# Signals & System

## * ① Introduction.

* Anything carries information is signal
* device or program which process the signal to form new signal is system

## * Characteristics of signal :-

→ function of more than one independent variable

→ Randomness

→ Bandwidth

## * Types of signals

### 1) Continuous Signal. (Analog Signal)

→ Continuous in both amplitude & time

### 2) Descrete signal

→ continuous in amplitude but descrete in time.

cont. signal $x(t)$ —✗— sampler — descrete signal $x[nT_s]$

conti. time $\xRightarrow{to}$ descrete time

* multiplexing is done in descrete signal.
* every descrete signal is not sampled version of C.S. (cont. signal)
* for multiplexing we do sampling.

### 3) Digital signal

→ It is descrete in both time & amplitude

we can also say descrete time & quantised amplitude.

conti. amplitude $\xRightarrow{to}$ descrete amplitude.

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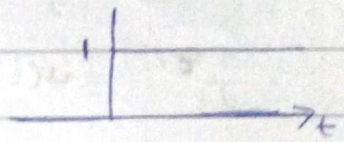
## * Standard Signals

### 1) Unit step function :-

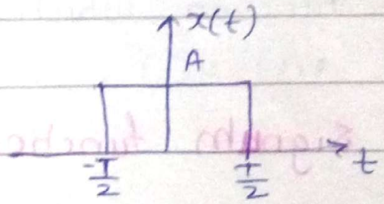
$$u(t) = \begin{cases} 1 \; ; & t > 0 \\ 0 \; ; & t < 0 \end{cases}$$




* bounded amp. value. i.e. 1
* at $t=0$ not defined but we may take $\frac{1}{2}$ at $(t=0)$

### 2) Rectangular function :-

$A\,\mathrm{rect}\left(\frac{t}{T}\right)$ or $A\Pi(t/T)$

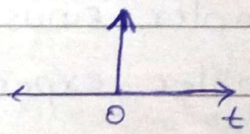



$$x(t) = A \; ; \; \frac{-T}{2} < t < \frac{T}{2}$$
$$= 0 \; ; \; \text{elsewhere}$$

* discontinuous at $-\frac{T}{2}$ & $\frac{T}{2}$
* ideal impulse $f^n$ not observed hence it is used to create impulse function

### 3) Continuous Impulse or Dirac Delta function.

$$\delta(t) = \begin{cases} \infty \; ; & t = 0 \\ 0 \; ; & t \neq 0 \end{cases}$$




* $\int_{-\infty}^{\infty} \delta(t)\,dT = 1$ * $\delta(-t) = \delta(t)$ * $\delta(\alpha t) = \frac{1}{|\alpha|}\delta(t)$

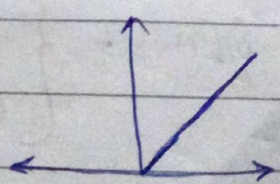
* $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$ if $x(t)$ is conti. at $t=t_0$ i.e. at $t=0$ $x(t)$ Amp. exis

$\Rightarrow t\,\delta(t) = 0$

* $$\int_{t_1}^{t_2} x(t)\,\delta(t-t_0)\,dt = \begin{cases} x(t_0) \; ; & t_1 \leq t_0 \leq t_2 \\ 0 \; ; & \text{elsewhere} \end{cases}$$

### 4) Unit Ramp function

$$r(t) = \begin{cases} t \; ; & t > 0 \\ 0 \; ; & t < 0 \end{cases}$$



* $r(t)$ at 0 is not defined.

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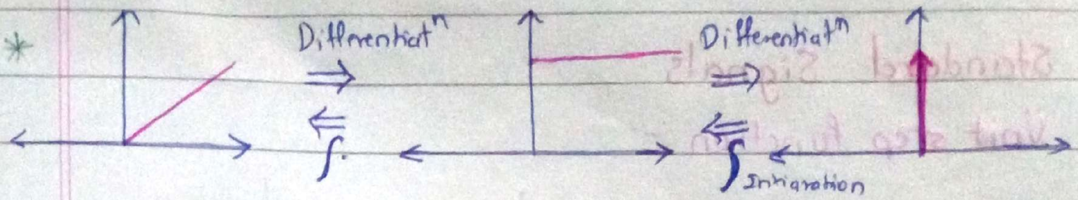



$$\frac{d}{dt} r(t) = u(t) \qquad \frac{d}{dt} u(t) = \delta(t)$$

$$r(t) = \int u(t)\,dt \qquad u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$

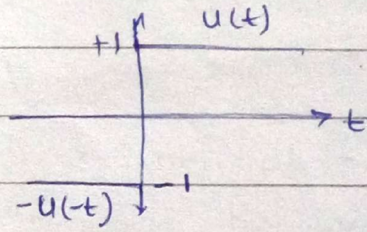
5) Singularity function

function which dont have higher order (2,3,...) derivatives

$r(t)$, $u(t)$, cos & sin. etc.

6) signum function

$$sgn(t) = u(t) - u(-t)$$
$$= 2u(t) - 1$$
$$(\because u(t) + u(-t) = 1)$$

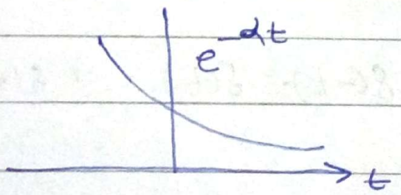


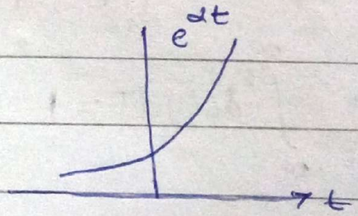

7) Exponential

① Real exponential ($e^{\sigma t}$)

② complex sinusoidal $e^{\pm j\omega_0 t}$

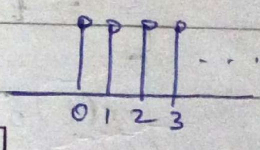
③ complex exponential $e^{st} = e^{(\sigma + j\omega)t}$ ①+②




(8) unit step sequence

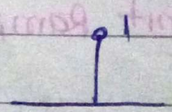
$$u[n] = \begin{cases} 1 ; & n \geq 0 \\ 0 ; & n < 0 \end{cases}$$




$$u[n] = \sum_{m=0}^{\infty} \delta[n-m] = \sum_{m=-\infty}^{n} \delta[m]$$

9) Descrete impulse or krander delta

$$\delta[n] = \begin{cases} n ; & n = 0 \\ 0 ; & n \neq 0 \end{cases}$$




* $\delta[kn] = \delta[n]$ scaled version of descrete impulse is same

$\delta[n] = u[n] - u[n-1]$

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## * Transformation.

1) Time-scalling $x(at)$ / $x[mn]$
2) Time-shift $x(t-t_0)$ / $x[n-n_0]$
3) Time-Reversal $x(-t)$ / $x[-n]$
4) Amplitude scaling. $k\,x(t)$ / $k\,x[n]$

① Prefer first shifting then scalling. (∵ because dej...)
② $y(t) = x(t-t_0)$
$y(2t) = x(2t-t_0)$
$\neq x[2(t-t_0)]$
③ $y(t) = x(2t)$
$y(t-t_0) = x[2(t-t_0)]$
$\neq x(2t-t_0)$

$x[mn] \Rightarrow$ when $m>1$ Decimation i.e. compression
when $m<1$ Interpolation i.e. expansion.

④ (1st interpolation then decimatn if we have to do both at a time)

## * Classification of signals.

1) Energy Signal & Power signal
⇓ energy is finite ⇓ power is finite.

$$E_{x(t)} = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2\,dt$$

$$E_{x[n]} = \lim_{N\to\infty} \sum_{n=-N}^{N} |x[n]|^2$$

$$P_{avg} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\,dt.$$

$$P_{avg} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

① $e^{-t}u(t)$
② $\delta(t+2)-\delta(t-2)$ ③ $\delta[n]$ i.e. descrete impulse

* if energy is finite $P_{avg} = 0$
* All time limited signal of finite amplitude are energy signal. eg 2

$E_{y(t)} = E'$ $E_{y(3t)} = \frac{E'}{3}$

$E_{A\,y(t)} = A^2E'$ $E_{A+y(t)} = \infty$

① $A\,u(t) \Rightarrow$ ~~const. amp. over infinite~~ time is Power signal.
② $A\cos(w_0t+\theta)$
③ $A e^{jw_0t}$
} all periodic signals are power signals.

* Power signals requires infinite energy
* Power s. + Energy s. = Power Signal.

There is no possible to have energy & power signal because they are mutually exclusive.

$x[n] = \alpha^n u[n] \Rightarrow |\alpha|<1 \Rightarrow$ Energy signal
$|\alpha| = 1 \Rightarrow$ Power signal
$|\alpha| > 1 \Rightarrow$ Neither Energy Nor power (NENP)

* N.E.N.P. $\Rightarrow$ $\left|\frac{1}{t}\right|$, $t\,u(t)$, $e^{-3t}$, $\delta(t)$ i.e. continuous impulse.
as $t\to\infty$ amp $\to\infty$ then N.E.N.P.

## 2) Even signal & odd signal

| Even signal | Odd signal |
|---|---|
| $x(t) = x(-t)$ | $x(t) = -x(-t)$ |

if $x(t) = a(t) + j\,b(t)$, then

| | |
|---|---|
| E.C. [Even conjugate] | O.C. (odd conjugate) |
| $x(t) = x^*(-t)$ | $x(t) = -x^*(-t)$ |
| * symmetric about y axis | Antisymmetric about y axis. |
| $\int_{-a}^{a} x_e(t) = 2\int_0^a x_e(t)\,dt$ | area under odd signal is 0. |
| $x_e(t) = \frac{x(t)+x(-t)}{2}$ | $x_o(t) = \frac{x(t)-x(-t)}{2}$ |

let e → +ve, o → -ve, apply & remember

* e + e = e  o + o = o  e + o = N.E.N.O.

e · e = e  o · o = e  o · e = o

diff. of even = odd  $\int$ of odd = even

$\int$ of even = odd  diff. of odd = even.

* $E_{x(t)} = E_{even} + E_{odd}$.

## 3) Periodic signal & Non periodic Signal or Aperiodic signal.

for C.T. $x(t) = x(t+T)$

steps for finding time period of $x_1(t) + x_2(t) + x_3(t) \dots$

1) Identify individual time periods denoting $T_1, T_2, T_3, \dots$ etc

2) Calculate $T_1/T_2$, $T_1/T_3$, $T_1/T_4$, .... etc.

3) If the ratio of step ② is rational the overall signal is periodic

4) calculate LCM of denominator of step 2.

5) $T = (L.C.M.)\,T_1$

OR alternate method is GCD method.

for D.T. $x[n] = x[n+N]$

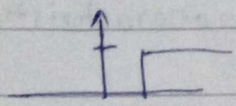
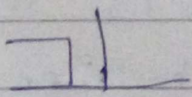
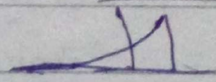
$$\boxed{\frac{\omega_0}{2\pi} = \frac{m}{N}}$$ compare $m$ & $N$

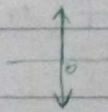
$N$ as Time period $m$ no. of cycles in $N$.

for a complex signal draw waveform to conclude periodicity.

4) Causal signal & anti causal signal & Non-causal.

pure +ve $t>0$ — pure -ve $t<0$ — both sided $t<0$ & $t>0$

* Dublet $\delta^{\cdot}(t)$ $\Rightarrow$  area under duplet is 0

$$\int x(t)\, \delta^n(t-t_0)\, dt = (-1)^n \left.\frac{d^n x(t)}{dt^n}\right|_{t=t_0}$$

[n is order of derivative]

$$\Rightarrow \int_{t_1}^{t_2} x(t)\, \delta(t-t_0)\, dt = x(t_0)$$

## * Systems

linear, stable, Invertible $\Rightarrow$ amp. dependent systems.

T.I., causal, static $\Rightarrow$ Time dependent systems.

### 1) linear system

given $\rightarrow$ $y(t) = f\{x(t)\}$

cond $\rightarrow$ ① $y_1(t) = f\{x_1(t)\}$ ② $y_2(t) = f\{x_2(t)\}$

③ $y_f(t) = f\{x_1(t) + x_2(t)\}$

if $y_1(t) + y_2(t) = y_f(t)$ system is linear.

also ① $\alpha y_1(t) = f\{\alpha x(t)\}$ ② $\alpha y(t) = \alpha f\{x(t)\}$

① & ② equals then system is linear.

* Unknown signal $x(t)$ can make system non linear.
* product of two known signal is non linear.
* Sampler i.e. sample $\{x(t)\}$ is linear
* Quantiser is non linear. since it dependeds on amp.

