Problem 0003

Problem

The prime factors of 13195 are 5, 7, 13 and 29.

What is the largest prime factor of the number 600851475143?

Solution

Variables and Functions

- Let n be the integer that should ultimately be factorised into the primes that compose it.
- $P(o)$ returns an array of prime numbers using the Sieve of Eratosthenes method - repeatedly casting out multiples of primes.
  - Let o be the exclusive upper limit for values of terms of $P(o)$.
  - Let f be an array of boolean values which acts as the "sieve".
  - Let i be an integer which represents the position within the sieve.
  - Let j be an integer which is used to represent positions in the sieve which are multiples of i.
- $F(m, s)$ factorises an integer.
  - Let m be the integer to be factorised.
  - Let s be an array of prime numbers, where $\sqrt{n}$ is the inclusive upper limit for values of terms of s.
  - Let p be a term in s.

Approach

Initially, n is defined as the number to be ultimately factorised. n is passed, along with $P(\sqrt{n})$ into the *recursive* function $F(m, s)$.

$F(m, s)$ iterates through terms of s. If the remainder from the division of $\frac{m}{p}$ (where p is the current term of s) is 0, p is a *prime* factor of m (and by extension n). p is returned along with the output of $F\left(\frac{m}{p}, s\right)$.

This function repeatedly divides by prime numbers until m is a prime number, which is known when either m is equal to p, or when $m \bmod p$ has not output 0 for any term of s. At this point, m itself is returned, and the array of prime factors is passed up the function call stack to the top level.

Code

The code to produce this solution is in `solution.py`.

Output

For $n = 13195$, the program outputs $[5, 7, 13, 29]$, as displayed in the problem text.

For $n = 600851475143$, the program outputs $[71, 839, 1471, 6857]$, with the greatest prime factor being 6857. This is correct.