Week 1 Webinar Notes and Discussed Problems

64-bit Binary Number System

$(10)_{10}$

$$= \underbrace{00000 \ldots 000}_{60 \text{ 0's}}\underline{1010}$$

Hexadecimal

Octal

Decimal (Base = 10)	Binary (Base = 2)	Octal (Base = 8)	Hexadecimal (Base = 16)
0	0	0	0 → 0000
1	1	1	1 → 0001
2		2	2 → 0010
3		3	3 → 0011
4		4	4 → 0100
5		5	5 → 0101
6		6	6 → 0110
7		7	7 → 0111
8			8 → 1000
9			9 → 1001
			10 → A → 1010
			11 → B → 1011
			12 → C → 1100
			13 → D → 1101
			14 → E → 1110
			15 → F → 1111

$$(129)_{10} = 10^0 * 9 + 10^1 * 2 + 10^2 * 1$$

$$= 1 * 9 + 10 * 2 + 100 * 1$$

$$= 9 + 20 + 100$$

$$= 129$$

$$(41)_{10} = (101001)_2 = (51)_8 = (29)_{16}$$

$$= (56)_7$$

2	41	
2	20	1
2	10	0
2	5	0
2	2	1
2	1	0
	0	1

8	41	
8	5	1
	0	5

16	41	
16	2	9
	0	2

7	41	
7	5	6
	0	5

$$(000\underbrace{1}\,\underbrace{0011})_2 = (13)_{16}$$

8-bit Unsigned Binary Number System:

$(5)_{10} = (00000101)_2$

$[0, 2^8 - 1]$

$= [0, \underbrace{255]}_{2^8 = 256 \text{ Numbers}}$

unsigned int x;

$[0, 2^{32} - 1]$

=

2-bit BNS

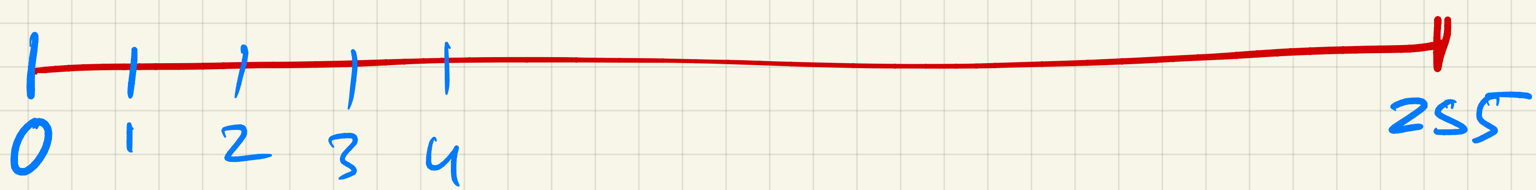
00	$\rightarrow$	0
01	$\rightarrow$	1
10	$\rightarrow$	2
11	$\rightarrow$	3

4 Numbe $= 2^2$

$[0, 3]$

$= [0, 2^2 - 1] = [0, 4-1]$

$= [0, 3]$



8-bit Unsigned BNS

$(255)_{10} + (7)_{10} =$

$$\begin{array}{rl} & 1\,1\,1\,1\,1\,1\,1 \\ (255)_{10} = & (1\,1\,1\,1\,1\,1\,1\,1)_2 \\ + & \\ (7)_{10} = & (0\,0\,0\,0\,0\,1\,1\,1)_2 \\ \hline \cancel{1} & (0\,0\,0\,0\,0\,1\,1\,0)_2 = (6)_{10} \end{array}$$

2

10

$3 = 11$

$2 \,|\, 3$

$1 \quad 1$

2

$x + y = 2$

$$
\begin{array}{r}
{\scriptstyle 1\ \ 1\ \ \ } \\
125 \\
689 \\
\hline
814
\end{array}
$$

$5+9=14$

20 | 14

1 4

10 | 11

1 1

8-bit Signed Binary Number System:

1 □ □ □ □ □ □ □

MSB = Most Significant Bit

LSB = Least Significant Bit

$$
\begin{array}{lcr}
 & & \overset{1}{0}1\overset{1}{0}1 \\
(5)_{10} & = & 0\;000\;0101 \\
(-5)_{10} & = & 1\;000\;0101 \\
\hline
 & & \underline{1}\;\;1\;000\;1010
\end{array}
= (-10)_{10}
$$

$$
\begin{array}{r}
5 \\
+\;-5 \\
\hline
0
\end{array}
$$

$(0)_{10} = 0000\;0000$

$(-0)_{10} = 1\;000\;0000$

8-bit 2's Complement Binary Number System:

Decimal to 8-bit 2's Complement BNS:

