Passive Solar House Report

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1 Background

The term 'Passive Solar House' refers to a house constructed to use the heat emitted by the Sun as the main source of heat. The house's design facilitates the heating process and also regulates the interior temperature. Using the Sun as a heat source allows us to conserve the energy we would spend on heating the house using other methods such as oil, electricity, or gas. Passive solar houses are becoming increasingly relevant as people are becoming more conscious about their carbon footprint and impact on the environment. A passive solar house is also cost effective for the same reasons (US Department of Energy).

The key principles of such a system include the proper placement of windows (ideally facing south and at an angle of 30 degrees for Boston), making effective use of insulating materials (to retain heat in the house), absorbing materials (dark surfaces which absorb heat), and the incorporation of a thermal mass/heat storage unit (to store heat).

Our group aims to optimize the material properties and find the ideal thicknesses to use for both the heat storage unit (the floor) and the insulation (the walls). After building a base model, we will create a parameter sweep to test different thickness combinations of these materials and investigate the effects they have on the internal air temperature of our passive solar house.

2 Modeling

2.1 House Geometry Explained

For this project, we decided to implement the house geometry presented in the assignment instructions (figure 1). This house consists of fiberglass walls (insulation), recycled ceramic flooring tiles (heat absorber and thermal mass), and one double paned window. Besides the thicknesses of the floor and walls (which we investigate using the parameter sweep we designed in our model), we use the given dimensions for our passive solar house.

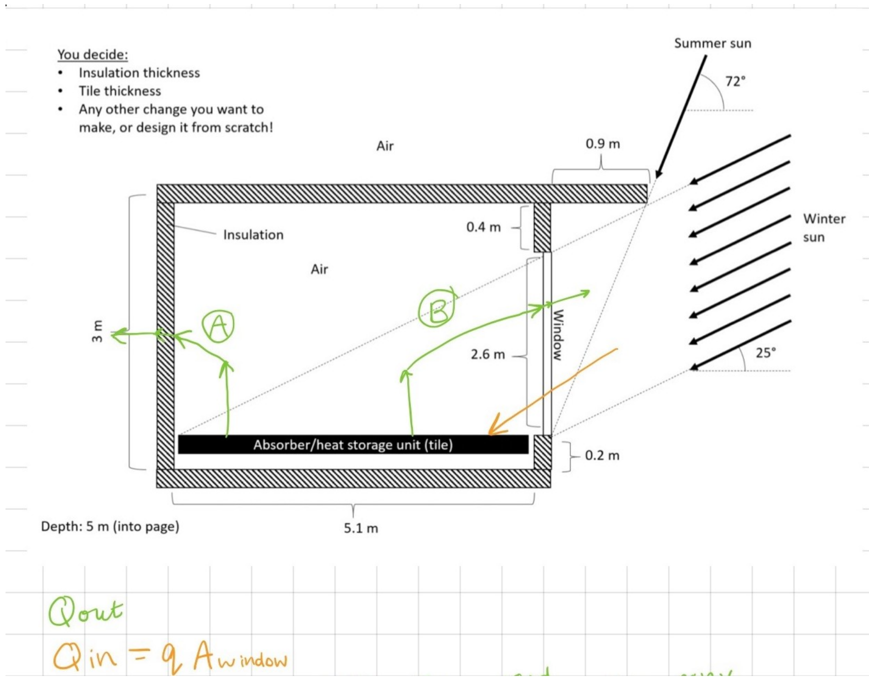


Figure 1: House Dimensions and Paths of Heat Flow

2.2 Assumptions

We assumed that heat storage only occurs in the floor, considering recycled ceramic (and oak) have a significantly greater heat capacity than the other materials in the house. We also assume the floor and internal air are each at a uniform temperature. The house is also built around the assumption that the walls are perfect insulators and that a negligible amount of air flows into our out of the house. The impact of this assumption is further discussed in section 3 Optimization. Our model neglects all solar radiation into the house except through the window, and assumes that all solar radiation that goes through the clear glass of the window is entirely absorbed by the floor. To simplify our thermal resistor network calculation, we assume that air flows both below and above the heat storage unit (the floor), as displayed in the diagram in Figure 1, meaning that the floor is suspended and therefore convects heat into the air above and below. This approximation is not realistic for a home in Massachusetts. However, it significantly simplifies our mathematical model since we can then incorporate the path of heat from the heat storage unit through the bottom of the house as the same as heat going from the heat storage unit through the perfectly insulating surfaces above the heat storage unit. These assumptions are reasonable based on the goal of the project and the house design given.

