Definition. Given $u \in \mathbb{N}^n$ with $\|u\|_1 = md$, let us define a partition of u by d as a sequence of vectors $v_i \in \mathbb{N}^n$, $\|v_i\|_1 = d$, $i \in \{1 \dots m\}$, such that $\sum_{i=1}^m v_i = u$.

Example. For $u = [1,3,2]$ and $d = 3$, there are 6 partitions:

- $[0,1,2] + [1,2,0]$ and $[1,2,0] + [0,1,2]$
- $[0,2,1] + [1,1,1]$ and $[1,1,1] + [0,2,1]$
- $[0,3,0] + [1,0,2]$ and $[1,0,2] + [0,3,0]$

Problems.

1. Express the number of partitions $\phi(u,d)$.
2. Give an efficient algorithm for the computation of all solutions.

Note. Converting an S-patch into a four-sided S-patch involves a computation in the order of

$$\sum_u \phi(u,d),$$

where n is the number of sides, d is the depth of the original S-patch, and $m = n - 2$. The summation goes through all possible values of $u \in \mathbb{N}^n$ with $\|u\|_1 = md$, the number of which is

$$\binom{n + md - 1}{md}.$$