$r(t) = A\sin(\omega t \pm \theta) \Rightarrow c(t) = A \cdot \underset{\text{magnitude of T.F.}}{M} \sin(\omega t \pm \theta \pm \underset{\text{angle of T.F.}}{\phi})$$

* 2^{nd} order system





① $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$ $\qquad$ ② $\frac{V_o(s)}{V_i(s)} = \frac{1/LC}{s^2 + s\left(\frac{R}{L}\right) + \frac{1}{LC}}$

comparing ① & ②.

$\omega_n = \frac{1}{\sqrt{LC}}$ rad/sec $\qquad \xi = \frac{R}{2}\sqrt{\frac{C}{L}}$ damping ratio

ratio of energy lost to energy stored.

$\xi\omega_n$ = Damping factor / actual damping.

* 2^{nd} order system Response depends on ξ only.
* 2^{nd} order system is stable for all the +ve values of ξ (i.e. $\xi > 0$)
  $\because$ poles lies in left of s plane. $\quad \xi < \infty$

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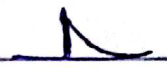
* damped, undamped, overdamped etc. graphs are generated from terms in $c(t)$.

basically two terms are there ① transient ⇒ exponential term

② steady state ⇒ sinusoidal or constant term.

if only e^{-at} (i.e.) decaying term ⇒  overdamped

if e^{-at} with sinusoidal term then ⇒  damped oscillation.

if sinusoidal term only ⇒  ⇒ undamped oscillation.

if e^{+at} (rising exponential) ⇒ 

if e^{+at} with sinusoidal ⇒  ⇒ overdamped.

* ξ is damping ratio which gives damping characteristics & pole locations.

cc *** τ (time constant) it is ∞ at $s=0$ $\because s = \frac{1}{\tau}$

if $\tau \to \infty$ (i.e. pole nearer to $j\omega$ axis) i.e. $s \to 0$

System Response is very slow. ($\xi = 0$) on $j\omega$ axis

$\therefore$ when $\xi = 0$ $\tau = \infty$ $s = 0$ system Response is slowest.

system is marginally stable with undamped oscillations since poles on $j\omega$ axis. value of pole on $j\omega$ axis i.e. imaginary value gives oscillation frequency.

& real part of pole is always related to time constant which is get from exponential term.

$\therefore$ exponential term gives real part i.e. time constant

$\therefore$ Sinusoidal term gives imaginary part i.e. freq. of oscillations

as poles ~~appr~~ $s \to \infty$, $\tau \to 0$ system is fast. & more stable ($\xi >> 1$)

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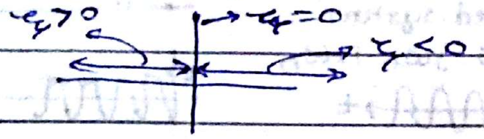
Now for 2^{nd} order system damping is depending on ξ value. following points are important

* ~~ω_n ... freq of oscillation~~ ξ damping ratio.



system is stable for $\xi > 0$
marginal stable for $\xi = 0$
unstable for $\xi < 0$

* When $\xi = 1$ $\quad \underline{s = -\omega_n}$ pole is left side $-\omega_n$ $\therefore T = \frac{1}{\omega_n}$ ($\omega_d = 0$)

$\xi = 0$ $\quad \omega_d = \omega_n$ pole on $j\omega$ axis ω_n freq. of osillat'

* When $\xi \uparrow$ from -1 to 1 2^{nd} order pole path is a circle with radius ω_n (ξ value gives pole location)

* $\xi \uparrow$ from 0 to 1 2^{nd} order pole moves from $j\omega$ axis towards left of real axis ~~and~~ through a path of radius ω_n &

$\therefore T \downarrow$ses, settling time $\downarrow$ses, $\omega_d \downarrow$ses $\therefore$ damped oscilation $\downarrow$ses.

as $\omega_d \downarrow$ses $t_r, t_d, t_p \uparrow$ses, system become fast relatively & relative stability $\uparrow$ses.

* When $\xi \geq 1$ & increases. then we get only real axis poles. i.e. two poles on real axis. ~~...move... as $\xi \uparrow$ from~~

$\xi = 1$ $\quad s = -\omega_n$ ... $\xi \uparrow$ from ... one pole is move towards origine and another pole is move towards ∞.

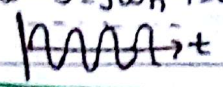
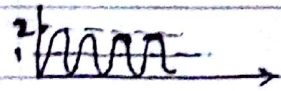
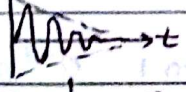
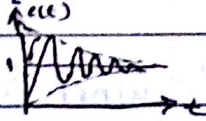
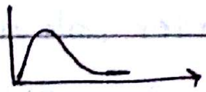
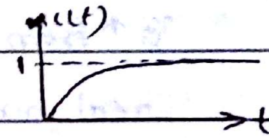
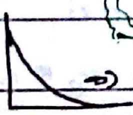
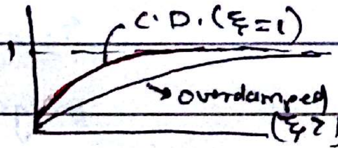
the pole which is moving towards ∞ is ISP $\therefore$ can neglected since it wont affect the system but the pole moving toward origine is dominent and having following effects

$\therefore$ at $\xi > 1$ $\quad T \uparrow$ ses. $\therefore$ settling time $\uparrow$ses. ω_d becomes 0 $\because$ pole on real axis

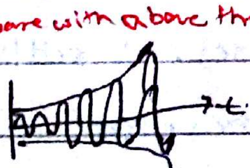
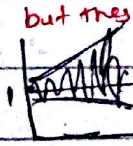
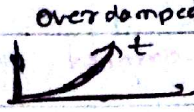
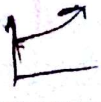
$\therefore$ stability of the system is $\downarrow$ses. ~~stable~~

* $T_{\text{undamped}} (\xi = 0) > T_{\text{overdamped}} (\xi > 1) > T_{\text{underdamped}} (\xi < 1) > T_{\text{critically damped}} (\xi = 1)$

undamped	overdamped	underdamped	critically damped
"∞"	$\frac{1}{\xi\omega_n - \omega_n\sqrt{\xi^2 - 1}}$	$\frac{1}{\xi\omega_n}$	$\frac{1}{\omega_n}$
marginally stable (poles on $j\omega$ axis)	slow responce (poles on real axis)	medium responce	fast responce

ξ Value	Polelocation	Impulse Responce	unit step responce.
$\xi=0$ marginal stable	imaginary axis $s=j\omega_n$	$c(t)=\omega_n \sin\omega_n t$ undamped. system $T=\infty$ $s=j\omega_n$ $f=\omega_n$ 	$c(t)=1-\cos\omega_n t$ — 11 — 
$0<\xi<1$ stable	left side of s plane having both real & imag. part. i.e. complex pole $s=-\xi\omega_n+j\omega_n\sqrt{1-\xi^2}$	$c(t)=\frac{\omega_n}{\sqrt{1-\xi^2}}\sin(\omega_n\sqrt{1-\xi^2})\cdot e^{-\xi\omega_n t}$ damped or underdamped.  $\tau=\frac{1}{\xi\omega_n}$ $f=\omega_d=\omega_n\sqrt{1-\xi^2}$	$c(t)=1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left[\omega_n\sqrt{1-\xi^2}+\tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]t$ 
$\xi=1$ stable	$s=-\omega_n$ on real axis	$c(t)=\omega_n^2 t e^{-\omega_n t}$  criticallydamped. $\tau=\frac{1}{\omega_n}$ $f=\omega_d=0$	$c(t)=1-e^{-\omega_n t}[\omega_n t+1]$ 
$\xi>+1$ stable	poles on real axis. $s=-\xi\omega_n\pm\omega_n\sqrt{\xi^2-1}$	$c(t)=\underbrace{K_1 e^{-(\xi\omega_n-\omega_n\sqrt{\xi^2-1})t}}_{\text{D.P.} \Rightarrow \text{affect responce}}-\underbrace{K_2 e^{-(\xi\omega_n+\omega_n\sqrt{\xi^2-1})t}}_{\text{ISP}}$  ⇒ due to D.P overdamped. system $\tau=\frac{1}{\xi\omega_n-\omega_n\sqrt{\xi^2-1}}$ minus due to DP nearjω axis $F=0$ rad/sec.	$c(t)=1-K_1 e^{-(\xi\omega_n-\omega_n\sqrt{\xi^2-1})t}-K_2 e^{-(\xi\omega_n+\omega_n\sqrt{\xi^2-1})t}$  C.D.($\xi=1$) overdamped ($\xi>1$)

Rest cases on RHS compare with above three cases for τ & f but they are unstable

$-1<\xi<0$ (unstable)	Poles on L.H.S of s plane complex conjugate	 overdamped.	
$\xi=-1$ (unstable)	on real axis $s=\omega_n$		
$\xi\geq1$ (unstable)	two poles.		

* time domain specification.

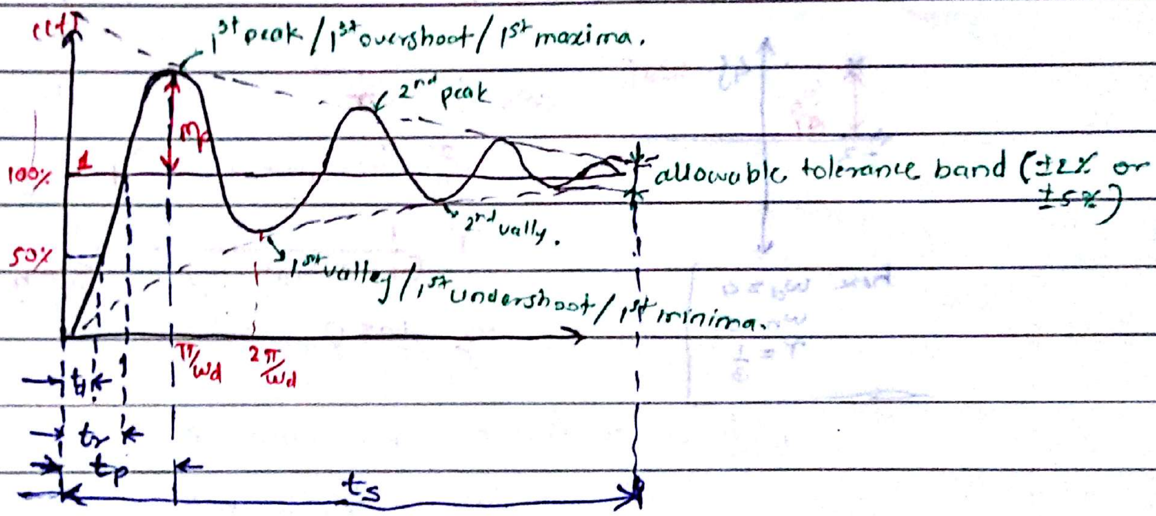
* for time domain specification we use unit step i/p

	transient	steadystate	stability	
impulse	✓	×	✓	practically not exist
step	✓	✓	✓	✓ exist
ramp	✓	✓	×	} unbounded ∴ unstable
parabolic	✓	✓	×	

* underdamped system with ξ 0.4 to 0.7 is best for time domain analysis. because it is stable, having less t_r (rise time) & less settling time (t_s)

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left[\omega_n\sqrt{1-\xi^2} + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]t$$

*



* delay time → 0 to 50% $\quad t_d = \left(\frac{1+0.7\xi}{\omega_n}\right)$ sec.

