

# $G^1$ hole filling with S-patches made easy

Péter Salvi

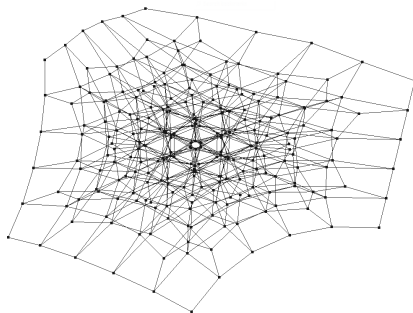
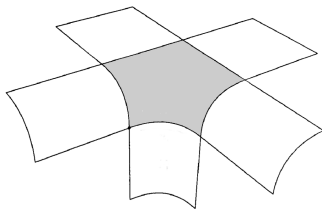
Budapest University of Technology and Economics

KÉPAF 2019

Debrecen, January 28-31

# S-patches

- Generalization of Bézier  $\triangle$ 
  - Any number of sides
- Beautiful theory
- Too many control points
  - Automatic placement?
- Suitable for hole filling!



6-sided quintic S-patch  
with 252 control points

# Generalizing Bézier triangles

- Degree  $d$  Bézier triangle:

$$T(\lambda) = \sum_{\mathbf{s}} P_{\mathbf{s}} \frac{d!}{s_1! s_2! s_3!} \lambda_1^{s_1} \lambda_2^{s_2} \lambda_3^{s_3} = \sum_{\mathbf{s}} P_{\mathbf{s}} B_{\mathbf{s}}^d(\lambda)$$

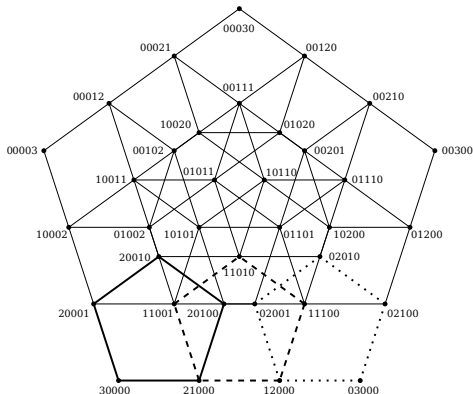
- $\mathbf{s} = (s_1, s_2, s_3)$  with  $s_i \geq 0$  and  $s_1 + s_2 + s_3 = d$
- $\lambda = (\lambda_1, \lambda_2, \lambda_3)$  barycentric coordinates relative to the domain triangle
- Depth  $d$  S-patch with  $n$  sides:

$$S(\lambda) = \sum_{\mathbf{s}} P_{\mathbf{s}} \frac{d!}{\prod_{i=1}^n s_i!} \prod_{i=1}^n \lambda_i^{s_i} = \sum_{\mathbf{s}} P_{\mathbf{s}} B_{\mathbf{s}}^d(\lambda)$$

- $\mathbf{s} = (s_1, s_2, \dots, s_n)$  with  $s_i \geq 0$  and  $\sum_{i=1}^n s_i = d$
- $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)$  generalized barycentric coordinates relative to the  $n$ -sided domain polygon

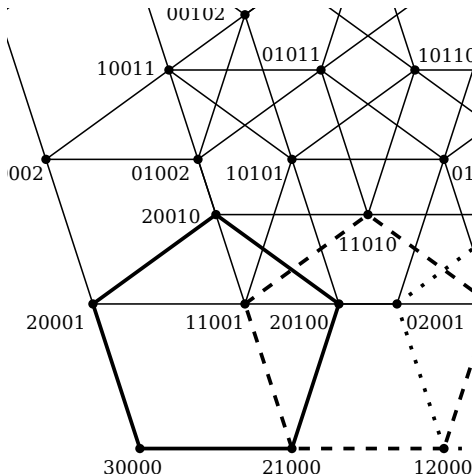
# Control structure

- Bézier curve boundaries
  - Depth  $\approx$  degree
- Adjacent CPs have shifted labels
  - E.g. from 21000:
    - 30000
    - 11001
    - 20100
    - 12000
- Consecutive shifting defines *panels*
- $G^1$  continuity can be set by fixing the panels



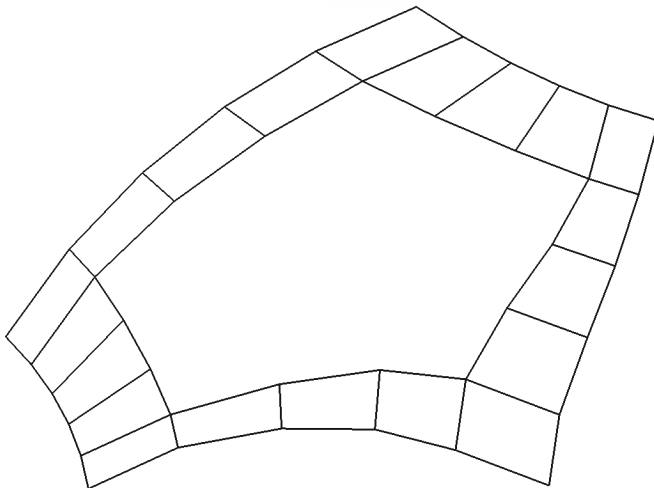
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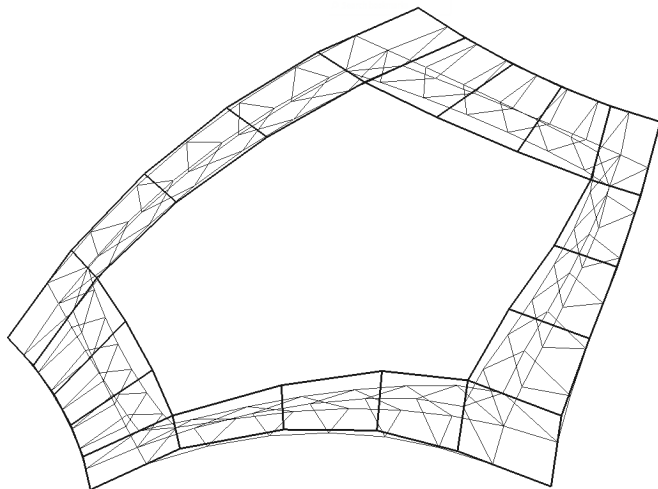
Example: five-sided hole with quintic boundaries

