NESTED ZERO INFLATED GENERALIZED POISSON REGRESSION FOR FIFA WORLD CUP 2022

LORENZ A. GILCH

ABSTRACT. This article is devoted to the forecast of the FIFA World Cup 2022 via nested zero-inflated generalized Poisson regression. Our regression model incorporates the Elo points of the participating teams, the location of the matches and the of team-specific skills in attack and defense as covariates. The proposed model allows predictions in terms of probabilities in order to quantify the chances for each team to reach a certain stage of the tournament. We use Monte Carlo simulations for estimating the outcome of each single match of the tournament, from which we are able to simulate the whole tournament itself. The model is fitted on all football games of the participating teams since 2016 weighted by date and importance. Validation with previous tournaments and comparison with other Poisson models are given.

1. Introduction

Football is a typical low-scoring game and single matches are frequently decided through single events during the match. While several factors like extraordinary individual performances of single players or the whole team, individual errors, injuries, refereeing errors or just lucky coincidences are hard to forecast, each team has its strengths and weaknesses (e.g., attack and defense skills) and most of the results reflect these different qualities of the teams. Following this idea we derive probabilities for the exact result of a single match between two participating teams, which involves the following four ingredients for both teams:

- Elo ranking
- Attack strength
- Defense strength
- Location of the match

The complexity of the tournament with billions of different outcomes makes it very difficult to obtain accurate estimates for the probabilities of certain events. Therefore, we do not aim on forecasting the exact outcome of the tournament, but we want to make the discrepancy between the participating teams quantifiable and we want to measure the chances of each team to reach certain stages of the tournament or to become world champion. In particular, since the groups are already drawn and the tournament structure for each team (in particular, the possible ways to the final) is set, the idea is to measure whether a team has a rather simple or hard way to the final.

Date: November 3, 2022.

Key words and phrases. FIFA World Cup 2022; football; forecast; ZIGP; regression; Elo.

Let us give some background on modelling results of football matches. Several statistical models have been proposed in the literature for the prediction of the outcome of football matches. They can be divided into two classes. The first class, the result-based model, models directly the probability of a game outcome in terms of win/loss/draw, while the second class, the score-based model, targets on the prediction of the exact match score. In this article we use the second approach since the match score is a non-neglecting, very important factor in the group phase of the world championship and it also implies a model for the first class. When two teams have the same amount of points after having finished the group stage of the FIFA World Cup 2022 then the goal difference (and not the result of the direct duel!) determines which team will be ranked better than the other one. This may very likely result in a stronger opponent in the round of last 16 for the worse ranked team. In fact, as we have seen in former World Cups or in other tournaments, in most cases the goal difference is a crucial criterion which team will become group winner, runner-up or will be eliminated in the group stage. This observation underlines the importance and necessity of estimating the exact score of each single match and not only the outcome (win/loss/draw).

There are several models for modelling the exact scores of football matches and most of them involve a Poisson model. The easiest model assumes independence of the goals scored by each team and that each score can be modeled by a Poisson regression model; see, e.g., Lee (1997). Bivariate Poisson models, where the scores (i.e., the number of goals scored by both teams in a match) are modelled via two dependent Poisson distributed random variables, were proposed by Maher (1982) and extended by Dixon & Coles (1997) and Karlis & Ntzoufras (2003). A short overview on different Poisson models and related models like generalized Poisson models or zero-inflated models are given in Zeileis et al. (2008) and Chou & Steenhard (2011). Different similar models based on Poisson regression of increasing complexity (including discussion, goodness of fit and comparing them in terms of scoring functions) were analysed and used in Gilch & Müller (2018) for the prediction of the FIFA World Cup 2018. Gilch (2019) used a nested Poisson regression model for the Africa Cup of Nations 2019, Africa's continental championship, for predicting the hardto-predict Africa Cup. Beyond the mentioned articles above forecast of football matches (in particular, Poisson regression based forecasts) has been studied in large variety also in recent years, amongst others Groll et al. (2021), Wheatcroft (2021), H.R. Azhari (2018), Saraiva et al. (2016). Let me also remark the work of Joe & Zhu (2005) for a detailed comparison of generalized Poisson distribution and negative Binomial distribution, another class of possible distributions for modellung football scores.

Possible covariates for the above models may be divided into two main categories: those containing "prospective" informations and those containing "retrospective" informations. The first category contains other forecasts, especially bookmakers' odds, see e.g. Leitner et al. ("2010"), Zeileis et al. (2012) and references therein. This approach relies on the fact that bookmakers have a strong economic incentive to rate the result correctly and that they can be seen as experts in the matter of the forecast of sport events. However, their forecast models remain undisclosed and rely on information that is not publicly available. The second category contains only historical match data and no other forecasts. Models based on the second category allow to explicitly model the influence of objective, team-specific

characteristics which will serve as covariates (in particular, strength and weakness in attack and defense). Although there is a lot of data collected during matches (e.g., pass accuracy, team's mileage, etc.), only a very limited number of data is publicly available which can be used as possible covariates for forecast models. Groll et al. (2015) performed a variable selection on various covariates and found that the three most significant retrospective covariates are the FIFA ranking followed by the number of Champions league and Euro league players of a team. However, at the time of this analysis the composition and the line ups of the teams have not been announced and hence the number of Champions/Euro league players as covariates are not available. This is the reason why our model will be based on a team ranking only.

As a quantitative measure of the participating team strengths in this article, we use the World Football Elo ranking, which is publicly available under

We do not make use of the FIFA ranking (which is a simplified Elo ranking since July 2018), because the calculation of the FIFA ranking changed over time and the Elo ranking is more widely used in football forecast models. See also Gásques & Royuela (2016) for a discussion on this topic and a justification of the Elo ranking. Our model will be based on this Elo ranking and matches of the participating teams since 2016, where we additionally take the location of matches into account.

In this article we follow the retrospective approach and we present a nested generalized Poisson regression model with zero-inflation for the prediction of the scores of single matches, where the model is solely based on the Elo ranking and matches of the participating teams since 2016, where we additionally take the location of matches into account Since the FIFA World Cup 2022 is a complex tournament, involving important effects such as, e.g., group draws (e.g., see Deutsch (2011)) and dependences of the different matches, Monte-Carlo simulations are used to forecast the whole course of the tournament. For a more detailed summary on statistical modeling of major international football events, see, e.g., Groll et al. (2015) and references therein. As we will see later our proposed model shows a good fit, the obtained forecasts are conclusive and outperform classical Poisson models in many cases; furthermore, our model gives quantitative insights in each team's individual chances to proceed to certain stages of the tournament.

The paper is organized as follows: in Section 2 we present our nested zero-inflated generalized Poisson regression model. In Section 2 we validate the proposed model by comparing the simulation results with the simulation results of classical Poisson regression models in view of tournaments from the past. The simulation results for the FIFA World Cup 2022 will be presented in Section 4. Finally, in Section 5 we discuss our model and in Section 6 we give concluding remarks.

2. The Nested ZIGP Model

2.1. **Preliminaries.** The simulation of the whole FIFA World Cup 2022 is based on the simulation of the *exact* result of each single match which is modeled as $G_A:G_B$, where G_A (G_B , respectively) is the number of goals scored by team A (by team B, respectively).