* $(17)_{10} = (00010001)_{\text{8-bit 2's complement BNS}}$

2	17	
2	8	1
2	4	0
2	2	0
2	1	0
	0	1

* $(-17)_{10} = (11101111)_{\text{8-bit 2's cmplm BNS}}$

step-1: $(17)_{10} = 00010001$

step-2: $\sim 17 = 11101110$

step-3: $\sim 17 + 1 =$

$$
\begin{array}{r}
11101110 \\
+\ 00000001 \\
\hline
11101111
\end{array}
$$

8-bit 2's Complement BNS to Decimal :

$$(\underset{7}{0}\underset{6}{0}\underset{5}{0}\underset{4}{1}\underset{3}{0}\underset{2}{0}\underset{1}{1}\underset{0}{0})_{\text{8-bit 2's Complement BNS}} = (18)_{10}$$

$$= 2^0 * 0 + 2^1 * 1 + 2^2 * 0 + 2^3 * 0 + 2^4 * 1 + 2^5 * 0 + 2^6 * 0 + 2^7 * 0$$

$$= 0 + 2 * 1 + 0 + 0 + 16 * 1 + 0 + 0 + 0$$

$$= 18$$

$$(\underset{7}{1}\underset{6}{0}\underset{5}{0}\underset{4}{1}\underset{3}{0}\underset{2}{0}\underset{1}{1}\underset{0}{0})_{\text{8-bit 2's Complement BNS}} = (-110)_{10}$$

$$= 2^0 * 0 + 2^1 * 1 + 2^2 * 0 + 2^3 * 0 + 2^4 * 1 + 2^5 * 0 + 2^6 * 0 + (-1) * 2^7$$

$$= 0 + 2 + 0 + 0 + 16 + 0 + 0 - 128$$

$$= 18 - 128 = -110$$

8-bit 2's Complement BNS

$[-2^7, 2^7-1] = [-2^{8-1}, 2^{8-1}-1]$

$= \underbrace{[-128, 127]}_{256 \text{ Numbers}}$

0000 0000

$2^7 = 128 - 1$
$= 2^7 - 1$

1000 0000

$= (-1) * 2^7 + 0$

$= -128$

int x;

4 bytes

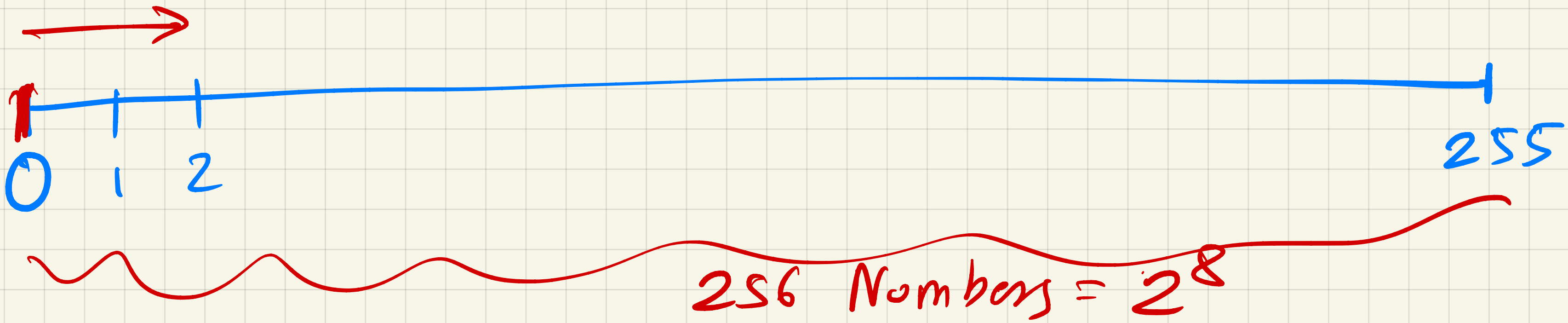
$= 4 * 8$

$= 32$ bits

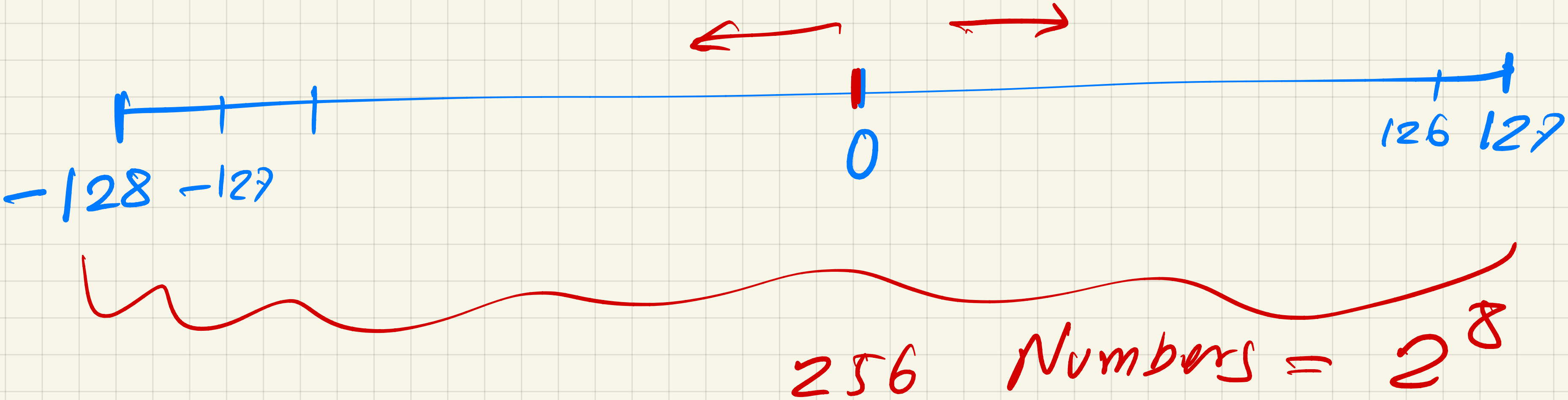
$[-2^{32-1}, 2^{32-1}-1]$

$= [-2^{31}, 2^{31}-1]$

8-bit Unsigned BNS



8-bit 2's Complement BNS:



Exercise 1: Express each statement in logic using the variables:

p: It is windy.
q: It is cold.
r: It is raining.

