Problem 0004

Problem

A palindromic number reads the same both ways. The largest palindrome made from the product of two 2-digit numbers is $9009 = 91 \cdot 99$.

Find the largest palindrome made from the product of two 3-digit numbers.

Solution

Variables and Functions

- Let $P()$ be a function which returns the largest palindrome made from the product of two 3-digit numbers.
  - Let a be an integer.
  - Let b be an integer.
  - Let c be an integer.
  - Let n be an integer, which may be either 5 digits with the pattern $abcba$, or 6 digits with the pattern $abccba$.
  - Let x be an integer that n is divided by.
  - Let j be the inclusive lower limit for x.
  - Let k be the inclusive upper limit for x.

Approach

Each palindromic number is generated through a nested loop in which values of a, b and c are decreased with an increment of 1. Due to the nature of the formation of n, any values of a, b or c will generate a palindromic number.

Due to the fact that n is calculated through multiplying two 3-digit numbers, the maximum value for n is $999 \cdot 999 = 998001$. The largest 6-digit palindromic number less than 998001 is 997799. Therefore, before any division of n, a check of $n \leq 997799$ can be made to avoid un-necessary computation.

Since both factors of n must be 3 digits,

$$100 \leq \frac{n}{x} \leq 999$$

Therefore,

$$\frac{n}{999} \leq x \leq \frac{n}{100}$$

In terms of j and k, this is shown as

$$j \leq x \leq k$$

Often, the result of $\frac{x}{999}$ or $\frac{x}{100}$ will be a floating point value. Therefore, both of these values must be rounded appropriately. As j is the minimum value for x, it must be rounded up. Therefore, $j = \left\lceil \frac{n}{999} \right\rceil$.

k is the maximum value for x, so $k = \left\lfloor \frac{n}{100} \right\rfloor$.

Sometimes, k will be greater than 999. *Both* factors of n must be 3 digits, so in this case k will be set equal to 999. Similarly, if $j < 100$, j will be set to 100.

Now the divisor, x, is iterated through from j to k. If any values of x divide evenly into n, this means that x is one of the factors. We have already confirmed that x is 3 digits, and that $\frac{n}{x}$ is 3 digits. Therefore, n is a palindrome that is composed from the product of two 3-digit numbers. Since the code began with the largest possible values of n and worked downwards, the first valid value of n found is the largest palindrome that can be made from the product of two 3-digit numbers.

Code

The code to produce this solution is in `solution.py`.

Output

The code outputs 906609, which is correct.