Homework # 2

Question # 5

A.

1. Exercise 1.12.2, sections b, e

B.

$$\begin{array}{l} p \rightarrow (q \wedge r) \\ \neg q \\ \hline \therefore \neg p \end{array}$$

line	statement	justification
1	$p \rightarrow (q \wedge r)$	h1
~~2~~	~~$\neg p \vee (q \wedge r)$~~	~~conditional identity~~
2.	$\neg q$	h2
3.	$\neg q \vee \neg r$	Addition, line 2
4.	$\neg (q \wedge r)$	de Morgan's Law
5.	$\neg p$	Modus Tollens

E.

$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \neg q \\ \hline \therefore r \end{array}$$

line	statement	justification
1	$p \vee q$	H1
2.	$q \vee p$	Commutative law
3.	$\neg q$	H3
4.	p	~~disjust~~ disjunctive syllogism, lines 2,3
5.	$\neg p \vee r$	H2
6.	$\neg\neg p$	double negation law
7.	r	disjunctive syllogism, lines 5,6

$\neg p \equiv p$ $\quad \neg p \to q$ $\quad \therefore \neg p \to q$ $\quad \neg p \equiv q$ $\quad \neg q \vee q$ $\quad q \to q$

$\neg(\neg p) \vee q$ $\quad p \equiv -q$ $\quad -q \vee q$ $\quad -p$

2. Ex 1.12.3, section C

Ans

$$\begin{array}{l} p \vee q \\ -p \\ \hline \therefore q \end{array}$$

line	statement	justification
1	$p \vee q$	H1
2.	$\neg p \to q$	conditional identity
3.	$-p$	H2
4.	q	modus ponens, lines 2,3

3. Ex 1.12.5, sections c, d

~~c. c = I will buy a new car j = I will get a job~~
~~h = I will buy a new house~~

c. c = I buy a new car j = I get a job
h = I buy a new house

$$\begin{array}{l} (c \wedge h) \to j \\ \neg j \\ \hline \therefore \neg c \end{array}$$

line	statement	justification
1		

$\boxed{\text{Invalid!! } \to C = T \quad J = F \quad H = F}$

$$\begin{array}{l} (T \wedge F) \to F \\ T \\ \hline \therefore F \end{array} \qquad \begin{array}{l} T \\ T \\ \hline F \end{array}$$

Question # 6 Ex 2.4.1, section d; Ex 2.4.3, section b

2.4.1 D. The product of 2 odd integers is an odd integer

If 2 integers are odd, then their product is also odd

Proof: Let x and y be odd integers. We shall prove that xy is odd

a. since x and y are odd, $x = 2k+1$ and $y = 2j+1$ for some integers k, j.

b. $x \cdot y = (2k+1) \cdot (2j+1)$
$= 4kj + 2k + 2j + 1$
$= 2(2kj + k + j) + 1$

∴ since ~~k is a~~ j and k are integers, $(2kj + k + j)$ will be an integer ~~m~~ m.

∴ since $x \cdot y = 2m+1$, where $m = 2kj + k + j$, $x \cdot y$ is odd

2.4.3 B. If x is a real number and $x \leq 3$, then $12 - 7x + x^2 \geq 0$

Proof:

a. assume x is a real number and $x \leq 3$. We shall prove that $x^2 - 7x + 12 \geq 0$.

b. $x^2 - 7x + 12 \geq 0$ can be factored into $(x-3)(x-4) \geq 0$

c. since $x \leq 3$ we know that
$x - 3 \leq 0$ and $x - 4 \leq 0$

d. Since both x and y will be negative or zero, we know that $(x-3)(x-4) \geq 0$ since two negatives are always positive. ■

~~D. C = I will buy a new house~~

D. C = I buy a new car j = I get a job
h = I buy a new house

$$\begin{array}{l} (C \wedge h) \rightarrow j \\ \neg j \\ h \\ \hline \therefore \neg C \end{array}$$

line	statement	justification
1.	$(C \wedge h) \rightarrow j$	H1
2.	$\neg j$	H2
3.	$\neg(C \wedge h)$	~~pod~~ modus tollens (line 1&2)
4.	$\neg C \vee \neg h$	de Morgan's law
5.	$(\neg h) \vee \neg C$	commutative law
6.	h	H3
7.	$\neg(\neg h)$	double negation law
8.	$\neg C$	disjunctive syllogism, lines 5, 7

