

# Toy EDM simulation: key physics

Samuel Grant

September 16, 2025

## 1 Introduction

This document outlines the key equations used in `toy-edm`: a simple muon electric dipole moment (EDM) simulation for the Fermilab Muon  $g-2$  experiment. All calculations are done in the muon rest frame, where rest frame quantities are indicated by an asterisk.

## 2 Fundamental parameters

### 2.1 Physical constants

$$c = 299\,792\,458\,\text{m s}^{-1} \tag{1}$$

$$e = 1.602\,176\,634 \times 10^{-19}\,\text{C} \tag{2}$$

$$\hbar = 1.054\,571\,817\,6 \times 10^{-34}\,\text{J s} \tag{3}$$

$$m_\mu = 105.658\,374\,5\,\text{MeV}/c^2 = 1.883\,53 \times 10^{-28}\,\text{kg} \tag{4}$$

### 2.2 Muon anomalous magnetic moment

The anomalous magnetic moment is given by

$$a_\mu = \frac{g-2}{2} = 1\,165\,920\,715 \times 10^{-12} \tag{5}$$

and the  $g$ -factor is

$$g = 2(1 + a_\mu) = 2.002\,331\,84. \tag{6}$$

Please see the latest results from the Fermilab Muon  $g-2$  experiment for more information [FNAL Muon  $g-2$  2025].

### 2.3 Magic momentum

At the magic momentum, the Lorentz factor is

$$\gamma_{\text{magic}} = \sqrt{1 + \frac{1}{a_\mu}} = 29.3, \tag{7}$$

and the corresponding speed factor is

$$\beta_{\text{magic}} = \sqrt{1 - \frac{1}{\gamma_{\text{magic}}^2}} = 0.9994. \tag{8}$$

### 3 $g - 2$ spin precession

#### 3.1 Cyclotron angular frequency (rest frame)

The cyclotron frequency in the muon rest frame is

$$\vec{\omega}_c^* = \frac{eB}{m_\mu} \quad (9)$$

for  $B = 1.45$  T, giving

$$\omega_c^* = 1.233 \times 10^9 \text{ rad s}^{-1} = 196.3 \text{ MHz}. \quad (10)$$

In the laboratory frame, this is reduced by the Lorentz factor

$$\omega_c = \frac{\omega_c^*}{\gamma_{\text{magic}}} = 4.21 \times 10^7 \text{ rad s}^{-1} = 6.70 \text{ MHz}. \quad (11)$$

#### 3.2 Anomalous precession angular frequency

The anomalous precession frequency is defined as the difference between the spin precession frequency and the cyclotron frequency, given by

$$\vec{\omega}_a = a_\mu \frac{eB}{m_\mu} = a_\mu \vec{\omega}_c, \quad (12)$$

which is frame independent at the magic momentum. This gives an anomalous precession frequency of

$$\omega_a = 1.438 \times 10^6 \text{ rad s}^{-1} = 0.229 \text{ MHz} \quad (13)$$

and a corresponding  $T_{g-2}$  period of

$$T_{g-2} = \frac{2\pi}{\omega_a} = 4.37 \text{ } \mu\text{s}. \quad (14)$$

## 4 EDM physics

#### 4.1 EDM precession angular frequency

The precession frequency induced by a non-zero EDM is

$$\vec{\omega}_\eta = \eta \frac{e}{2m_\mu} (\vec{\beta} \times \vec{B} + \frac{\vec{E}}{c}) \quad (15)$$

where  $\vec{\beta} \times \vec{B}$  is related to the motional electric field induced by the Lorentz transformation into the muon rest frame,  $E_{\text{motion}}$ , where

$$\frac{E_{\text{motion}}}{c} = \vec{\beta} \times \vec{B}. \quad (16)$$

This makes motional electric field in the muon rest frame

$$E_{\text{motional}} = \beta_{\text{magic}} B c = 4.34 \times 10^8 \text{ V m}^{-1}. \quad (17)$$

The total electric field also includes the contribution from the electrostatic quadrupoles (ESQs),  $E_{\text{ESQ}}$ , so that the total field is

$$E_{\text{total}} = E_{\text{motion}} + E_{\text{ESQ}}, \quad (18)$$

where the field from vertical ESQ plates running at 20 kV, separated by 10 cm<sup>1</sup> is

$$E_{\text{ESQ}} = V/d = 20 \times 10^3 / 10^{-2} = 2 \times 10^6 \text{ V m}^{-1} \quad (19)$$

which is a factor of  $\approx 200$  less than the motional field, and is neglected in this simulation. The EDM precession frequency is then

$$\vec{\omega}_\eta = \eta \frac{e}{2m_\mu} \beta_{\text{magic}} B c \quad (20)$$

## 4.2 EDM tilt angle

The muon EDM is given by

$$\vec{d}_\mu = \eta \frac{e}{2mc} \vec{s} \quad (21)$$

where  $\eta$  is the dimensionless coupling parameter, given by

$$\eta = \frac{2m_\mu c d_\mu}{e\hbar/2} = \frac{4m_\mu d_\mu c}{e\hbar} \quad (22)$$

for spin-1/2 particles.

The EDM causes the spin precession axis to tilt out of the horizontal plane by an angle  $\delta^*$ , in the rest frame, which is given by the ratio of the EDM and  $g-2$  precession frequencies

$$\tan \delta^* = \frac{\omega_\eta}{\omega_a} = \frac{\eta \beta_{\text{magic}}}{2a_\mu}, \quad (23)$$

The tilt angle in the lab frame is reduced by the Lorentz factor, so that

$$\tan \delta = \frac{\tan \delta^*}{\gamma_{\text{magic}}} \quad (24)$$

For small angles,  $\tan \delta \approx \delta$ . To extract the EDM from a measured tilt angle, the above formulas may be rearranged to give

$$d_\mu = \frac{e\hbar a_\mu}{2m_\mu c \beta_{\text{magic}}} \tan \delta^* \quad (25)$$

For example, for  $d_\mu = 5.4 \times 10^{-18} \text{ e} \cdot \text{cm}$ , the expected rest frame tilt is  $\delta^* = 49.6 \text{ mrad}$ , and the lab frame tilt is  $\delta = 1.69 \text{ mrad}$ .

## 5 Spin evolution

### 5.1 Rest frame spin components

The spin vector evolution in the muon rest frame:

$$S_x(t) = \cos(\omega_a t) \quad (26)$$

$$S_z(t) = \sin(\omega_a t) \quad (27)$$

$$S_y(t) = \delta \sin(\omega_a t) \quad (\text{EDM contribution}) \quad (28)$$

---

<sup>1</sup>An estimate!.

## 5.2 The vertical angle

The angle is of the polarisation vector with respect to the horizontal plane is given by

$$\theta_y = \sin^{-1} \left( \frac{s_y}{\sqrt{S_x^2 + S_y^2 + S_z^2}} \right) \quad (29)$$

For small EDM effects,  $\theta_y \approx S_y \approx \delta \sin(\omega_a t)$ .

## 6 Frame transformations

### 6.1 laboratory frame conversion

To convert rest frame quantities to lab frame:

$$\delta = \frac{\delta^*}{\gamma_{\text{magic}}} \quad (30)$$

$$\omega_c = \frac{\omega_c^*}{\gamma_{\text{magic}}} \quad (31)$$

$$\omega_a = \omega_a^* \quad (\text{at magic momentum}) \quad (32)$$