$\sin\{x(t)\}$, $|x(t)|$, $x(t) + \text{constant}$ (incremental linear), $\text{signum}\{x[n]\}$

$\text{median}\{x[n]\}$, $x^*[n]$, $\text{Real}\{x[n]\}$ (complex), $3^{x[n]}$

$\dfrac{x[n]}{x[n-1]}$ these all are Non-linear systems.

* two system connected in series OR two system connected in parallel both systems may or may not linear

2) Time - Invarient {shift - invarient}

Time - varient {shift - varient}.

- A system is T.I. if i/p o/p characteristic donot change with time.

given → $y(t) = f\{x(t)\}$

cond → ① $y_1(t) = f\{x(t-t_0)\}$ any replace $x(t) \to x(t-t_0)$

② $y_2(t) = f\{x(t-t_0)\}$ replace all t by $(t-t_0)$

① = ② then time invarient

$e^{-x(t)}, x^2(t), \frac{d\,x(t)}{dt}, \frac{d^2 y(t)}{dt^2} \to$ Time Invarient

$t\,x(t), x(t)\cos\omega_0 t, x(\alpha t), \left(\frac{dy}{dt}\right)^2 \to$ Time varient.

modulator $\quad \alpha \neq 1$

*

$y(t) = \begin{cases} x(t) & ; x(t) > 0 \\ 0 & ; x(t) < 0 \end{cases}$ } T.I. but Non linear since amp. change

$y(t) = \begin{cases} x(t) & ; t > 0 \\ 0 & ; t < 0 \end{cases}$ } T.V. but linear.

$\frac{d\,y(t)}{dt} + t\,y(t) + x^2(t) \Rightarrow$ T.V. N.L.

$\frac{d^2 y(t)}{dt^2} + 4y(t) = 3x(t) \Rightarrow$ T.I. linear.

* in any diff equat$^n$ if all coeff. are fixed only with linear element that is an LTI system.

*** In LTI system whatever change from $x_1(t)$ to $x_2(t)$ same changes from $y_1(t)$ to $y_2(t)$.

3) Causal & Non-causal system.

Non Anticipative

A system is causal if the present o/p depends on present i/p and the past i/p but not future values of i/p

$y(t) = f\{x(t)\}$ or $f\{x(t-t_0)\}$ or $f\{x(t)\}$ or both ⇒ causal

$y(t) = f\{x(t+t_0)\} \Rightarrow$ Non causal.

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Causal $\left\{ \begin{array}{l} (3t+1)x(t) \;,\; |x(t)| \;,\; \sin\{x(t)\} \\ 2x[n]+3u[n+1] \;,\; \overset{\text{Accumulator}}{\sum_{k=-\infty}^{n} x[k]} \;, \\ y'(t+4)+2y(t) = x(t-2) \end{array} \right.$

Non causal $\left\{ \begin{array}{l} x\{\sin t\} \;,\; \int_{-\infty}^{2t} x(\tau)\,d\tau \\ \sum_{k=0}^{n} x[k] \;,\; \sum_{k=n-n_0}^{n+n_0} x[k] \;,\; \frac{1}{2m+1}\sum_{k=-m}^{+m} x[n-k] \end{array} \right.$

moving average system

$$y[n] = \sum_{k=n_0}^{n} x[k] \quad \left\} \begin{array}{l} n_0 \le n \Rightarrow \text{causal} \\ n_0 > n \Rightarrow \text{Non Causal.} \end{array} \right.$$

## 4) Static system (or memory less) & Dynamic System (or memory).

$y(t) = f\{x(t)\}$ static

$y(4) - f\{x(t^2)\}$ dynamic

* Any diff. term due to energy storing element is Dynamic.
eg. $V_L = L\frac{di}{dt}$

* All static or memoryless system are causal but converse can't say.

$e^{-(t+5)}x(t)$, $2x(t)+3$, $g[n+2]x[n]$ $\Rightarrow$ static

$x[n^2]$., $\frac{d}{dt}x(t)$, $\int x(t)\,dt$ $\Rightarrow$ Dynamic.

## 5) stability (stable & Unstable)

BIBO $\longrightarrow$ stable  BIUBO $\rightarrow$ Unstable

if $|x(t)| \le m_x < \infty$ then $y(t) \le m_y < \infty$ $\Rightarrow$ stable.

OR $|y(t)| = |x(t)|^2 = \text{finite}^2 = \text{finite}$ the stable.

stable $\}$ $u(t)$, $x^2(t)$, $x(t-4)$, $e^{x[n]}$, $\sum_{k=n-n_0}^{n+n_0} x[k]$

un-stable $\left\{ \begin{array}{l} \text{impulse, ramp}, \int_{-\infty}^{t} x(\tau)\,d\tau \;,\; \int_{-\infty}^{t} x(\tau)\cos w_0\tau\,d\tau \\ \frac{d}{dt}x(t) \;,\; \sum_{k=n_0}^{n} x[k] \end{array} \right.$

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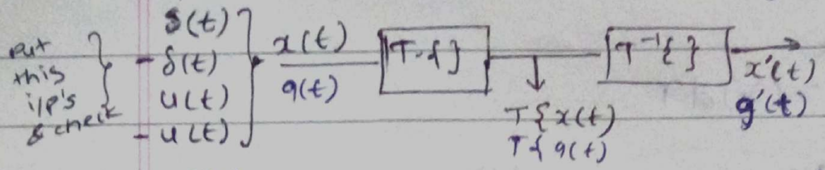
* Invertible or Inverse System.

two diffrent i/p's for a given system should not produce same o/p then system is Invertible




$x'(t) \neq g'(t)$

$x(t)$    $x[n]$    for $x(t)$    $x[n]$ for desorete

* $\int$ & $\Sigma$ are invertible but $\frac{d}{dt}$ & diffrence are non Invertible

$$x(t-4), \int_{-\infty}^{t} x(\tau)\,d\tau, \sum_{k=-\infty}^{n} x[k] \quad \text{invertible}$$

$$y(t+4), \frac{dy(t)}{dt}, y[n]-y[n-1] \quad \Leftarrow \text{inverse of respective}$$

$$x^2(t), \frac{d}{dt}x(t), n\,x[n], x[n].x[n-3] \quad \text{Non invertible fn.}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} \qquad \sum_{k=0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$\int \sin(ax)\,dx = \frac{-\cos ax}{a} \qquad \sum_{k=0}^{+\infty} a^k = \frac{1}{1-a} \quad |a|<1$$

$$\int \cos(ax)\,dx = \frac{\sin ax}{a} \qquad \sum_{k=1}^{\infty} k a^k = \frac{a}{(1-a)^2} \quad |a|<1$$

$$\int \frac{1}{a^2+b^2x^2}\,dx = \frac{1}{ab}\tan^{-1}\left[\frac{bx}{a}\right] \qquad \sum_{k=1}^{N} k = \frac{N(N+1)}{2}$$

$$\int x\sin(ax)\,dx = \frac{\sin(ax)-ax\cos(ax)}{a^2} \qquad \sum_{k=1}^{N} k^2 = \frac{N(N+1)(2N+1)}{6}$$

$$\int x\cos(ax)\,dx = \frac{\cos(ax)+ax\sin(ax)}{a^2}$$

$$\int_0^{\infty} e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}, \quad a>0$$

$$\int_0^{\infty} \operatorname{sinc} x\,dx = \int_0^{\infty} \operatorname{sinc}^2 x\,dx = \frac{1}{2}$$

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## LTI system

An LTI system is always Represented w.r.t impulse response i.e (if i/p is impulse o/p is impulse response)

- SIFTIGN :- any signal can be produced as combinations of impulses.

- convolution :-

Convolution is nothing but zero state response (i.e all initial conditions are zero) for a given i/p. Sliding one signal over the 1st folded & then shifted version of other signal.

step 1: limit of $y(t)$ => sum of lower lim. < t < sum of ~~higher~~ upper lim

step 2: change axis from "t" to $\tau$ & draw signal

step 3: folding one signal

step 4: shift that signal.

step 5: taking each case multiplication.

Step 6: Integration with proper limits.

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \text{ OR}$$

areas

$$\boxed{A_y = A_x \cdot A_h} = \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau.$$

* $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$ → Product property

* $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0) = x(t_0)$ → shifting property

* $x(t) * \delta(t-t_0) = x(t-t_0)$ → convolution property of impulse.

* $\frac{d}{dt} x(t) * h(t) = \frac{d}{dt} y(t)$ hence $x^m(t) * h^n(t) = y^{(m+n)}(t)$

⇒ $\int x_1(t) * \frac{dx_2(t)}{dt} = x_1(t) * x_2(t)$   m & n - order of diff$^n$ } (2-1-8)

** $x(t-\alpha) * h(t-\beta) = y(t-\alpha-\beta)$

* $x(\alpha t) * h(\alpha t) = \frac{1}{|\alpha|}\, y(\alpha t)$ ⇒ $x(-t) * h(-t) = y(-t)$

** $u(t) * u(t) = r(t) = t\, u(t)$

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* convolution of two unequal length rectangle is trapezium if length of rectangle is same then triangle. (2.1.9a)
* convolution of general signal with periodic impulse train is periodic repetation of general signal. signals are seperated by duration depends on signal length and impulse duration. (2.1.8)

* $u(n) * u(n) = (n+1)\,u(n)$

* $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h(n-k)$

| | samples |
|---|---|
| i/p | m |
| IR | n |
| o/p | m+n-1 |

* $\sum_{m=0}^{N} \alpha^m = \frac{1-\alpha^{N+1}}{1-\alpha}$

convolution :- (descrete) 2methods

1) diagonally addition
2) sum by column method

} (2.1.14)

de-convolution :-

1) sum by column method
2) division method (2.1.14)
3) Redursive division method

Periodic convolution :- (or circular convolution)

when $x[n]$ has periodicity.

| Ordinary convolution | Periodic convo. |
|---|---|
| $x[n] \rightarrow m$ samples | $x_p[n] \rightarrow m$ |
| $h[n] \rightarrow N$ | $h_p[n] \rightarrow N$ |
| $y[n] \rightarrow M+N-1$ | $y_p[n] \rightarrow \max[m,n]$ |

① circular convolut$^n$ obtain. from ordinary convolution (2.1.16)
linear conv + aliasing = Periodic convo.

② other method is matrix method. (2.1.16)

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## * LTI systems (in impulse Response)

1) Causal LTI system

Before application of i/p as a impulse at $t=0$ we can not expect the o/p as a impulse response before $t=0$

i.e. $h(t) = 0$ for $t<0$

$h[n] = 0$ for $n<0$

2) Stability of LTI system

$\int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty$ i.e. finite → condition for stability

—"— for descrete

3) static or memory less (ML) of LTI system

$h(t) = k\,\delta(t)$

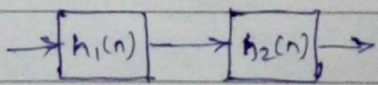
or $h(t) = 0$ for $t \neq 0$ , same for descrete

$e^{-4t} u(t+3)$ → N.C. & stable

$a^n u[n-1]$ → causal & Unstable for $a \geq 1$

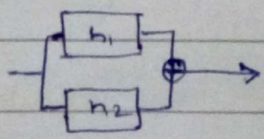
causal & stable for $a<1$

$e^{\alpha t} u(t) + e^{\beta t} u(-t)$ → $\alpha < 0, \beta > 0$ stable

Series LTI — 

$\equiv h_1(n) * h_2(n)$

↳ convolution (multiplication)

llel LTI — 

$\equiv h_1(n) + h_2(n)$

→ addition

4) Invertibility & Inverse

$h(t) * h_{inv}(t) = \delta(t)$

$h(n) * h_{inv}[n] = \delta[n]$

if given $y(n) = f\{x[n]\}$ to get I.R. $h(n)$ put $x[n] = \delta[n]$

$h(n) = f\{\delta[n]\}$ $\because x[n] = \delta[n]$

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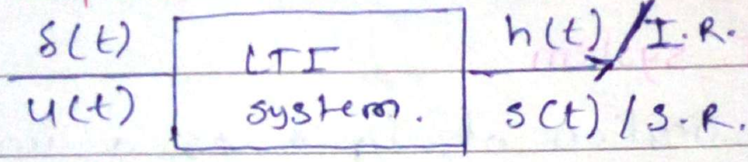
## Step Response




$$\delta(t) = \frac{d}{dt} u(t) \qquad \delta(n) = u[n] - u[n-1]$$

$$h(t) = \frac{d}{dt} s(t) \qquad h[n] = s[n] - s[n-1]$$

$$s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau \qquad s[n] = \sum_{k=-\infty}^{\infty} h[k]$$

* Differentiation of step response is impulse response

* $\int$ of impulse responce is step responce.