Hence, simulating each single match allows us to simulate the course of the whole tournament. Even the most probable tournament outcome has a probability very close to zero to be actually realized. Hence, deviations of the true tournament outcome from the model's most probable one are not only possible, but most likely. However, simulations of the tournament yield estimates of the probabilities for each team to reach certain stages of the tournament and allow to make the different team's chances quantifiable.

We are interested to give quantitative insights into the following questions:

- (1) Which team has the best chances to become new world champion?
- (2) How big are the probabilities that a team will win its group or will be eliminated in the group stage?
- (3) How big is the probability that a team will reach a certain stage of the tournament?
- 2.2. **Involved data.** The main idea is to predict the exact outcome of a single match based on a generalized Poisson regression model which incorporates the following covariates:
 - Elo ranking of the teams
 - Attack and defense strengths of the teams
 - Location of the match (either one team plays at home or the match takes place on neutral ground)

At this point let me remark that the national team of Qatar will be the single team which plays at home. However, for estimating the different attack and defense skills of all participating teams, we will take into account all matches since 2016; of course, for estimating the regression parameters from historic match data it will be crucial whether a match was played on neutral playground or not.

We use an Elo rating system, see Elo (1978), which includes modifications to take various football-specific variables (like home advantage, goal difference, etc.) into account. The Elo ranking is published by the website eloratings.net, from where also all historic match data was retrieved. We give a quick introduction to the formula for the Elo ratings: let Elobefore be the Elo points of a team before a match; then the Elo points Eloafter after the match against an opponent with Elo points Elo_{Opp} is calculated as follows:

$$Elo_{after} = Elo_{before} + K \cdot G \cdot (W - W_e),$$

where

- K is a weight index regarding the tournament of the match (World Cup matches have weight 60, while continental tournaments have weight 50, etc.)
- G is a number taking into account the goal difference:

$$G = \begin{cases} 1, & \text{if the match is a draw or won by one goal,} \\ \frac{3}{2}, & \text{if the match is won by two goals,} \\ \frac{11+N}{8}, & \text{where } N \text{ is the goal difference otherwise.} \end{cases}$$

• W is the result of the match: 1 for a win, 0.5 for a draw, and 0 for a defeat.

ullet W_e is the expected outcome of the match calculated as follows:

$$W_e = \frac{1}{10^{-\frac{D}{400}} + 1},$$

where $D = \text{Elo}_{\text{Defore}} - \text{Elo}_{\text{Opp}}$ is the difference of the Elo points of both teams.

The Elo ratings on 30 October 2022 for the top 6 participating nations in the FIFA World Cup 2022 (in this rating) were as follows:

Brazil	Argentina	Spain	Netherlands	Belgium	France
2169	2141	2045	2040	2025	2005

Our model will be fitted using all matches of the participating teams between 1 January 2016 and 30 October 2022. In addition, the historic match data is weighted according to the following criteria:

- Importance of the match according to FIFA weights
- Time depreciation

In order to weigh the historic match data for the regression model we use the following date weight function for a match m:

$$w_{\text{date}}(m) = \left(\frac{1}{2}\right)^{\frac{D(m)}{H}},$$

where D(m) is the number of days ago when the match m was played and H is the half period in days, that is, a match played H days ago has half the weight of a match played today. Here, we choose the half period as $H = 365 \cdot 3$ days = 3 years; compare with Ley et al. (2019).

For weighing the importance of a match m, we use the match importance ratio in the FIFA ranking which is given by

$$w_{\rm importance}(m) = \begin{cases} 4, & \text{if m is a World Cup match,} \\ 3, & \text{if m is a continental championship/Confederation Cup match,} \\ 2.5, & \text{if m is a World Cup or continental qualifier/Nations League match,} \\ 1, & \text{otherwise.} \end{cases}$$

The overall importance of a single match from the past will be assigned as

$$w(m) = w_{\text{date}}(m) \cdot w_{\text{importance}}(m).$$

2.3. Nested Zero-Inflated Generalized Poisson Regression. We present a nested regression approach for estimating the probabilities of the exact result of single matches. For this purpose, we model the number of goals scored by a team in a single match as a random variable which follows a zero-inflated generalised Poisson-distribution (ZIGP). Generalised Poisson distributions generalise the Poisson distribution by adding a dispersion parameter; additionally, a point measure at 0 is added, since the event that no goal is scored by a team typically is a special event. We recall the definition that a discrete

random variable X follows a Zero-Inflated Generalized Poisson distribution with Poisson parameter $\mu > 0$, dispersion parameter $\varphi \geq 1$ and zero-inflation $\omega \in [0, 1)$ if

$$\mathbb{P}[X=k] = \begin{cases} \omega + (1-\omega) \cdot e^{-\frac{\mu}{\varphi}}, & \text{if k=0,} \\ (1-\omega) \cdot \frac{\mu \cdot \left(\mu + (\varphi-1) \cdot k\right)^{k-1}}{k!} \varphi^{-k} e^{-\frac{1}{\varphi} \left(\mu + (\varphi-1)x\right)}, & \text{if } k \in \mathbb{N}; \end{cases}$$

compare, e.g., with Consul (1989) and Stekeler (2004). If $\omega = 0$ and $\varphi = 1$, then we obtain just the classical Poisson distribution. The advantage of ZIGP is now that we have an additional dispersion parameter. We also note that

$$\mathbb{E}(X) = (1 - \omega) \cdot \mu,$$

$$Var(X) = (1 - \omega) \cdot \mu \cdot (\varphi^2 + \omega \mu).$$

The idea is now to model the number of scored goals of a team by a ZIGP distribution, whose parameters depend on the opponent's Elo ranking and the location of the match. Moreover, the number of goals scored by the weaker team (according to the Elo ranking) does additionally depend on the number of scored goals of the stronger team.

We now explain the regression method in detail. Consider a match between two teams A and B whose result we want to estimate in terms of probabilities. In the following we will always assume that A has higher Elo score than B. This assumption can be justified, since usually the better team dominates the weaker team's tactics. Moreover the number of goals the stronger team scores has an impact on the number of goals of the weaker team. For example, if team A scores 5 goals it is more likely that B scores also 1 or 2 goals, because the defense of team A lacks in concentration due to the expected victory. If the stronger team A scores only 1 goal, it is more likely that B scores no or just one goal, since team A focusses more on the defense and tries to secure the victory.

