CISC102 - Discrete Math I

July 2018

Assignment 3 - Problem 3

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Prove that

$\forall n \geq 1$,

$\frac{1}{1*2} + \frac{1}{2*3} + \frac{1}{3*4} + ... + \frac{1}{n(n+1)} = \frac{n}{n+1}$

Proof by induction

Base Case: n = 1

$\rightarrow \frac{1}{1(1+1)} = \frac{1}{1+1}$

$\rightarrow \frac{1}{2} = \frac{1}{2}$

$\therefore LHS = RHS \implies$ the statement is true for the Base Case

Inductive Hypothesis: Assume the statement is true for some integer $k \geq 1$

That is, $\frac{1}{1*2} + \frac{1}{2*3} + \frac{1}{3*4} + ... + \frac{1}{k(k+1)} = \frac{k}{k+1}$

Now consider the case where n = k+1 (the next integer)

$\frac{1}{1*2} + \frac{1}{2*3} + \frac{1}{3*4} + ... + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+1+1)} = \frac{k+1}{k+1+1}$

$\rightarrow (\frac{1}{1*2} + \frac{1}{2*3} + \frac{1}{3*4} + ... + \frac{1}{k(k+1)}) + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$

Here we can substitute the inductive hypothesis for the bracketed section

$\rightarrow \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$

Multiply both sides by (k+1)(k+2)

$\rightarrow k(k+2) + 1 = (k+1)(k+1)$

Expand

$\rightarrow k^2 + 2k + 1 = k^2 + k + k + 1^2$

Simplify

$\rightarrow k^2 + 2k + 1 = k^2 + 2k + 1$

$\therefore LHS = RHS \implies$ the Inductive Hypothesis is true

Putting the Base Case and the Inductive Hypothesis together, we conclude that the statement $\frac{1}{1*2} + \frac{1}{2*3} + \frac{1}{3*4} + ... + \frac{1}{n(n+1)} = \frac{n}{n+1}$ is true, $\forall n \geq 1$ ■