* Rise time → 10 to 90% $\quad t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)}{}$ sec

$$t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d} \text{ sec.}$$

* Peak time → 0 to peak of time response $\quad t_p = \frac{n\pi}{\omega_d} \quad n = 1$ 1st overshoot.

* Overshoot → diff. btn time response at peak to steady state value. $\quad m_p = [c(t_p) - c(\infty)]$

* % Peak overshoot → normalise diff. betn —||— $\quad \% m_p = \left[\frac{c(t_p) - c(\infty)}{c(\infty)}\right] \times 100\%$

c(∞) for unit step i/p = 1

* for 1st peak n=1, for 2nd peak n=3 (for 1st valley n=2)(2nd vally n=4) $\quad \% m_p = e^{-(n\pi\xi/\sqrt{1-\xi^2})} \times 100\%$

* settling time (t_s) → 0 to tollerance band → ±5% $t_s = 3\tau$; ±2% $t_s = 4\tau$; ±0% $t_s = 5\tau$ $\quad \left(\tau = \frac{1}{\xi\omega_n}\right)$

* time period of oscillation → $T_{osc} = \frac{2\pi}{\omega_d}$ sec $\quad T_{os} = 2t_p$ sec. $\quad$ * No. of oscillatn before reaching SS. $\Rightarrow \frac{t_s}{T_{osc}} = \frac{t_s}{2t_p}$

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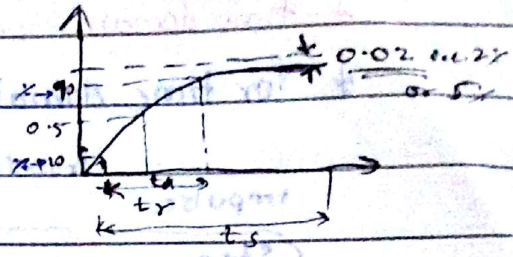
* for exponentially rise time responce



$c(t) = k(1 - e^{-t/\tau})$

$t_s = 4\tau$ if ±2% tollerance

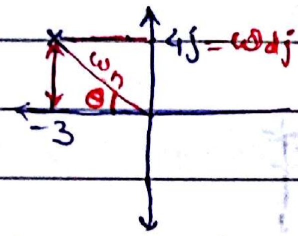
$0.5 = (1 - e^{-t_d/\tau})$ $K=1$ ∵ t_d at 0.5 find t_d.

$t_r = 2.2 \times \tau$. this formula valid for exponential rise response.

No peaks are exist ∴ No peak time no peak overshoot.

* given $c(t) = ke^{-3t}\sin 4t$ (impulse response)

here −3 indicates real part 4 indicates imagenary part.



$\omega_d = \sqrt{\omega_n^2 - \left(\frac{1}{\tau}\right)^2}$ or $\omega_d = \omega_n\sqrt{1-\xi^2}$

$\omega_n^2 = (\text{real part of pole})^2 + (\text{imag. part of pole})^2$

$\tau = \frac{1}{(\text{pole location})} = \frac{1}{\xi\omega_n}$

$\xi = \cos\theta$

here $\omega_d = 4$
$\omega_n = 5$
$\tau = \frac{1}{3}$

Now we can find time specifications for unit step i/p. using these values of ξ, ω_n, ω_d, etc

* $\xi = 0$ peak overshoot = 100% $\xi = 1$ Peak overshoot = 0%

∴ as $\xi \uparrow$ from 0 to 1 %M_p decreases from 100% to 0%

* Imp points of timedomain specifications where (pole variation) ↑ pole structure

① * As Real part of pole constant & img. part ↑ses

→ Time constant τ remains constant hence settling time is same for all poles.

→ $\omega_d \uparrow$ hence t_d, t_r, t_p ↓ses.

→ $\xi \downarrow$ hence % of peak overshoot is ↑ses.

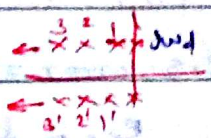
large peak overshoot indicates system is less stable & more oscillatory

optimum Range of peak overshoot is 5% to 25%

if exceeds 25% system becomes less relatevely stable

if below 5% system responce becomes very slow. $\boxed{\therefore 5\% \text{ to } 25\%}$

②* Now if poles moving towards left



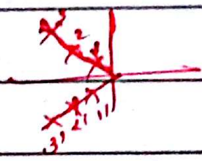
→ τ ↓ses , t_s ↓ses ω_d constant

→ t_p constant , $t_d = \frac{1+0.7\xi \uparrow}{\omega_n \uparrow}$, $t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d}$

→ $\xi \uparrow$ more relatively stable

$\xi \uparrow$ ses, $\cos(\theta \downarrow)$ ⇒ % mp ↓ses ∴ more relative stable.

③* ξ const = $\cos\theta$ ⇒ constant



τ ↓ses.

θ constant ξ = consta. ∴ % peak overshoot is constant.

τ decrea t_s ↓ses. t_d, t_r, t_p ↓ses (∵ ω_d ↑ses)

SO * When ω_d ↑ses t_d, t_r, t_p ↓ses ~~or Pole move~~

* ξ ↑ses % peak overshoot ↓ses.

* τ ↑ses t_s ↑ses.

* Poles move upword in $j\omega$ axis ω_d ↑ses τ constant

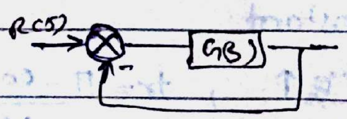
* Poles move to left from $j\omega$ axis horizontally τ ↓ses ω_d constant

(SSE) error is ⇒

* **Steady state errors :-** (deviation of o/p from the i/p)

(s.s. error ⇒ the error at $t \to \infty$)

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,E[s]$$



(UFS) $\frac{E(s)}{R(s)} = \frac{1}{1+G(s)} \Rightarrow E(s) = \frac{R(s)}{1+G(s)}$

* steady state error depends on ① type of i/p ② type of system G(s)

* SSE are require to calculate for only closed loop stable system

* SSE are valid only for unity FeedBack System (UFS).

if N.U.F.S. given then convert to UFS. & then find SSE

	Step	Ramp	Parabolic
i/p r(t)	$A\,u(t)$	$At\,u(t)$	$\frac{At^2}{2}u(t)$
e_{ss}	$\frac{A}{1+K_p}$	$\frac{A}{K_v}$	$\frac{A}{K_a}$
errors const.	K_p - position error const.	$K_v \Rightarrow$ velocity error const	K_a - acce error const
eq^n s.	$\lim_{s\to 0} G(s)$	$\lim_{s\to 0} s[G(s)]$	$\lim_{s\to 0} s^2[G(s)]$

$\boxed{e_{ss}} \Rightarrow$	(0) s^0 step	(1) s^1 ramp	(2) s^2 parabolic
Type 0	$\frac{A}{1+K_p}$ (positional)	∞	∞
Type 1	0	$\frac{A}{K_v}$ (velocity)	∞
Type 2.	0	0	$\frac{A}{K_a}$ (acceleration)

Type = i/p ⇒ constant / type > i/p ⇒ 0 / type < i/p ⇒ ∞

For e_{ss} A & K Required

K = calculate from G(s) = $\frac{\text{coeff. in numerator multiplication}}{\text{multiplication of denominator coeff.}}$

A = calculate from (i/p) comparing with (i/p) → equation on upperside

* If given a T.F. verify whether it is OL or CL. ∵ e_{ss} is valid only for CL stable system.

if OL.T.F. given convert to C.L.T.F by $\frac{G}{1+G}$ → for checking stability of CLS check stability.

check stability by R.H. criteria or other method to find R.H.S poles.

if system is CL & stable find e_{ss} or e_{ss} can not find.

Note * Check options before giving ans.

but To find e_{ss} take G(s) only OLTF to find K (don't do $\frac{G}{1+G}$ & take TF) X don't

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* if CLTF, BD or SFG is given.

$$e_{ss} = \lim_{t\to\infty} [r(t) - c(t)]$$

$$e_{ss} = \lim_{s\to 0} s[R(s) - C(s)] = \lim_{s\to 0} sR(s)\,[1 - CLTF]$$

* for unit ramp i/p $A=1$, unit step i/p $A=1$

* To find responce at $t\to\infty$ find $Y(s)$

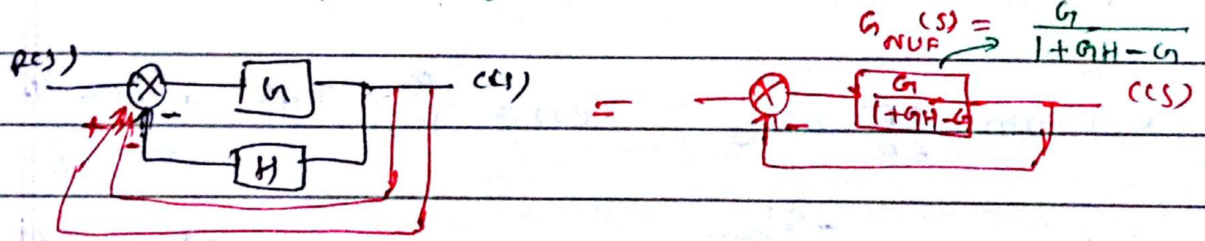
$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\,Y(s)$$

* steady state o/p (c_{ss}) mean $\lim_{t\to\infty} c(t) = \lim_{s\to 0} s[C(s)]$

* The Feed Forword path are used in the system to reduce the effect of noise and disturbance in the system.

