G^1 hole filling with S-patches made easy

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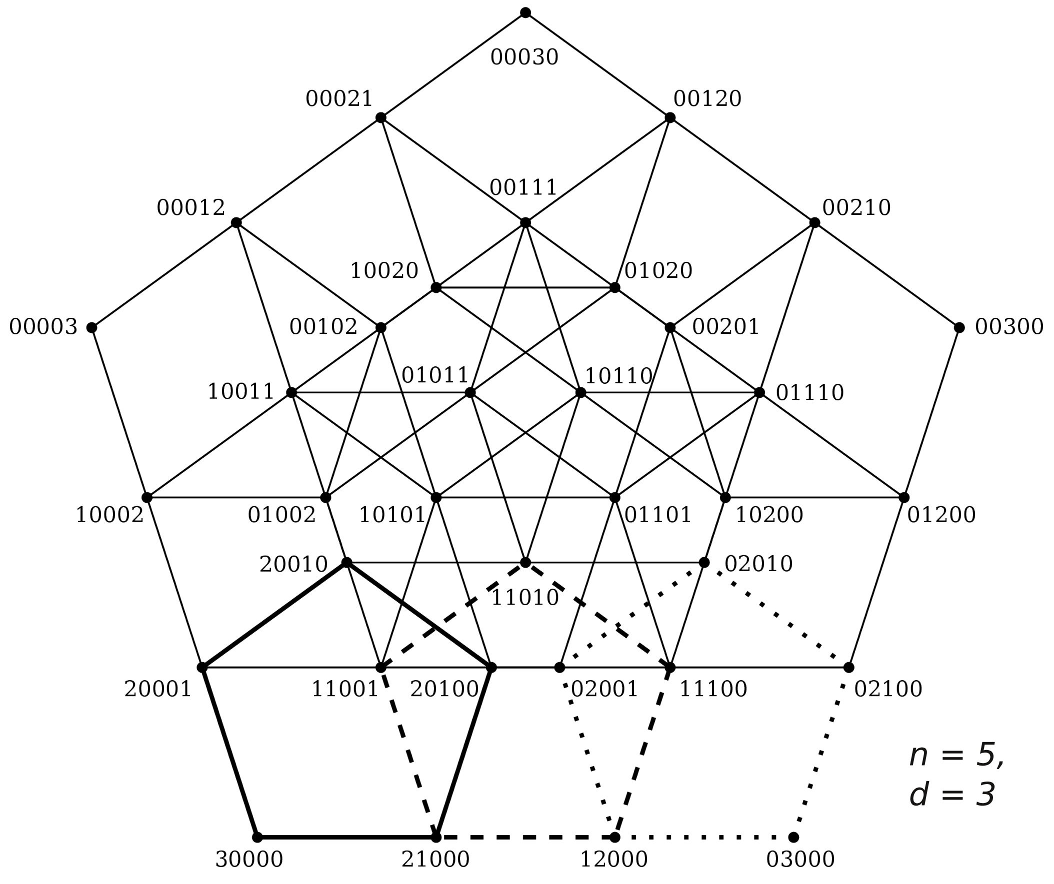
S-patch [1]

S-patches generalize Bézier triangles to any number of sides. An n-sided S-patch is created by embedding its domain polygon into an (n-1)-dimensional Bézier simplex. Its control points have labels of n nonnegative integers, summing to the *depth* (d) of the patch. Adjacent control points have shifted labels, forming *panels* at the sides.



Boundary constraints

The boundaries of an S-patch are Bézier curves of degree d, with the same control points. This means that C^0 continuity is given. Two S-patches are G^1-continuous, when:
 (i) all panels are affine images of the domain
 (ii) opposite panels are in the same plane
Thus only one point is unknown in each panel.

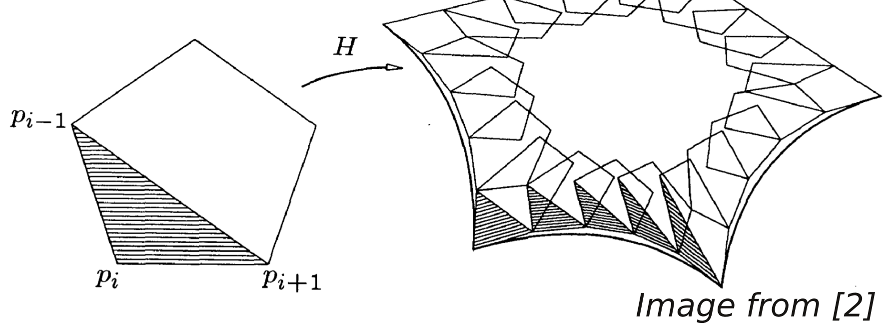
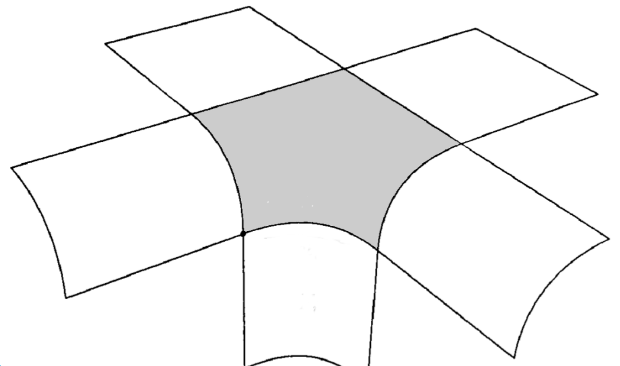