## Step 1. Ribbons (pairs of Bézier curves)



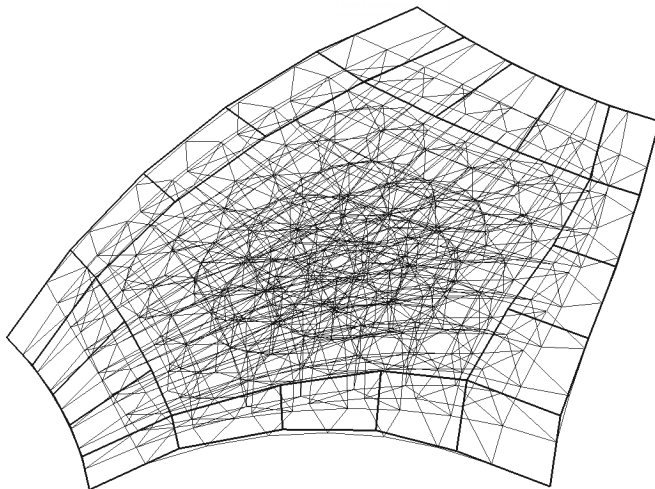
Example: five-sided hole with quintic boundaries

## Step 2. Boundary panels ( $G^1$ continuity constraints)



Example: five-sided hole with quintic boundaries

## Step 3. Interior control points (discrete biharmonic system)





Thank you for your attention.

See you at the poster!



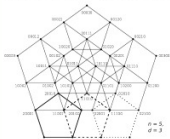
## $G^1$ hole filling with S-patches made easy

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### S-patch [1]

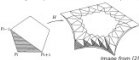
S-patches generalize Bézier triangles to any number of sides. An  $n$ -sided S-patch is created by embedding its domain polygon into an  $(n+1)$ -dimensional Bézier simplex. Its control points have labels of  $n$  nonnegative integers, summing to the depth ( $d$ ) of the patch. Adjacent control points have shifted labels, forming panels at the sides.



### Boundary constraints

The boundaries of an S-patch are Bézier curves of degree  $d$ , with the same control points. This means that  $C^1$  continuity is given. Two S-patches are  $G^1$ -continuous, when:

- all panels are affine images of the domain
- opposite panels are in the same plane
- thus only one point is unknown in each panel.



Assuming that twist compatibility is satisfied, this problem can be solved by an S-patch of depth  $d+3$  (see the paper for details).

### Motivation

S-patches are mathematically beautiful, but their control network becomes intricate even for surfaces of moderate complexity. For example, a six-sided quintic patch has 252 control points.



For such a large number of control points, automatic placement is the only option. In this paper, we show that S-patches are particularly suitable for filling multi-sided holes with tangential continuity to the adjacent surfaces. The boundary curves of the loop to be filled are assumed to be polynomial. This problem has already been investigated in [2], but explicit equations were given only for the quadratic and cubic cases.



### "Bi-harmonic" S-patch

Hole filling is done in 3 steps:

- Create the defining ribbons from the boundary constraints (two Bézier curves on each side, always twist-compatible)
- Generate the boundary panels based on the ribbons
- Set the interior control points by a bi-harmonic mask. As the S-patch control structure is not a grid, masks rely on the adjacency relations shown by the control point labels. The bi-harmonic mask is the harmonic mask applied to itself.

```
biharmonicMask(i):
  mask = 0 for all indices
  for (j,vj) in harmonicMask(i):
    for (k,wk) in harmonicMask(j):
      mask[k] += vj * wk
  return mask
```



### Results



### Discrete Coons patch [3]

$C^1$  Coons patches are quadrilateral transfinite surfaces, defined solely by the boundary curves. It has been shown, that when these are polynomial, the resulting surface will be a Bézier surface.



The interior control points of this patch are the solutions of the discrete Euler-Lagrange PDE, defined by the mask shown on the right. Different masks give rise to different surfaces, with various geometric properties.



### Harmonic mask

Another interesting mask is the one corresponding to the Laplace PDE for harmonic equation. The result resembles (and under very special conditions, is) a minimal surface.



Minimal-like surfaces are usually not what the designer wants. It may be better to use a bi-harmonic mask, as it is connected to the variation of curvature, but this 5x5 mask needs two fixed control point rows.

### References

- Ch. T. Loop, T. D. DeRose, *A multisided generalization of Bézier surfaces*, ACM Transactions on Graphics 8(3), pp. 204-234, 1989.
- Ch. T. Loop, T. D. DeRose, *Generalized B-spline surfaces of arbitrary topology*, in: 12th Conference on Computer Graphics and Interactive Techniques, SIGGRAPH, pp. 347-356, 1990.
- G. Farin, D. Hansford, *Discrete Coons patches*, Computer Aided Design 16(7), pp. 691-700, 1999.