* SSE calculated for CL System with Unity Feedback only.
if N.U.F. system is given convert to U.F. system

$$G_{NUF}(s) = \frac{G}{1+GH-G} \Rightarrow \text{O.L.T.F. of C.L. System}$$



use to Calculate e_{ss}

* (also to find type & order of NUF system use $G_{NUF} = \frac{G}{1+GH-G}$ & find Type & order)

* $T.F. = \frac{K}{1+\tau s} e^{-Ds}$ k = static gain, τ = time constant, D = transport delay

* i/p is sinusoidal o/p is also sinusoidal with extra magnitude & phase angle ^(or) magnitude obtained from T.F. & also put $s=j\omega$ in T.F. & get ω value from i/p now calculate mag. & angle

* K is spring constant M is mass of instrument then
  $\xi = 2\sqrt{KM}$ to prevent overshoot in step response

* type increases s.s.e. ↓ses. & vice versa
  introducing integral in F.W. path ↑ses Type of system ⇒ e_{ss} ↓ses
  lly diff. T.F. i.e. F.P. Gain ↓ses type hen ↑ses e_{ss}.

* ISE (integral of squared error) = $\int_0^\infty e^2(t)\,dt$ $e(t)$ = ILT of $E(s)$

$$\frac{E(s)}{R(s)} = \frac{1}{1+GH} \quad (H=1) \quad \because \text{unity feedback}$$

* (Response at $t \to \infty$ means final value)

* T.F. of Thermocouple : $\frac{C(s)}{R(s)} = \frac{K}{1+\tau s}$ ~~& $r(t) = Rt\,u(t)$~~
  ramp/rate

* as system type ↑ses s.s.e. is eliminated
* with Feedback τ ↓ses & hence response decay quickly
* Transient response depends on system only
  s.s. response depends on input only.
* presence of Non-linearities in system introduces s.s.e.

* 1 rpm = $\frac{2\pi}{60}$ rad/s $e_{ss} = \frac{R}{K_v}$ Type 1 order 2 I/P R = 10 rpm
  $R = \frac{\pi}{3}$

* for $m_p = e^{-\xi\pi/\sqrt{1-\xi^2}}$ * IR → $c(t) = -4e^{-t} + 6e^{-2t}$
  Not in %   for $t \geq 0$ U.S.R. $c(t) = \int_0^t -4e^{-t} + 6e^{-2t}$
  Solve

* $\frac{E(s)}{X(s)} = \frac{1}{1+GH}$ $e_{ss} = \lim_{s \to 0} s.E(s)$

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* stability.

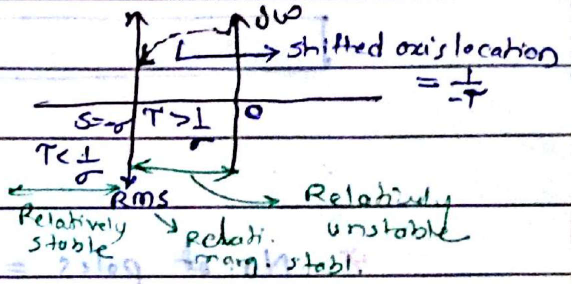
* LTI system is said to be stable if
  ① Bounded I/P gives Bounded O/P.
  ② if I/P is '0' O/P must be 0 irrespective of initial conditions.

absolutely stable :- stable for all values of system parameters.

conditional stable :- stable for certain range of system parameters

Marginal, critical, limitedly stable :- Bounded I/P & O/P maintains constant amplitude and frequency. non-repeated poles on $j\omega$ axis.

* Relative stability :- applicable for stable system only
using Relative stability concept
we can find τ, t_s, & t_{ss}.



* For Finding CL s/m stability by using RH criteria we need characteristic equation $(1+G(s)H(s))$
whereas to find stability by techniques like RL, BP, NP we require a OLTF of Unity or non-unity feedback s/m.

* R-H (Routh-Hurwitch) Criterion.

→ main purpose is to find number of poles on Right hand side.
→ To find close loop system stability.
→ To find no. of CL poles lies on left, right or Imag. axis of s-plane
→ To find range of K-values of CL s/m stability.
→ To find K-value for s/m to become marginal stable or undamped
→ To find Natural frequency or undamped oscillations.
→ To find relative stability & hence from relative stability constant we can find τ, t_s, t_{ss}

characteristic eq^n = $a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots\dots a_{n-1} s^1 + a_n s^0 = 0$

s^n	a_0	a_2	$a_4 \dots$
s^{n-1}	a_1	a_3	$a_5 \dots$
s^{n-2}	$\frac{(a_1 a_2 - a_0 a_3)}{a_1}$	$\frac{(a_1 a_4 - a_0 a_5)}{a_1}$	$\dots$
$\vdots$			
s^0	a_0		

+/−, +/−, +/−, +/− } stable

+, −, +, −, −, + } unstable

Condⁿ for s/m stability in R-H criteria.

① all coeff in 1st coulumn have same sign & no coeff in 1st colmn 0

② if any sign change occurs in 1st column s/m unstable.

no. of signchange in 1st clmn = no. of poles on RHS of S plane.

* $\boxed{as^2 + bs + c = 0}$ a,b,c > 0 or < 0 ⇒ s/m stable. ∵ same sign

b = 0, ac < 0 or ac > 0 ⇒ marginal stable.

* $\boxed{as^3 + bs^2 + cs + d = 0}$ bc > ad ⇒ s/m stable

bc = ad ⇒ marginal stable → take even powers $bs^2 + d = 0$ ∵ $s = \pm j\sqrt{\frac{d}{b}}$

bc < ad ⇒ unstable.

~~freqⁿ of oscillatⁿ~~ ⇒ $\boxed{\omega_n = \sqrt{\frac{d}{b}}}$

* No. of poles = highest power of chara. eqⁿ.

No of poles in RHS = no. of sign changes.

* Case 1 if any elem in a 1st coulm is 0 replace it by positive constant ξ & continue process. finally substitute ξ = 0 & check the sign changes.

* Case 2 if all the elements in a row becomes 0. find AE (Auxillary eqⁿ) of above row take diff. of AE. w.r.t. s. then take coeffs of Result (diff AE coeff) & put on 0 row. & continue the process

→ The row of zero occures when poles are symmetrical about origine. i.e. A.E should consist only even power of s terms ∵ roots are symm. about origine.

→ The AE gives no. of poles on jω axis. s/m is marginally stable

→ in RH tabular form if only once Row of zero occures & all the coefficients in the 1st colmn is +ve then the s/m is marginally stable. ∵ poles on jω axis & non repeated.

→ if sign changes occures below AE there should be symmetrical pole in RHS & LHS of the splane.

if sign change in the above of AE no. summetrical pole in the left to the pole placed in the right side.

* no. of time ~~repeated~~ zero row occurs in R.H. table given no. of time repeated poles ~~are on jω axis~~. sym. about origine.
if only one time zero row Non-repeated ~~poles on jω axis~~.
sym. poles about origine.

* if only one zero row occures dosnt mean poles on jω axis **.

* for conditional stable system
find K value from eqn of K in 1st column comparing with 0
(expresn of K) > 0 stable < 0 unstable = 0 marg. stable then we find ω_n for = 0

* To find relative stablity first find whether given system is stable or not ∵ relative stability only given for stable system.
stabity about $s = -a$ replace s by $(z-a)$ and find RH table & conclude the Relative stability.

* limitations of RH criteria.
1) By using RH criteria we cant identy fy the nature of the system. becz we cant determine the exact locatn of the pole
2) RH criteria not applicable for exponential or sign cosine terms. ∵ it gives infinite series. ∴ RH criteria applicatable for only finite no. of ~~poles~~ terms
But using RH criteria we can get approximate soln to the exponential term. by neglecting higher order terms i.e $e^{-sT} = (1-sT)$

* Root locus.

→ To find CL s/m stability.

→ To find range of K-values for s/m stability.

→ t.f. K values for s/m to be marginal stable

→ find natural freq^n of oscillat^n when s/m is marginal stable

→ K values to become s/m undamped, underdamped, CD, OD

→ To find relative stability. RL moving left more relatively stable else RL branch moving RHS side less relatively stable.

→ Best method to find absolute stability.

* Root locus means close loop poles path by varying k value from 0 to ∞.

* We can't draw root locus by characteristic eq^n bcz as order ↑ses finding roots is difficult.

∴ To draw R.L. we use O.L.T.F. But for the stability analysis we use C.L.T.F.

* Relation Bet^n OLTF & CLTF - poles & zeros

→ closed loop poles are nothing but open loop poles & open loop zeros. multiplied by const factor of system gain K. $\boxed{D(s) + KN(s) = 0}$

① K=0 closed loop poles = openloop poles.

② K=∞ closed loop poles = openloop zeros.

[⇒ K↑ses [0→∞] ⇒ O.L poles → O.L zeros

K↓ses [∞→0] ⇒ O.L zeros → O.L poles]

0 to ∞ 0 to ∞

* To draw R.L no. of poles = no. of zeros

if zeros are less consider zero's are ∞.

R.L. starts at poles and ends at zeros.

* angle condition. (A.C.)

-ve feed back chara. $Eq^n = 1 + G(s)H(s) = 0$

$\angle G(s)H(s)$ = odd multiples of $\pm 180 = \pm[2q+1]180^\circ$

$q = 0, 1, 2 \ldots$

Direct R.L. 180° Rules.

+ve feedback chara. $eq^n = 1 - G(s)H(s) = 0$

$\angle G(s)H(s)$ = even multiples of $\pm 180 = (2q)180^\circ$ $q = 0, 1, 2$.

Inverse R.L complementary RL. 0° Rules

* Purpose of Angle condition to check any point lies on R.L or not. i.e. All pts on R.L. must satisfy angle condition.

Put s value in G(s) H(s) of given pt & find $\angle$ if 180° Rule ✓ if 0° Rule ✗

* Magnitude condition. (M.C.)

$|G(s)H(s)| = 1$ at any pt on R.L.

purpose is to find s/m gain at any pt.

if pt is given find if pt is on RL or not by 2 condition then by mag. condt^n $|G(s)H(s)| = 1$ find value K at that point.

following points are imp for the construction of R.L.

① Symmetrical :- RL is symmetrical about Real axis.

BP is no. symm. ∵ it plotted on semilog paper which is nonlinear

NP is symm.

② No. of root locus branches or no. of loci :-

No. of R.L. branch is max. no. of (Poles or zeros)

③ R.L. branches :-

A point to exist on R.L the sum of the poles & zeros on RHS of the pt. should be odd

Never apply A.C. & M.C. for poles & zeros.