2.3 Methodology

In our model, we consider the dimensions of the floor, walls, and windows, the conductivity of fiberglass insulation, the density, heat capacity, and volume of the recycled ceramic tiles, and the heat transfer coefficients of all surfaces (indoor, outdoor, and double paned window) to be constant. Besides the thickness of the floor, the thickness of the walls, and the quantities for oak (an additional floor material we tested), the values for the constants were given in the project assignment. We chose to define these constants at the beginning of our code to be used throughout the model.

We used the sinusoidal function $T(t) = -3 + 6\sin\frac{2\pi t}{24\times 60\times 60} + 3\pi/4$, where T is temperature in Celsius and t is time in seconds, to represent the air temperature outside the house since it fluctuates based on time of day. This equation was given to us in the project description document.

Next, we calculated the thermal resistances for each stage of heat flow, and created a thermal resistance network to calculate heat flow from the heat storage (the floor) through other parts of the house (inside air, walls (insulators), the window) to the outside air. Figure 2 includes a visual representation of the thermal resistor network and the related resistor equations for heat flow from the floor to the external air. This resistor network is extremely important, as it is the basis for our ODE that allows us to solve for the temperature of the floor. Table 1 demonstrates the role of each resistor in defining the paths of heat flow for the house.

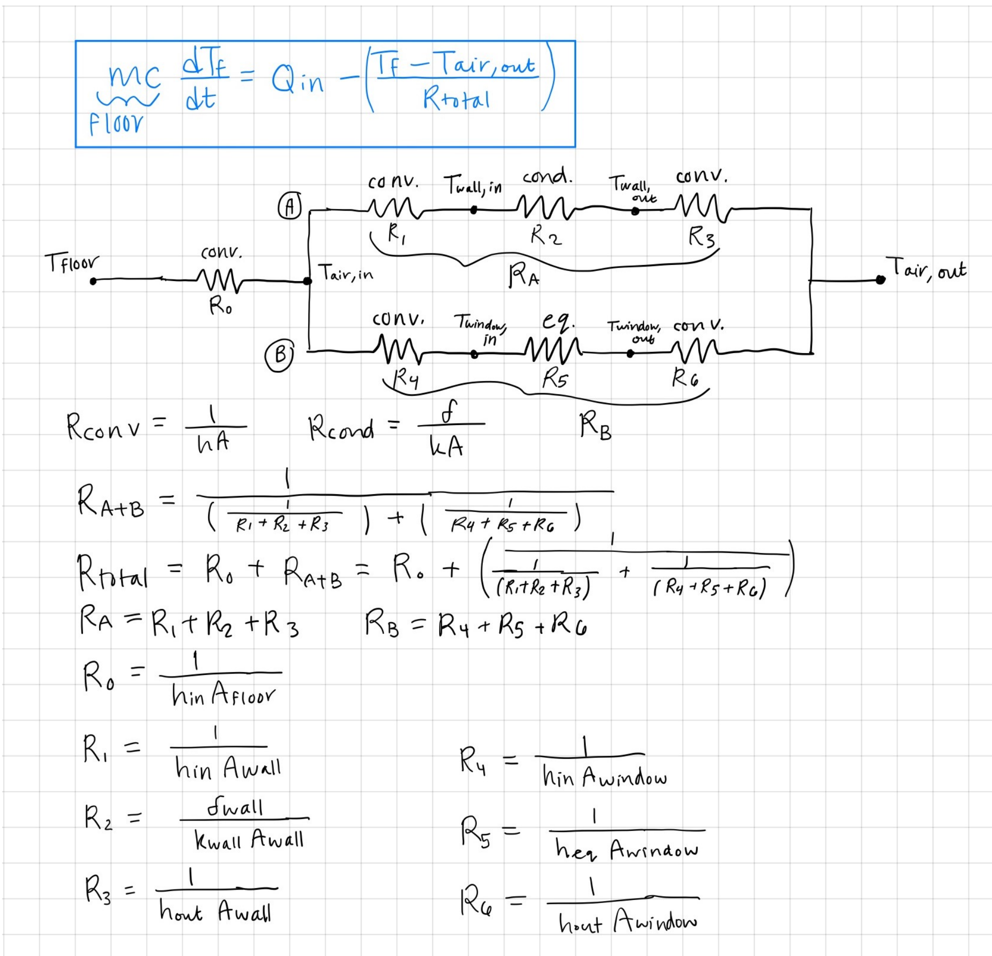


Figure 2: Resistor Network and Resistor Equations

Thermal models can be represented as resistor circuits because of the way heat flows. In our house, there is one path for heat to travel from the floor to the air inside. Once the heat has reached the air in the house, there are two possible paths: through the window or through the insulator (the wall). This fork is represented in Figure 2 by the portion of the circuit that is in parallel. These paths are represented by branches with resistors in series because there are no further path deviations possible for the individual materials. The forks converge at $T_{air,out}$ because all heat ultimately flows to the air surrounding the house regardless of the path it took.

The heat flow from the floor to the external air resistances is the sum of the following resistances, where resistances may be calculated based on the type of heat flow, in this case conduction or convection. Conduction refers to heat transfer from an object to another object by direct contact. Meanwhile convection refers to heat transfer from the surface of a material to a fluid (i.e. the ceramic floor to the air in the house). Table 1 shows the breakdown of our defined thermal resistances and the type of heat transfer for each.