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* Non-minimum phase T.F. is defined as T.F. which has zeros in the right half of s-plane.

* T.F. of zero order hold system is $H(s) = \frac{1}{s}[1-e^{-Ts}]$

  $\therefore h(t) = u(t) - u(t-T)$ — IR of hold system.

* in convolution if h(t) graph given with limts & also x(t) if ask to find out y(t) at t = k.

  then $y(k) = \int_0^k x(\tau)h(2-\tau)d\tau$ (see class notes for detail)

* steady state value of $y(t) = y(\infty)$ let $y_{ss}$

  to find value of $t_1$ at which $y(t)$ = 99% of its steady state value $y(\infty)$

  $y(t_1) = 99\% \, y_{ss}$ solve & get $t_1$.

* $H(f) = \frac{5}{1+j10\pi f} = \frac{5}{0.2+j2\pi f}$ Comparing $e^{-at}u(t) = \frac{1}{a+j2\pi f}$ if H(f) is given

  $\therefore h(t) = 5e^{-0.2t}u(t)$

* steady state o/p Nothing but F.V.T.

* modulus operation is non-linear.

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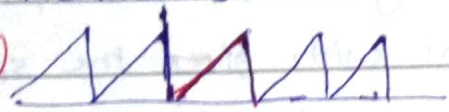
# Doubts & Concepts

## 1) fourier Series

it is the approximation of signal with various harmonics in terms of sine or cos function.

$x(t)$



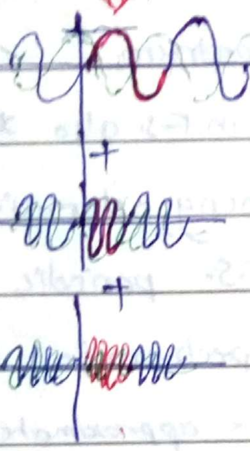
$x(t) = a_0 + a_1\cos\omega_0 t + a_2\cos\omega_0 t \ldots$
$+ b_1\sin\omega_0 t + b_2\sin\omega_0 t \ldots$

⇓ fourier series

harmonics { blue-sine + green-cos.

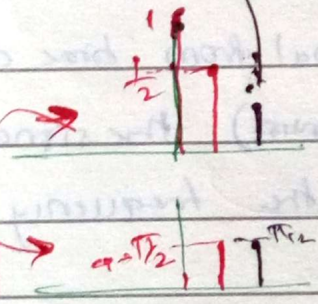
↓ magnitude of equivalent is magnitude of 1st harmonic

& phase betn them ↓ phase of 1st harmonic




approximation of $x(t)$ signal

Spectrum of fourier series :-




for first harmonic

summary ⇒ i.e. a that shap($\Delta$) is approximated interms of sin & cos terms i.e. fourier series

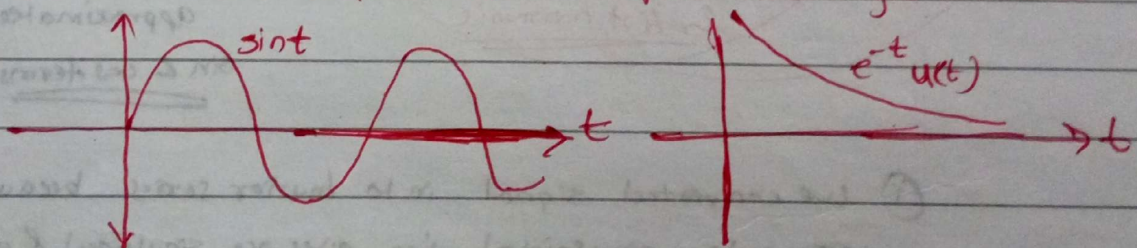
1. we converted signal in to fourier series because for LTI system sinusoidal i/p gives o/p sinusoidal & we can analyse the signal.
2. & all periodic signals can represent in unique form.

* Fourier transform

in fourier series we can approximate signal into sine & cosine wave. but for any periodic signal & we get the descrete spectrum. but it limited only for periodic signal now, fourier transform is the tool wich gives the spectrum of the signal wich is also wehell for non-periodic signal & which is continuous spectrum.

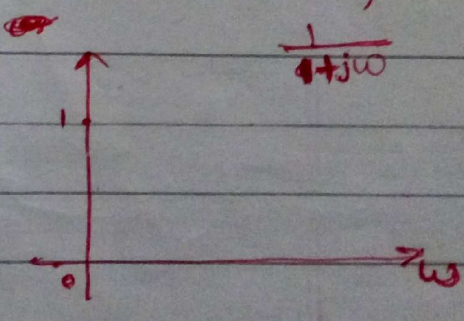
the spectrum in FS is descrete in FS. also $x(t)$ to spectrum means time domain to frequency domain but specifically we can say that in F.S. periodic signal in time domain to the descrete spectrum in frequency domain. & spectrum in F.T. means we are approximated the signal from time domain to frequency domain (i.e. spectrum) the signal is ~~sinusoidal or~~ periodic or nonperiodic to the frequency domain wich is continuous.

Time domain means the function of signal ie. variation of the signal with respect to change in time




Q how the transformation done

frequency domain means the function of signal i.e. variation or behavariour of the signal wich varies w.r.t frequency change

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3) $\omega, \omega_0, t, f, t_0$ etc. formulaes from $\omega$ to f to

→ $\omega = 2\pi f$

(A) for continuous

when signal given → $x(t) = \cos 50\pi t + \sin 60\pi t$.

consider $\cos 50\pi t$ compare with $\cos \omega t$.

$\therefore \omega = 50\pi$ let $\omega_1 = 50\pi$ $T = \frac{2\pi}{\omega} = \frac{1}{25}$ let $T_1 = \frac{1}{25}$

$\|^{ly}$ $\sin 60\pi t \rightarrow \sin \omega t$

$\therefore \omega = 60\pi$ let $\omega_2 = 60\pi$. $\|^{ly}$ let $T_2 = \frac{1}{30}$

$\frac{T_1}{T_2} = \frac{6}{5}$ } = Rational means total signal is periodic

if her four terms are there then $T_1, T_2, T_3, T_4$

$\frac{T_1}{T_2}, \frac{T_1}{T_3}, \frac{T_1}{T_4}, \ldots$

lcm - $T_2, T_3, T_4$

lcm × $T_1$ = total signal T

LCM of denominators here only 5

$\therefore$ Lcm = 5

$\text{Lcm} \times T_1 = 5 \times \frac{1}{25} = \boxed{\frac{1}{5}}$ total T of signal $x(t)$

$\omega = \frac{2\pi}{\frac{1}{5}} = 10\pi$

Total $\omega$ of signal is $10\pi$

consider it $\omega_0$

$\therefore$ Now $x(t) = \cos 5 \cdot 10\pi t + \sin 6 \cdot 10\pi t$

fundamental frequency

harmonics

* Now summary of shortcut

just take $\text{GCD}(\overset{\omega_1}{50\pi}, \overset{\omega_2}{60\pi}) = \overset{\omega_0}{10\pi} \Rightarrow (5, 6)$ harmonics

$T = \frac{2\pi}{\omega_0} = \frac{1}{5}$

for descret. periodicity condition is $x(n) = x(n+N)$ fundamental period

eg. $x(n) = \sin\left[\frac{3\pi n}{5}\right]$ ⇒ here compare with ~~sinwn~~ ($\sin \omega_0 n$)

continuous sinusoidal & complex sinusoidals are periodic for any value of $\omega_0$

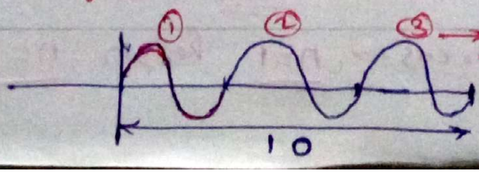
but the equivalent descrete form are periodic if $\omega_0/2\pi$ ratio is rational

i.e. $\frac{m}{N}$ - ratio is rational

→ Comparing with $\sin \omega_0 n$. $\omega_0 = 3\pi/5$ $\therefore$ Now $\frac{\omega_0}{2\pi} = \frac{3\pi/5}{2\pi} = \frac{3}{10} = \frac{m}{N}$

rational

$\therefore$ periodic




It means ⟸ 3 cycles per 10 period

m cycles for N period

eg

$x(n) = \sin(\frac{\pi}{3}n) + \cos(\frac{\pi}{4}n)$

$\frac{\omega_0}{2\pi} = \frac{\pi/3}{2\pi} = \frac{1}{6} \Rightarrow$ here $m=1$, $N=6$

$\frac{\omega_0}{2\pi} = \frac{\pi/4}{2\pi} = \frac{1}{8} \Rightarrow$ here $m=1$, $N=8$

$\} \; \mathrm{Lcm}(6,8) = 24$

~~Lcm(N₁, N₂)~~ Total N for $x(n)$ is 24 → fundamental period for $x(t)$

if signal is $2\cos\left(\frac{150\pi n}{200} + 30°\right)$

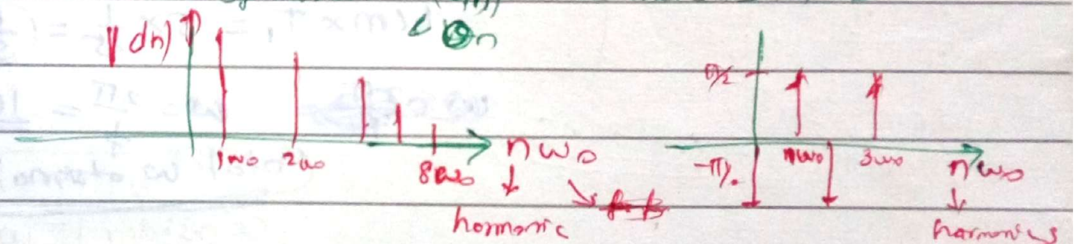
consider $\frac{\omega_0}{2\pi} = \frac{150\pi/200}{2\pi} = \frac{3}{8}$ $\quad N=8$

In Fourier series

* $\sin\omega t = \sin n\omega_0 t$ or $\cos\omega t = \cos n\omega_0 t$

$n$ → harmonics, $\omega_0$ → fundamental freq^n req^d

whenever draw spectrum $(d_n)$ & $\theta_n$, x axis is $n\omega_0$



for even function phase $\theta_n$ always 0 or ±180

for odd funct^n phase $\theta_n$ always ±90

if $x(t)$ given as → $\sin n\omega_0 t + \frac{1}{3}\sin 3\omega_0 t + \frac{1}{5}\sin 5\omega_0 t + \ldots$

here, $\omega_0$ is funda. freq. & 1, 3, 5 → harmonics

if $x(t)$ given as → $10\cos(10\pi t + \pi/2) + 4\sin(30\pi t + \pi/8)$

$10\cos(10\pi t + \pi/2) \Rightarrow$ comparing $\omega t = 10\pi t$

$\omega_1 = 10\pi$

$2\pi f = 10\pi \rightarrow f_1 = 5\,Hz$

~~here $\omega_0$ ... harmonics not present~~

~~Since $n\omega_0 = 10\pi$ ...~~

$4\sin(30\pi t + \pi/8) \Rightarrow$ comparing $\omega t = 30\pi t$ $\quad \omega_2 = 30\pi$

$f_2 = 15\,Hz$.

GCD $10\pi, 30\pi = 10\pi \Rightarrow \omega_0$

∴ for cos → $n=1$ for sin $n=3$

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(C)

~~x(t)~~ ~~ω=2πf~~ diff betn $X(f)$ & $X(\omega)$

## * conclusion (1)

in formula $X(\omega) \Rightarrow X(2\pi f)$ only exclude $2\pi$

$\Rightarrow X(\omega) \xleftrightarrow{\text{equivalent}} X(f)$

but if ω is outside function $X(\omega)$ eg. $j\omega X(\omega)$ or $e^{j\omega}X(\omega)$

then replace ω by 2πf does not exclude 2π

$\Rightarrow j\omega X(\omega) \Longrightarrow j2\pi f\, X(f)$

$e^{j\omega} X(\omega) \Longleftrightarrow e^{j2\pi f} X(f)$

## * Conclusion (2)

$2\pi\delta(\omega) \Longleftrightarrow \delta(f)$

$\Rightarrow \pi\delta(\omega) \Longleftrightarrow 0.5\,\delta(f)$

$\pi\delta(\omega + 2\pi a) \Longleftrightarrow 0.5\,\delta(f+a)$

(D)

for the properties $(t - t_0)$ it is constant which denotes that signal shift for constant value of t

lly for frequency domain,

$(\omega - \omega_0)$ it is constant frequency of shifting not fundamental frequency

* to avoid confusion. take factor α mostly taken in formula pg no. 88.