Denote by G_A and G_B the number of goals scored by teams A and B. Both G_A and G_B are assumed to be ZIGP-distributed: G_A follows a ZIGP-distribution with parameters $\mu_{A|B}$, $\varphi_{A|B}$ and $\omega_{A|B}$, while G_B follows a ZIGP-distribution with parameters $\bar{\mu}_{B|A}$, $\bar{\varphi}_{B|A}$ and $\bar{\omega}_{B|A}$. These parameters are now determined as follows:

- (1) The number of G_A can be seen as the number \hat{G}_A of scored goals from A (strength of attack of A) or as the number \check{G}_A of goals against of team B (strength of defense of B). In the following we model G_A from both points of view and take the average values of the parameters for modelling G_A .
 - (a) In the first step we model the strength of team A in terms of the number of scored goals \hat{G}_A in dependence of the opponent's Elo score Elo_B and the location of the match $\text{loc}_{A|B}$, which is defined as

$$\operatorname{loc}_{A|B} = \begin{cases} 1, & \text{if A plays at home,} \\ 0, & \text{if the match takes place on neutral playground,} \\ -1, & \text{if B plays at home.} \end{cases}$$

The parameters of the distribution of \hat{G}_A are modelled as follows:

$$\log \mu_{A}(\text{Elo}_{B}) = \alpha_{0}^{(1)} + \alpha_{1}^{(1)} \cdot \text{Elo}_{B} + \alpha_{2}^{(1)} \cdot \log_{A|B},$$

$$\varphi_{A} = 1 + e^{\beta^{(1)}},$$

$$\omega_{A} = \frac{\gamma^{(1)}}{1 + \gamma^{(1)}},$$
(2.1)

where $\alpha_0^{(1)}, \alpha_1^{(1)}, \alpha_2^{(1)}, \beta^{(1)}, \gamma^{(1)}$ are obtained via ZIGP regression. Recall again that \hat{G}_A is a model for the scored goals of team A, which does *not* take into account the specific defense skills of team B.

(b) Teams of similar Elo scores may have different strengths in attack and defense. To take this effect into account we model the number \check{G}_A of goals team B receives against a team of higher Elo score Elo_A using a ZIGP distribution with mean parameter ν_B , dispersion parameter ψ_B and zero-inflation parameter δ_B as follows:

$$\log \nu_{B}(\text{Elo}_{A}) = \alpha_{0}^{(2)} + \alpha_{1}^{(2)} \cdot \text{Elo}_{A} + \alpha_{2}^{(2)} \cdot \log_{|A},$$

$$\psi_{B} = 1 + e^{\beta^{(2)}},$$

$$\delta_{B} = \frac{\gamma^{(2)}}{1 + \gamma^{(2)}},$$
(2.2)

where $\alpha_0^{(2)}, \alpha_1^{(2)}, \alpha_2^{(2)}, \beta^{(2)}, \gamma^{(2)}$ are obtained via ZIGP regression. Recall that the number of scored goals of team A can be seen as the goals against \check{G}_A of team B.

(c) Team A shall in average score $(1-\omega)\cdot\mu_A(\mathrm{Elo}_B)$ goals against team B (modelled by \hat{G}_A), but team B shall receive in average $(1-\omega_B)\cdot\nu_B(\mathrm{Elo}_A)$ goals against (modelled by \check{G}_A). As these two values rarely coincide we model the numbers of goals G_A as a ZIGP distribution with the average parameters

$$\mu_{A|B} := \frac{\mu_A(\text{Elo}_B) + \nu_B(\text{Elo}_A)}{2},$$

$$\varphi_{A|B} := \frac{\varphi_A + \psi_B}{2},$$

$$\omega_{A|B} := \frac{\omega_A + \delta_B}{2}.$$

(2) The number of goals G_B scored by B is assumed to depend on the Elo score Elo_A , the location $\text{loc}_{B|A}$ of the match and additionally on the outcome of G_A . Hence, we model G_B via a ZIGP distribution with Poisson parameters $\bar{\mu}_{B|A}$, dispersion $\bar{\varphi}_{B|A}$ and zero inflation $\bar{\omega}_{B|A}$ satisfying

$$\log \bar{\mu}_{B|A} = \alpha_0^{(3)} + \alpha_1^{(3)} \cdot \text{Elo}_A + \alpha_2^{(3)} \cdot \log_{|A} + \alpha_3^{(3)} \cdot G_A,
\bar{\varphi}_{B|A} := 1 + e^{\beta^{(3)}},
\bar{\omega}_{B|A} := \frac{\gamma^{(3)}}{1 + \gamma^{(3)}},$$
(2.3)

where the parameters $\alpha_0^{(3)}, \alpha_1^{(3)}, \alpha_2^{(3)}, \alpha_3^{(3)}, \beta^{(3)}, \gamma^{(3)}$ are obtained by ZIGP regression.

(3) The result of the match A vs. B is simulated by realizing G_A first and then realizing G_B in dependence of the realization of G_A .

Example: For better comprehension, we provide an example and consider the FIFA World Cup 2022 group match France vs. Denmark, which takes place in Qatar: France has 2005 Elo points while Denmark has 1971 points. Against a team of Elo score 1971 France is assumed to score without zero-inflation in average

$$\mu_{\text{France}}(1936) = \exp(2.472632 - 0.0010679575 \cdot 1971 + 0.2724600768 \cdot 0) = 1.444391$$

goals, and France's zero inflation is estimated as

$$\omega_{\rm France} = \frac{e^{-4.058738}}{1 + e^{-4.058738}} = 0.0169776.$$

Therefore, France is assumed to score in average

$$(1 - \omega_{\text{France}}) \cdot \mu_{\text{France}}(1971) = 1.419869$$

goals against Denmark. Furthermore, we obtain

$$\varphi_{\text{France}} = 1 + e^{-11.846181} = 1.000007.$$

Vice versa, Denmark receives in average without zero-inflation

$$\nu_{\text{Denmark}}(2005) = \exp(-4.205890 + 0.0021582919 \cdot 2005 - 0.371076439 \cdot 0) = 1.129173$$

goals, and the zero-inflation of Denmark's goals against is estimated as

$$\delta_{\text{Denmark}} = \frac{e^{-11.151799}}{1 + e^{-11.151799}} = 0.0000143.$$

Hence, Denmark receives in average

$$(1 - \delta_{Denmark}) \cdot \nu_{Denmark}(2005) = 1.129157$$

goals against when playing against an opponent of Elo strength 2005. We obtain also

$$\psi_{\text{Denmark}} = 1 + e^{-13.407282} = 1.000002.$$

Therefore, the number of goals, which France will score against Denmark, will be modelled as a ZIGP distributed random variable with mean

$$\mu_{\text{France}|\text{Denmark}} = \left(1 - \frac{\omega_{\text{France}} + \delta_{\text{Denmark}}}{2}\right) \cdot \frac{\mu_{\text{France}}(1971) + \nu_{\text{Denmark}}(2005)}{2} = 1.27585,$$

dispersion parameter

$$\varphi_{\text{France}|\text{Denmark}} = \frac{\varphi_{\text{France}} + \psi_{\text{Denmark}}}{2} = 1.000005$$

and zero-inflation

$$\omega_{\mathrm{France}|\mathrm{Denmark}} = \frac{\omega_{\mathrm{France}} + \delta_{\mathrm{Denmark}}}{2} = 0.008495965.$$

The average number of goals, which Denmark scores against a team of Elo score 2005 provided that G_A goals against are received, is also modelled by a ZIGP distributed random variable with parameters

$$\bar{\mu}_{\text{Denmark}|\text{France}} = \exp(3.118465 - 0.0013932201 \cdot 2005 - 0.03989474 \cdot G_A + 0.051954905 \cdot 0);$$

e.g., if $G_A = 1$ then $\bar{\mu}_{Denmark|France} = 1.32998$. Furthermore, we obtain

$$\begin{split} \bar{\varphi}_{\text{Denmark}|\text{France}} &= 1 + e^{-7.425663} = 1.000596, \\ \bar{\omega}_{\text{Denmark}|\text{France}} &= \frac{e^{-2.942621}}{1 + e^{-2.942621}} = 0.05008642. \end{split}$$

Remark: We note that the presented dependent approach may also be justified through the definition of conditional probabilities:

$$\mathbb{P}[G_A = i, G_B = j] = \mathbb{P}[G_A = i] \cdot \mathbb{P}[G_B = j \mid G_A = i] \quad \forall i, j \in \mathbb{N}_0.$$

For a comparision of this model in contrast to similar Poisson models, we refer to Gilch (2019) and Gilch & Müller (2018). All calculations were performed with R (version 4.0.3) and the ZIGP package. In particular, the presented model generalizes the model used in Gilch (2019) by adding a dispersion parameter, zero-inflation and a regression approach which weights historical data according to importance and time depreciation.