Table 1: Breakdown of Thermal Resistances

Resistance Name	**Sub-Resistance**	**From**	**To**	**Type of Heat Flow**
R0	R0	Floor	Internal Air	Convection
RA (R1 + R2 + R3 in series)	R1	Internal Air	Internal Wall Surface	Convection
	R2	Internal Wall Surface	External Wall Surface	Conduction
	R3	External Wall Surface	External Air	Convection
RB (R4 + R5 + R6 in series)	R4	Internal Air	Internal Window Surface	Convection
	R5	Internal Window Surface	External Window Surface	Convection*
	R6	External Window Surface	External Air	Convection

*The project description document provided an equivalent coefficient of heat transfer for a double paned window that allows it to be treated as one resistor and the mode of heat transfer as convection.

Using the thermal resistance network, the resistor values, and our calculation of the total resistance for the thermal system, we had the last piece needed to develop an Ordinary Differential Equation (ODE) to present the relationship between heat flow, temperature, time, and material properties. Putting this relationship into ODE form was advantageous for representing all possible heat flows in one equation and allowing us to solve for temperature of specific materials/fluids, which was the primary goal of our model. We used the rearranged equations for rate of heat transfer for convection and conduction, which is the form $\mathrm{Q} = \frac{\Delta T}{R_{\text{total}}}$ to represent the rate of heat leaving the house. The rate of heat from the Sun entering the house was provided as heat flux (or the rate of heat flow per unit area) $q = -361 \cos \frac{\pi t}{12 \times 3600} + 224 \cos \frac{\pi t}{6 \times 3600} + 210\ \mathrm{W/m^2}$, which we multiplied by the area of the window to calculate the overall rate of heat into the house. Knowing $\mathrm{Q_{in}}$ and $\mathrm{Q_{out}}$ gave us the necessary components to use conservation of energy, where $\frac{dU}{dt} = \mathrm{Q_{in}}$ - $\mathrm{Q_{out}}$. Since we also know that conservation of energy $\frac{dU}{dt} = \mathrm{mc}\Delta T$, the two equations can be set equal to each other to define a direct relationship between change in temperature over time and rate of heat flow, $\mathrm{mc}\Delta T = \mathrm{Q_{in}}$ - $\mathrm{Q_{out}}$. This mathematical representation of the physical principles of conservation of energy in terms of temperature and rate of heat flow led us to define the ODE we used in our model, where the instantaneous change in temperature over change in time is defined in terms of temperature.