$(t - t_0) \Leftrightarrow (t - \alpha)$

$(\omega - \omega_0) \Leftrightarrow (\omega - 2\pi\alpha)$

Time shift. $x(t-\alpha) \longleftrightarrow e^{-j\omega\alpha} X(\omega)$.

frequency shift $e^{j2\pi\alpha t} x(t) \longleftrightarrow X(\omega - 2\pi\alpha)$

for shifting ⇒ from one domain to other domain

Extraordinarily factor of exponential with corresponding domain.

→

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}.$$

if $x(t)$ is shifted by $t_o$

$\therefore t \Rightarrow t - t_o$

$$\int_{-\infty}^{\infty} x(t-t_o)\, e^{-j\omega(t-t_o)}$$

$$= e^{+j\omega t_o} \int_{-\infty}^{\infty} x(t-t_o)\, e^{-j\omega t}$$

in integration derivation we can derive the multiplier

So. above statement of conversion from one domain to other is valid, the multiplicant factor is constant ~~e~~

i.e. $e^{j\omega_o t}\, x(t)$

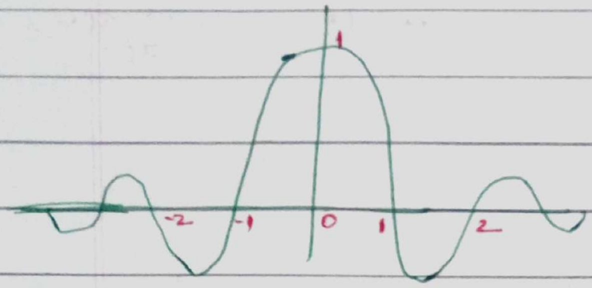
time $f^n$ only $\omega_o$ is constant $\therefore$ total $f^n$ is in time domain

$||^{ly}$ $e^{-j\omega t_o}\, X(\omega)$

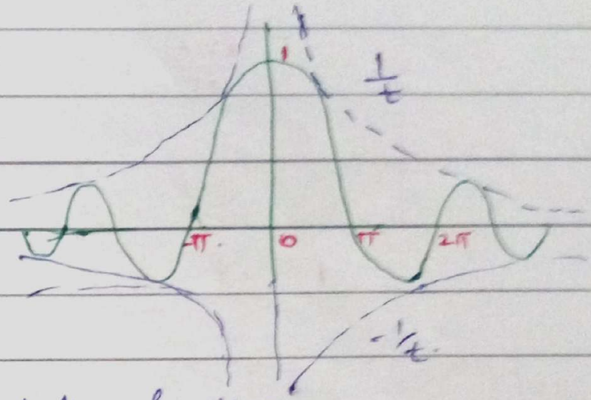
$\rightarrow$ $\omega$ funct$^n$ $t_o$ is constant $\therefore$ total $f^n$ is in freq. domain.

Q) sinc fn & its transformation from one domain to other

$$\mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}$$




$$Sa(x) = \frac{\sin x}{x}$$




also called filtering or interpolation function.

If we consider & compare with $\sin t$. $\sin t = 0$ at $0, \pi, 2\pi, \dots n\pi$.

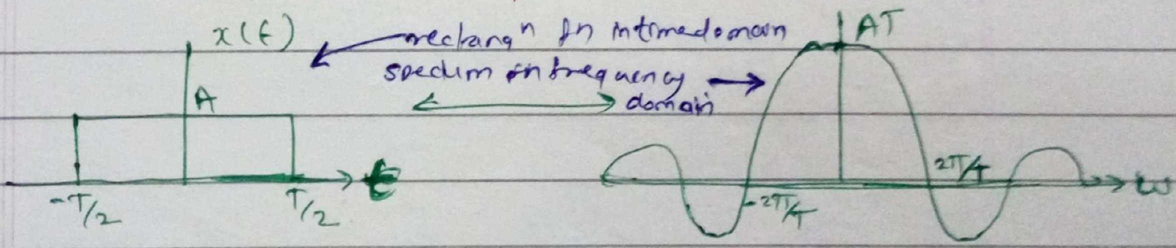
then ~~...~~ $\mathrm{sinc}\, x$ is 0 at $\pi, 2\pi \dots n\pi$ but not at 0

$\sin t$ magnitude constant $\quad$ $\mathrm{sinc}\, t$ magnitude is decaying

$\therefore$ at $\boxed{t=0}$ $\boxed{\mathrm{sinc}\, t = 1}$ as $\boxed{\sin t = 0}$

fourier transform of sinc function is rectangular pulse

or FT of Rectangular pulse is sinc function

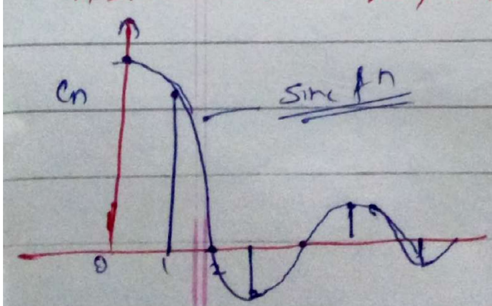



$$A\,\mathrm{rect}(t/T) \longleftrightarrow AT\,\mathrm{sinc}\left(\frac{\omega T}{2\pi}\right) = AT\,\mathrm{sinc}\left(\frac{\omega}{2\pi/T}\right)$$

If we find F.S. for rectangular periodic function then


($-\pi/2$ to $\pi/2$)

Cn spectrum is coming like




$$AT\, Sa\left(\frac{\omega T}{2}\right)$$

$$\mathrm{sinc}\left(\frac{\omega}{2\pi/T}\right) = \frac{\sin \pi \frac{\omega}{2\pi/T}}{\pi \frac{\omega}{2\pi/T}}$$

$$= \frac{\sin \frac{\omega T}{2}}{\frac{\omega T}{2}}$$

remember conversion both direction

$$\boxed{A\,\mathrm{rect}(t/T) \longleftrightarrow AT\, Sa\left(\frac{\omega T}{2}\right)}$$

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ramp $\underset{\int}{\overset{\text{diff}}{\rightleftarrows}}$ rect $\underset{\int}{\overset{\text{diff}}{\rightleftarrows}}$ impulse

$x(t) \longleftrightarrow X(\omega)$

$\frac{dx(t)}{dt} \longleftrightarrow j\omega X(\omega)$

$x(t) \longleftrightarrow X(\omega)$

$jt\, x(t) \longleftrightarrow \frac{d}{d\omega} X(\omega)$

F.T. of $\underline{\underline{\delta(t) = 1}}$

* RMS value of a wave having Period $T$

$$\text{RMS} = \sqrt{\frac{1}{T}\int_0^T (x(t))^2\, dt}$$

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# fourier Series.

* fourier series is an approximation process where a non sinusoidal waveform is converted into sinusoidal waveform

* One period to many frequency.

* Signal to have fourier series orthagonality is a must condition.

$x_1(t)$ & $x_2(t)$ are orthogonal if $\int_{t_1}^{t_2} x_1(t)\, x_2(t)\, dt = 0$

if $= 1$ then ortho normal *

* T.F.S.

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$a_0 = \frac{1}{T}\int_0^T g(t)\,dt \Rightarrow \frac{\cancel{\text{Area of } g(t) \text{ over a period}}}{\text{fundamental period.}}$$

D.C. value i.e. average value

$$a_n = \frac{2}{T}\int_0^T g(t) \cos n\omega_0 t\, dt \qquad b_n = \frac{2}{T}\int_0^T g(t) \sin n\omega_0 t\, dt.$$

To get peak value of fundamental component.

* Polar form :-

$$g(t) = d_0 + \sum_{n=1}^{\infty} d_n \cos(n\omega_0 t + \theta_n)$$

Comparing with T.F.S.

$d_0 = a_0$

$a_n = d_n \cos\theta_n$

$b_n = d_n \sin\theta_n$

$|d_n| = \sqrt{a_n^2 + b_n^2}$ → magnitude spectrum

$\theta_n = \tan^{-1}\left(-\frac{b_n}{a_n}\right)$ → phase spectrum

Amplitude spectrum

(for odd $f^n$ $\theta_n = \pm \pi/2$
for even $f^n$ $\theta_n = \pm 180$)

* maximum power of the signal is concentrated at lower range of frequencies.

* fourier Series spectrum is descrete

| | | $a_0$ | $a_n$ | $b_n$ |
|---|---|---|---|---|
| even | $g(t) = g(-t)$ | ? | ? | 0 |
| odd | $g(t) = -g(-t)$ | 0 | 0 | ? |
| Half wave | $g(t) = -g(t \pm T/2)$ | 0 | ? n=odd<br>0 n=even | ? n=odd<br>0 n=even |

* E.F.S. exponential or complex fourier series.

$$g(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

TFS to EFS

$C_0 = a_0$

$$C_n = \frac{a_n - jb_n}{2}$$

$$= \frac{1}{T}\int_0^T g(t)\, e^{-j\omega_0 n t}\, dt$$

$$C_{-n} = \frac{a_n + jb_n}{2}$$

EFS to TFS.

$a_0 = C_0$

$a_n = C_n + C_{-n}$

$jb_n = C_{-n} - C_n$

* −ve frequency is selected in EFS spectrum.

& magnitude at $(n\omega_0 = 1)$ in EFS = (mag. at $n\omega_0 = 1$) + (mag at $n\omega_0 = -1$) in TFS.

but phase only for +ve frequency in TFS. same as EFS.

(Pg 63. Q. 15 & 16)

* convergence of FS.

TFS → $a_0, a_n, \& b_n$ ] $< \infty$ Dirichlet condition
EFS → $C_0 \& C_n$

1. $\int_0^T |x(t)|\, dt < \infty$
2. $x(t)$ may have finite number of maxima & minima within time period "T"
3. number of discontinuties also finite. in 'T'

* Properties of F.S.

1) linearity : $x_1(t) \longleftrightarrow c_n$ } F.S. coeff of $x_1(t)$ is $c_n$

$x_2(t) \longleftrightarrow d_n$ } —— " —— $x_2(t)$ is $d_n$

then $\alpha x_1(t) + \beta x_2(t) \longleftrightarrow \alpha c_n + \beta d_n$.

2) Time shift: $x(t-t_0) \longleftrightarrow c_n e^{-jn\omega_0 t_0}$

$x(t) \leftrightarrow c_n$

3) Frequency shift: $x(t) e^{j\omega_0 m t} \longleftrightarrow c_{n-m}$

*** Shifting in one domain is multiplication of exponential term in other domain.

4) Time Scaling: $x(t) \leftrightarrow c_n$ the $x(\alpha t) \leftrightarrow c_n$

time compressing by $\alpha$ changes frequency from $\omega_0$ to $\alpha\omega_0$

5) Differentiation in time: $\frac{dx(t)}{dt} \longleftrightarrow (jn\omega_0) c_n$ ...

① do diff. of $x(t)$ number of times until we left with Impulse function

② then using shifting property of impulse & time diff. property. find $c_n$

③ in this method dc value is not taken care of so find DC value separately by $\boxed{\frac{\text{area under one period}}{\text{period}}}$

6) Parseval's theorem:

The total average power in a periodic signal equal to the sum of squared amplitude of each harmonic

$x(t) \longleftrightarrow c_n$ then $\frac{1}{T}\int_0^T |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |c_n|^2$

* maximum energy or power of any signal is always there in the low frequency region.

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* System with periodic i/p & o/p s.

* If we apply i/p as $x(t) = e^{j\omega t}$ then o/p is $y(t) = e^{j\omega t} H(\omega)$

  $H(\omega)$ is known as frequency response

* knowing $H(\omega)$ we can determine whether the system amplifies or attenuates a given sinusoidal component & how much phase shift added.

* response $y(t)$ of an LTI system for periodic system $x(t)$

$$y(t) = \sum_{n=-\infty}^{\infty} C_n H(n\omega_0) e^{-jn\omega_0 t}.$$

i/p coeff $\rightarrow C_n$ $\Rightarrow$ O/p coeff $\rightarrow C_n H(n\omega_0)$

* for an LTI system

  whenever i/p is sinusoidal o/p is definately sinusoidal

  IIly for exponential i/p exponential o/p.

* Signal analysis (frequency component frequency analysis) } F.S. & F.T.

  System analysis (circuit analysis by transferfunction etc) } L.T. & Z.T.