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④ Assymtotes :- Asymptotes are R.L. branches approaching to ∞

No. of Assymptotes $n = P - Z$

angle of assymptotes $\theta = \frac{(2q+1)180°}{P-Z}$ $q = 0, 1, 2, \ldots, P-Z-1$

Assymptotes are symmetrical about Real axis.

⑤ Centroid : intersection of assymtotes on Real axis

$$\text{Centroid} = \frac{\text{Sum of real parts of poles} - \text{Sum of real parts of zeros}}{P - Z}$$

→ centroid may or may not be on R.L. branch

- from centrol we draw assymptotes with an angle of assympotes.

⑥ Break point: pt at wich two or more poles meets

Root Locus branches enters or leaves the real axis with

an angle of $\pm\frac{180°}{n}$ when n is no. of poles at the break point

BAP → Break away point :- RL leaves Real axis

BIP → Break in point :- RL enters in Real axis.

→ minimum 1 break away pt btn two adjusent pole if RL branch is exist in btwn them

→ minimum 1 break in pt btn two adjucent zero if RL branch exist in betwn the zeros.

→ left most side of left most zero if exist RL branch then at least one break in point would be there

→ lly if there exis RL branch of left most side of left most pole then minimum one breakaway point.

⑧ To find Break point

① characteristic eqn $G(s)H(s)$

② Rearrange eqn in the form of $k = f(s)$

③ diff. k w.r.t. s & make it equal to 0

④ roots of $\frac{dk}{ds} = 0$ gives Break point

⑤ take valid break point according to R.L. branch.

* (Denominator of OLTF diff. w.r.t. s & equat to 0 find values of s

⑦ Intersection point with imaginary axis.

① form characteristic eqn

② Write RH criteria Tabular form.

③ find K-marginal value.

④ form the AE equation

⑤ Root of AE equation gives valid and invalid intersection pt.

for valid intersection pt. with imag. axis K marginal value is +ve

⑧ Angle of departure and arrival

Angle of departure is calculated at complex conjugate poles

Angle of arrival (departure) is calculated at complex conjugated zeros.

* Angle of departure (ϕ_d) → it gives the ∠ at which poles leaves from initial position i.e. R.L. branch starts at an angle of ϕ_d (at complex pole)

$$\phi_d = 180^\circ + \angle GH = 180 - \phi \quad [\text{Where } \phi = \Sigma\phi_p - \Sigma\phi_z]$$

* Angle of arrival (ϕ_a) → it gives ∠ at which poles terminated at complex zeros. i.e R.L. branch ends at an angle ϕ_a (in complex zero)

$$\phi_a = 180^\circ - \angle GH = 180 + \phi \quad [\text{where } \phi = \Sigma\phi_p - \Sigma\phi_z].$$

(while finding angle put s = value of pole at which ∠ is finding)

* Method to draw Root locus:

1. Identify the real axis root locus branches & break points
2. find the angle asymtotes and centroid (if required)
   find angle of departure & arrival.
3. Vary k values from 0 to ∞ & identify the path from pole to zero. . The R.L. is symmetrical about real axis. & all the poles must reaches to zero.

* find nature of system OD, CD, UD, undamped etc. to get K value for diff. nature we require K value at Break point then conclude nature of system w.r.t. k value

* at break point s/m is CD. to find K at CD use magnitude condn.

* step by step procedure

① Locate poles & Zeros in Splane.

② Identify no. of branches & starting poin(poles) & ending points of branches.

③ find centroid & assymptotes.

④ Break points (if necessary)

⑤ intersection point on jω axis.

⑥ angle of departure if complex poles or arrival if complex zeros

⑦ conclude for range of K values & nature of system.

* To find poles & zeros use OLTF.

* To find Break point & to find intersection pt on jω axis (i.e. $K_{marginal}$) use characteristic eqn of closeloop T.F.

* to find angle of departure or arrival use T.F. G(s) H(s) OLTF

* when no. poles = no. of zeros centroid & asymptotes not required.

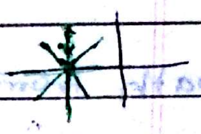
* When B.P. = centroid all poles are symmetrical abt B.P.

CLTF for this case is $\frac{1}{(s+BP)^4}$ (see example)

centroid

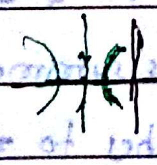
* σ = Real part of complex pole.

if Symmetric 1B.P.



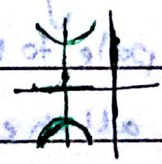
if complex poles are very near to Real axis

1BP + 2BP.



complex poles are away from real axis

1B.P. 1 pair of complex BAP



* whenever complex poles are very close to real axis as compared to imaginary axis then no. of B.P. on real axis increases.

* spirule is used to draw root locus & calibrate it in terms of variable parameter.

* Bode plot:

→* To draw frequency Responce of OLTF.

→ To find close loop system stability

→ To find gain margine phase margine, gain cross over frequency & phase crosscover frequency.

→ To find Relative stability by using gain margine and phase margine.

gain margin large more relatively stable but s/m is low.

if gm & pm is less then less relatively stable but s/m becomes oscillatory.

∴ gm & pm should be optimum gm ⇒ 5 to 10 dB pm ⇒ $30°$ to $40°$ (range)

Bodeplot = ① magnitude plot + ② phase plot.

magnitude plot :→ Decade (10 times change in frequency) $\omega_2 = 10\omega_1$ decade means 20dB/decade

* Procedure to draw the bodeplot.

① S is replaced by $j\omega$ to convert it to frequency domain.

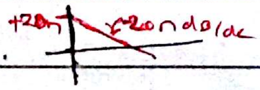
② write magnitude convert it in to dB, mag. in dB is $20\log|G(j\omega)H(j\omega)|$

③ find the phase angle by using $\tan^{-1}\left[\frac{\text{imaginary part}}{\text{Real part}}\right]$

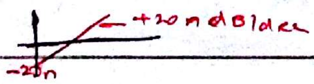
④ Vary 'w' from minimum to max value. & draw mag. & phase plot.

* shift in the magnitude plot depends on the K value. whereas the phase plot is independent of K value.

* n poles at origine ⇒ mag. plot starts with mag. +20n with slope of −20n dB/decade.



* n zeros at origin ⇒ mag plot starts with mag −20n with slope of +20n dB/decade



* $\frac{K}{s^n}$ same as n poles at origine but graph is shifted by $20\log K$ upward. if K=1 mag. plot pass throug $\omega = 1$

axis of magnitude plot is 0dB line.

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* n-infinite poles $\Rightarrow H(s) = \frac{1}{(sT+1)^n}$

mag. in dB $= -20n \log \sqrt{(j\omega T)^2 + 1}$

$\angle\phi_{actual} = -n\tan^{-1}(\omega T)$

* n-infinite zeros $\Rightarrow H(s) = (sT+1)^n$

mag. in dB $= 20n \log \sqrt{(j\omega T)^2 + 1}$

$\angle\phi_{actual} = n\tan^{-1}(\omega T)$

* corner freq^{n}cy $= \frac{1}{T}$ freq. at which slop is changing.

< corner freq. slope of pole & zero is 0 dB $\angle\phi = 0°$.

> corner freq. slope of pole is ~~0 dB~~ $-20n$ dB/dec

slope of zero is $+20n$ dB/de.

(n is repeated no. of poles)

$\angle\phi$ for poles $= -90°n$ $\angle\phi$ for zeros $+90°n$.

* Error $= \underbrace{\{\text{Actual value}\}}_{(T.F.)} - \underbrace{\{\text{Assymptotic value}\}}_{(B.P)}$

→ The error at corner freq. is very large. eigthe above or below corner frequency the error decreases symmetrically below one decade whatever the error exist after 1 decade. the error is same. error is symm. about corner freq.

* When f^n is given convert it in to standard Transfer f^n

$$\frac{K(1+sT_a)(1+sT_b)\cdots}{(1+sT_1)(1+sT_2)}$$ & invers of T gives cornerfrequencies.

*eg.

* change in slop is slope of no. of finite poles / finite zeros at perticular corner frequency. × by ±20. -20 for pole, +20 for zero

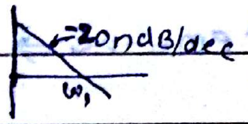
$\frac{k(1+\frac{s}{3\omega})^m}{(1+\frac{s}{2})^n}$ change in slope at $\omega=2$ is $-20n$ dB/dec
change in slope at $\omega=3$ is $+20m$ db/dec

* Slope of line betⁿ the two corner frequency from ω_1 to ω_2: consider T.F. up to ω_1 only, get diff. betⁿ poles & zeros. (d)
the slop = ±20d (+ if z>p, - if p>z)

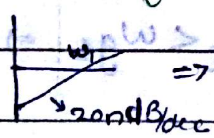
* Slope around corner frequency :- around ω_1 means
① below ω_1 (exclude ω_1 & above freq. terms) in T.F. get slope
② above ω_1 (include only less that ω_1 & ω_1 also)

* To find T.F from magnitude plot.
① observe the initial slope it gives the poles & zeros at origine.
② find change in slope at each corner frequency. if change is +ve consider finite zeros ($\frac{slope}{20}$) no. of zeros. if slope -ve then poles.
③ find K value by using known magnitude at known frequency.
To find K value take initial slope. if we know mag. for some corner freq. take transfer fⁿ up to that corner frequency and then take log eqⁿ & get K value.

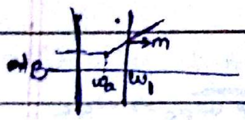
eg



$\Rightarrow k=(\omega_1)^n$

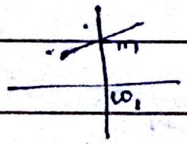


$\Rightarrow k=(\omega_1)^{-n}$



$m = 20\log k + 20\log\frac{\omega_1}{\omega_0}$

*



$m = 20\log k + 20\log\omega_1$

* $\text{slope} = \frac{dm}{d\log\omega} = \frac{m_2-m_1}{\log\omega_2-\log\omega_1}$

* log base in these calculation is 10 * dont take ln.

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* minimum phase s/m : s/m in which all the finite poles and finite zero's lies in the left of splane.
it gives phase angle less than or equal to $-90°$

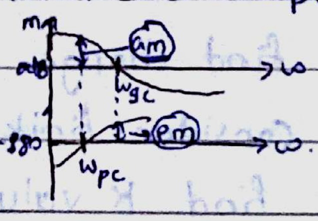
* All pass sytem: all poles lie's in left & all zeros are lies in right.
$m = 1 \quad \phi = \pm 180°$.

