Toy EDM simulation: key physics

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September 18, 2025

1 Introduction

This document outlines the key equations used in `toy-edm`: a simple muon electric dipole moment (EDM) simulation for the Fermilab Muon g-2 experiment. All calculations are done in the muon rest frame, where rest frame quantities are indicated by an asterisk.

2 Fundamental parameters

2.1 Physical constants

$$c = 299\,792\,458\,\mathrm{m\,s^{-1}} \tag{1}$$
$$e = 1.602\,176\,634 \times 10^{-19}\,\mathrm{C} \tag{2}$$
$$\hbar = 1.054\,571\,817\,6 \times 10^{-34}\,\mathrm{J\,s} \tag{3}$$
$$m_\mu = 105.658\,374\,5\,\mathrm{MeV/c^2} = 1.883\,53 \times 10^{-28}\,\mathrm{kg} \tag{4}$$

2.2 Muon anomalous magnetic moment

The anomalous magnetic moment is given by

$$a_\mu = \frac{g-2}{2} = 1\,165\,920\,715 \times 10^{-12} \tag{5}$$

and the g-factor is

$$g = 2(1 + a_\mu) = 2.002\,331\,84. \tag{6}$$

Please see the latest results from the Fermilab Muon $g-2$ experiment for more information [FNAL Muon $g-2$ 2025].

2.3 Magic momentum

At the magic momentum, the Lorentz factor is

$$\gamma_{\mathrm{magic}} = \sqrt{1 + \frac{1}{a_\mu}} = 29.3, \tag{7}$$

and the corresponding speed factor is

$$\beta_{\mathrm{magic}} = \sqrt{1 - \frac{1}{\gamma^2_{\mathrm{magic}}}} = 0.9994. \tag{8}$$

3 $g-2$ spin precession

3.1 Cyclotron angular frequency (rest frame)

The cyclotron frequency is

$$\vec{\omega_c} = \frac{eB}{m_\mu \gamma_{\text{magic}}} \tag{9}$$

for $B = 1.45\,\text{T}$, giving

$$\omega_c = 4.21 \times 10^7\,\text{rad}\,\text{s}^{-1} = 6.70\,\text{MHz}. \tag{10}$$

3.2 Anomalous precession angular frequency

The anomalous precession frequency is defined as the difference between the spin precession frequency and the cyclotron frequency, given by

$$\vec{\omega_a} = a_\mu \frac{eB}{m_\mu} = a_\mu \vec{\omega_c}, \tag{11}$$

which is frame independent at the magic momentum. This gives an anomalous precession frequency of

$$\omega_a = 1.438 \times 10^6\,\text{rad}\,\text{s}^{-1} = 0.229\,\text{MHz} \tag{12}$$

and a corresponding T_{g-2} period of

$$T_{g-2} = \frac{2\pi}{\omega_a} = 4.37\,\mu\text{s}. \tag{13}$$

4 EDM physics

4.1 EDM precession angular frequency

The precession frequency induced by a non-zero EDM is

$$\vec{\omega_\eta} = \eta \frac{e}{2m_\mu}(\vec{\beta} \times \vec{B} + \frac{\vec{E}}{c}) \tag{14}$$

where $\beta \times B$ is related to the motional electric field induced by the Lorentz transformation into the muon rest frame, E_{motion}, where

$$\frac{E_{\text{motion}}}{c} = \vec{\beta} \times \vec{B}. \tag{15}$$

This makes motional electric field in the muon rest frame

$$E_{\text{motional}} = \beta_{\text{magic}} Bc = 4.34 \times 10^8\,\text{V}\,\text{m}^{-1}. \tag{16}$$

The total electric field also includes the contribution from the electrostatic quadrupoles (ESQs), E_{ESQ}, so that the total field is

$$E_{\text{total}} = E_{\text{motion}} + E_{\text{ESQ}}, \tag{17}$$

where the field from vertical ESQ plates running at 20 kV, separated by 10 cm[1] is

$$E_{\text{ESQ}} = V/d = 20 \times 10^3/10^{-2} = 2 \times 10^6\,\text{V}\,\text{m}^{-1} \tag{18}$$

which is a factor of ≈ 200 less that the motional field, and is neglected in this simulation. The EDM precession frequency is then

$$\vec{\omega_\eta} = \eta \frac{e}{2m_\mu} \beta_{\text{magic}} Bc \tag{19}$$

[1] An estimate!.

4.2 EDM tilt angle

The muon EDM is given by

$$\vec{d_\mu} = \eta \frac{e}{2mc} \vec{s} \tag{20}$$

where η is the dimensionless coupling parameter, given by

$$\eta = \frac{2m_\mu c d_\mu}{e\hbar/2} = \frac{4m_\mu d_\mu c}{e\hbar} \tag{21}$$

for spin-1/2 particles.

The EDM causes the spin precession axis to tilt out of the horizontal plane by an angle δ^*, in the rest frame, which is given by the ratio of the EDM and $g-2$ precession frequencies

$$\tan\delta^* = \frac{\omega_\eta}{\omega_a} = \frac{\eta\beta_{\text{magic}}}{2a_\mu}, \tag{22}$$

The tilt angle in the lab frame is reduced by the Lorentz factor, so that

$$\tan\delta = \frac{\tan\delta^*}{\gamma_{\text{magic}}} \tag{23}$$

For small angles, $\tan\delta \approx \delta$. To extract the EDM from a measured tilt angle, the aboves formulas may be rearranged to give

$$\boxed{d_\mu = \frac{e\hbar a_\mu}{2m_\mu c\beta_{\text{magic}}} \tan\delta^*} \tag{24}$$

For example, for $d_\mu = 5.4 \times 10^{-18}\,\text{e}\cdot\text{cm}$, the expected rest frame tilt is $\delta^* = 49.6\,\text{mrad}$, and the lab frame tilt is $\delta = 1.69\,\text{mrad}$.

5 Spin evolution

5.1 Rest frame spin components

The spin vector evolution in the muon rest frame:

$$S_x(t) = \cos(\omega_a t) \tag{25}$$
$$S_z(t) = \sin(\omega_a t) \tag{26}$$
$$S_y(t) = \delta\sin(\omega_a t) \quad \text{(EDM contribution)} \tag{27}$$

5.2 The vertical angle

The angle is of the polarisation vector with respect to the horizontal plane is given by

$$\theta_y = \sin^{-1}\left(\frac{s_y}{\sqrt{S_x^2 + S_y^2 + S_z^2}}\right) \tag{28}$$

For small EDM effects, $\theta_y \approx S_y \approx \delta\sin(\omega_a t)$.

6 Frame transformations

6.1 laboratory frame conversion

To convert rest frame quantities to lab frame:

$$\delta = \frac{\delta^*}{\gamma_{\text{magic}}} \tag{29}$$

$$\omega_c = \frac{\omega_c^*}{\gamma_{\text{magic}}} \tag{30}$$

$$\omega_a = \omega_a^* \quad \text{(at magic momentum)} \tag{31}$$