* For halfwave Rectifier

  average/dc value $a_0 = \frac{V_m}{\pi}$ $V_{rms} = \frac{V_m}{2}$ peak value of fundamental harmonics $a_1 = \frac{V_m}{2}$

  for full wave rectifier

  $a_0 = \frac{2V_m}{\pi}$ $V_{rms} = \frac{V_m}{\sqrt{2}}$ $a_1 = \frac{-4V_m}{3\pi}$

* $e^{\pm j\omega t} \sin(10\pi t)$ is periodic with period $10\pi$ $\qquad$ $e^{-7t} \sin\omega_0 t$ $\rightarrow$ N.P.

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## * fourier Transform (FT)

* fourier Series (FS) only for periodic but FT for both periodic & Non-periodic signal.
* FS spectrum is descrete while FT spectrum is continuous.

F.T. or spectrum of $x(t) = X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

I.F.T. is $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$

convergance of F.T.

→ F.T. defined for stable & energy signal.

→ F.T. power signal is defined as approx. to energy signal or impulse function are permitted.

→ F.T. not defined for absolutely integrable & square integrable signals.

$\omega = 2\pi f \qquad d\omega = 2\pi df$

$2\pi\delta(\omega) = \delta(f) \qquad 2\pi\delta(2\pi f) = 2\pi \frac{1}{2\pi}\delta(f) = \delta(f)$

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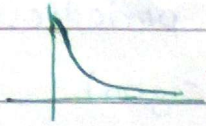
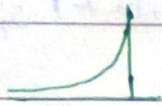
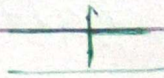
## * F.T. of standard signals.

Decaying expo. $e^{-at}u(t)$  $\longleftrightarrow \dfrac{1}{a+j\omega}$ ; $a>0$.

Increasing expo. $e^{at}u(-t)$  $\longleftrightarrow \dfrac{1}{a-j\omega}$

C.T. impulse $\delta(t)$  $\longleftrightarrow 1$ 

$1 \longleftrightarrow 2\pi\delta(\omega)$

Rectangular $A\,\mathrm{rect}(t/T)$, $A\Pi(t/T)$ $\longleftrightarrow AT\,Sa\left(\frac{\omega T}{2}\right)$ or $AT\,\mathrm{sinc}\left(\frac{\omega T}{2\pi}\right)$

$\therefore\ \mathrm{rect}(t) \longleftrightarrow \mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$

$\mathrm{tri}(t) \longleftrightarrow \mathrm{sinc}^2\left(\frac{\omega}{2\pi}\right)$

$\mathrm{sinc}(t) \longleftrightarrow \mathrm{rect}\left(\frac{\omega}{2\pi}\right)$

$\mathrm{sgn}(t) \longleftrightarrow \frac{2}{j\omega}$

$t e^{-at}u(t) \leftrightarrow \dfrac{1}{(a+j\omega)^2}$

$e^{-a|t|} \leftrightarrow \dfrac{2a}{a^2+\omega^2}$

$u(t) \longleftrightarrow \pi\delta(\omega) + \dfrac{1}{j\omega}$

$\cos(2\pi\alpha t) \leftrightarrow \pi\left[\delta(\omega+2\pi\alpha) + \delta(\omega-2\pi\alpha)\right]$

$\sin(\underbrace{2\pi\alpha}_{\omega_0} t) \leftrightarrow j\pi\left[\delta(\omega+\underbrace{2\pi\alpha}_{\omega_0}) - \delta(\omega-\underbrace{2\pi\alpha}_{\omega_0})\right]$

$e^{-\pi t^2} \leftrightarrow e^{-\omega^2/4\pi}$

$\leftrightarrow e^{-\pi f^2}$ gaussian fn.

$$e^{-at}\cos(2\pi\beta t)u(t) \leftrightarrow \frac{(a+j\omega)}{(a+j\omega)^2 + (2\pi\beta)^2}$$

$$e^{-at}\sin(2\pi\beta t)\,u(t) \longleftrightarrow \frac{2\pi\beta}{(a+j\omega)^2 + (2\pi\beta)^2}$$

$$\sum_{n=-\infty}^{\infty} \delta(t-nT) \longleftrightarrow \frac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$$

$$x_p(t) = \sum_{k=-\infty}^{\infty} X(k)\, e^{j2\pi k f_0 t} \longleftrightarrow \sum_{k=-\infty}^{\infty} 2\pi X(k)\,\delta(\omega - k\omega_0)$$

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# * fourier transform properties.

consider $\Rightarrow$ $x(t) \longleftrightarrow X(\omega)$

1) Similarity or duality : $X(t) \longleftrightarrow 2\pi x(-\omega)$

2) Time scaling : $x(\alpha t) \longleftrightarrow \frac{1}{|\alpha|} X\left(\frac{\omega}{\alpha}\right)$

3) folding : $x(-t) \longleftrightarrow X(-\omega)$

4) Time shift : $x(t-t_0) \longleftrightarrow e^{-j\omega t_0} X(\omega)$ } $t \to \omega$ $e^{-t_0}$

5) frequency shift : $e^{j\omega t_0} \xrightarrow{2\pi t} e^{j2\pi\omega_0 t} \longleftrightarrow X(\omega - 2\pi t_0)$ } $\omega \to t$ OR $e^{+\omega_0}$

$e^{j2\pi\alpha t} \longleftrightarrow X(\omega - 2\pi\alpha) \leftrightarrow X(f-\alpha)$

6) convolution : $x(t) * h(t) \longleftrightarrow X(\omega) \cdot H(\omega)$

7) Multiplication : $x(t)\, h(t) \longleftrightarrow \frac{1}{2\pi} X(\omega) * H(\omega)$
or frequency convolution

8) modulation : $x(t) \cos(\omega t_0) \longleftrightarrow 0.5\left[X(\omega + 2\pi t_0) + X(\omega - 2\pi t_0)\right]$

9) Derivative : $\frac{d}{dt} x(t) \longleftrightarrow j\omega X(\omega)$

10) Times-t : $-j\omega x(t) \longleftrightarrow 2\pi \frac{d}{d\omega} X(\omega)$
or freq. differentiatn $\quad t x(t) \longleftrightarrow j \frac{d}{d\omega}(X(\omega))$

11) Integration : $\int_{-\infty}^{t} x(t)\,dt \longleftrightarrow \frac{1}{j\omega} X(\omega) + \pi X(0)\delta(\omega)$

12) Conjugation : $x^*(t) \longleftrightarrow X^*(-\omega)$

13) correlation : $x(t) ** y(t) \longleftrightarrow X(\omega) Y^*(\omega)$

14) Autocorrelation : $x(t) ** x(t) \longleftrightarrow X(\omega) X^*(\omega) = |X(\omega)|^2$

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## * fourier transform theorems & imp points.

$$x(t) \longleftrightarrow X(\omega)$$

1) Rayleigh's Energy thm : or Parseval's Power thm.

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega.$$

2) Central ordinates

$$x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, d\omega.$$

$\boxed{OR}$

$$X(0) = \int_{-\infty}^{\infty} x(t) dt$$

3) Plancherel's thm:

$$\int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, Y^*(\omega)\, d\omega.$$

* impulse in one domain become constant in another domain.
* F.S. & F.T. are approximations
* Rectangle in one domain become sap or sinc $f^n$ in other domain.
* any problem in which exponential, sine, cosine term is there, then go for frequency shift property or modulation property.
* differentiation in time property always miss dc component in other domain.

$$\frac{d}{dt} x(t) \xleftrightarrow{FT} j\omega X(\omega) \quad \text{but} \quad X(\omega) \neq \frac{FT\left\{\frac{d}{dt} x(t)\right\}}{j\omega}.$$

* differentiation w.r.t. one variable corresponds to multiplication by other variable
* if spectrum is multiply by exponential causes time shift.

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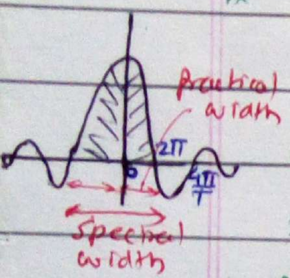
* fourier transform of guasian function is guasian function
* fourier transform of impulse train is impulse train.

⇒ ∴ quasian $f^n$ & impulse train are same in both domain

guasian * guasian = guasian.

* Integration property taking care of dc component where diff. property miss dc component.
* integration ⇒ Smoothing diff ⇒ sharpning.

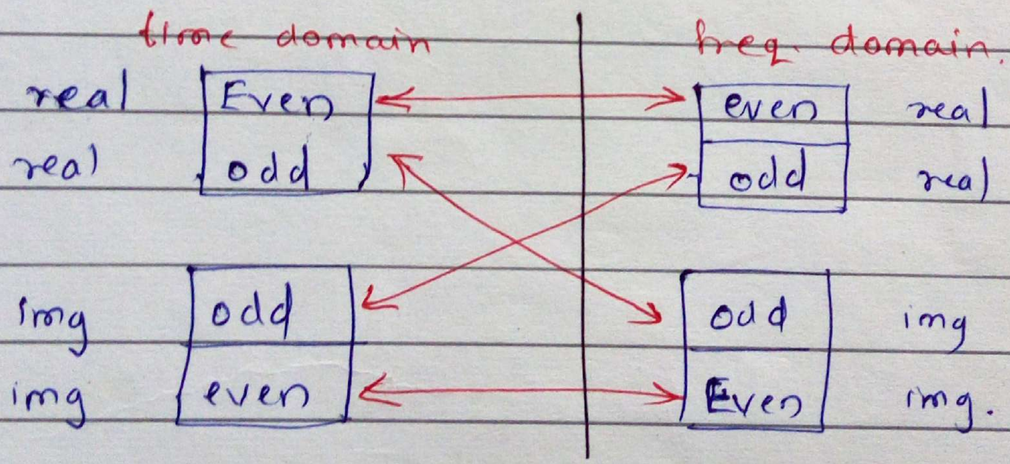



* Maximum energy is concentrated in central lobe (main lobe)

∴ maximum energy is there in lower frequencies.

* null to null bandwidth is $\frac{2\pi}{T}$
* Bandwidth always related to +ve. frequency here $\frac{2\pi}{T}$
* A signal can not be time limitted & Band limited simultaneously

i.e. Narrow band in one domain becomes wide band in other domain

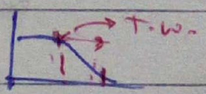



* compression in one domain becomes expansion in other domain.

* convolution of two non periodic signal may or may not periodic
* i/p known, response known get the o/p then → convolution property
* energy relation known Reiyleigh's thm.

Date: / /

* Noise is coming from outside of s/m ; unpredictable signal
* distortion is caused by s/m component; deterministic signal
* if magnitude is not constant ; magnitude or amp. distortion
  if phase is not linear; phase distortion.
* every ideal filter is Non causal & unstable
* transition (T.W.) width of the filter decides order of filter




  for higher order T.W. ↓ses.
* phase delay :- (carrier delay)
  occuring at a single frequency which is due to carrier.

$$t_p(\omega) = -\frac{\theta(\omega)}{\omega}$$

* Group delay :- (envelop delay)
  Occuring at a group of narrow band frequencies which is due to envelope of msg signal.

$$t_g(\omega) = -\frac{d\,\theta(\omega)}{d\omega}$$

* for linear phase channel $\theta(\omega) = -\omega t_0$ i.e. $t_p = t_g = \text{const.}$

* $y(t) = \frac{1}{100}\cos(100t - 10^{-6})\cos(10^6 t - 1.56)$ $\quad t_g = 10^{-8}$ $t_p = 1.56\times 10^{-6}$

$x(t) = \cos 20t + \cos 220t$ $\quad y(t) = \overbrace{1\cos\left(20t - \frac{\pi}{100}\right) + 2\cos\left(220t - \frac{\pi}{2}\right)}^{\text{amp. distortion}}$

$$\underbrace{20\left(t - \frac{\pi}{200}\right) - \quad 220\left(t - \frac{\pi}{440}\right)}_{\text{phase distortion.}}$$

* Hilbert Transform.

* Acts like high pass filter
* it is not actually a transform from one domain to other it just adds extra phase.

$$x(t) \longrightarrow \boxed{\begin{matrix} H.T. \\ \frac{1}{\pi t} \end{matrix}} \xrightarrow{\hat{x}(t)} \Rightarrow \hat{x}(t) = x(t) * \frac{1}{\pi t}$$

in freq. responce $\hat{X}(\omega) = X(\omega) \cdot [-j\,sgn(\omega)]$

$$\Rightarrow \frac{\hat{X}(\omega)}{X(\omega)} = -j\,sgn(\omega)$$

$$= -j \,;\, \omega > 0$$

$$= +j \,;\, \omega < 0$$

* $\displaystyle\int_{t_1}^{t_2} x(t)\,\hat{x}(t)\,dt = 0 \Big\}$ $\therefore$ <u>orthogonal</u>

* $\cos \omega_0 t \longrightarrow \sin \omega_0 t$  * $\sin \omega_0 t \longrightarrow -\cos \omega_0 t$
* $e^{j\omega_0 t} \longrightarrow -j\, e^{j\omega_0 t}$
* $\delta(t) \longrightarrow \frac{1}{\pi t}$  H/T of H(T) i.e $\frac{1}{\pi t}$ is $\delta(t)$

* $x(t) \longrightarrow \boxed{HT} \longrightarrow \boxed{HT} \longrightarrow -x(t)$

$x(t) \cos \omega_0 t \longrightarrow x(t) \sin \omega_0 t.$

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* co-relation

* without folding convolution means co-relation.

