

# JOHN SPINA HW7 P1

We can use an unsorted array. Inserting 2 elements will take  $O(1)$  time since we can just add them onto the end. To perform half delete we can find the median and partition the array in half in  $O(n)$  time, or  $dn$  on average. So we always need at least  $dn$  in the bank so that we can perform a half-delete.

$n$	2	4	6	8	10
$C_i$	2	2	2	2	2
$\hat{C}_i$	$2+4d$	$2+4d$	$2+4d$	$2+4d$	$2+4d$
bank	$4d$	$8d$	$12d$	$16d$	$20d$

We are inserting two elements at a time in  $O(1)$  time so that costs us 2, and we always need  $2dn$  in the bank for a half-delete, so  $\hat{C}_i = 2+4d$ . When running half-delete

we take  $dn$  time for  $\frac{n}{2}$  elements which is  $2d$  per element.

$$\phi = \text{size}, C_i = k|A_{i-1}|, \phi(A_i) = 2k|A_i|, \phi(A_{i-1}) = 2k|A_{i-1}|$$

$$|A_i| = \frac{1}{2}|A_{i-1}|$$

$$\hat{C}_i = C_i + \phi(A_i) - \phi(A_{i-1}) = k|A_{i-1}| + 2k|A_i| - 2k|A_{i-1}| = k|A_{i-1}| + k|A_{i-1}| - 2k|A_{i-1}|$$

$$= 0. \text{ Amortized cost is } 0 \text{ for } m \text{ operations, so they run in } O(m) \text{ time and } O(1) \text{ (constant) amortized time.}$$