Image from [2]

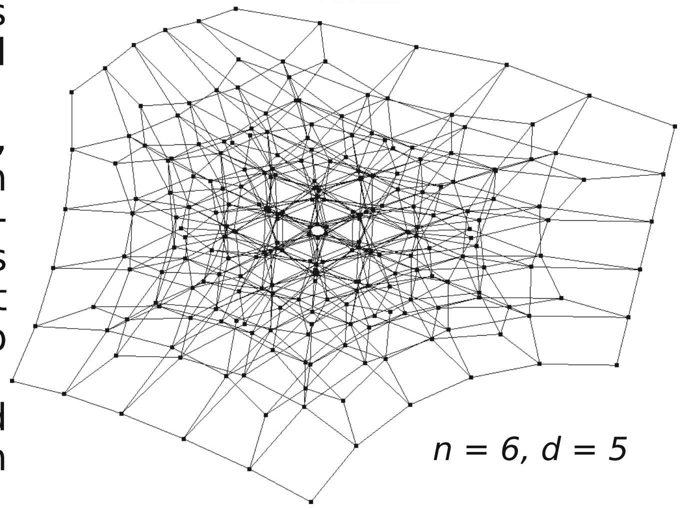
Assuming that twist compatibility is satisfied, this problem can be solved by an S-patch of depth d+3 (see the paper for details).

Motivation

S-patches are mathematically beautiful, but their control network becomes intricate even for surfaces of moderate complexity. For example, a six-sided quintic patch has 252 control points.



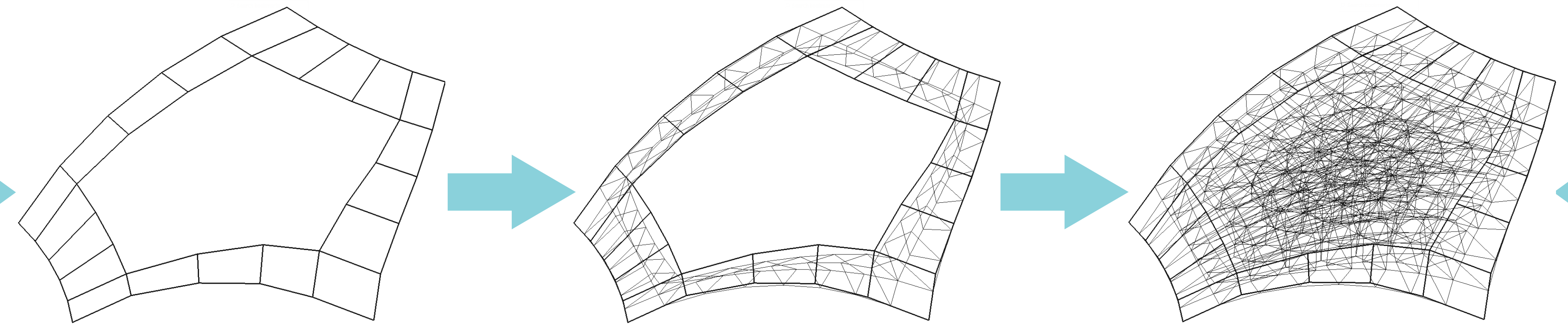
For such a large number of control points, automatic placement is the only option. In this paper, we show that S-patches are particularly suitable for filling multi-sided holes with tangential continuity to the adjacent surfaces. The boundary curves of the loop to be filled are assumed to be polynomial. This problem has already been investigated in [2], but explicit equations were given only for the quadratic and cubic cases.