We calculated the internal air temperature by treating the heat flow from the floor to the external air as a voltage divider. Using this method, we can solve for the internal air temperature without setting up an additional ODE. This process is shown in Figure 3. The temperatures of the floor, internal air, and external air are the voltages, the equivalent resistances explained above are the resistors, and the heat flow is the current. Since all of the heat flows from the floor to the outside air, we know that the "current" from the floor to the internal air is equivalent to the "current" from the internal air to the external air, and set these equal to each other. We can then use circuit laws to solve for the internal air temperature (shown in Figure 3).

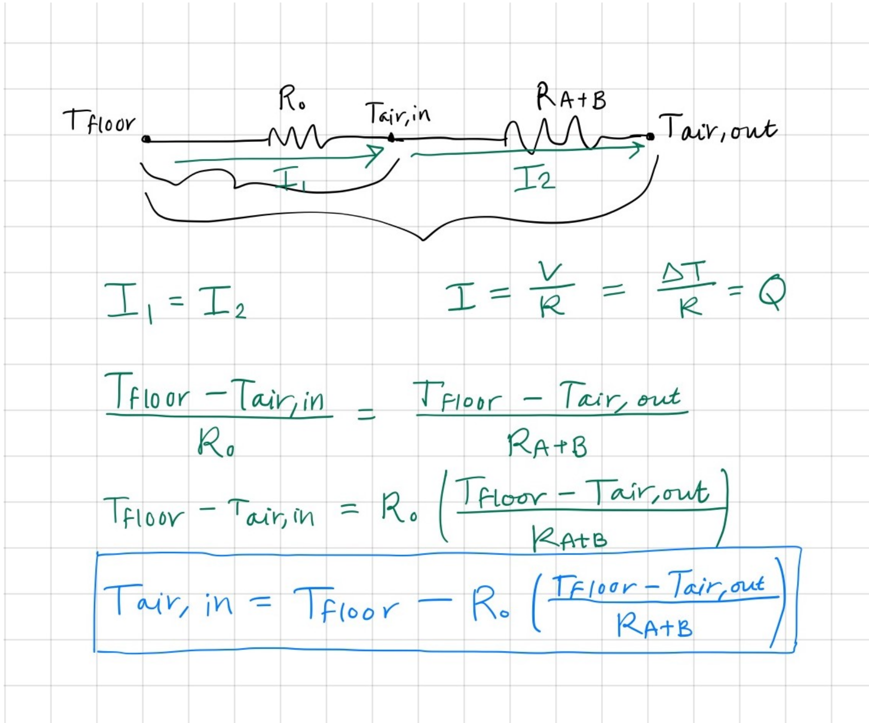


Figure 3: Voltage Divider to solve for internal air temperature

Our final ODE is $\frac{dT_f}{dt} = \frac{Q_{in} - \frac{T_f - T_{air,out}}{R_{total}}}{m_{floor}c_{floor}}$ where the resistances are defined above, m_{floor} is the mass of the floor and c_{floor} is the specific heat of the floor. Subbing in for R_{total} (and using the equation for resistors in parallel) gives us a final ODE of $\frac{dT_f}{dt} = \frac{Q_{in} - \frac{T_f - T_{air,out}}{R0 + (\frac{1}{RA} + \frac{1}{RB})^{-1}}}{m_{floor}c_{floor}}$. The linear equation we use to solve for internal air temperature is $T_{air,in} = T_{floor} - R0(\frac{T_{floor} - T_{air,out}}{(\frac{1}{RA} + \frac{1}{RB})^{-1}})$.

Once we had equations to model the behaviour of heat transfer in our house, we created a Matlab script with our known constant values and the equations we generated, and used the ode45 function to solve for the temperature of the floor (floor temperature was the unknown value in the ODE we found for our house). The floor temperature values outputted by the ode45 function over a set period (thirty days in our model) were plugged into the linear equation to solve for inside air temperature over the same period. In order to optimize our model, we created a parameter sweeping system to iterate through possible combinations of fiberglass wall thicknesses and heat storage unit/recycled ceramic tile thicknesses. This operation is discussed in more detail in section 3 Optimization. The resulting temperature values for the floor and inside air are stored in separate matrices and all resistors are reset for each new thickness we examine.