5 B.

1. Ex ~~1.B.3~~ 1.13.3, section b

~~$\exists x (P(x) \vee Q$~~

$$\begin{array}{l} \exists x (P(x) \vee Q(x)) \\ \exists x \neg Q(x) \\ \hline \therefore \exists x P(x) \end{array}$$

$P(a) \vee Q(a)$ T
$Q(b)$ T
F

this table show arguement is invalid

	P	Q
a	F	T
b	F	F

~~|	P	Q
a	T	F
b	F	T |~~

2. Ex 1.13.5, sections d, e

D.

$M(x) =$ x missed class

$D(x) =$ x got a detention

$\forall x\,(M(x) \rightarrow D(x))$

Penelope is a student

$\neg M(\text{Penelope})$

$\therefore \neg D(\text{Penelope})$

	M	D
a	F	T
b	F	T

line	statement	justification
1	$\forall x\,(M(x) \rightarrow D(x))$	H1
2.	Penelope is a student	H2
3.	$M(\text{Penelope}) \rightarrow D(\text{Penelope})$	Universal instantiation, 1, 2
4.	$\neg M(\text{Penelope}) \vee D(\text{Penelope})$	conditional identity
5.	$\neg M(\text{Penelope})$	H3

~~Invalid: when M(a) = T and D(~~

Invalid: when $M(a) = F$ and $D(a) = T$ all hypotheses are true, but the conclusion is false

$\neg(M \vee D) \vee \neg A$

$(\neg M \wedge \neg D) \vee \neg A$

E. $M(x) =$ x missed class $D(x) =$ x got a detention

$A(x) =$ x got an A

$\forall x ((M(x) \vee D(x)) \rightarrow \neg A(x))$

Penelope is a student in the class

~~Penelope got an A~~ $A(\text{Penelope})$

$\therefore$ ~~Penelope~~ $\neg D(\text{Penelope})$

M	D	A
F	T	F

line	statement	justification
1	$\forall x (M(x) \vee D(x) \rightarrow \neg A(x))$	H1
2.	Penelope is student	H2
3.	$M(P) \vee D(P) \rightarrow \neg A(P)$	universal generalization, lines 1, 2
4.	$\neg(M(P) \vee D(P)) \vee \neg A(P)$	conditional identity
~~3~~	~~$\neg(M(P) \wedge \neg D(P))$~~	
5.	$(\neg M(P) \wedge \neg D(P)) \vee \neg A(P)$	de Morgan's law

line	statement	justification
1	$\forall x ((M(x) \vee D(x)) \rightarrow \neg A(x))$	H1
2.	P is a student in class	H2
3.	$(M(P) \vee D(P)) \rightarrow \neg A(P)$	universal gen., lines 1,2
4.	$\neg(M(P) \vee D(P)) \vee \neg A(P)$	conditional identity
5.	$(\neg A(P)) \vee \neg(M(P) \vee D(P))$	commutative law
6.	$A(P)$	H3
7.	$\neg(\neg A(P))$	Double negation ~~identity~~ law
8.	$\neg(M(P) \vee D(P))$	Disjunctive syllogism, lines 5, 7
9	$\neg M(P) \wedge \neg D(P)$	de Morgan's law
10	$\neg D(P) \wedge \neg M(P)$	commutative law
11	$\neg D(P)$	Addition, line 10

$p \to c \equiv \neg c \to \neg p$

Question #7 Ex 2.5.1, section d; Ex 2.5.4, sections a, b; Ex 2.5.5, section c

2.5.1 D For every integer n, if n^2-2n+7 is even, then n is odd

$D(n) = n$ is even

$$\forall n\,(D(n^2-2n+7) \to \neg D(n)) \equiv \forall n\,(D(n) \to \neg(D(n^2-2n+7)))$$