* Auto correlation & cross co-relation

applications :- ① Matched filter. $h(t) = x[-(t-T)]$

② Hidden periodicity ⇒ used in music.

③ Template matching in image processing.

④ for comparission.

ACF (Auto correlat$^n$ f$^n$)

① Energy $R_x(\tau) = \int_{-\infty}^{\infty} x(t)\, x^*(t-\tau)\, dt.$ — * no meaning

② Power $R_x(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)\, x(t-\tau)\, dt.$

$x(t+\tau)$ also can ∵ ACF is even f$^n$ of $\tau$

Properties of ACF

* ACF is even f$^n$ of $\tau$ ∴ $R(\tau) = R(-\tau)$
* ACF at origine indicates either energy or power in the signal.
* Max value of ACF occures at origine. $R_x(\tau) \le R_x(0)$
* $R_x(\tau) = x(\tau) * x(-\tau)$
* F.T. of ACF is called ESD or PSD $R_x(\tau) \longrightarrow S_x(\omega)$
* for LTI system if $Y(\omega) = X(\omega) \cdot H(\omega)$ then.

$|Y(\omega)|^2 = |X(\omega)|^2 |H(\omega)|^2 \Rightarrow S_y(\omega) = S_x(\omega)\, |H(\omega)|^2$

o/p spectral density — i/p spectral density.

* spectral density = |mag|$^2$

* Area under spectral density is energy or power.

* $\left.\begin{array}{l} A\cos(\omega_0 t + \theta) \\ \text{OR } A\sin(\omega_0 t + \theta) \end{array}\right\} R_x(\tau) = \frac{A^2}{2} \cos\omega_0 t$

* for energy signal go by convolution in F.T. method.

for Energy at o/p / msv at o/p $= \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{o/p S.D. } d\omega.$

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* F.T. of periodic signal.

F.T. of a periodic signal consists of sequence of equidistant impulse located at harmonic frequencies of the signal.

$$x_p(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \Rightarrow X_p(\omega) = \sum_{n=-\infty}^{\infty} 2\pi c_n \delta(\omega - \omega_0)$$

* ① get $c_n$
② get i/p spectrum
③ get F.T. ④ get o/p spectrum ⑤ take IFT.

* Sampling theorem.
  - → Ideal
  - → Practical ($\omega_0 \geq 2\omega_m$, $f_s \geq 2f_m$)
    - → Natural
    - → flat-top

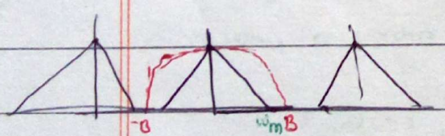
* Ideal Sampling.

$$x_\delta(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$X_\delta(\omega) = \frac{1}{T_s} \sum_{n=\infty}^{\infty} X(\omega - n\omega_0)$$

* $\omega_0 > 2\omega_m$ → oversampling → guard band $= \omega_0 - 2\omega_m$.
$\omega_0 = 2\omega_m$ → critical sampling → guard band $= 0$.
$\omega_0 < 2\omega_m$ → under sampling → aliasing problem

* Critical sampling rate is known as Nyquist Rate. $NR = 2\omega_m$




freq. at o/p of sampler $= (\omega - n\omega_0)$, $n \to -\infty$ to $\infty$

$= f_m \pm n f_s$

$\omega_0 = 2\pi f_s$  $T_s = \frac{1}{f_s}$

* Natural sampling :- In natural sampling n is decided by $c_n$

* Flat-top sampling:

* Because of maintaining constant amp level we introduce amplitude distortion of $T\,\mathrm{sinc}\left(\frac{\omega T}{2\pi}\right)$ & phase delay of $\frac{-\omega T}{2}$ which is known as aperture effect.

* To cancel this, Flat top sample signal is applied to an equiliser $\left(\frac{1}{|H(\omega)|}\right)$

* $X(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} x(\omega - n\omega_0)$

$y(t) =$ . . . . . sampled signal. $O/p = \frac{y(t)}{T_s}$ eg. pg. 105

* O/p at $t = 0$

$$y(0) = \int_{-\infty}^{\infty} Y(f)\,df.$$

$$e^{-\pi t^2} \xleftrightarrow{F.T.} e^{-\pi f^2} \Big\} \text{aussian fn.}$$

$$e^{-\frac{\pi t^2}{4}} \xleftrightarrow{F.T.} 2e^{-4\pi f^2}$$

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* Laplace transform.

* $L\{x(t)\} = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt = X(s)$   $s = \sigma + j\omega$.

when $\sigma = 0$.  $X(s) = F.T.\{x(t)\}$

also we can say $X(s) = F.T.\{x(t)\, e^{-\sigma t}\}$

$-\infty$ to $\infty$ → bilateral transform   $0$ to $\infty$ → unilateral.

Region of convergence of L.T. (ROC)

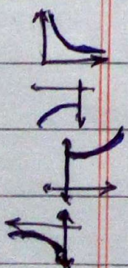
$X(s) < \infty$

for any condition to have Laplace T. condition is

$$\int_{-\infty}^{\infty} |x(t)\, e^{-\sigma t}|\, dt < \infty \rightarrow \text{Necessary condit}^n.$$

* for a finite length signal ROC is complete S plane. i.e. ROC is convergable to S plane.
* ROC of L.T. consist of lines parallel to $j\omega$ axis.
* ROC do not contain any pole
* for stability in L.T. ROC must include $j\omega$ axis.
* ROC is not decided by ~~poles~~ zeros
* to get signal plot of $\pm e^{\pm at} u(\pm t)$ do take ~~[illegible]~~ points ① at $t=0$ ② at $t=\infty$ ③ at $t=-\infty$ and plot according to $u(\pm t)$ $\because (e^{\infty} = \infty ;\ e^{-\infty} = 0 \} ; -e^{\infty} = -\infty)$

* $\sigma$ $>/<$ $Re(\pm a)$ taken according to graph is plotted. if $x(t)$ graph is stable Then ROC must contain $j\omega$ axis ∴ $>/<$ sign taken accordingly to have $j\omega$ axis in ROC.

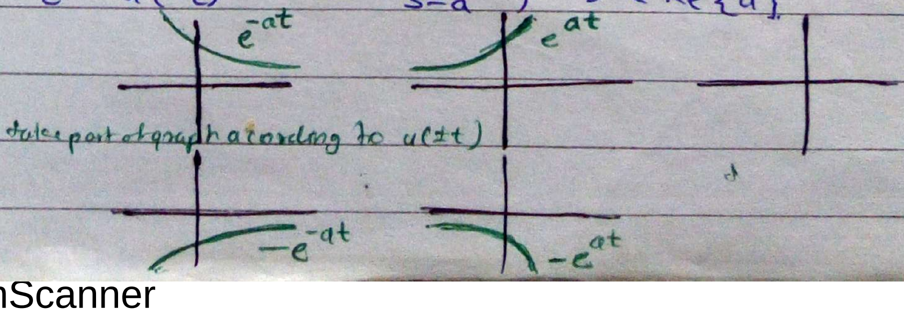


$e^{-at} u(t) \longleftrightarrow \frac{1}{s+a}$ ; $\sigma > Re\{-a\}$

$-e^{-at} u(-t) \longleftrightarrow \frac{1}{s+a}$ ; $\sigma < Re\{-a\}$

$e^{at} u(t) \longleftrightarrow \frac{1}{s-a}$ ; $\sigma > Re\{a\}$

$-e^{at} u(-t) \longleftrightarrow \frac{1}{s-a}$ ; $\sigma < Re\{a\}$



* imp. formulaes in L.T.

* $e^{-at} \Leftrightarrow \frac{1}{s+a}$ ; $e^{at} \Leftrightarrow \frac{1}{s-a}$ ; $u(t) \Leftrightarrow \frac{1}{s}$

* $\underbrace{t\,u(t)}_{=r(t)} \longleftrightarrow \frac{1}{s^2}$

if * $\left[ (t-a)u(t) = tu(t) - a\,u(t) \right.$

$\left. t\,u(t-a) = (t-a+a)\,u(t-a) \right]$

* $\sin at \longleftrightarrow \frac{a}{s^2+a^2}$ $\cos at \Leftrightarrow \frac{s}{s^2+a^2}$ $\cosh at \Leftrightarrow \frac{s}{s^2-a^2}$

* $\frac{t^{n-1}}{(n-1)!} e^{-at} u(t) \longleftrightarrow \frac{1}{(s+a)^n}$ * $t^n \leftrightarrow \frac{n!}{s^{n+1}}$ $n > -1$

* $t^n f(t) \Rightarrow (-1)^n \frac{d^n}{ds^n} F(s)$

* $\int_0^t f(t)\,dt^n = \frac{1}{s^n} F(s)$

* $f(t)\delta(t-a) = e^{-as} f(a)$

* $\overset{\rightarrow \text{derivative}}{f^n}(t) = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - f^{(n-1)}(0)$

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* Properties of L.T.

① Linearity :- if $x_1(t) \longleftrightarrow X_1(s)$ with ROC $= R_1$

$x_2(t) \longleftrightarrow X_2(s)$ with ROC $= R_2$

Then $a x_1(t) + b x_2(t) \longleftrightarrow a X_1(s) + b X_2(s)$ ROC $= R_1 \cap R_2$

② time shifting :-

$$x(t-t_0) \longleftrightarrow e^{-st_0} X(s) \; ; \; ROC = R.$$

③ shift in s domain :-

$$x(t) e^{s_0 t} \longleftrightarrow X(s-s_0) \qquad ROC = R + Re(s_0)$$

④ time Reversal :-

$$x(-t) \longleftrightarrow X(-s) \qquad ROC = -R$$

⑤ differentiation in time :-

$$\frac{d\,x(t)}{dt} \longleftrightarrow sX(s) \qquad ROC = R$$

⑥ Diff. in s-domain.

$$t\,x(t) \longleftrightarrow -\frac{d}{ds} X(s)$$

⑦ Convolution in time

$$x_1(t) * x_2(t) \longleftrightarrow X(s) H(s) \qquad ROC = R_1 \cap R_2$$

⑧ integration in time

$$\int_{-\infty}^{t} x(\tau)\, d\tau \longleftrightarrow \frac{X(s)}{s}$$

⑨ Integration in frequensy

$$\frac{x(t)}{t} \longleftrightarrow \int_{s}^{\infty} X(s)\, ds.$$

unilateral L.T. :- $X(s) = \int x(t) e^{-st} dt.$ (only onsided signal is token)

diff. in time :- $\frac{d}{dt} x(t) = sX(s) - x(0)$

$$\frac{d^2}{dt^2} x(t) = s^2 X(s) - s x(0) - x^{+1}(0)$$

⑩ initial & final value thm :-

$$x(0) = \lim_{s \to \infty} s X(s) \qquad x(\infty) = \lim_{s \to 0} s X(s)$$

* before using initial thm very whethe $X(s)$ is strictly proper (power of Num < power of deno) or not. by division if not make it strickly proper & apply initial value ~~term~~ thm to strictly proper term only.

f. v. thm ← only of ploles in left plane → * except $s=0$

* final value thm is not applicable if poles are on $j\omega$ axis or right side of s plane or for sinusoidal signal. i.e. if $x(\infty) \to$ indeterminate

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* Partial ~~differential~~ fractions

* $Y(s) = \frac{4}{(s+2)(s+1)^3} = \frac{A}{(s+2)} + \frac{B}{(s+1)^3} + \frac{C}{(s+1)^2} + \frac{D}{s+1}$ — (*)

1. $A = (s+2)\,Y(s)\Big|_{s=-2}$ $\quad B = (s+1)^3\,Y(s)\Big|_{s=-1}$

$C = \frac{d}{ds}\left\{(s+1)^3\,Y(s)\right\}\Big|_{s=-1}$ $\quad D = \frac{1}{2!}\frac{d^2}{ds^2}\left\{(s+1)^3\,Y(s)\right\}\Big|_{s=-1}$

2. find A & B as above put $s=0$ (other than $-2$ & $-1$) get eq$^n$ of C & D again put $s=1$ (other than $-2$ & $-1$) get another eq$^n$ put in eq$^n$ (*).
No. two eq$^n$ & two unknown. get C & D.