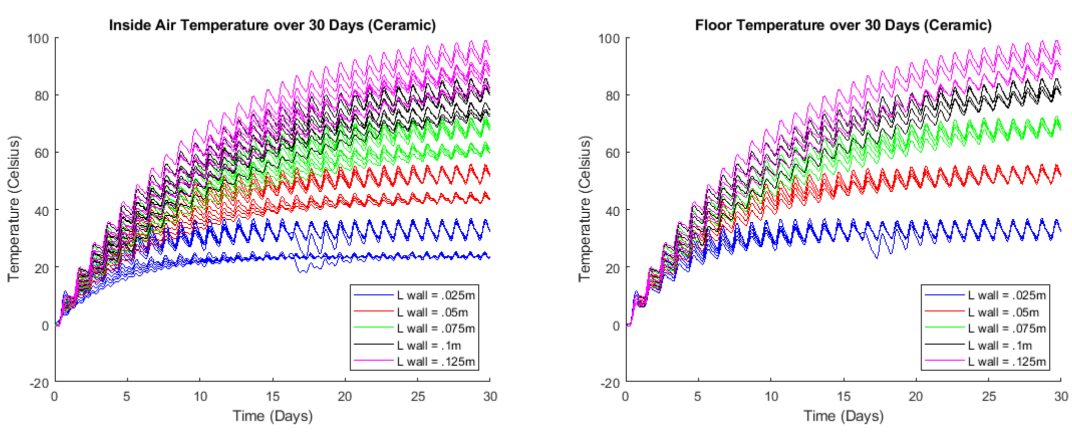
We then plotted the temperature values for all possible storage unit and insulation thickness combinations. A subset of these results are also plotted to allow us to choose the best combination to optimize our model (Based on plots, we focused on $L_{wall} = 0.025$m, which is explained in section 3 Optimization).

3 Optimization

3.1 Process

To investigate which combination of the thickness values of both the heat storage unit (the floor) and the insulation (the wall) would yield a comfortable inside air and floor temperature, we optimized these values using a two parameter sweep to test all possible combinations for a range of thicknesses. We chose a range of five possible thickness values for the floor and five for the wall. These arrays were incorporated into a for loop nested in a while loop, where the while loop iterated through wall thicknesses and the for loop iterated through floor thicknesses. In this system, one wall thickness value is assigned, constants that use that value are recalculated, and then ode45 is used within the for loop to solve for floor temperature at every floor thickness value in the range with the selected wall thickness. The resulting floor temperatures for each combination are stored in a matrix and used to calculate the inside air temperature for the same thickness combinations. Once all combinations with the selected wall thickness have been calculated, the while loop increments and a new wall thickness is chosen to be paired with all possible floor thicknesses again. This process repeats until all combinations of wall and floor thicknesses have been used to calculate the floor and inside air temperatures.

Those resulting temperatures are stored in one large matrix for each location (one for floor temperature and one for inside air temperature), which is then used to plot the results. Separate plots are generated for the floor temperature and the wall temperature and are displayed below (Figure 4). Both follow the same system of sorting the results based on wall thickness, as there is a correlation between the thickness of the insulation (the wall) and the temperature of both the floor and the air in the house. These plots allowed us to visualize our results and select a wall thickness value. As the graphs below display, an insulation thickness of 0.025m creates the most comfortable internal air temperature for all the possible thicknesses of the floor. Thus, we chose the insulation thickness of 0.025m to use for our passive solar house.



(a) Temperature of internal air

(b) Temperature of floor

Figure 4: Temperatures for sweeping wall thicknesses

We then plotted the floor and inside air temperatures using the wall thickness we selected from the plots above (0.025m) and organized the resulting graph based on the floor thickness value (see below). The results provided us with the evidence to select a floor thickness that best met the requirements of our house. The mathematical behavior of these values is discussed below.

Based on this graph, we can see that a floor thickness of 0.4m provides the optimal internal air temperature.

