G^1 hole filling with S-patches made easy

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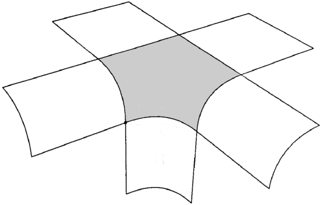
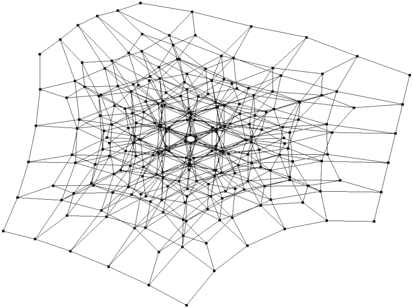
S-patches

- Generalization of Bézier △
  - Any number of sides
- Beautiful theory
- Too many control points
  - Automatic placement?
- Suitable for hole filling!





6-sided quintic S-patch
with 252 control points

Generalizing Bézier triangles

- Degree d Bézier triangle:

$$T(\lambda) = \sum_{\mathbf{s}} P_{\mathbf{s}} \frac{d!}{s_1!s_2!s_3!} \lambda_1^{s_1} \lambda_2^{s_2} \lambda_3^{s_3} = \sum_{\mathbf{s}} P_{\mathbf{s}} B_{\mathbf{s}}^d(\lambda)$$

  - $\mathbf{s} = (s_1, s_2, s_3)$ with $s_i \geq 0$ and $s_1 + s_2 + s_3 = d$
  - $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ barycentric coordinates relative to the domain triangle

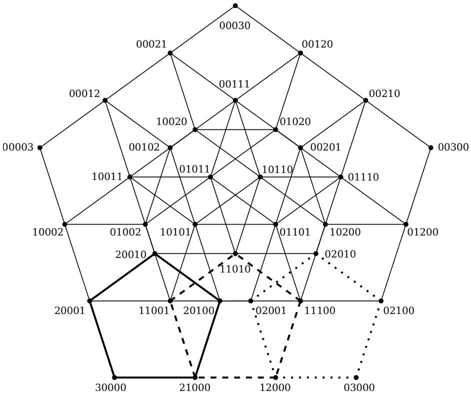
- Depth d S-patch with n sides:

$$S(\lambda) = \sum_{\mathbf{s}} P_{\mathbf{s}} \frac{d!}{\prod_{i=1}^n s_i!} \prod_{i=1}^n \lambda_i^{s_i} = \sum_{\mathbf{s}} P_{\mathbf{s}} B_{\mathbf{s}}^d(\lambda)$$

  - $\mathbf{s} = (s_1, s_2, \ldots, s_n)$ with $s_i \geq 0$ and $\sum_{i=1}^n s_i = d$
  - $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ generalized barycentric coordinates relative to the n-sided domain polygon

Control structure

- Bézier curve boundaries
  - Depth ≈ degree
- Adjacent CPs have shifted labels
  - E.g. from 21000:
    - 30000
    - 11001
    - 20100
    - 12000
- Consecutive shifting defines *panels*
- G^1 continuity can be set by fixing the panels



5-sided cubic control net

Control structure

- Bézier curve boundaries
  - Depth ≈ degree
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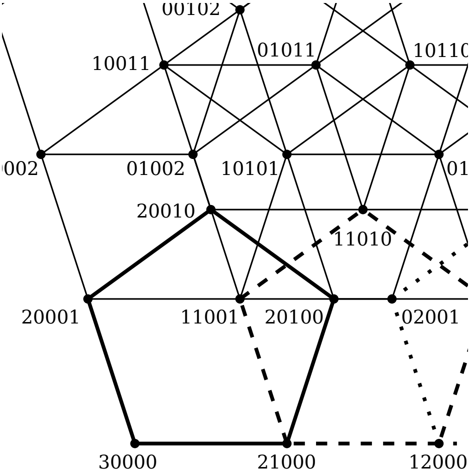
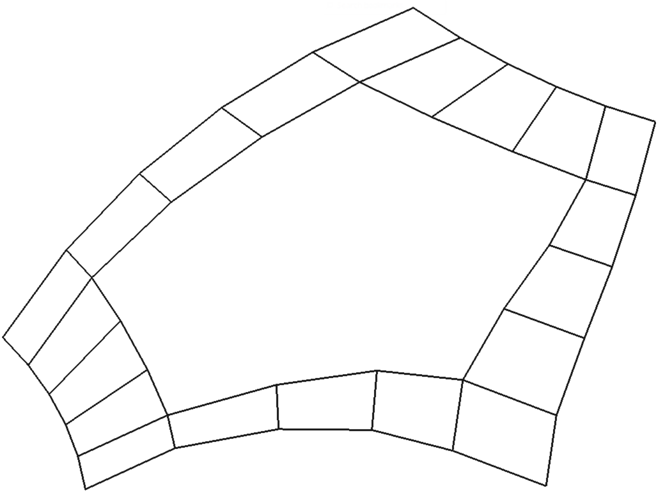
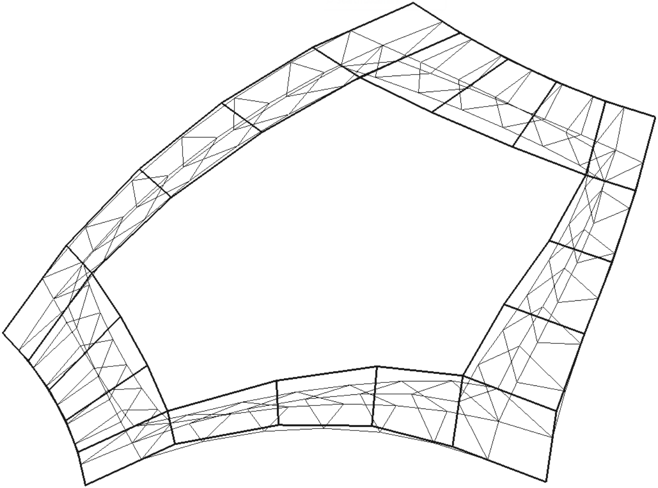


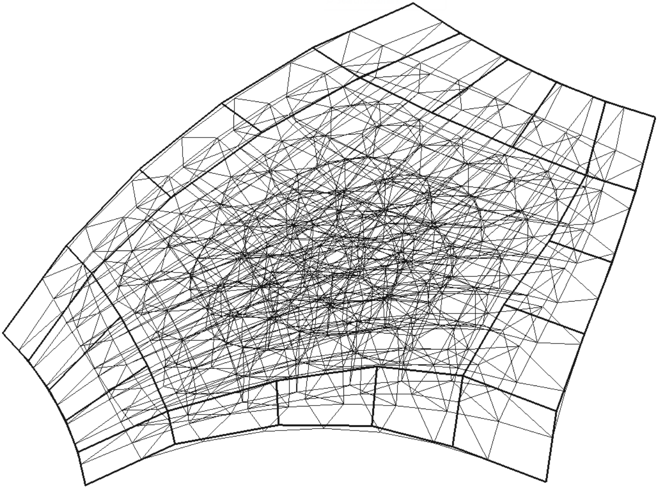
Step 1. Ribbons (pairs of Bézier curves)



Step 2. Boundary panels (G^1 continuity constraints)



Step 3. Interior control points (discrete biharmonic system)



Thank you for your attention.

See you at the poster!