

Take the following implicit equation for a cylinder:

$$f(x, y, z) = \sqrt{x^2 + y^2} - 1.$$

Its gradient is

$$\nabla f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2}} \cdot (x, y, 0) = \frac{(x, y, 0)}{\|(x, y, 0)\|},$$

so obviously $\|\nabla f(x, y, z)\| = 1$. The Hessian (which is consequently the same as the curvature tensor) is

$$H(x, y, z) = \begin{pmatrix} y^2 & -xy & 0 \\ -xy & x^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot (x^2 + y^2)^{-3/2},$$

for which $\text{tr}(H) = 1/\sqrt{x^2 + y^2}$. For a point on the cylinder, we know that $x^2 + y^2 = 1$, so the mean curvature is

$$\frac{\text{tr}(H)}{2} = \frac{1}{2}.$$

For the Gaussian, first compute the square of the Hessian:

$$H^2(x, y, z) = \begin{pmatrix} x^2y^2 + y^4 & -xy^3 - yx^3 & 0 \\ -xy^3 - yx^3 & x^2y^2 + x^4 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

from which $\text{tr}(H) = x^4 + 2x^2y^2 + y^4 = (x^2 + y^2)^2$, which is 1 for a point on the cylinder. Now the Gaussian is

$$\frac{\text{tr}(H)^2 - \text{tr}(H^2)}{2} = \frac{1 - 1}{2} = 0.$$