3. limit method.

* $\int_{-\infty}^{t} y(\lambda)\, e^{-3(t-\lambda)}\, u(t-\lambda)\, d\lambda = y(t) * e^{-3(t)}\, u(t)$

* $F_1(s)\, F_1^*(s) = |F_1(s)|^2$

| Zero state Respone (ZSR) | Zero i/p Response ZIR |
|---|---|
| * Forced Responce | * natural Responce. |
| * all initial condit$^n$ taken to be 0 | * all initial conditions are considered |
| * i/p is applied | * i/p is not taken. |

* o/p has constant value of 1 $\Rightarrow y(t)\Big|_{t\to\infty} = 1$ F.V.T. (final value thm.)
dont take H(s) as it is when above condit$^n$ is given first verify FVT & modify accordingly to H(s). (eg. pg. 122 Q.5)

* L.T. of pure sinusoidal signal is not possible.
L.T. of $\sin t\, u(t)$ exist but <u>not $\sin t$</u>.

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## * Causality & stability.

* causality means Right side of splane
* stability mean ~~[illegible]~~ jw axis included in ROC

causal – RHS    anticausal – LHS    non causal – both sided

* I/p ROC → $\boxed{\begin{matrix} H(w) \\ ROC \end{matrix}}$ → o/p ROC

H(w) ROC = intersection of I/p ROC & o/p ROC
= i/p ROC ∩ o/p ROC

* if i/p is Anticausal o/p is non causal then sym should be causal.

## * L.T. of switched periodic signals:-

$$X_p(s) = \frac{\int_0^T x(t)\, e^{-st}\, dt}{1 - e^{-sT}}$$

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## * DTFT (Descrete time F.T).

CTFT → ① $\omega \rightarrow -\infty$ to $\infty$ ② Non periodic continuous spectrum

DTFT → ① $\omega \rightarrow -\pi$ to $\pi$ ③ Periodic continuous spectrum.

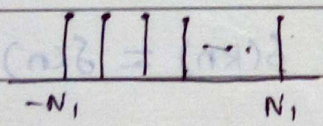
$$\underbrace{X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}_{\text{CTFT}} \Rightarrow \underbrace{X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}}_{\text{DTFT}}.$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega.$$

## * Periodicity

DTFT is periodic spectrum. $\left|X(e^{j(\omega+2\pi k)})\right| = Xe^{j\omega}$.

* $a^n u[n] \Leftrightarrow \dfrac{1}{1-ae^{-j\omega}}$

*  $\longleftrightarrow \dfrac{\sin \omega(N_1+\frac{1}{2})}{\sin(\omega/2)} = \dfrac{\sin \omega\frac{(2N_1+1)}{2}}{\sin \frac{\omega}{2}}$

$$\sum_{m=0}^{N} \alpha^m = \frac{1-\alpha^{N+1}}{1-\alpha}.$$

* $\delta(n-n_0) \longleftrightarrow e^{-j\omega n_0}$
* $\delta(n) \longleftrightarrow 1$

* Spectrum at $\omega = 0$ (i.e $Y(e^{j0})$) means in time domain area under the signal (i.e $\sum_{-\infty}^{\infty} y(n)$)

* $\sum_{n=-\infty}^{\infty} x(n)(-1)^n = X(e^{j\pi})$ / High freq. gain. $= H(e^{j\pi})$

$\sum_{n=-\infty}^{\infty} x(n) = X(e^{j0})$ / D.C. gain $= H(e^{j0})$

* Properties :- $x(n) \longleftrightarrow X(e^{j\omega})$

1) Time shift :→ $x(n-n_0) \longleftrightarrow e^{-j\omega n_0} X(e^{j\omega})$

2) frequency shift :-

$$x(n)\, e^{j\omega_0 n} \longleftrightarrow X\left(e^{j(\omega-\omega_0)}\right)$$

* Time scalling :-

$$x(n/k) \longleftrightarrow X(e^{j\omega k})$$

* frequency diff. :-

$$-jn\,x(n) \longleftrightarrow \frac{d}{d\omega} X(e^{j\omega})$$

$$n\,x(n) \longleftrightarrow j\frac{d}{d\omega} X(e^{j\omega})$$

* Convolution :-

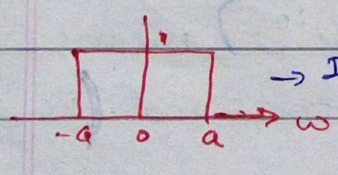
$$x(n) * h(n) \longleftrightarrow X(e^{j\omega}) \cdot H(e^{j\omega})$$

* Parseval's Relation :-

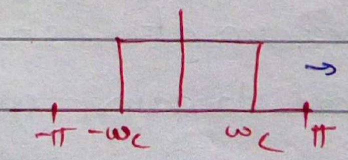
$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int |X(e^{j\omega})|^2 \, d\omega.$$

* $\delta(\alpha t) = \frac{1}{|\alpha|}\delta(t)$ but $\delta(kn) = \delta(n)$

* In DTFT the spectrum is repeated since it is periodic ∴ spectrum $-\pi$ to $\pi$ is imp. & taken into consideration bcz $\pi$ to $3\pi$ (or $-3\pi$ to $-\pi$ will be the same as $-\pi$ to $\pi$




→ Ideal Analog filter




→ Ideal digital filter.

* if DTFT spectrum is rectangular then in time domain it is sa or sinc function.

* $x[3n+1]$ $\quad$ $\cancel{x[3(n+\frac{1}{3})]}$ $\quad$ $x[n+1]$ then $x(3n+1)$

$$e^{-j\omega(-1)} X(e^{j\omega}) \qquad e^{j\omega/3} X(e^{j\omega/3})$$

* $$j\frac{d}{d\omega}\left[\frac{1}{1-ae^{-j\omega}}\right] = \frac{ae^{-j\omega}}{(1-ae^{-j\omega})^2}$$

* $$\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 2\pi \sum_{n=N_1}^{N_2} |-jn\,x(n)|^2$$ by parsevals relation

($N_1 - N_2$ length of signal)

* Some imp formulae of DTFT.

* $a^n u[n] \longleftrightarrow \frac{1}{1-ae^{-j\omega}}$ $\quad$ * $e^{j\pi} = -1$

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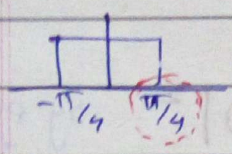
* When we go for spectrum analysis of DTFT i.e. taking magnitude and phase. both combinely gives conjugate Symmetry i.e. if Mag. spectrum is even $f^n$ phase spectrum must be odd $f^n$.

* $$\sum_{n=-\infty}^{\infty} (-1)^n x(n) = X(e^{j\pi})$$

* 

$\xleftrightarrow{IFT}$ $\dfrac{\sin\frac{\pi}{4}n}{\pi n}$ $\xrightarrow{\text{formulate}}$ ?

* as in continuous differentiation is called as HPF.

$x(n)-x(n-1)$ Here in descrete difference is called as HPF.

$x(n)+x(n-1)$ * $ll_y$ $\int$ in continuous & $\Sigma$ or summation in descrete LPF.

$x(n)$ * Addition of both gives Allpass filter

*

* $Y(0) = X(0) \cdot H(0)$ spectrum at $\omega=0$ is area under the signal.

$= \Sigma|x(n)| \; \Sigma|h(n)|$

* Differentiation of phase responce is group delay.

$$t_g(\omega) = -\frac{d\,\theta(\omega)}{d\omega}$$

* FIR filters have linear phase. i.e. IR may be symmetric or Anti symetric.

$\theta(\omega) = -\alpha\omega$ $\alpha \Rightarrow n$ at which signal is symmetrical.

energy in $x(n) = \dfrac{\sin\omega_c n}{\pi n} \Rightarrow \dfrac{\omega_c}{\pi}$

$$\sum_{-\infty}^{\infty} x[n]y[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\,Y(e^{j\omega})\,d\omega$$ modified form of parsvals thm.

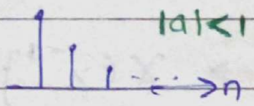
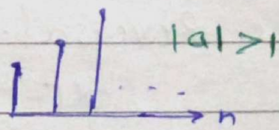
* $\int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega \Rightarrow$ Apply IDTFT & area concep. $\Rightarrow$ $2\pi x[0]$

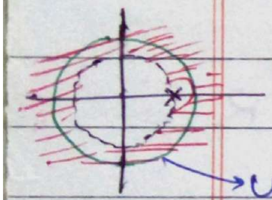
* maximum value in Auto corelat$^n$ at origne is nothing but energy (i.e.) Parsevals thm $\Rightarrow$ $\Sigma|x(n)|^2$

## * Z-transform.

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}. \qquad z = re^{j\omega}$$

* Discrete time counterpart of L.T. is ZT.
* In L.T. stability is defined w.r.t. jω axis in Z.T. stability is defined w.r.t. unit circle.
* Z.T. calculated on unit circle is DTFT.
* DTFT defined for stable system only whereas Z.T. defined for unstable signal also.

* $a^n u[n] \Rightarrow$  



$$a^n u[n] \longleftrightarrow \frac{1}{1-az^{-1}} \text{ OR } \frac{z}{z-a} \; ; \; |z| > |a| \Rightarrow ROC$$

→ unit circle Ref. for stability if ROC includes unit circle then stable else not.

**ROC** { Rightsided (in L.T.) ⇒ Outsided (in Z.T.)
Left sided (in L.T.) ⇒ Insided (in Z.T.)

$$-a^n u[-n-1] \longleftrightarrow \frac{+a^{-1}z}{1-a^{-1}z} \text{ or } \frac{+z}{z-a} \; ; \; |z| < |a| \Rightarrow ROC.$$

$x(n) = \delta(n) \Rightarrow X(z) = 1 \Rightarrow$ ROC entire zplane

$x(n) = \delta(n-1) \Rightarrow X(z) = z^{-1} \Rightarrow$ ROC entire zplane except $z = 0$

$x(n) = \delta(n+1) \Rightarrow X(z) = z \Rightarrow$ ROC entire zplane except $z = \infty$.

* ROC exclude point where Z.T. is ∞.
also for finite length signal ROC is entire zplane except 0 & ∞. ⇒ ROC = $0 < |z| < \infty$

* $u(n) - u(n-10) \Rightarrow$ signal from $0 \le n \le 9$

* No common ROC means No Z.T.
* for an anticausal signal poles should lie outside the unit circle for stability.

## * Properties of Z-T.

Linearity : $x_1(n) \longleftrightarrow X_1(z)$ ROC $R_1$

$x_2(n) \longleftrightarrow X_2(z)$ ROC $R_2$

$a x_1(n) + b x_2(n) \longleftrightarrow a X_1(z) + b X_2(z)$ ROC = $R_1 \cap R_2$

time shift :

$$x[n-n_0] \longleftrightarrow z^{-n_0} X(z) \quad \text{ROC} = R$$

Exponential multiplications or scalling in Z-domain.

$$a^n x[n] \longleftrightarrow X(z/a) \quad \text{ROC} = |a|R.$$

time Reversal.

$$x[-n] \longleftrightarrow X(z^{-1}) \quad \text{ROC} = 1/R.$$

Differentiation in Z-domain:

$$n\,x[n] \longleftrightarrow -z\frac{d\,X(z)}{dz} \quad \text{ROC} = R.$$

Convolution in time:

$$x[n] * h[n] \longleftrightarrow X(z)H(z) \quad \text{ROC} = R_1 \cap R_2$$

Time Scalling property:

$$x[n/k] \longleftrightarrow X[z^k]$$

but we know $\delta[kn] = \delta[n]$

Accumulation :

$$\sum_{k=-\infty}^{n} x[k] \longleftrightarrow \frac{X(z)}{1-z^{-1}} \quad \text{ROC} = R \cap |z|>1$$

Initial value thm : $x(0) = \lim_{z\to\infty} X(z)$

final value thm : $x(\infty) = \lim_{z\to 1} (1-z^{-1})X(z)$

or $x(\infty) = \lim_{z\to 1} (z-1)X(z)$.

causality verification in UZT. $\lim_{z\to\infty} X(z) \neq \infty$

(i.e. when NUM power not greater than DEN. power then system is causal.)

> when poles lying outside the unit circle F.V.T. is not applicable.