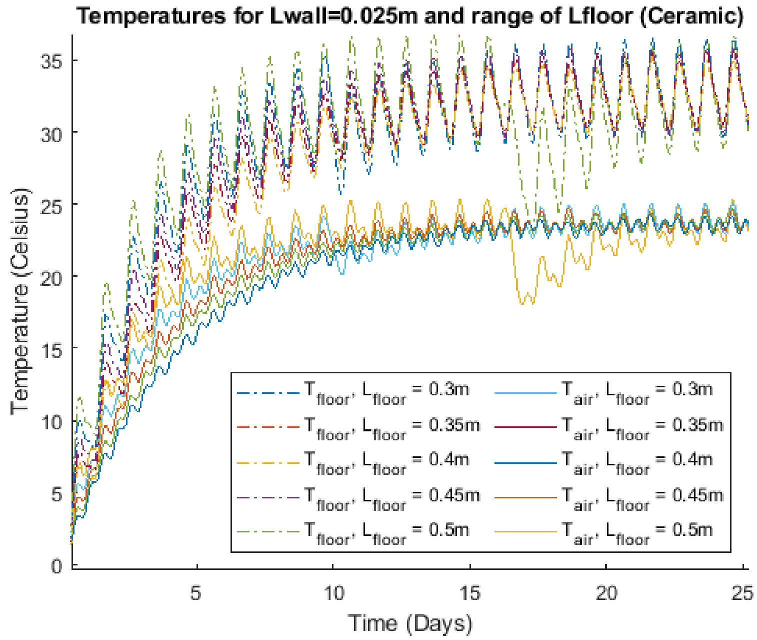
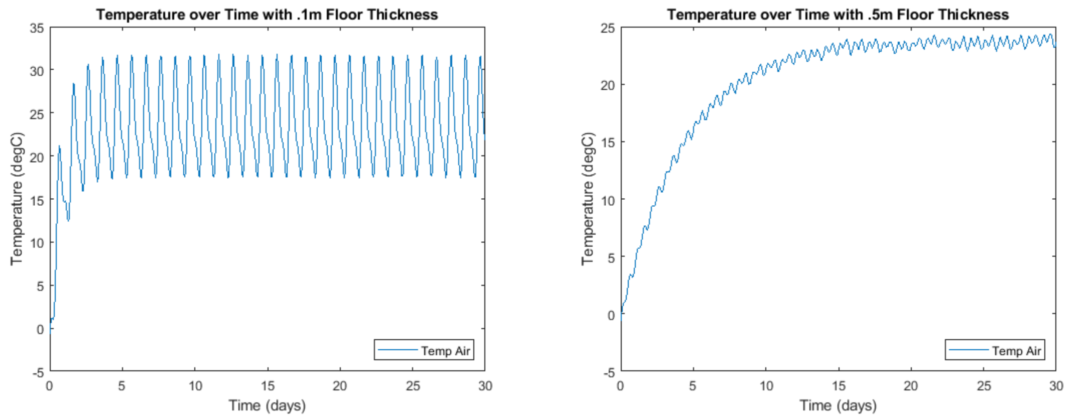


Figure 5: Temperatures of Internal Air and Floor from Sweeping Floor thickness with wall thickness of 0.025m

3.2 Heat Storage Unit Thickness

With a constant insulation thickness, the thickness of the heat storage unit (in the case of our house, this is just the floor) has a few impacts. As the thickness of the floor increases, the house takes longer to reach equilibrium. This makes sense, as the heat storage unit would take longer to heat up as it has a larger heat capacity. With a very small floor thickness (Figure 6a), the house reaches its equilibrium quickly (in about 3 days), however, the internal air temperature fluctuates significantly more. This is because the floor does not have a large enough heat capacity to keep the house warm at night when the sun is not providing significant solar radiation. The plots below show the internal air temperature for two different floor thicknesses and a constant insulation thickness of 0.025m. As displayed, at 0.1m floor thickness the internal air temperature has a large amplitude because there is less material to store the heat (Figure 6a), therefore a small heat capacity resulting in large fluctuations during the day and night. With a 0.5m floor thickness (Figure 6b), the internal air temperature has a smaller amplitude as the thicker floor can store more heat because it has a larger heat capacity. This makes the internal air temperature more stable and less influenced by the oscillation of solar radiation provided by the sun during the days and nights. Figure 6 displays the results of using floor thickness on the extreme end of our range in order to make its impact clear.