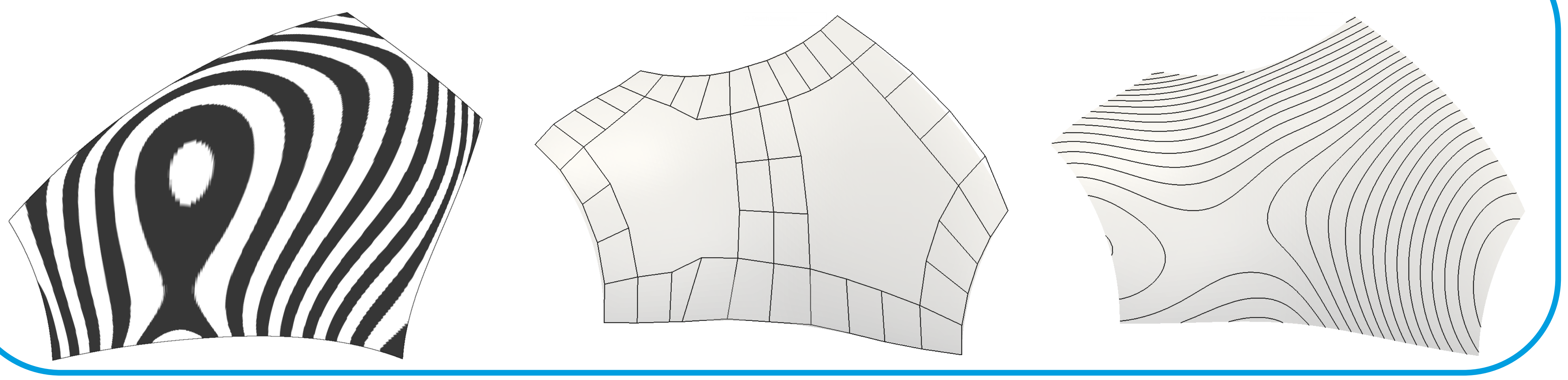
"Biharmonic" S-patch

Hole filling is done in 3 steps:
 (i) Create the defining *ribbons* from the boundary constraints [two Bézier curves on each side, always twist-compatible]
 (ii) Generate the boundary panels based on the ribbons
 (iii) Set the interior control points by a biharmonic mask
As the S-patch control structure is not a grid, masks rely on the adjacency relations shown by the control point labels.
The biharmonic mask is the harmonic mask applied to itself.

```
biharmonicMask(i):
  mask = 0 for all indices
  for (j,wj) in harmonicMask(i):
    for (k,wk) in harmonicMask(j):
      mask[k] += wj * wk
  return mask
```

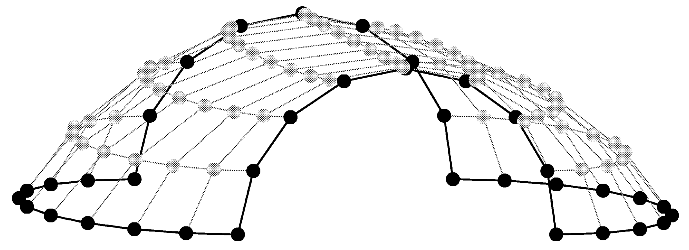


Results



Discrete Coons patch [3]

C^0 Coons patches are quadrilateral transfinite surfaces, defined solely by the boundary curves. It has been shown, that when these are polynomial, the resulting surface will be a Bézier surface.



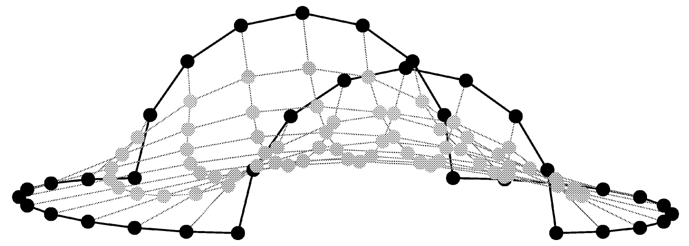
The interior control points of this patch are the solutions of the discerete Euler-Lagrange PDE, defined by the mask shown on the right. Different masks give rise to different surfaces, with various geometric properties.

-1	2	-1
2	●	2
-1	2	-1

Harmonic mask

Another interesting mask is the one corresponding to the Laplace PDE (or harmonic equation). The result resembles (and under very special conditions, *is*) a minimal surface.

0	1	0
1	●	1
0	1	0



Minimal-like surfaces are usually not what the designer wants. It may be better to use a biharmonic mask, as it is connected to the *variation* of curvature, but this 5x5 mask needs two fixed control point rows.

References

[1] Ch. T. Loop, T. D. DeRose, ***A multisided generalization of Bézier surfaces***. ACM Transactions on Graphics 8(3), pp. 204-234, 1989.

[2] Ch. T. Loop, T. D. DeRose, ***Generalized B-spline surfaces of arbitrary topology***. In: 17th Conference on Computer Graphics and Interactive Techniques, SIGGRAPH, pp. 347-356, 1990.

[3] G. Farin, D. Hansford, ***Discrete Coons patches***. Computer Aided Design 16(7), pp. 691-700, 1999.