2.4. Goodness of Fit Tests. We check goodness of fit of the ZIGP regressions in (2.1), (2.2) and (2.3) for all participating teams. For each team \mathbf{T} , we calculate the following χ^2 -statistic from the list of historic matches:

$$\chi_{\mathbf{T}} = \sum_{i=1}^{n_{\mathbf{T}}} \frac{(x_i - \hat{\mu}_i)^2}{\hat{\mu}_i},$$

where $n_{\mathbf{T}}$ is the number of matches of team \mathbf{T} since 2016, x_i is the number of scored goals of team \mathbf{T} in match i and $\hat{\mu}_i$ is the estimated ZIGP regression mean in dependence of the opponent's historical Elo points.

2.4.1. ZIGP Regression for \hat{G}_A . Concerning the regression in (2.1) we observe that almost all teams have a very good fit. In Table 1 the p-values for the top 6 teams are given.

Team	Brazil	Argentina	Spain	Netherlands	Belgium	France
<i>p</i> -value	0.74	0.75	0.15	0.88	0.95	0.31

TABLE 1. Goodness of fit test for the ZIGP regression in (2.1) for the top 5 teams.

Only Ghana and Costa Rica have a low p-value of less than 0.05; all other teams have a p-value of at least 0.14, most have an even much higher p-value. Since Costa Rica and Ghana belong to the weaker teams, these low p-values cause only limited effects on simulation results. The scored goals (red points) of Spain from historic match data together with the estimated averages μ_{Spain} (black line) and the standard error (shaded grey area) are given in Figure 1.

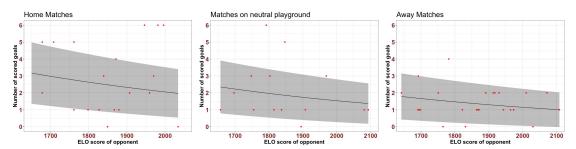


FIGURE 1. ZIGP regression (2.1) for Spain.

2.4.2. ZIGP Regression for \check{G}_A . Concerning the regression in (2.2) which models the number of goals against we calculate an analogous χ^2 -statistic for each team. The p-values for the top 6 teams are given in Table 2.

-	Team	Brazil	Argentina	Spain	Netherlands	Belgium	France
1	<i>p</i> -value	0.69	0.005	0.77	0.82	0.16	0.86

TABLE 2. Goodness of fit test for the ZIGP regression in (2.2) for the top 5 teams.

Let us remark that three countries have a very poor p-value, namely Argentina, Canada and Uruguay (0.032); all other countries have p-values of at least 0.9, most of them even much higher. Concerning Argentina's bad fit we remark that Argentina's regression (2.1) shows a good fit and limits the effect of Argentina's regression in (2.2). Furthermore, Canada probably won't play a big role during the evolution of the tournament. The goals againt (red points) of Brazil from historic match data together with the estimated averages ν_{Brazil} (black line) and the standard error (shaded grey area) are given in Figure 2.

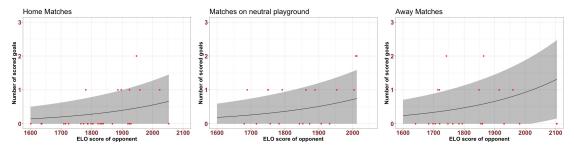


FIGURE 2. ZIGP regression (2.2) for Brazil.

2.4.3. ZIGP Regression for G_B . Finally, we test the goodness of fit for the regression in (2.3) which models the number of goals against of the weaker team in dependence of the number of goals which are scored by the stronger team; see Table 3.

Team	Spain	Argentina	Germany	Portugal	Senegal
<i>p</i> -value	0.65	0.08	0.51	0.67	0.86

TABLE 3. Goodness of fit test for the Poisson regression in (2.3) for some of the teams.

Very few teams like Qatar or the United States have p-values less than 0.05, which arise mainly from very few outliers, while the p-values of the majority of the other teams suggest reasonable fits. For instance, the regression (2.3) for Wales, when playing away, facing a stronger opponent and when receiving exactly 2 goals against, is plotted in Figure 3.

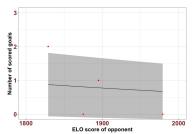


FIGURE 3. ZIGP regression (2.3) for Wales' away matches against stronger opponents, who score exactly 2 goals.

- 2.5. Further Poisson Models. In order to measure the quality of the proposed ZIGP model we compare it with other standard Poisson models. These are described in the following
- 2.5.1. Independent Poisson Regression. In this model we assume both G_A and G_B to be independent Poisson distributed variables with Poisson rates $\lambda_{A|B}$ and $\lambda_{B|A}$. We estimate the Poisson rates $\lambda_{A|B}$ and $\lambda_{B|A}$ via Poisson regression with the Elo scores of A and B and the location $loc_{A|B}$ of the match as covariates. These rates are determined as follows:
 - (1) The first step models the number of goals \hat{G}_A scored by team A playing against a team with a given Elo score Elo_B. The random variable \hat{G}_A is modeled as a Poisson distribution with parameter μ_A . The parameter μ_A as a function of the Elo rating Elo_B of the opponent B is given as

$$\log \mu_A(\text{Elo}_B) = \alpha_0 + \alpha_1 \cdot \text{Elo}_B + \alpha_2 \cdot \log_{A|B}$$
 (2.4)

where α_0, α_1 and α_2 are obtained via Poisson regression.

(2) Analogously to our ZIGP model, we model the number of scored goals of team A also as the number of goals against of team B when facing an opponent with Elo score Elo_A. We model this number as a Poisson distribution with parameter ν_B in terms of a function in the Elo rating Elo_A and the location of the match $\log_{|A|}$:

$$\log \nu_B(\text{Elo}_A) = \beta_0 + \beta_1 \cdot \text{Elo}_A + \beta_2 \cdot \log_{|A|}, \tag{2.5}$$

where the parameters β_0 , β_1 and β_2 are obtained via Poisson regression.