If n is even then n^2-2n+7 is odd

Proof:

a. since n is ~~odd~~ even, $n = 2k$ for some integer k

b. plugging in, ~~$(2k+1)^2 - 2(2k+1)+7$~~

$$= (2k)^2 - 2(2k) + 7$$
$$= 4k^2 - 4k + 7$$
$$= 2(2k^2 - 2k + 3) + 1$$

c. since k is an integer, $(2k^2-2k+3)$ is also an integer, m

d. $\therefore$ ~~since 2m+1~~ n^2-2n+7 is in the form of $2m+1$ which is an odd integer

2.5.4 A For every pair of real numbers x and y, if $x^3 + xy^2 \leq x^2y + y^3$, then $x \leq y$

Assume $x > y$ where x and y are real numbers. We will prove that $x^3 + xy^2 > y^3 + x^2y$

Proof:

$$x^3 + xy^2 - x^2y - y^3 > 0$$
$$= x^2(x-y) + y^2(x-y) > 0$$
$$= (x-y)(x^2+y^2)$$

since $x > y$, $(x-y) > 0$ and $x^2+y^2 > 0$ for all real x, y and therefore $x^3 + xy^2 > y^3 + x^2y$ if $x > y$.

2.5.4 B For every pair of real numbers x and y, if $x+y > 20$, then $x > 10$ or $y > 10$.

Assume $x \leq 10$ AND $y \leq 10$, where x and y are real numbers. We will prove that $x+y \leq 20$.

a. $x+y-20 \leq 0$

b. since $x \leq 10$ and $y \leq 10$, $x+y-20$ can be no greater than 0.

$\therefore$ $x+y \leq 20$ if $x \leq 10$ and $y \leq 10$ ~~for~~ where x and y are real numbers

2.5.5 C For every nonzero real number x, if x is irrational, then $\frac{1}{x}$ is also irrational.

Contrapositive proof

Assume that $\frac{1}{x}$ is rational for all nonzero real numbers. We will prove that x is rational.

$\frac{1}{x} = \frac{a}{b}$ where a and b are integers

cross multiplying gives $b = ax$ or $x = \frac{b}{a}$

since a and b are integers and $a \neq 0$, $\frac{b}{a}$ is rational.

$\therefore$ x is rational

Question # 8 Ex 2.6.6 sections c, d

2.6.6 C. The average of three real numbers is ~~greater than~~ $\geq$ at least one of the numbers.

Proof by contradiction: The average of three real numbers is $<$ ~~all of the numbers.~~ each of the numbers.

$\therefore$

$$\frac{a+b+c}{3} < a \wedge \frac{a+b+c}{3} < b \wedge \frac{a+b+c}{3} < c$$

$$= \cancel{3}\left(\frac{a+b+c}{\cancel{3}}\right) < a+b+c$$

$$= a+b+c < a+b+c$$

This cannot be possible since sum of 3 numbers can't be less than the sum of those 3 numbers.

$\therefore$ The average of 3 ~~act~~ real numbers must be $\geq$ at least one of the numbers.

D. There is no smallest integer.

Proof by contradiction: Assume there is a smallest integer r.

~~Let's subtract 1 from r to get r-1.~~

~~r-1<r, but subtracting 1~~

Let's subtract 1 from r to get r-1. Since r-1<r ~~we~~ we know r-1 is smaller than r.

This contradicts the assumption that r is the ~~smallest~~ smallest integer.

$\therefore$ There is no smallest integer.

Question #9

Ex 2.7.2, section b

B. Case 1: x and y are ~~even~~ even.

(i) since x and y are even there ~~[illegible]~~ are integers j and k such that $x = 2k$ and $y = 2j$.

(ii) ~~pub~~ plugging in for $x+y$ gives $2k + 2j$
$= 2(k+j)$

(iii) since k and j are integers, $(k+j)$ ~~are also integers~~.
is also an integer. We will call $m = (k+j)$
$\therefore$ $x+y = 2m$ ~~which~~ and there for $x+y$ is even

case 2: x and y are odd

(i) since x and y are odd, there exists integers k and j such that $x = 2k+1$ and $y = 2j+1$

(ii) Plugging in for $x+y$ gives $(2k+1) + (2j+1)$

$= 2k + 2j + 2$

$= 2(k+j+1)$

(iii) since k and j are integers, $(k+j+1)$ is also an integer.
We will call $m = (k+j+1)$
$\therefore$ $x+y = 2m$. Therefore $x+y$ is also even.