FVT only valid for poles inside unit circle except simple pole at $z = \pm 1$

* some imp. formulae

* $a^n u[n] \longleftrightarrow \frac{1}{1-az^{-1}}$ ROC : $|z| > |a|$

* $\delta(n) \leftrightarrow 1$ entire Z plane

* $e^{j\pi/2} = j$ , $e^{-j\pi/2} = -j$

* $u[n] \longleftrightarrow \frac{1}{1-z^{-1}}$ OR $\frac{z}{z-1}$

* $z = e^{j\omega}$ when $r=1$ $\omega = 0 \Rightarrow z = 1$ $\omega = \pi/2 \Rightarrow z = j$
  $\omega = \pi \Rightarrow z = -1$

*( apply this to transfer $f^n$ of z to identify which type filter it is)

$$\frac{d\,x(t)}{dt} \xleftrightarrow{L.T.} S\,X(s)$$

$$x(n) - x(n-1) \xleftrightarrow{Z.T.} (1-z^{-1})\,X(z)$$

| L.T. | | Z.T. |
|---|---|---|
| $s$ | $\Longleftrightarrow$ | $(1-z^{-1})$ |
| $\frac{1}{s}$ | $\Longleftrightarrow$ | $\frac{1}{(1-z^{-1})}$ |

UZT (unilateral Z transform) $X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$

Left shift : $x(n+1) \longleftrightarrow zX(z) - z\,x(0)$

Right shift : $x(n-1) \longleftrightarrow z^{-1}X(z) + x(-1)$

$x(n-2) \longleftrightarrow z^{-2}X(z) + z^{-1}x(-1) + x(-2)$

$u(t) * u(t) = r(t)$ or $t\,u(t)$

but $u(n) * u(n) = (n+1)\,u(n)$

$h(n) * h_{inv}(n) = \delta(n) \Rightarrow H(z)\,H_{inv}(z) = 1$ } valid for LTI system.

* $a^n \rightarrow \frac{z}{z-a}$ $\sin n\theta \rightarrow \frac{z\sin\theta}{z^2 - 2z\cos\theta + 1}$

$u[n] \rightarrow \frac{z}{z-1}$ $\cos n\theta \rightarrow \frac{z(z-\cos\theta)}{z^2 - 2z\cos\theta + 1}$

* Z Inverse

$\frac{z}{z-a} \rightarrow a^n$ $\frac{z^2}{(z-a)^2} \rightarrow (n+1)a^n$ $\frac{1}{z-a} \rightarrow a^{n-1}u(n-1)$

$\frac{1}{(z-a)^2} \rightarrow a^{n-2}(n-1)\,u(n-2)$

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* $x(-t) \xleftrightarrow{L.T.} X(-s)$  ROC = −R  in Laplace T.
$x(-t) \xleftrightarrow{Z.T} X(z^{-1})$  ROC = $\frac{1}{R}$  in Z-T.
∴ Reflection in time domain ⟷ inversion in Z-domain

* $3^n u[-n] = \left(\frac{1}{3}\right)^{-n} u[-n]$ Now use time reversal.
* $y[n]$ at $n=k$ is coefficient of $z^{-k}$

* poles & zeros are reciprocal to each other in Z-domain
OR numerator polynomial is mirror image of denominator poly. that T.F. Represent All pass filter eg. $H(z) = \frac{\alpha + z^{-1}}{1+\alpha z^{-1}}$

$\frac{\alpha}{1}$ $\frac{1}{\alpha}$ } mirror image.

zero at $z = -\frac{1}{\alpha}$  Pole at $z = -\alpha$

* Analog filter can not have linear phase.
Digital filter can have linear phase so we design all pass filter to get linear phase.
so whenever non linear phase just cascade APF digital filter to have linear phase.

* Butterworth & chebeshew → Analog filter non linear phase.

* Some times we get values at $z=1$ & $z=-1$ while identifying the LPF (see eg. 15 pg 148) then go for normal procedure.
we get −ve value.

* $y[n] = x[n] + x[n-1]$ → LPF $\xrightarrow{Z.T.}$ $(1+z^{-1})$
$y[n] = x[n] - x[n-1]$ → HPF $\xleftrightarrow{Z.T.}$ $(1-z^{-1})$

* ROC concept is there in L.T. & Z.T. Not in F.T. & DFT. because FT & DFT deal with stable system only but L.T. & Z.T. system may or may not stable

* Degital freq. $(\omega) = \frac{2\pi f}{f_s}$ → analog freq. / → sampling freq.  (eg. 19 pg 149)

* For T.F. to be bothe causal & stable all poles must lie inside the unit circle ~~Basic versa~~
Not depend on zeros.

* minimum phase means poles and zeros inside unit circle.
* Non-minimum phase system means one or more poles or zeros can lies out side the unit circle.

* Proper rational transfer $f^n$ & its inverse are both causal & stable. if and only if poles & zeros inside unit circle.

* $$H_{inv}(z) = \frac{1}{H(z)} \neq H(z^{-1})$$

for better stability or fast Responce in z domain locate the pole very near to the origine wheare as in the s-plane place pole away from the ~~s-plane~~ jw axis towards left half of the splane.

if $H(s) = \frac{1}{s+\alpha} \xrightarrow{Z.T.} \frac{1}{1 - e^{\alpha T_s} z^{-1}}$ ($T_s \Rightarrow$ sampling period)

$T_s = \frac{1}{f_s}$

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## * Realization structures.

IIR filter | FIR filter.

* Used for magnitude approximat$^n$ | * used for linear phase respence

* $y(n) = \sum_{k=0}^{M} k_n x(n-k) - \sum_{k=1}^{N} a_k y(n-k)$ | * $y(n) = \sum_{k=0}^{\infty} b_k x[n-k]$

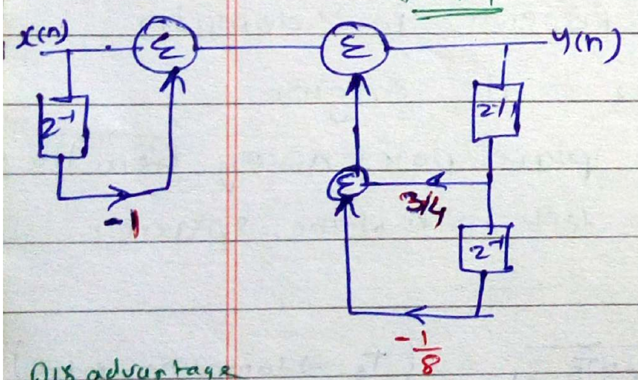
Specificat$^n$ → $\delta_p$ → pass band $\delta_s$ → stop band $\omega_p$ → passband gain
$\omega_s$ → stop band gain

* for the given specifications IIR requires less order compare to FIR.

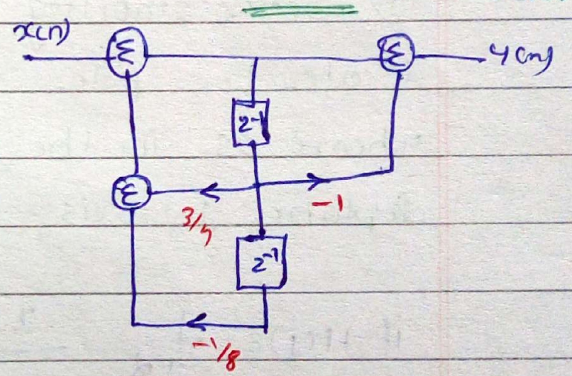
* FIR filters are generally stable. IIR not gaurantee.

eg. $y[n] = x[n] - x[n-1] + \frac{3}{4} y(n-1) - \frac{1}{8} y(n-2)$

form 1




form 2 or canonical

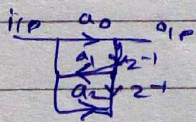



D/s advantage
more delays
more memory.
Execution time is more

order of D.E. or T.F. = No. of delays.
∴ here = 2

* If we interchange the direction of branch arrows - i/p & o/p. the T.F. remains same i.e. flow graph Reversal thm.

* $\frac{1}{1 - 0.72z^{-1} + 0.13z^{-2}} \Rightarrow$




$a_0 = 1$
$a_1 = 0.7$
$a_2 = -0.13$
} change in sign.

* To implement $n^{th}$ order cascaded section by taking minimum $2^{nd}$ order section the number of possible pairing are $\left(\frac{n}{2}!\right)^2$

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Descrete $\longleftrightarrow$ Periodic
(D) (P)

Continuous $\longleftrightarrow$ Non-periodic
(C) (NP)

| | t-d | f-d. |
|---|---|---|
| C.F.S. | Periodic & continuous | Descrete & N.P. |
| C.T.F.T. | conti. & NP | N.P. & conti. |
| D.T.F.T. | descrete & NP | Periodic & conti. |
| D.F.T. | Periodic & descrete | descrete & periodic. |

| | | i/p | o/p |
|---|---|---|---|
| for conti signal | CTFT. | $e^{j\omega t}$ | $e^{j\omega t}\,\underbrace{H(\omega)}_{\text{frequency Responce (FR)}}$ |
| | L.T. | $e^{st}$ | $e^{st}\,H(s)$ Transfer fn. (TF) |
| for descrete signal | DTFT | $e^{j\omega_0 n}$ | $e^{j\omega_0 n}\,H(e^{j\omega_0})$ (F.R) |
| | Z.T. | $z^n$ | $z^n\,H(z)$ (T.F.) |

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$H(s) \quad s = j\omega$

$A\cos(\omega_0 t + \theta) \rightarrow \boxed{H(\omega) = |H(\omega)| \angle H(\omega)} \rightarrow y_{ss}(t) = A|H(\omega_0)| \cos(\omega_0 t + \theta + \angle H(\omega_0))$

$A\cos(\omega_0 n + \phi) \rightarrow \boxed{H(e^{j\omega}) = |H(e^{j\omega})| \angle H(e^{j\omega})}$

$H(z) \quad z = e^{j\omega} \quad y_{ss}[n] = A|H(e^{j\omega_0})| \cos(\omega_0 n + \phi + \angle H(e^{j\omega_0}))$

| T.F. | F.R. |
|---|---|
| $s$ | $\rightarrow j\omega$ |
| $z$ | $\rightarrow e^{j\omega}$. |

$h(t) \xleftrightarrow{CTFT} H(\omega) \qquad h[n] \xleftrightarrow{DTFT} H(e^{j\omega})$

$h(t) \xleftrightarrow{L.T.} H(s) \qquad h[n] \xleftrightarrow{Z.T.} H(z)$

Differential / Difference

$\Downarrow \qquad \Downarrow$

$\underbrace{H(\omega)}_{F.R.} = \dfrac{Y(\omega)}{X(\omega)} \qquad \underbrace{H(e^{j\omega})}_{F.R.} = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})}$

$\underbrace{H(s)}_{T.F.} = \dfrac{Y(s)}{X(s)} \qquad \underbrace{H(z)}_{T.F.} = \dfrac{Y(z)}{X(z)}$

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CTFT $x(t-t_0) \longrightarrow e^{-j\omega t_0} X(\omega)$

LT $x(t-t_0) \longrightarrow e^{-st_0} X(s)$

DTFT $x[n-n_0] \longrightarrow e^{-j\omega n_0} X(e^{j\omega})$

ZT $x[n-n_0] \longrightarrow Z^{-n_0} X(Z)$

| CTFT | DTFT |
|---|---|
| $-jt \rightarrow \frac{d}{d\omega}$ | $-jn \rightarrow \frac{d}{d\omega}$ |
| $-t \rightarrow \frac{d}{ds}$ | $n \rightarrow -z\frac{d}{dz}$ |
| LT | ZT |

* for idial filter

for practical filter values varies small

| | | | |
|---|---|---|---|
| continuous | $\omega \rightarrow 0$ | $\omega \rightarrow \infty$ | $\omega = \omega_c$ |
| Descrete | $\omega \rightarrow 0$ | $\omega \rightarrow \infty$ | |
| LPF | 1 | 0 | $1/\sqrt{2}$ |
| HPF | 0 | 1 | $1/\sqrt{2}$ |
| BPF | 0 | 0 | 1 |
| BSF | 1 | 1 | 0 |

eg. $H(s) = \frac{s}{s^2+s+1}$

$$\left.\begin{array}{l}\lim_{s\to 0} H(s) = 0 \\ \lim_{s\to\infty} H(s) = 0\end{array}\right\} \text{B.P.F}$$

for descrete $s \rightarrow 0$ & $s \rightarrow \pi$

$e^{i\theta} = \cancel{\sin\theta + i\cos\theta}\ \cos\theta + i\sin\theta$

$e^{\theta} = \cancel{\cos i\theta +}\ \cosh\theta + i\sinh\theta$

$e^{j\pi} = -1$ $\qquad e^{j0} = 1 = e^{0}$

$e^{\pi} = 1$

$x = y$

$x^2 = xy$

$x^2 - y^2 = xy - y^2$

$(x+y)(x-y) = y(x-y)$

$2y = y$

$2 = 1$