(3) Team A shall in average score $\mu_A(\text{Elo}_B)$ goals against team B, but team B shall receive in average $\nu_B(\text{Elo}_A)$ goals against. Hence, we model the numbers of goals G_A as a Poisson distribution with parameter

$$\lambda_{A|B} = \frac{\mu_A(\mathrm{Elo}_B) + \nu_B(\mathrm{Elo}_A)}{2}.$$

Analogously, we set

$$\lambda_{B|A} = \frac{\mu_B(\mathrm{Elo}_A) + \nu_A(\mathrm{Elo}_B)}{2}.$$

For each team, the regression parameters $\alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1$ and β_2 are estimated from historical match data. The match A vs. B is then simulated using two *independent* Poisson random variables G_A and G_B with rates $\lambda_{A|B}$ and $\lambda_{B|A}$.

2.5.2. Bivariate Poisson Regression. A weakness of the independent Poisson model is that the number of goals G_A and G_B are realized independently, which is a rather less reasonable assumption. Therefore, we consider also a bivariate Poisson regression approach. First, recall the definition of a bivariate Poisson distribution: let X_1, X_2, X_0 be independent Poisson distributed random variables with rates $\lambda_1, \lambda_2, \lambda_0$. Define $Y_1 = X_1 + X_0$ and $Y_2 = X_2 + X_0$. Then (Y_1, Y_2) is bivariate Poisson distributed with parameters $(\lambda_1, \lambda_2, \lambda_0)$. In particular, Y_i is Poisson distributed with rate $\lambda_i + \lambda_0$ and $Cov(Y_1, Y_2) = \lambda_0$.

In the following we model (G_A, G_B) as a bivariate Poisson distributed random vector for every couple A, B separately. The main idea is to perform one regression over all matches of team A and to estimate the average number of scored goals of team A and its opponent in terms of his Elo strength Elo_B . Then we perform another regression over all matches of team B and estimate the expected number of goals of B and the average goals against of B when playing against a team of Elo strength Elo_A . Hereby, we use the same notation as in previous models with μ_T being the Poisson rate for the average of scored goals of any team T, while ν_T is the average number of goals against of team T. The model uses the following regression approach:

(1) For each World Cup participating team T, we estimate the parameters

$$(\lambda_1, \lambda_2, \lambda_0) = (\mu_{\mathbf{T}}, \nu_{\mathbf{T}}, \tau_{\mathbf{T}})$$

from the viewpoint of team \mathbf{T} , where we only take into account the matches of team \mathbf{T} . The parameters shall depend on the Elo strength Elo_O of an opponent team \mathbf{O} and on the location $\text{loc}_{\mathbf{T}|\mathbf{O}}$ of the match. To this end, we use the following Poisson regression model:

$$\log \mu_{\mathbf{T}}(\text{Elo}_{\mathbf{O}}) = \alpha_{1,0} + \alpha_{1,1} \cdot \text{Elo}_{\mathbf{O}} + \alpha_{1,2} \cdot \text{loc}_{\mathbf{T}|\mathbf{O}},$$

$$\log \nu_{\mathbf{T}}(\text{Elo}_{\mathbf{O}}) = \alpha_{2,0} + \alpha_{2,1} \cdot \text{Elo}_{\mathbf{O}} + \alpha_{2,2} \cdot \text{loc}_{\mathbf{T}|\mathbf{O}},$$

$$\log \nu_{\mathbf{T}}(\text{Elo}_{\mathbf{O}}) = \alpha_{3,0}$$
(2.6)

That is, the estimated expected number of scored goals of team **T** against a team of Elo strength Elo_O at location $loc_{\mathbf{T}|\mathbf{O}}$ is given by $\mu_{\mathbf{T}}(Elo_{\mathbf{O}}) + \tau_{\mathbf{T}}$, while the estimated expected number of scored goals of a team with Elo score Elo_O and location $loc_{\mathbf{T}|\mathbf{O}}$ against **T** is given by $\nu_{\mathbf{T}}(Elo_{\mathbf{O}}) + \tau_{\mathbf{T}}$.

(2) In order to estimate the Poisson rates $(\lambda_1, \lambda_2, \lambda_0)$ for the match result (G_A, G_B) we can use the regression coefficients both of A and B in the following way: λ_1 may be estimated either by considering all matches of team A and calculating $\mu_A(\text{Elo}_B)$ or by considering all matches of team B and calculating $\nu_B(\text{Elo}_A)$, which corresponds to the goals against of team B (that is, the number of scored goals of team A against B). Therefore, we estimate λ_1 as the mean of $\mu_A(\text{Elo}_B)$ and $\nu_B(\text{Elo}_A)$. Analogously, we estimate λ_2 as the mean of $\mu_B(\text{Elo}_A)$ and $\nu_A(\text{Elo}_B)$ and λ_0 also as the mean of the covariances τ_A and τ_B . That is,

$$\lambda_{1} = \frac{\mu_{A}(\text{Elo}_{B}) + \nu_{B}(\text{Elo}_{A})}{2},$$

$$\lambda_{2} = \frac{\mu_{B}(\text{Elo}_{A}) + \nu_{A}(\text{Elo}_{B})}{2},$$

$$\lambda_{0} = \frac{\tau_{A}(\text{Elo}_{B}) + \tau_{B}(\text{Elo}_{A})}{2}.$$

Finally, we assume that (G_A, G_B) is bivariate Poisson distributed with parameters $(\lambda_1, \lambda_2, \lambda_0)$.

Remarks:

• In (2.6) we estimate $\tau_{\mathbf{T}}$ to be a constant for each team. If we add Elo_O as an additional covariate for $\tau_{\mathbf{T}}$, that is,

$$\log \tau_{\mathbf{T}}(\mathrm{Elo}_{\mathbf{O}}) = \alpha_{3.0} + \alpha_{3.1} \mathrm{Elo}_{\mathbf{O}},$$

then the AIC increases for most of the teams, and thus we omit $Elo_{\mathbf{O}}$ as covariate.

• For the simulation, we used the bivpois-package of Karlis and Ntzoufras.

3. Validation of the ZIGP Model

In this section we want to compare the forecast quality of our nested ZIGP model with the standard Poisson models from Subsection 2.5. For this purpose, we made simulations for the past FIFA World Cups 2010, 2014 and 2018 and the EURO 2016 and 2020, Europe's continental championship, and compared the obtained forecast with the real outcomes of the those tournaments.

3.1. Scoring Functions. In order to compare simulations results with the real outcomes of tournaments in the past we use standard scoring functions. We introduce the following notation: let \mathbf{T} be a participating national team. Then define:

$$\operatorname{result}(\mathbf{T}) = \begin{cases} 1, & \text{if } \mathbf{T} \text{ became champion,} \\ 2, & \text{if } \mathbf{T} \text{ went to the final but didn't win the final,} \\ 3, & \text{if } \mathbf{T} \text{ went to the semifinal but didn't win the semifinal,} \\ 4, & \text{if } \mathbf{T} \text{ went to the quarterfinal but didn't win the quarterfinal,} \\ 5, & \text{if } \mathbf{T} \text{ went to the round of last 16 but didn't win this round,} \\ 6, & \text{if } \mathbf{T} \text{ went out of the tournament after the round robin.} \end{cases}$$

E.g., for the World Cup 2014 we obtain result(Germany) = 1, result(Argentina) = 2, or result(Italy) = 6. For every World Cup participant \mathbf{T} we set the simulation result probability as $p_i(\mathbf{T}) := \mathbb{P}[\text{result}(\mathbf{T}) = i]$.