* Non-minimum phase system: one or more poles or zeros or both lies on right. this system gives more negative angle at $\omega = \infty$

$$\boxed{\phi_{NMPS} = \phi_{MPS} + \phi_{ALPS}.}$$

* Gain cross over frequency :- ω_{gc} is a freq^n at which magnitude $|G(s)H(s)| = 1$ or 0 dB. the phase value at ω_{gc} is called phase margine. (PM)

* Phase cross over frequency :- ω_{pc} is a freq. at which phase $\angle$ is $-180°$. the gain value at ω_{pc} is called gain margine (GM)



* $GM_{dB} = -20\log|G(j\omega)H(j\omega)|\Big|_{\omega=\omega_{pc}}$ dB

* $PM = 180° + \angle GH(j\omega)\Big|_{\omega=\omega_{gc}}$.

when $\boxed{\omega_{pc} > \omega_{gc}} \Rightarrow GM > 1$ & PM +ve $\Rightarrow \boxed{\text{s/m stable}}$
+dB
$\boxed{\omega_{pc} = \omega_{gc}} \Rightarrow \underset{0\,dB}{GM} = 1$ & $PM = 0° \Rightarrow \boxed{\text{marginal stable}}$
$\boxed{\omega_{pc} < \omega_{gc}} \Rightarrow GM < 1$ then & $PM = -ve \Rightarrow \boxed{\text{Unstable.}}$
$= -ve$ dB

* $G(s)H(s) = \frac{1}{s} \Rightarrow \omega_{pc} = \infty$ (constant phase of $-90°$) $\Rightarrow$ stable
$\hookrightarrow \omega_{pc} >> \omega_{gc}$
$= \frac{1}{s^2} \Rightarrow \omega_{pc} = \omega_{gc}$ (constant phase of $-180°$) $\Rightarrow$ marg. stable
$\hookrightarrow \omega_{pc} = \omega_{gc}$
$= \frac{1}{s^3} \Rightarrow \omega_{pc} = 0$ (constant phase of $-270°$) $\Rightarrow$ Unstable.
$\hookrightarrow \omega_{pc} << \omega_{gc}$.

Actually ω_{pc} doesn't exist

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* ~~Plotting~~ complex bode plots.

* n complex poles $G(s)H(s) = \left(\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}\right)^n$

put $s=j\omega$ & take magnitude

we get ⇒ $M = \left[\frac{1}{\sqrt{[1-(\frac{\omega}{\omega_n})^2]^2+[2\xi(\frac{\omega}{\omega_n})]^2}}\right]^n$

in dB $Mag_{dB(Actual)} = -20n\log\left[\sqrt{[1-(\frac{\omega}{\omega_n})^2]^2+[2\xi(\frac{\omega}{\omega_n})]^2}\right]$

(ω_n = corner freq^n)

$\phi_{actual} = -n\tan^{-1}\left[\frac{2\xi(\frac{\omega}{\omega_n})}{1-(\frac{\omega}{\omega_n})^2}\right]$

M correction at C. freq. = $\boxed{-20n\log 2\xi}$

ϕ correction at cor. freq = $\boxed{-90^\circ}$

* n complex zeros. Mag dB (only) same as above & ϕ too only change in sign.

zero M is M^{-1}_{poles}

M correction & ϕ correction also only change − to + here (+)

* correction at corner frequency depends on ξ

other than corner frequency depends on ξ and ω ~~in~~ mag. plot

Whereas in phase plot correction at cor. freq. is constant

other than corner freq. depends on ξ & ω.

* $\left(1+\frac{s}{20}+\frac{s^2}{100}\right)$ compare with $\left(\frac{s^2+2\xi\omega_n s+\omega_n^2}{\omega_n^2}\right)$

$= \left[1+\left(\frac{2\xi}{\omega_n}\right)s+\left(\frac{s}{\omega_n}\right)^2\right]$

& find ξ & ω_n

* B.W. is inversly ∝ to risetime but increases noise as BW ↑.

* Polar plots

→ To draw frequency response of CLTF.

→ To find CL s/m stability.

→ To find gain margine and phase margine

→ To find relative stability.

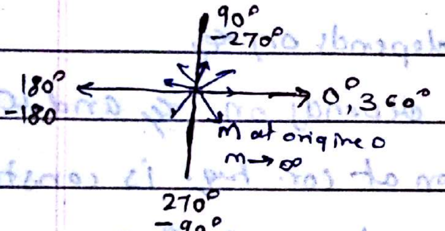
→ The polar plot is not a complete freq. plot. ∵ frequency range $0 \to \infty$

Nyquist plot is complete freqⁿ plot ∵ $-\infty$ to $+\infty$.

→ Polar plot is nothing but magnitude Vs phase plot.

To draw polar plot

std procedure: take various values of M & ϕ of $G(s)H(s)$ by putting various values of ω ranging from $0 \to \infty$ & plot graph.



$G(s)H(s) = \dfrac{K(s+2)}{(sT+1)}$ etc put $s = j\omega$.

$\angle K = 0°$

$\angle G(s)H(s) = \angle K + \angle \tan^{-1}\frac{\omega}{2} - \angle \tan^{-1}\left(\frac{\omega T}{1}\right)$

$|G(s)H(s)| = \dfrac{|K| \cdot \sqrt{(j\omega)^2 + 2^2}}{\sqrt{(j\omega T)^2 + 1^2}}$

another procedure: * if mag. at $\omega = 0$ ≷ mag at $\omega = \infty$ then go for following method to draw polarplot

① find the mag. & phase at $\omega = 0$ i.e. $m_1 \angle \phi_1$

②. find mag & phase at $\omega = \infty$ i.e. $m_2 \angle \phi_2$

③ ending direction → $\phi_1 - \phi_2$ ⇒ +ve → clockwise.
-ve → Anticlockwise.

④ starting direction considered to the transfer fⁿ. It should have only +ve sign. terms.

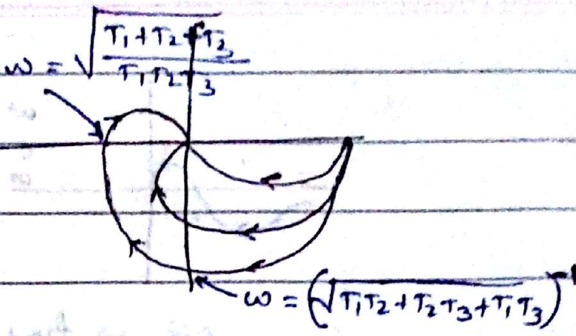
if finite poles is near to imag. then direction is CW ~~oox~~ → zero

if finite zero is near to the imag the direction is ACW ~~xxo~~ → ACW.

if zeros at origine → std procedure
if Z > P → std procedure

if P = Z first observe mag at 0 & ∞ then another procedure

* $GH(s) = \dfrac{K}{(sT_1+1)(sT_2+1)(sT_3+1)}$

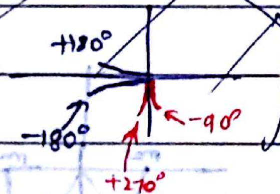


(addition of each finite pole shift ending angle by $-90°$)

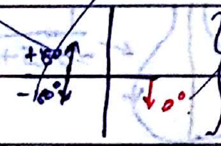
* for $\dfrac{1}{(sT_1+1)(sT_2+1)}$ starting ϕ = direct wise normal line. to the lin ϕ. If mag is not ∞

ending ϕ = direction wise parallel line coincide line towards the line.

line ending with llel or coinciding with 180° line

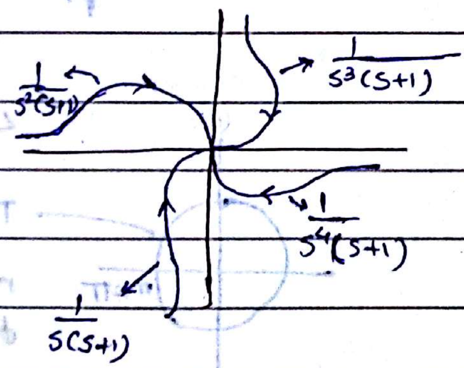


ending with 180°



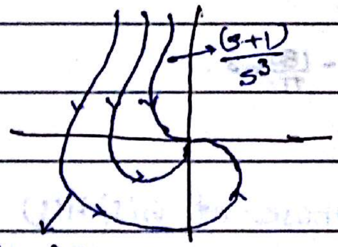
starting with 180°

*



(addition each pole at origine shift the total plot by $-90°$ in CW direction)

*

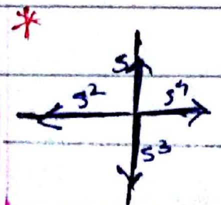


$\dfrac{(s+1)(s+2)(s+3)}{s^3}$

(addition of each finite zero shift the ending angle by $+90°$ in ACW directn)

* whenever transfer fn consist only poles or only zeros at origine. polar plot is angle line

*



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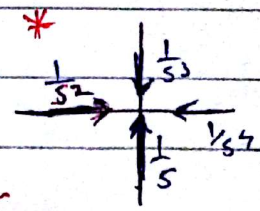
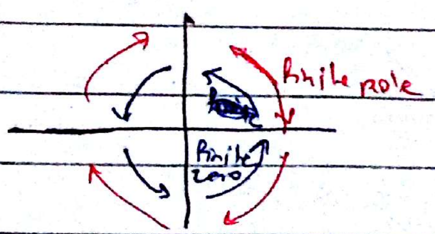


(if T.F. $= \dfrac{1}{s^2} \ldots$ plot starts from $-180°$ line)
& so on..



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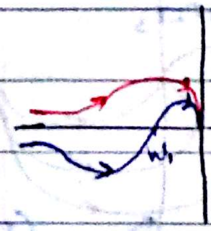


ω	m	ϕ
0	∞	-180°
∞	0	-270°

zero nearer to $j\omega$ axis

$$\frac{(s+1)}{s^2(s+2)(s+3)}$$

*starting direction ACW
ending direction CW.

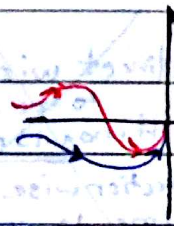
$$\frac{(s+3)}{s^2(s+1)(s+2)}$$

*starting direction CW.
ending direction CW.

pole nearer to $j\omega$ axis.