(a) It is windy and cold.

$p \wedge q$

(b) It is windy but not cold.

$p \wedge \neg q$

Exercise 2: Express each statement in logic using the variables:

c: I will return to college.
j: I will get a job.

$p \to q$

(a) If I return to college, then I won't get a job.

$c \to \neg j$

(b) I am not getting a job, but I am still not returning to college.

$\neg j \wedge \neg c$

(c) Not getting a job is a sufficient condition for me to return to college.

$\neg j \to c$

(d) I will return to college only if I won't get a job.

$c \to \neg j$

Exercise 3: Use the laws of propositional logic to prove the following:

$p \to q$
$= \neg p \vee q$

$\neg p \to (q \to r) \equiv q \to (p \vee r)$

$L.H.S = \neg p \to (q \to r)$

$\equiv \neg(\neg p) \vee (q \to r)$ [Conditional Identity]

$\equiv \neg\neg p \vee (\neg q \vee r)$ [Conditional Indentity]

$\equiv p \vee (\neg q \vee r)$ [Double negation]

$\equiv (p \vee \neg q) \vee r$ [Associative law]

$\equiv (\neg q \vee p) \vee r$ [Commutative law]

$\equiv \neg q \vee (p \vee r)$ [Associative law]

$2+3$
$= 3+2$

$\equiv q \to (p \vee r)$ [Conditional Indentity]

$\neg p \vee q$

$\equiv p \to q$

$$\overline{p \to q} \equiv \underline{\neg p \vee q}$$

Exercise 4: In the following question, the domain = (set of all integers). Determine if the statements are true or false.

(a) $\exists x\, (\underbrace{x + 2 = 1}_{P(x)})$

True, $x=-1$, $P(-1) = -1+2 = 1$

(b) $\exists x\, (x + x = 1)$

False

(c) $\forall x\, (\underbrace{x^2 - x \neq 0}_{P(x)})$

False, for $x=1$, $P(1) = 1^2 - 1 = 1 - 1 =$

(d) $\forall x\, (\underbrace{x^2 > 0}_{P(x)})$

False, for $x=0$, $P(0) = 0^2 = 0$

(e) $\exists x\, (\underbrace{x^2 > 0}_{P(x)})$

True, for $x=1$, $P(1) = 1^2 = 1 > 0$

Exercise 5: In the following question, the domain = (set of students at a university).
Define the following predicates:

E(x) = x is enrolled in the class
T(x) = x took the test

Translate the following English statements into a logical expression with the same meaning.

(a) Someone who is enrolled in the class took the test.

$\exists_x [E(x) \wedge T(x)]$

(b) All students enrolled in the class took the test.

$\forall_x [E(x) \rightarrow T(x)]$

(c) Everyone who took the test is enrolled in the class.

$\forall_x [T(x) \rightarrow E(x)]$

(d) At least one student who is enrolled in the class did not take the test.

$\exists_x [E(x) \wedge \neg T(x)]$

Exercise 6: In the following question, the domain = (set of real numbers). Determine whether each statement is true or false. Explain your answer.

(a) $\forall x \; \underbrace{\exists y \; (x + y = 0)}_{P(x)}$ → True

$x=1$: $\exists y(1+y=0) \Rightarrow$ True, $y=-1$

$x=-1$: $\exists y\,(-1+y=0) \Rightarrow$ True, $y=1$

$x=100$: $\exists y\,(100+y=0) \Rightarrow$ True, $y=-100$

(b) $\exists x \; \underbrace{\forall y \; (x + y = 0)}_{P(x)}$ → False

$x=1$; $\forall y\,(1+y=0) \Rightarrow$ False

$x=-10$: $\forall y\,(-10+y=0) \Rightarrow$ False

(c) $\exists x \; \underbrace{\forall y \; (x*y = y)}_{P(x)}$ → True

$x=1$: $\forall y\,(1*y=y) \equiv \forall y\,(y=y)$

Exercise 7: In the following question, the domain = (set of musicians in an orchestra).
The predicates S, B, and P are defined as:
S(x) = x plays a string instrument
B(x) = x plays a brass instrument
P(x, y) = x practices more than y

Give a quantified expression that is equivalent to the following English statements:

(a) There are no brass players in the orchestra.

(b) There is a brass player who practices more than all the string players.

(c) All the string players practice more than all the brass players.

(d) Exactly one person practices more than Sam.