In order to compare the different simulation results with the reality we use the following well-known scoring functions:

(1) Brier Score: The error of team T is defined as

$$\operatorname{error}_{\operatorname{BS}}(\mathbf{T}) := \sum_{j=1}^{6} (p_j(\mathbf{T}) - \mathbb{1}_{[\operatorname{result}(\mathbf{T}) = j]})^2.$$

The total error score is then given by

$$BS = \sum_{\mathbf{T} \text{ tournament participant}} \operatorname{error}_{BS}(\mathbf{T}).$$

(2) Rank-Probability-Score (RPS): The error of team T is defined as

$$\operatorname{error}_{\operatorname{RPS}}(\mathbf{T}) := \frac{1}{5} \sum_{i=1}^{5} \left(\sum_{j=1}^{i} p_j(\mathbf{T}) - \mathbb{1}_{[\operatorname{result}(\mathbf{T}) = j]} \right)^2.$$

The total error score is then given by

$$RPS = \sum_{\mathbf{T} \text{ tournament participant}} error_{RPS}(\mathbf{T}).$$

3.2. Comparison of Different Models. In Table 4 we compare the quality of our nested ZIGP model forecast with the simulation results from standard Poisson regression models from Subsection 2.5 in terms of the values of the scoring functions applied to the World Cups 2010, 2014 and 2018:

Year	Error function	ZIGP	BV	IP
2010	Brier Score	17.79	17.97	17.97
	Rank Probability Score	4.93	4.99	5.05
2014	Brier Score	20.18	22.33	22.10
	Rank Probability Score	5.06	5.52	5.48
2018	Brier Score	18.51	18.00	18.39
	Rank Probability Score	5.54	5.51	5.51

TABLE 4. Validation of ZIGP models compared with other Poisson regression models measured by different scoring functions for the World Cups 2010, 2014 and 2018 and the EURO 2016 and 2020. (BV=Bivariate Poisson, IP=Independent Poisson)

While our proposed ZIGP model clearly outperforms the standard Poisson regression model for the World Cups 2010 and 2014, the forecast for the World Cup 2018 is worse than the standard Poisson models. A closer investigation shows that the bad, unpredictable

performance of Germany (eliminated in the group stage with rather weaker teams after having finished the World Cup qualifiers with 10 out of 10 victories) had an heavy impact on the values of the scoring function. Furthermore, the ZIGP models still leads to similar values (compared to the values of the standard Poisson models) of the scoring functions for the World Cup 2018.

In Table 5 we compare the quality of our ZIGP model forecast with the simulation results of standard Poisson models for the EURO 2016 and 2020, Europe's continental championships.

Year	Error function	ZIGP	BV	IP
2016	Brier Score	17.52	23.27	23.25
	Rank Probability Score	5.28	5.97	5.98
2020	Brier Score	14.54	14.36	16.37
	Rank Probability Score	5.06	5.07	5.19

TABLE 5. Validation of ZIGP model compared with other Poisson regression models measured by different scoring functions for EURO 2016 and 2020. (BV=Bivariate Poisson, IP=Independent Poisson)

For the EURP 2016, we observe that our proposed ZIGP model clearly outperfoms the standard Poisson models. Concerning the EURO 2020, the forecast of the bivariate Poisson model is very slightly better than our ZIGP model w.r.t. the Brier Score, but not w.r.t. the rank probability score; the difference in quality of the forecasts seems to be non significant.

As a summary, saying that we do not expect to find a model which outperform other models in *each and every* case our proposed nested ZIGP model seems to be a reasonable improvement of standard Poisson models or may serve as an alternative model.

Finally, let me remark that the proposed *Nested Poisson Regression model* from Gilch (2019) and Gilch & Müller (2018) is in most cases also outperformed by the nested ZIGP model presented in this article.

4. FIFA World Cup 2022 Forecast

In this section we present the simulation results for the FIFA World Cup 2022, which allows us to answer the questions formulated in Section 2.1. For this purpose, we have simulated each single match of the FIFA World Cup 2022 via the presented nested ZIGP model from Subsection 2.3, which in turn allows us to simulate the whole FIFA World Cup 2022 tournament. After each simulated match we have updated the Elo ranking according to the simulation results. This honours teams, which are in a good shape during a tournament and perform maybe better than expected. Overall, we have performed 100.000 simulations of the whole tournament with R, where we reset the Elo ranking at the beginning of each single tournament simulation.

4.1. **Single Matches.** Since the basic element of our tournament simulation is the simulation of single matches, we visualize how to quantify the results of single matches. E.g., in Group E the highly interesting match between Spain and Germany takes place in Qatar. We present the probabilities for the result of this match according to our model in Figure 4: the most probable scores are a 1:1 draw or a 2:1 or 2:0 victory of Spain.

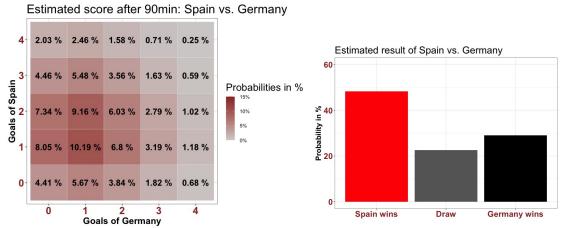


FIGURE 4. Probabilities for the score of the preliminary round match Spain vs. Germany (group E) in Qatar.

4.2. **Group Forecast.** In the following Tables 6-13 we present the probabilities for the group stage that teams become group winners, runner-ups, or are eliminated in the group stage.

In most groups there is a clear favourite team but it is uncertain – even among football experts – which team will be runner-up. The forecast quantifies and compares each team's chances to proceed to the round of last 16 in terms of probabilities.

In Group E the top match of the group stage will take place between Spain and Germany. Although it is not unlikely that the direct match between both teams ends with a draw, the forecasts prefers Spain as group winner due to Spain's strength in attack, which makes it more likely for Spain to win the group also by goal difference.