(a) Thin heat storage (0.1m)

(b) Thick heat storage (0.5m)

Figure 6: Temperature of internal air with differing heat storage unit thickness

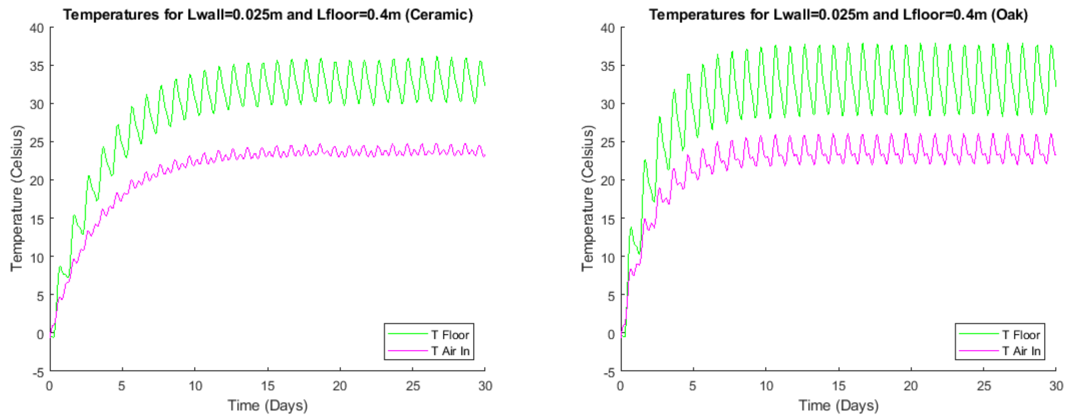
3.3 Insulation Thickness

Our parameter sweep (see Figure 4) system indicated that an insulation thickness of 0.025m created an optimal inside air temperature for our house (see subsection 3.1 Process), but upon deeper investigation we learned that insulation in Massachusetts is typically 0.09m to 0.15m thick (Energy.gov). This range of values comes from Massachusetts being in zone 5 out of 7 of the United States insulation map (where 7 is coldest), and the typical R value range of R-13 to R-21 (Home Depot). Based on the assumption in the model that the walls are perfect insulators, it is logical that our model requires a much lower insulation thickness and less insulation overall. If we were to model the walls as not perfect insulators, additional insulation would be necessary to maintain the same temperature inside the house, and the thickness would likely be in the widely accepted range, stated above.

4 Results and Discussion

4.1 Results

Figure 7a displays our passive solar house reaching equilibrium with a floor thickness of 0.4m (using recycled ceramic tiles) and insulation thickness of 0.025m (using fiberglass). Figure 7b displays the same but for oak flooring (another material we tried, discussed below). Temperature values for the floor and the inside air are displayed for one month, including the original heating of the house from an initial temperature equivalent to the outside air temperature at time = 0. As displayed in the plot, the inside air temperature reaches equilibrium in about ten days for both ceramic and oak flooring.

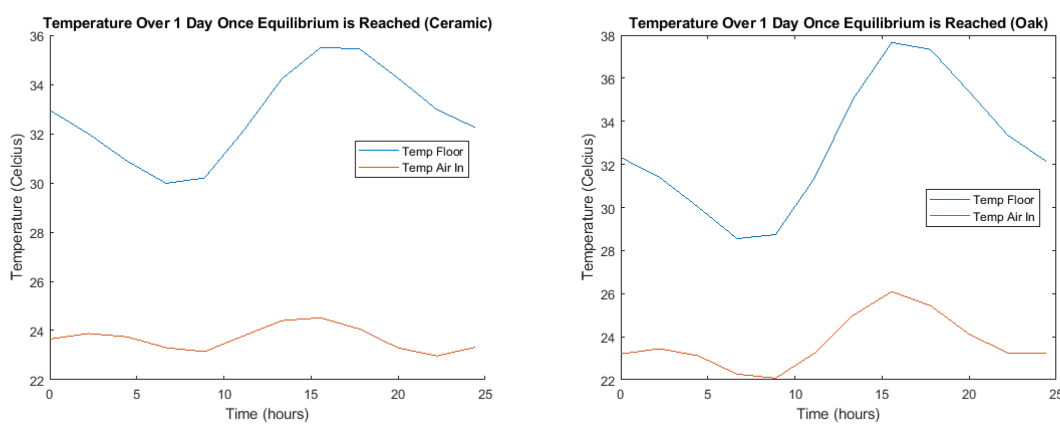


(a) recycled ceramic tile

(b) oak

Figure 7: Temperature of internal air over 1 month with differing flooring materials