OR ⇒ find intersection point with -180° line $\omega_1 = 1$ rad exist
$\omega = \pm j\sqrt{5}$ not exist

My *



ω	m	ϕ
0	∞	-180°
∞	0	-90°

zero nearer to $j\omega$ axis

$$\frac{(s+1)(s+2)}{s^2(s+3)}$$

starting D → ACW
ending D → ACW.

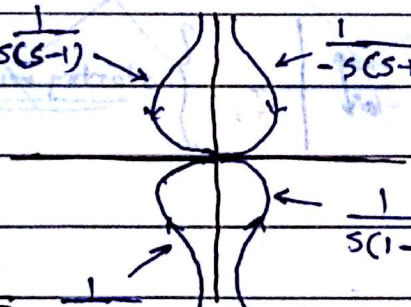
$$\frac{(s+2)(s+3)}{s^2(s+1)}$$

St. D. → CW.
E. D. → ACW.

pole nearer to $j\omega$ axis

*

$\frac{1}{s(s-1)}$

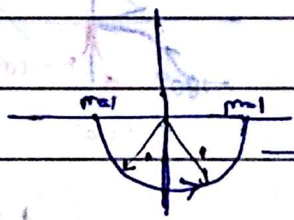


$\frac{1}{-s(s+1)} = \frac{1}{s(-s-1)}$

$\frac{1}{s(1-s)}$

$|m| = \frac{1}{\omega\sqrt{\omega^2+1}}$ ⇐ $\frac{1}{s(s+1)}$

$\angle = -\tan^{-1}\frac{\omega}{1} - 90^\circ$

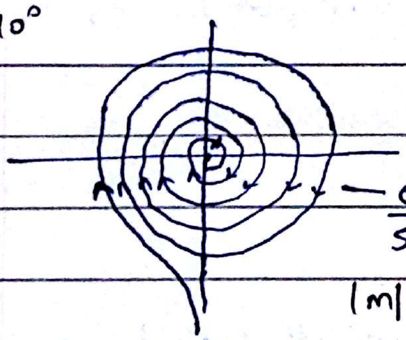


$\frac{s+10}{s-10}$

Here

$|m| = \frac{\sqrt{\omega^2+10}}{\sqrt{\omega^2+10}}$

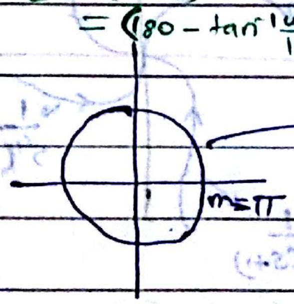
*** $\angle$ of $(s-10)$
$= \left(180 - \tan^{-1}\frac{\omega}{10}\right)$ $\angle = \tan^{-1}\frac{\omega}{10} - (180 - \tan^{-1}\frac{\omega}{10})$



$$\frac{e^{-s}}{s(s+1)}$$

$|m| = \frac{1}{\omega^2\sqrt{\omega^2+1}}$

$\angle\phi = -90^\circ - \tan^{-1}\frac{\omega}{1} - \frac{180}{\pi}\times\omega$.



$\pi e^{-0.255}$

$m = \pi$

$\phi = 0.255\,\omega \times \frac{180}{\pi}$.

* Polar plot is a magnitude & phase of $G(s)H(s)$

* Nyquist plot.

→ To draw freq. response of open loop tranfer function

→ To find number of closed loop poles in the right of s-plane.

→ To find the range of K-value for system stability.

→ To find GM, PM, ω_{gc} & ω_{pc}.

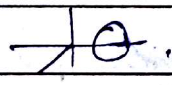
→ To find relative stability using GM & PM.

Nyquist stability criteria developed by using principal of arguments.

Principal of arguments→ It states that if there are poles and zeros are enclosed by the random selected closed path then the corresponding G(s)H(s) plane encircles origin with P-Z times i.e. $N = P - Z$.

* $N = P - Z = +2$ ACW +ve direction 

* $N = P - Z = -2$ CW -ve direction 

* $N = P - Z = 0$ 

* the principle of argument concept is applied to the total right of the s-plane with ∞ radius. call, Nyquist countour.

* Nyquist stability analysis is right of s-plane analysis.

* $N = P - Z$ here P → Poles of CE in the RHS of s plane.
i.e. OLTF. poles in the RHS of splane

* [1+G(s) H(s) ⇓ Characteristic equation] & Z → zeros of CE in the RHS of splane
i.e. CLTF. poles in the RHS of s-plane.

N ⇒ No. of encirclement about critical point
(critical pt ⇒ (−1 + j0))

To become C/L s/m stable there shouldn't be any CL pole in the RHS plane CL pole is nothing but Zero's of CE.

∴ $Z = 0$, $N = P$

* ∴ No. of encirclement about the critical point must be equal to poles of CE. which are nothing but O.L.T.F. poles

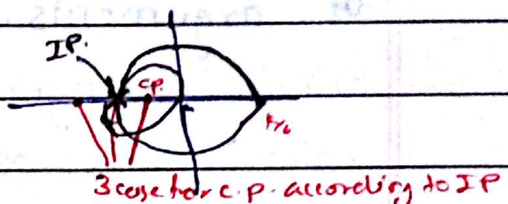
$N = P$ & $Z = 0$

While drawing N.P.

① draw polar plot of given TF $G(s)H(s)$ then take mirror image with opposite direction or direction must be continuous to form a loop.

② if in T.F. 'n' ~~open~~ poles at origin then encirclement of $180° \times n$ ~~~~ with ∞ radius. & direction of this must be continuos with above polar plot. see example in book note to clear.

* $\frac{K}{(s+1)(s+2)(s+3)}$ ⇒ N.P. ⇒



→ find I.P. at $-180°$ line like in polar plots.

→ for stability $N = P$. (here $P = 0$. ∴ N should be zero for stability.

∴ IP should < CP. for stability. find IP & conclude for 3 cases of K values

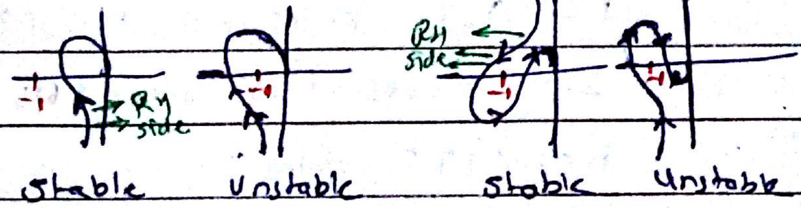
? (at IP = CP marginal stable)?

? * In any problem given polar diagram means there is no effect of infinite radius half circle on s/m stability.

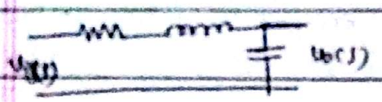
? * Whenever the polar plot intersects -180 line with magnitude less than 1 the s/m is stable. ∴ $\omega_{pc} > \omega_{gc}$.

OR if equal to 1 (IP = C.P.) then s/m is marg. stable ∵ $\omega_{pc} = \omega_{gc}$

OR if greater than 1 the s/m is unstable ∵ $\omega_{pc} < \omega_{gc}$.

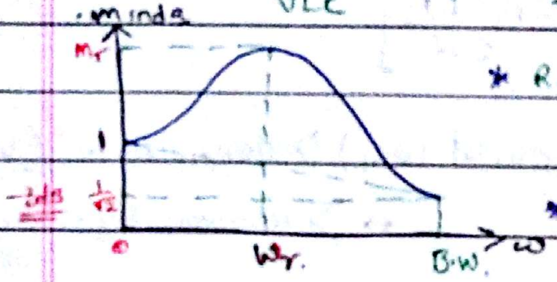
? 

* Frequency domain Specifications:



$$\frac{V_o(s)}{V_i(s)} = \frac{1/LC}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

$$\omega_n = \frac{1}{\sqrt{LC}} \qquad \xi = \frac{R}{2}\sqrt{\frac{C}{L}} \qquad \theta = \frac{1}{2\xi}$$



* Resonant freq^n $(\omega_r) = \omega_n\sqrt{1-2\xi^2}$ rad/sec

Freq^n at with max. magnitude.

* Resonant peak $M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$

max. magnitude at resonant frequency.

* Frequency domain specificat^n valid when $\xi < \frac{1}{\sqrt{2}}$

$\xi > \frac{1}{\sqrt{2}}$ there no resonant peak on resonant frequency.

$\because$ $\xi = \frac{1}{\sqrt{2}} \Rightarrow \omega_r = \omega_n\sqrt{1-2\xi^2} = \underline{\underline{0}} \Rightarrow \underline{\underline{M_r = 1}}$

* Bandwidth :-

$$B.W. = \omega_n\sqrt{1-2\xi^2 + \sqrt{2-4\xi^2+4\xi^4}}$$

range of freq^n at which mag. drops to $-3dB$ i.e. $\frac{1}{\sqrt{2}}$ from the max value at the low freq^n.

Calculation of Gain Margine & phase margine (T.F. given)

GM	P.M.
① find ω_{pc} using $\angle GH = -180$	① find ω_{gc} at $\lvert GH \rvert = 1$ i.e. $m = 1$
② find $m\rvert_{\omega_{pc}}$ (magnitude at ω_{pc})	② find $\angle GH$ (p.m.) at ω_{gc}.
③ $GM = \frac{1}{m\rvert_{\omega_{pc}}}$.	

$GM = -20\log m$ in dB

* Whenever plot/T.F. maintain less magnitude than 1 or less at all freq. at all the freq. range then $GM = PM = \infty$

* In this case ω_{gc}, ω_{pc} doesn't exist eg. $\frac{1}{(s+5)}$

① 1) $PM = \infty$ [1st priority] 2) None 2nd priority.

3) else $\omega_{gc} = 0 \longrightarrow$ & calculate PM.

* ~~SCCS~~ $\frac{1}{s} \Rightarrow \omega_{pc} = \infty$ $GM = \infty$ $\omega_{gc} = 1$ $PM = 90°$ stable

$\frac{1}{s^2} \Rightarrow \omega_{pc} = 1$ $GM = 1$ $\omega_{gc} = 1$ $PM = 0$ M.S.