Team	Group winner	Runner-up	Exit in preliminary round
Netherlands	61.2 %	23.2 %	15.6 %
Ecuador	17.6 %	31.6 %	50.8 %
Senegal	15.3 %	28.8~%	77.8 %
Qatar	5.8 %	16.4~%	78.8 %

Table 6. Probabilities for Group A

Team	Group winner	Runner-up	Exit in preliminary round
England	54.7 %	24.5~%	20.9~%
Wales	18.8 %	29.1~%	52.1~%
Iran	13.8 %	23.8 %	62.3~%
United States	12.6~%	22.5 %	64.9~%

Table 7. Probabilities for Group B

Team	Group winner	Runner-up	Exit in preliminary round
Argentina	75.2 %	17.6 %	7.2 %
Poland	13.9 %	37.9~%	48.2~%
Mexico	8.4 %	31.3~%	60.4~%
Saudi Arabia	2.4 %	13.2~%	84.3 %

Table 8. Probabilities for Group C

Team	Group winner	Runner-up	Exit in preliminary round
France	47.4 %	34.3 %	18.2 %
Denmark	42.8 %	36.5~%	20.8~%
Tunisia	5.2~%	15.1~%	79.8~%
Australia	4.7~%	14.2~%	81.3 %

Table 9. Probabilities for Group D

Team	Group winner	Runner-up	Exit in preliminary round
Spain	54.2 %	29.3 %	16.76 %
Germany	34.7 %	38.5~%	26.8~%
Japan	7.4~%	19.0 %	73.7~%
Costa Rica	3.9~%	13.1~%	82.9 %

Table 10. Probabilities for Group E

	Team	Group winner	Runner-up	Exit in preliminary round
ſ	Belgium	57.0 %	24.8 %	18.1%
	Croatia	24.4 %	34.5 %	41.2~%
	Morocco	11.4 %	22.7~%	65.8~%
	Canada	7.2 %	17.9~%	74.9~%

Table 11. Probabilities for Group F

Team	Group winner	Runner-up	Exit in preliminary round
Brazil	64.2 %	24.4~%	11.5~%
Switzerland	17.7 %	37.8~%	44.3~%
Serbia	17.6 %	34.3~%	48.2~%
Cameroon	0.4 %	3.6~%	96.0~%

Table 12. Probabilities for Group G

Team	Group winner	Runner-up	Exit in preliminary round
Portugal	59.8 %	27.7 %	12.4 %
Uruguay	30.0 %	44.3~%	25.8~%
South Korea	9.3~%	23.7~%	67.0 %
Ghana	0.8~%	4.3~%	94.8~%

Table 13. Probabilities for Group H

4.3. Playoff Round Forecast. Our simulations yield the following probabilities for each team to win the tournament or to reach certain stages of the tournament. The result is presented in Table 14. The ZIGP regression model favors Brazil and Argentina, followed by Belgium, Spain and the Netherlands. The current world champion France belongs also to the co-favourites, but the remaining teams have significantly less chances to win the FIFA World Cup 2022.

5. Discussion

In this section we want to discuss the presented ZIGP model and related models. In literature, one can find a great variety of scientific research papers which model the outcome of football matches by Poisson regression models. Most of these models use an independent or a bivariate Poisson regression approach. In the independent case there is only one single parameter to estimate which models the mean and variance simultaneously, while in the bivariate case one has an additional parameter which models only the correlation between the numbers of goals scored by both teams in a match. Multiplicative mixtures of distributions may lead to overdispersion. Therefore, it is desirable to use models having a variance function which is flexible enough to deal with overdispersion and underdispersion. This was the starting point to use an approach which allows additional flexibility of the variance. One natural model for this is the generalised Poisson model, which was suggested by Consul (1989) and where we have an additional parameter for modelling the variance in each regression. Additionally, we added a zero-inflation parameter in order to take into account the special value of 0 (that is, no goals are scored by a team during a match), which leads to a ZIGP distribution. This was the starting point of the present article. For more details on (zero-inflated) generalised Poisson regression we refer to Stekeler (2004) and Erhardt (2006). Of course, the ZIGP model we used is not the only natural candidate

Team	Champion	Final	Semifinal	Quarterfinal	Last 16	Prelim. Round
Brazil	17.3 %	27.3 %	43.5 %	67.8 %	88.6 %	11.5 %
Argentina	13.1 %	22.1~%	39.7~%	60.2~%	92.8~%	7.2~%
Belgium	10.8~%	19.3~%	31.1~%	51.6~%	81.8~%	18.1 %
Spain	10.2~%	17.7~%	30.1~%	52.4~%	83.5~%	16.6~%
Netherlands	10.0~%	17.9~%	33.9~%	61.3~%	84.4~%	15.6 %
France	8.4 %	16.8~%	32.1~%	51.7~%	81.8~%	18.2~%
Portugal	6.5~%	13.8~%	24.5~%	46.0~%	87.5~%	12.4~%
Denmark	6.1~%	13.5 %	27.9~%	46.9 %	79.3 %	20.8~%
Germany	3.6~%	8.8~%	17.8~%	39.1~%	73.2~%	26.8~%
England	3.5~%	9.4~%	22.0~%	45.3~%	79.2~%	20.9 %
Switzerland	2.1~%	5.8~%	12.5~%	28.6~%	55.6~%	44.3 %
Serbia	2.0~%	5.1~%	11.1~%	25.9~%	51.8~%	48.2~%
Croatia	1.7~%	4.7~%	10.5~%	25.9~%	58.9~%	41.2 %
Senegal	1.0 %	2.8~%	8.1 %	21.0~%	44.1~%	55.9~%
Uruguay	0.9 %	3.3~%	8.7~%	24.3~%	74.3~%	25.8~%
Ecuador	0.9 %	2.9~%	9.0~%	24.1~%	49.3~%	50.8~%
Poland	0.6~%	2.3~%	8.2~%	18.7~%	51.8~%	48.2 %
Morocco	0.4~%	1.2~%	3.6~%	11.8~%	34.1~%	65.8~%
Wales	0.2~%	1.2~%	5.0~%	18.1~%	47.9~%	52.1~%
Japan	0.2~%	0.8~%	2.3~%	8.1~%	26.4~%	73.7~%
Iran	0.1 %	0.6~%	3.1~%	12.8~%	37.6~%	62.3~%
United States	0.1 %	0.4~%	2.6~%	11.5~%	35.2~%	64.9 %
Mexico	0.1 %	0.8~%	4.1~%	10.9 %	39.7~%	60.4~%
Canada	0.1 %	0.5~%	1.8~%	7.0~%	25.1~%	74.9 %
South Korea	0.1 %	0.5~%	1.6~%	6.5~%	33.1~%	67.0 %
Costa Rica	0.1 %	0.2~%	0.9~%	4.3~%	17.1~%	82.9 %
Australia	0.0 %	0.2~%	1.3~%	4.3~%	18.9~%	81.3 %
Tunisia	0.0 %	0.2~%	1.2~%	4.2~%	20.3~%	79.8~%
Qatar	0.0 %	0.1~%	1.0~%	5.9~%	22.2~%	77.8~%
Saudi Arabia	0.0 %	0.1~%	0.8~%	3.3~%	15.6~%	84.3 %
Cameroon	0.0 %	0.0~%	0.1~%	0.6~%	4.0~%	96.0 %
Ghana	0.0 %	0.0~%	0.0~%	0.3~%	5.1~%	94.8~%

Table 14. FIFA World Cup 2022 simulation results for the teams' probabilities to proceed to a certain stage.

for modeling football matches, but it can be seen as another reasonable class of Poisson models which are suitable for forecasting results of football matches.

We now want to discuss our proposed model. As we have shown in Subsection 3 the proposed ZIGP model with weighted historical data seems to improve standard models in most cases. However, as we have seen for the World Cup 2018 outlier behaviour of some teams may be less efficient detected due to heavy weights of historic match data from the recent past. Hence, a possible weakness of the model could be the weights of historical

matches where we used a half period of three years (suggested by Ley et al. (2019)). But we have observed that sometimes non-weighting of historical match data does not necessarily lead to worse qualities of the forecast. Outperformance of some teams during a championship is still hard to detect.