Figure 8 shows the floor temperature and internal air temperature of our house over twenty four hours once it has reached equilibrium for both ceramic tiles (Figure 8a) and oak (Figure 8b). For ceramic, the minimum internal air temperature once equilibrium is reached is 22.0C and the maximum internal air temperature is 24.8C. The average internal air temperature is 23.5C, which equates to 74.3 F. Once equilibrium is reached, the minimum floor temperature is 28.6C, the max temperature is 37.7C, and the average temperature is 32.8C. For oak, the minimum internal air temperature once equilibrium is reached is 22.1C and the maximum internal air temperature is 26.1C. The average internal air temperature is 23.7C, which equates to 74.66 F. Once equilibrium is reached, the minimum floor temperature is 28.6C, the max temperature is 37.7C, and the average temperature is 32.8C. There was a very minimal impact of using ceramic as opposed to oak, as the average floor temperature is the same. The implications of these values are elaborated upon in subsection 4.2 Discussion. The internal air temperature is slightly warmer than the average home in the winter, but it is within the acceptable range given to us in the project description.



(a) recycled ceramic tile

(b) oak

Figure 8: Temperature of internal air over 1 day with differing flooring materials (once equilibrium has been reached)

We decided to investigate oak flooring (in addition to the given recycled ceramic tile material) to reduce the floor temperature to a comfortable temperature. Since oak is quite common (Riha), we replaced the recycled ceramic tiles with oak. Upon looking at the plots generated, we realized that replacing the floor material with oak resulted in minimal change to the floor temperature. We believe this result is a function of the material properties of recycled ceramic tile and oak. To elaborate, the density and specific heat of the recycled ceramic tile is 3000 kg/m^3 and 800 J/kg-K, respectively, while the density and specific heat of the oak wood is 750 kg/m^3 and 2000 J/kg-K. Because the order of magnitude of density and specific heat are swapped for the materials, the heat capacity values are functionally the same and therefore do not significantly impact our heat storage unit. We saw similar temperature values for both the floor and the inside air using both oak and recycled ceramic tile. From Figure 8, we can see that using recycled ceramic tile provides a slightly more comfortable internal air temperature.

4.2 Discussion

With regards to internal air temperature, a person would be fairly comfortable living in this house once it has reached equilibrium. It is a little warmer than a typical home in the winter, however, it is still a comfortable living temperature. The temperature of the floor, however, is too hot to stand on barefoot. This result is because the floor is the sole heat storage unit in the house, and there is no surface above the floor to protect people's feet. Therefore, if you were to walk barefoot (or with socks on) in this house you would burn your feet. Someone living in this house would probably need to have a separate pair of indoor shoes in order to protect their feet. Additionally, they would probably be a little sad that there is only one window in the house since windows are nice and make a space feel more enjoyable to spend time in, especially in the winter time when it can be too cold to go outside.

There are many future improvements we can make to both our model and our house overall. We could add a layer of a different flooring material over the current recycled ceramic tile floor so you can walk around the house without burning your feet. This approach would be best accomplished with a material with a lower/negligible heat capacity suspended over the current heat storage unit (the floor) so it will not store heat and therefore reach a temperature closer to the temperature of the air inside the house. In addition, the chosen material thicknesses in our model are ideal for the house in a cold climate in the winter. Creating a model for the summer when the outside temperature is warmer would be another important step to ensure that the house can maintain a comfortable temperature in both seasons. Given these additional considerations, we would investigate how alternative materials for both the heat storage unit and insulation change the resulting temperatures, and ideally would find materials and associated quantities that optimize the cost of raw materials while minimizing environmental impact.

5 References

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6 Appendix

See our GitHub Repository for our code and other project artifacts