$\frac{1}{s^3} \Rightarrow \omega_{pc} = 0$ $GM = 0$ $\omega_{gc} = 1$ $PM = -90°$. U.S

Center & Radius of

M circle :- $\left[\frac{M^2}{(1-M^2)}, 0\right]$ $\left[\left|\frac{M}{1-M^2}\right|\right]$

N circle :- $\left[-\frac{1}{2}, \frac{1}{2N}\right]$ $\left[\frac{\sqrt{N^2+1}}{2N}\right]$

* bandwidth (ω_b) > natural freq. of oscilatⁿ (ω_n) > damped osilatⁿ (ω_d) ↔
↔ > resonant freq. (ω_r)

* $G_1(s)$ has GM_1 & $G_2(s)$ has GM_2
$G_1(s)$ is more stable if $GM_1 > GM_2$.

* If we add zero to T.F. GM will ↑ses

* $GM = \frac{K_{max}}{\text{desired value of } K}$

* statespace analysis

* state gives the future behaviour of the s/m based on present i/p and past history.

* States variable depends on voltage across the capacitor & current through inductor. ∴ no. of state variable = sum of inductor & capacitor. OR order of diff. eq^n.

* Advantages :- (state space analysis)

→ applicable to dynamic system (Linear, non linear, TV, TIV.)

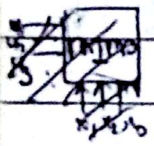
→ analysis is done by considering initial condit^n.

→ More accurate than T.F.

→ gives info about controllability & observability.

* Disadvantages → complex techniques, many computat^n required

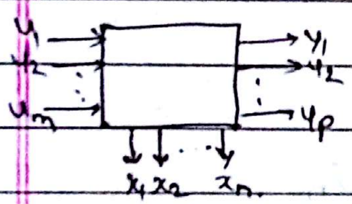
* standard form of the state model



$\dot{X} = AX + BU$ → state eq^n / dynamic eq^n

$Y = CX + DU$ → o/p eq^n.

A = state matrix
B = i/p matrix
C = o/p matrix
D = Tx matrix



$[U]_{m\times1}$ $[X]_{n\times1}$ $[Y]_{p\times1}$

$[A]_{n\times n}$ $[B]_{n\times m}$ $[C]_{p\times n}$ $[D]_{pm}$

∴ $[\dot{X}]_{n\times1}$ $[Y]_{p\times1}$

* four forms of state models

1. Controllable cannonical form
2. observable canonical form
3. diagonalization form or normal form.
4. Jordan canonical form.

* we can derive or find state modes from 4 types of (qtys) given data.

1. from differential eq^n
2. from transfer function
3. from signal flow graph
4. from Electrical network.

* state model to diff. equatⁿ *(by controllable cannonical form) (CCF)

1) $\dddot{y} + 3\ddot{y} + 5\dot{y} + 7y = 9u$

$y = x_1, \quad \dot{y} = \dot{x}_1 = x_2, \quad \ddot{y} = \dot{x}_2 = x_3, \quad \dddot{y} = \dot{x}_3$
① ②

(order = 3 ∴ 3 state space variable)

$\therefore \dot{x}_3 = 9u - 7x_1 - 5x_2 - 3x_3$ ③

$$\text{eq}^n \begin{cases} ① \\ ② \\ ③ \end{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -5 & -3 \end{bmatrix}}_{A_{CCF}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 9 \end{bmatrix}}_{B_{CCF}} u \quad \Big\} \text{ state eq}^n$$

$$[y] = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{C_{CCF}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \Big\} \text{ o/p. eq}^n$$

*(by observable cannonical form) conversions.

Acct, Bcct, Ccct are in above eg.

$A_{OCF} = (A_{CCF})^T \qquad B_{OCF} = \begin{bmatrix} b_4 \\ b_3 \\ b_2 \\ b_1 \end{bmatrix}$ where $B_{CCF} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$

$C_{OCF} = [c_4\ c_3\ c_2\ c_1]$, where $C_{CCF} = [c_1\ c_2\ c_3\ c_4]$

* state model eqⁿ to the transfer fⁿ.

$$\frac{Y(s)}{U(s)} = \frac{2s + 3}{s^2 + 5s + 6}, \quad s^n = \dot{x}_n$$

($\dot{x}_1 = x_2$, x_1; $\dot{x}_2$, $\dot{x}_1 = x_2$, x_1)

$\therefore u = \dot{x}_2 + 5x_2 + x_1 6 \Rightarrow \dot{x}_2 = 1u - 5x_2 - 6x_1$

$y = 2x_2 + 3x_1 \quad ; \quad \dot{x}_1 = x_2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

(with op. sign) → common factor in the numerator

$$y = \begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

(with same sign)

* diagonalization form.

$$\frac{Y(s)}{U(s)} = \frac{1}{(s+1)(s+2)(s+3)} \Rightarrow Y(s) = \frac{\frac{1}{2}U(s)}{(s+1)} - \frac{1\,U(s)}{(s+2)} + \frac{\frac{1}{2}U(s)}{(s+3)}$$

$Y(s) = x_1 + x_2 + x_3.$

$[y] = [1\ 1\ 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \\ -1 \\ \frac{1}{2} \end{bmatrix} u \begin{cases} ① \\ ② \\ ③ \end{cases}$$

Poles / eigen values. Partial fraction.

$x_1 = \frac{\frac{1}{2}u}{s+1}$

$s x_1 + x_1 = \frac{1}{2}u$

$s = \frac{d}{dt}$

$\therefore \dot{x}_1 + x_1 = \frac{1}{2}u$

① $\dot{x}_1 = \frac{1}{2}u - x_1$

||ly ② $\dot{x}_2 = -u - 2x_2$

||ly ③ $\dot{x}_3 = \frac{1}{2}u - 3x_3$

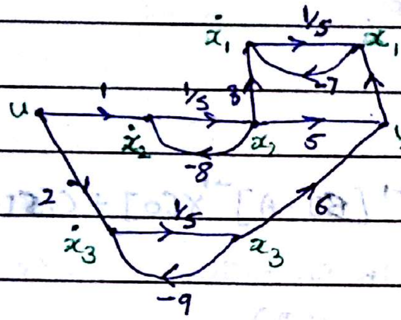
* Jordan canonical form

$$\frac{Y(s)}{U(s)} = \frac{1}{(s+5)^3(s+10)} = \frac{A'}{(s+5)} + \frac{B'}{(s+5)^2} + \frac{C'}{(s+5)^3} + \frac{D'}{(s+10)}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -5 & 1 & 0 & 0 \\ 0 & -5 & 1 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} u$$

$$[y] = [A' \;\; B' \;\; C' \;\; D'] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

*



* state model of SFG.

$\dot{x}_1 = 3x_2 - 7x_1$

* (only dummy) $\dot{x}_2 = -8x_2 + 1u$

$\dot{x}_3 = 2u - 9x_3$

$y = 4x_1 + 5x_2 + 6x_3$.

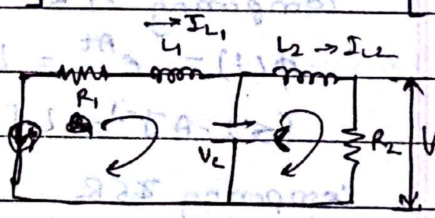
$\dot{x} \xrightarrow{1/s} x$

$s = \frac{d}{dt}$

$\frac{1}{s} = \int dt$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -7 & 3 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} u \qquad y = [4 \;\; 5 \;\; 6] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

* State model of Electrical N/W.



* select state variable as voltage across capacitor and current through inductor.
* No. of state variable = sum of inductor & capacitor
* write independent KVL & KCL eqns. S.V = V_2, I_{L1}, I_{L2}

(1) KCL $I_c = I_{L1} + I_{L2} \Rightarrow C\frac{dV_c}{dt} = I_{L1} - I_{L2} \Rightarrow \dot{V}_c = \frac{I_{L1}}{C} - \frac{I_{L2}}{C}$ — (1)

(2) KVL1 $V_i = R_1 I_{L1} + L_1\frac{dI_{L1}}{dt} + V_c \Rightarrow \dot{I}_{L1} = \frac{-R_1}{L_1} I_{L1} + \frac{V_i}{L_1} - \frac{V_c}{L}$ — (2)

(3) KVL2 $V_c = L_2\frac{dI_{L2}}{dt} + I_{L2}R_2 \Rightarrow \dot{I}_{L2} = \frac{V_c}{L_2} - \frac{I_{L2}R_2}{L_2}$ — (3)

$V_0 = I_{L2}R_2$ — (4)

$$\therefore \begin{bmatrix} \dot{V}_c \\ \dot{I}_{L1} \\ \dot{I}_{L2} \end{bmatrix} = \begin{bmatrix} 0 & 1/C & -1/C \\ -1/L & -R_1/L_1 & 0 \\ 1/L_2 & 0 & -R_2/L_2 \end{bmatrix} \begin{bmatrix} V_c \\ I_{L1} \\ I_{L2} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \\ 0 \end{bmatrix} V_i \Bigg\}$$ state eqn from (1) (2) (3)

$V_0 [0 \;\; 0 \;\; R_2] \begin{bmatrix} V_c \\ I_{L1} \\ I_{L2} \end{bmatrix}$ from (4)

* Transfer f^n from state model

$$T.F = C\frac{Adj[SI-A]B}{|SI-A|} + D. \quad (1)$$

$$* \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \underbrace{\begin{bmatrix}-2 & -3\\ 4 & 2\end{bmatrix}}_{A}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underbrace{\begin{bmatrix}3\\ 5\end{bmatrix}}_{B}u$$

$$[y] = \underbrace{[1 \;\; 1]}_{C}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \qquad D=0$$

$$[SI-A] = \begin{bmatrix}s+2 & 3\\ -4 & s-2\end{bmatrix} \Rightarrow Adj\,A = \begin{bmatrix}s-2 & -3\\ 4 & (s+2)\end{bmatrix}$$

Use above formula (1) to get T.F.

* Sol^n to the state eq^n.

$$\dot{X} = AX + BU \Rightarrow x(t) = \underbrace{L^{-1}[SI-A]^{-1}X[0]}_{\text{ZIR due to IC}} + \underbrace{L^{-1}[SI-A]^{-1}BU[S]}_{\text{ZSR due to i/p}}$$

(Zero i/p Responce due to initial condit^n)

$$x(t) = e^{At}\underbrace{x(0)}_{ZIR} + \underbrace{\int_0^t e^{A(t-T)}BU(T)\,dT}_{ZSR}$$

comparing ZIR.

$$\phi(t) = e^{At} = L^{-1}[SI-A]^{-1} \rightarrow \text{STM state transit}^n \text{ Mx}$$

$$[SI-A]^{-1} = L[\phi(t)]$$

comparing ZSR.

$$\int_0^T \phi(t-T)BU(T)\,dT = L^{-1}[\phi(s)BU(s)]$$

$$\therefore x(t) = e^{At}x(0) + L^{-1}[\phi(s) * BU[s]]$$

Properties of STM.