The goodness of fit for the teams seems to be reasonable but can be improved certainly by adding additional parameters (e.g., dependence not only on the opponents Elo strength but also on the opponents continental origin, etc.). However, this would go beyond the scope of this article.

Our proposed ZIGP model combines one team's attack strength and the opponent's defense skills in order to model the number of scored goals of one team. The weaker team's number of scored goals depends even on the number of scored goals of the stronger team. In combination with the use of ZIGP distributions this is a novelty in modelling football scores. Once again, this approach can be seen as a first stage in developing more accurate models based on these ideas. The aim of this article is to strengthen the use of ZIGP models for forecasting football scores, but further work has to be done in order to get improved models based on ZIGP regression models.

For further discussion on adaptions and different models, we refer once again to the discussion section in Gilch & Müller (2018) and Gilch (2019).

6. Conclusion

A team-specific zero-inflated generalized Poisson regression model for the number of goals in football matches facing each other in international tournament matches has been used for quantifying the chances of the teams participating in the FIFA World Cup 2022. This model includes the Elo points of the teams as covariates and the location of the matches. The regression is based on historical match data since 2016. The fitted model was used for Monte-Carlo simulations of the FIFA World Cup 2022. According to our simulation, the Brazil (followed by Argentina) turns out to be the top favorite for becoming new world champion. The current world champion France belongs only to the co-favourites. Moreover, for each team probabilities of reaching different stages of the tournament are calculated.

A major part of the statistical novelty of the presented work lies in the construction of the nested ZIGP regression model. Validation of the proposed ZIGP model on tournaments between 2010 and 2020 showed a good fit and an outperformance of standard Poisson models in many cases.

References

Chou, N.-T. & Steenhard, D. (2011), 'Bivariate count data regression models – a SAS[®] macro program', SAS Global Forum 2011 - Paper 355-2011 pp. 1–10.

Consul, P. (1989), Generalized Poisson distributions: Properties and Applications, Statistics, textbooks and monographs v. 99, New York, M. Dekker.

- Deutsch, R. C. (2011), 'Looking back at South Africa: Analyzing and reviewing the 2010 FIFA world cup', *CHANCE* **24**(2), 15–23.
 - URL: https://doi.org/10.1080/09332480.2011.10739859
- Dixon, M. J. & Coles, S. G. (1997), 'Modelling association football scores and inefficiencies in the football betting market', *Journal of the Royal Statistical Society. Series C (Applied Statistics)* **46**(2), 265–280.
 - URL: http://www.jstor.org/stable/2986290
- Elo, A. E. (1978), The rating of chessplayers, past and present, Arco Pub., New York. URL: http://www.amazon.com/Rating-Chess-Players-Past-Present/dp/0668047216
- Erhardt, V. (2006), Verallgemeinerte Poisson und Nullenueberschuss Regressionsmodelle mit regressiertem Erwartungswert, Dispersions- und Nullenueberschuss-Parameter und eine Anwendung zur Patentmodellierung, Master's thesis, Technical University of Munich.
- Gásques, R. & Royuela, V. (2016), 'The determinants of international football success: A panel data analysis of the elo rating*', Social Science Quarterly 97(2), 125–141. URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/ssqu.12262
- Gilch, L. A. (2019), 'Prediction Model for the Africa Cup of Nations 2019 via Nested Poisson Regression', African Journal of Applied Statistics 6(1), 599–615.
- Gilch, L. A. & Müller, S. (2018), 'On Elo based prediction models for the FIFA Worldcup 2018', Technical Report, Number MIP-1801, Department of Informatics and Mathematics, University of Passau, Germany.
- Groll, A., Hvattum, L. M., Ley, C., Popp, F., Schauberger, G., Eetvelde, H. V. & Zeileis, A. (2021), Hybrid machine learning forecasts for the UEFA EURO 2020. arXiv: https://arxiv.org/abs/2106.05799.
- Groll, A., Schauberger, G. & Tutz, G. (2015), 'Prediction of major international soccer tournaments based on team-specific regularized poisson regression: An application to the fifa world cup 2014', Journal of Quantitative Analysis in Sports 11(2), 97–115. URL: https://ideas.repec.org/a/bpj/jqsprt/v11y2015i2p97-115n1.html
- H.R. Azhari, Y. Widyaningsih, D. L. (2018), 'Predicting final result of football match using poisson regression model', *Journal of Physics: Conference Series* 1108(1), 1–5.
- Joe, H. & Zhu, R. (2005), 'Generalized Poisson distribution: the property of mixture of Poisson and comparison with negative binomial distribution', *Biometrical Journal* 47(2), 219–229.
 - URL: https://onlinelibrary.wiley.com/doi/abs/10.1002/bimj.200410102
- Karlis, D. & Ntzoufras, I. (2003), 'Analysis of sports data by using bivariate Poisson models', Journal of the Royal Statistical Society. Series D (The Statistician) **52**(3), 381–393.
 - URL: http://www.jstor.org/stable/4128211
- Lee, A. J. (1997), 'Modeling scores in the Premier League: Is Manchester United really the best?', CHANCE 10(1), 15–19.
 - URL: https://doi.org/10.1080/09332480.1997.10554791
- Leitner, C., Zeileis, A. & Hornik, K. ("2010"), 'Forecasting sports tournaments by ratings of (prob)abilities: A comparison for the EURO 2008', *International Journal of Forecasting* **26**(3), 471 481. Sports Forecasting.
 - URL: http://www.sciencedirect.com/science/article/pii/S0169207009001459

- Ley, C., de Wiele, T. V. & Eetvelde, H. V. (2019), 'Ranking soccer teams on basis of their current strength: a comparison of maximum likelihood approaches', *Statistical Modelling* 19, 55–77.
- Maher, M. J. (1982), 'Modelling association football scores', *Statistica Neerlandica* **36**(3), 109–118.
 - **URL:** https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1467-9574.1982.tb00782.x
- Saraiva, E., Suzuki, A. & Filho, C. (2016), 'Predicting football scores via poisson regression model: applications to the national football league', Communications for Statistical Applications and Methods 23(4), 297–319.
- Stekeler, D. (2004), Verallgemeinerte Poissonregression und daraus abgeleitete zeroinflated und zero-hurdle Regressionsmodelle, Master's thesis, Technical University of Munich.
- Wheatcroft, E. (2021), 'Forecasting football matches by predicting match statistics', *Journal of Sports Analytics* **7**(2), 77–97.
- Zeileis, A., Kleiber, C. & Jackman, S. (2008), 'Regression models for count data in r', Journal of Statistical Software, Articles 27(8), 1–25.
 - **URL:** https://www.jstatsoft.org/v027/i08
- Zeileis, A., Leitner, C. & Hornik, K. (2012), History repeating: Spain beats Germany in the EURO 2012 final, Working Papers in Economics and Statistics 2012-09, University of Innsbruck, Innsbruck.

URL: http://hdl.handle.net/10419/73891

Lorenz A. Gilch: Universität Passau, Innstrasse 33, 94032 Passau, Germany

 $Email\ address$: Lorenz.Gilch@uni-passau.de URL: http://www.math.tugraz.at/ \sim gilch/