Module I

Differential Equations and Tiny Houses

Chapter 1

Module 1 mini-project: Passive solar tiny house for Massachusetts

Design your dream tiny home for a fossil-fuel-free future!

This is a partner project with a joint final report.

The project will allow you to practice thermal modeling, setting up and solving ODEs, and communicating design choices and data analysis. You can even design an awesome house that you would want to live in. Please be creative and have fun!

1.1 Design requirements

- Size: 100-400 sq. ft.
- Reasonably comfortable in winter in Boston climate (about 17-25 °C air temperature indoors)
- No direct sunlight through south-facing windows at noon in summer

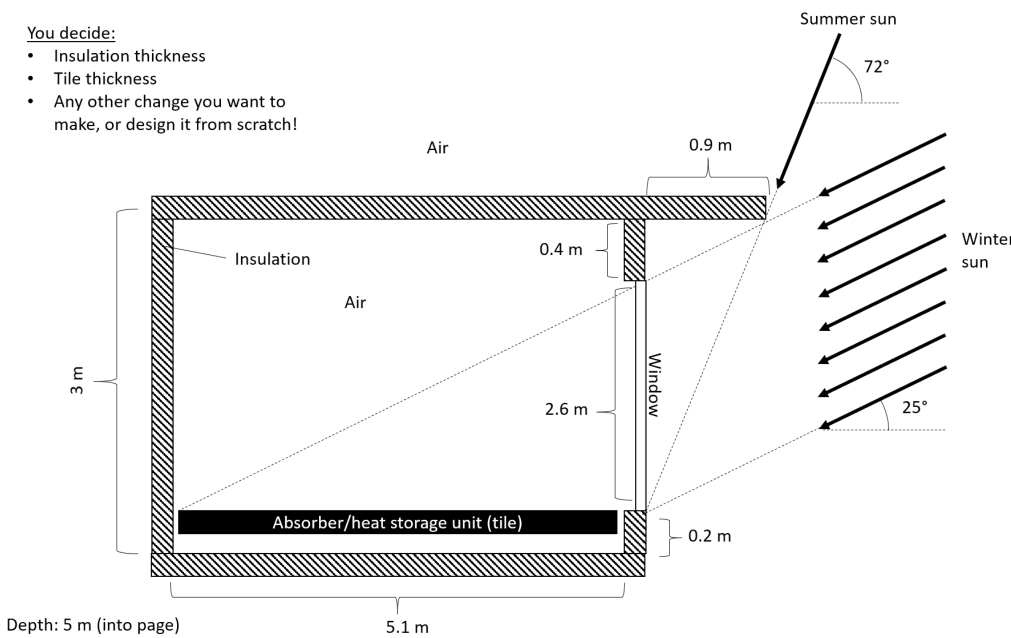
1.2 Design choices

You must choose a desirable insulation thickness and heat storage unit thickness. Other decisions are optional.

- Insulation thickness (required)
- Heat storage unit thickness (required)
- Insulation material (base case material: fiberglass)
- Heat storage unit material (base case material: recycled ceramic tile)
- Other materials
- Window placement
- House shape
- Window orientation (facing south recommended)
- Style, layout, color, interior design, landscape design, etc.

1.3 What we will give you

- A house template, including dimensions you can use:



- Equivalent heat transfer coefficient of double-paned window ($h_{eq} = 0.7$ W/m^2-K) (meaning $\dot{Q}_{window} = h_{eq} A_{window} (T_{innersurface} - T_{outersurface})$.
- Conductivity of fiberglass insulation (0.04 W/m-K)
- Density and heat capacity of tile for energy storage ($\rho = 3000$ kg/m^3, c=800 J/kg-K)
- Typical heat transfer coefficients for indoor surfaces (15 W/m^2-K) and outdoor surfaces (30 W/m^2-K)
- Air temperature: choose to model as:
  - Constant ($T = -3°$C) or
  - Sinusoidal ($T(t) = -3 + 6 \sin \frac{2\pi t}{24*60*60} + 3\pi/4$ (t is in seconds))
- Normal solar flux through south-facing window:$q = -361 \cos(\pi t/(12 * 3600)) + 224 \cos(\pi t/(6 * 3600)) + 210$ (q in W/m^2; t is in seconds). The heat absorbed by the absorber/heat storage unit ($\dot{Q_{in}}$) is the product of the normal solar flux and the window area, A: $\dot{Q_{in}} = qA$
- Angle of sun at noon in winter (24° from horizontal) and summer (72° from horizontal)
- Assumptions you *can* make:
  - Heat capacity of the heat storage unit is much greater than the building envelope or the air inside (however, you are welcome to design and model a house that has significant heat storage in the envelope or other parts).
  - Heat storage unit is at a spatially uniform temperature

- A negligible amount of air flows into or out of house
- Neglect radiation except solar radiation through windows
- All solar radiation hitting window is absorbed by heat storage unit, which is at uniform temperature
- Air flows both below and above the heat storage unit (as if it was suspended–this is not physically very realistic, but reasonably simplifies the model)
- Heat loss from the underside of the house matches that of the rest of the house (as if it was on stilts)

1.4 Levels of challenge

The essential task is to optimize the design of a small passive solar house to be comfortably warm in the Northeast. In the simplest implementation, you can:

- Use given house shape and materials (you might consider starting with 0.05 m-thick tile and insulation and determining optimal tile and insulation thickness by informed trial and error)
- Model the house as having a single lumped heat capacity (e.g., a high-heat-capacity storage unit and negligible-heat-capacity walls)
- Build on your previous work (or posted solutions) modeling a 2-heat-capacity house
- Model outside air temperature as constant
- Clearly explain the physics and math behind the equations you solve
- Use MATLAB to numerically find for the long-term periodic air temperature in the house
- Turn in a brief report on time

We highly recommend that you first do the essential task above and document your work before adding complexity to your project. (Last year, some teams added a lot of complexity up front and ended up spending much more time than desired.)

If you have created and documented a simple, working model, you are welcome to add levels of fidelity/challenge/creativity, such as:

- Customize house shape or materials
- Model outdoor air temperature as sinusoidal, or use real weather data
- Remove some assumptions and create a more complex model
- Design the house's interior or landscape
- Model performance in summer

1.5 *Deliverables*

- A working but not necessarily optimized MATLAB model of your house (Assignment 8) (one submission per person)
- A final report on the design and performance of your optimized house (Assignment 10) (one per pair)
- Give feedback on another group's report and reflect on learning (Assignment 11) (one submission per person)

Due dates of all assignments are on Canvas.