(1) $\phi[0] = I$ (Identity m^x) (2) $\phi^K[t] = [e^{At}]^K = e^{AKt}$.

(2) $\phi^{-1}[t] = \phi(-t)$ (3) $\phi(t_1+t_2) = \phi(t_1)\cdot\phi(t_2)$

(5) $\phi(t_2-t_1)\cdot\phi(t_1-t_0) = \phi(t_2-t_0)$

If state model is homogeneous sol^n is ZIR

if state model is Non-homogeneous sol^n is ZIR + ZSR.

* Controllability & observability.

* If it is possible to transfer the initial states to the desired state in a finite time period by the control i/p. the it is controllable.

e.g. in SFG from i/p to each and every state variable if path is exist then s/m is controllable else not.

* Kalman's test for controllability $Q_c = [B \; AB \; A^2B \ldots A^{n-1}B]$

Rank of Q_c = Rank of A : $|Q_c| \neq 0$.

* A s/m is said to be observable if it is possible to determine initial states of the systems by observing the o/p in a finite time interval.

(Kalman's test for observability) $Q_o = [C^T \; A^TC^T \; (A^T)^2C^T \ldots (A^T)^{n-1}C^T]$

Rank Q_o = Rank A $= \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^{n-1} \end{bmatrix}$

$|Q_o| \neq 0$.

* The pole/zero cancellation makes system
  1) uncontrollable but observable
  2) controllable but unobservable
  3) Uncontrollable & unobservable

  but not make controllable & observable.

* Gibbson's test for controllability & observability.

$$\dot{x}(t) = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 2 \\ 1 & -3 \end{bmatrix} u(t)$$

Non zero controllable

Non zero. controllable

$$y(t) = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} x(t)$$

observable → Non. zero, observable.

* compensators & controllers

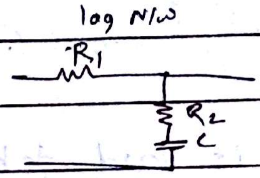
* compensators.

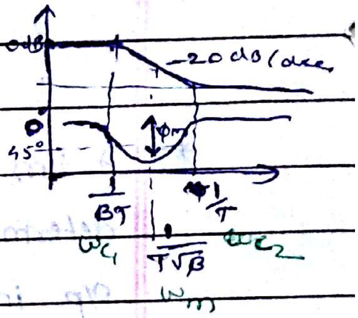
* lag compensator:

→ A compensator having the characteristic of lag network is called a lag compensator.

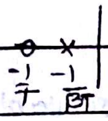
→ It result in large improvement in steady state performance but results in slower responce due to reduced B.W.

→ It is essentially LPF.





$$T.F. = \frac{s+Z_c \ (1/T)}{s+P_c \ (1/\beta T)}$$



∴ max phase lag $= \omega_m = \sqrt{\omega_{c_1}\omega_{c_2}} = \frac{1}{T\sqrt{\beta}}$

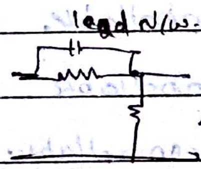
∴ Max lag angle $= \phi_m = \tan^{-1}\frac{1-\beta}{2\sqrt{\beta}}$

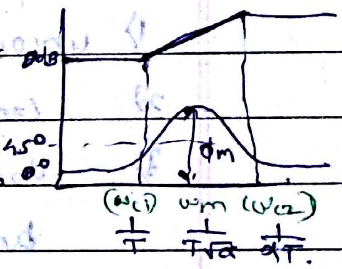
* Lead compensator

→ A compensator having characteristics of lead NtWk.

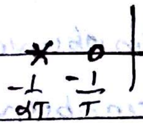
→ It appreciably improves the transient responce. it increases the B.W. & improves speed of the response, also reduce amount of overshoot

→ basically H.P.F.





$$T.F = \frac{s+Z_c \ (1/T)}{s+P_c \ (1/\alpha T)}$$



∴ max phase lead $= \omega_m = \sqrt{\omega_{c_1}\omega_{c_2}} = \frac{1}{T\sqrt{\alpha}}$

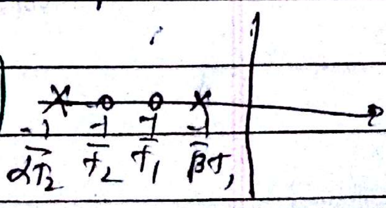
∴ max phase lead angle ⇒ $\phi_m = \tan^{-1}\left(\frac{1-\alpha}{2\sqrt{\alpha}}\right)$; $\alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m}$

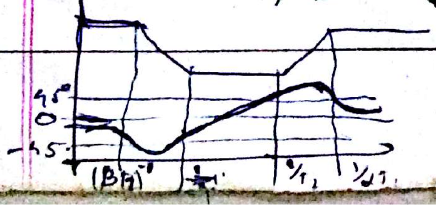
* lag lead compensator.

→ having characteristics of both lag & lead N/ω.

→ It improves both transient & steady state responce.

$$TF = \frac{(s+1/T_1)(s+1/T_2)}{(s+1/\beta T_1)(s+1/\alpha T_2)}$$ where $[\beta > 1 \ ; \ 0 < \alpha < 1]$





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* Controllers.

① Proportional controller : $G_c(s) = K_P$.

→ used to vary the transient responce. it is usually amp^n with gain K_P.

② Integral controller $G_c(s) = K_I/s$

→ used to ↓se s.s. error. by ↑sing type of system.

→ but dis-ad. → stability ↓ses.

③ Derivative controller $G_c(s) = K_D s$

→ used to ↑se stability stability ↑ses by adding zeros

→ but dis-adv. → s.s. error ↑ses. ∵ type of system ↓ses.

④ (P-I) controller (Proportion-Integral) → $G_c(s) = K_P + \frac{K_I}{s}$

→ used to ↓se s.s. error without effecting stability ∵ poles are at origine and zero is added.

→ Type of s/m ↑ses ∴ s.s. error ↓ses.

→ It acts as LPF. B.W. ↓ses and t_r ↑ses (∴ transient responce becomes slow)

→ ξ improves hence reducing max overshoot.

→ filters out high frequencies.

⑤ P-D controller $G_c(s) = K_D s + K_P$.

→ used to improve stability without effecting s.s. error.

∵ type is not changed & zero is added.

→ improving ξ & reducing max overshoot.

→ PD is HPF. Hence BW ↑se, t_r, t_s ↓ses (transient time responce improved)

→ improves GM, PM & M_r

→ Noise enter at high frequency.

→ Possibly requiring relatively large capacitor in circuit.

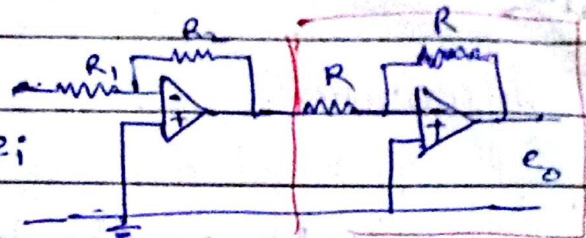
⑥ PID controller $G_c(s) = K_P + K_I/s + K_D \cdot s$.

It is used to ↓se e_{ss} & ↑se stability ∵ pole at origine & zero added.

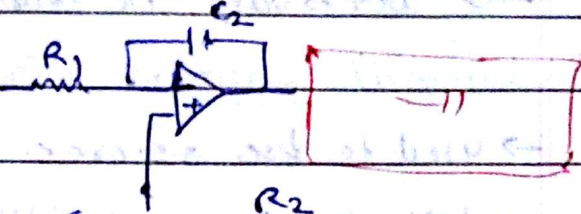
* Opamp ckts

* Control action — * $G(s) = \frac{E_o(s)}{E_i(s)}$

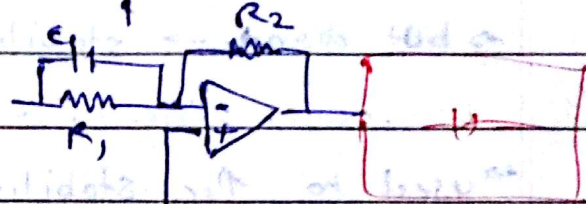
P — $\frac{R_2}{R_1}$



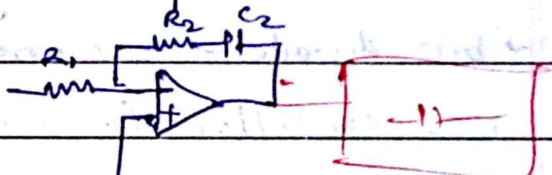
I — $\frac{1}{R_1 C_2 S}$



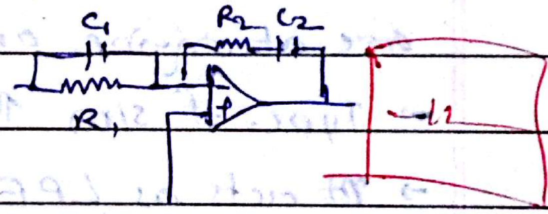
PD — $\frac{R_2}{R_1}(R_1 C_1 s+1)$



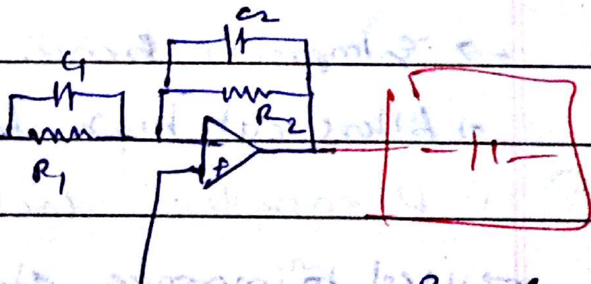
PI — $\frac{R_2}{R_1}\,\frac{R_2 C_2 s+1}{R_2 C_2 S}$



PID — $\frac{R_2}{R_1}\,\frac{(R_1 C_1 s+1)(R_2 C_2 S+1)}{R_2 C_2 S}$



Lead or lag — $\frac{R_2}{R_1}\,\frac{(R_1 C_1 S+1)}{(R_2 C_2 S+1)}$



lag-lead — $\frac{R_4}{R_3}\,\frac{[(R_1+R_3)C_1 S+1](R_2 C_2 S+1)}{(R_1 C_1 S+1)[(R_2+R_4)C_2 S+1]}$