“The Big Picture”: An Aggregate and Dynamic Economic Model

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Economic modeling entails the reliance on measurable parameters that can be used in combinations to explain a behavior at scale, thus reducing the random nature of markets and providing a deterministic outlook that acts as a basis for informed decision making by participants. The challenge that arises is to connect sector-level information to the broader macro picture due to the lack of common parameters between the 2 sets of descriptive approaches. On the one hand, an economy’s performance is driven by its sector-level output which in turn is informed by policies that prioritize the state of demand. For instance, monetary policy controls for unemployment and inflation that both start off at the sector-level before impacting the economy as a whole. On the other hand, the narrative surrounding the supply side of the economy revolves around the motivation for highly capitalized corporations to increase their profit margins and manage their complex supply chains. These 2 axes also play out in fiscal policy, as well as in the communication of institutional investors on the “ratings” of corporate financial instruments.

The question that arises is whether we are able to build a comprehensive model that ties sector-level performance with its impact on the steady state value of macro variables. The intent is for it to be a framework used to make investment decisions by providing a reference frame for capital, productivity and consumption to identify any drift. In this paper, we implement the model at the broad level of the U.S. economy. However, it can also be applied in more detail to each sector and each interest rate environment. The focus here, nonetheless, is not on the investment approach but rather the building blocks of the underlying quantitative models.

I. Aggregate Analysis

The literature is rich with theoretical models that explain the decision-making of firms in various competitive environments but very few have attempted to revitalize the tradition of economic planning via Input-Output analysis to explain variations between predicted and actual factor shares of production at various levels of detail in an economy. Fisher, 2014 presents an overview of how singular value decomposition can be used to do just that by constructing sector-specific technology matrices that allow us to measure both direct and indirect factor use throughout the production chain. As a result, we are able to identify the sectors that are most reliant on capital (or labor) starting only with annual input-output (IO) tables produced by the Bureau of Economic Analysis from 1997 onward.

We begin by organizing the IO tables into two key groups: n=71 economic sectors and f=5 primary factors of production. The factors include capital, labor, indirect taxes, non-comparable imports, and scraps. This separation ensures that we can isolate how much each sector relies on different input types.

From the sectoral data, we construct an augmented input coefficient matrix A of dimensions n×(n+f). This matrix includes normalized inputs used by each sector from both other sectors and the primary factors of production. Each entry in A represents the input requirement per unit of output.

To capture the total (direct and indirect) input requirements, we compute the Leontief inverse $(I - A)^{-1}$, where I is the identity matrix of size n×n. The inverse captures the propagation of input demands through the production network and reflects how a unit increase in final demand affects overall input use.

Separately, we form the matrix B of size f×(n+f) which maps the input requirements for each of the f primary factors of production across all sectors and inputs. This matrix is used to track the total demand for each factor per unit of gross output.

We then derive the Leontief matrix L by multiplying the factor matrix B with the Leontief inverse:

$$L = B * (I - A)^{-1}$$

The matrix L has dimensions f × n and represents the total factor requirements—both direct and indirect—for producing one unit of final output in each sector.

The technology matrix T is obtained by transposing L:

$$T = L'$$

This results in an n × f matrix, where each row corresponds to a sector or input, and each column corresponds to a factor of production. The matrix captures the total (direct and indirect) factor usage embedded in each unit of final demand across the economy.

To extract dominant patterns in factor usage and reduce dimensionality, we apply Singular Value Decomposition to the technology matrix T. Specifically, we decompose T as:

$$T=U\Sigma V'$$

where U and V are orthogonal matrices containing the left and right singular vectors, respectively, and Σ is a diagonal matrix of singular values. The columns of U represent principal modes of variation across sectors, while the rows of V' highlight how each factor contributes to those modes.

This decomposition enables us to isolate the dominant latent structures that explain most of the variance in factor usage. Importantly, the singular vectors associated with the largest singular values can be interpreted as the most influential capital-intensive patterns shared across sectors. SVD also helps to mitigate noise in the data and allows us to focus our analysis on a lower-dimensional subspace that captures the essential differences in capital reliance.

Appendix I contains a reduced form technology matrix, narrowed down to capital-intensive industries only (defined as those with a capital share of output greater than 0.5). This information allows us to compare the evolution of value added over the years (1997 onwards) across these industries. Value-added is defined as the contribution of each industry to overall GDP, in other words the difference between an industry's gross output and the cost of its intermediate inputs.

Appendix II shows that most capital-intensive industries contribute less than 2% each in value added to the U.S. economy, with broadcasting/telecom, and federal reserve banks/credit intermediation contributing slightly more than that. The only sizeable contribution among capital-intensive industries remains real estate with almost 13% of value added. Furthermore, it appears that the magnitude of value added has shrunk amongst most capital-intensive industries with the exception of publishing industries, data processing/internet publishing federal reserve banks/credit intermediation and nonresidential real estate. It is important to note the directional shift in weighting the U.S. GDP's reliance on capital-intensive industries.

II. Dynamic Analysis

To extend the scope of our study, we need fundamental parameters that capture the macro picture in a dynamic framework. To accomplish this task, we turn to Judd & Maliar, 2011 and their generalized stochastic simulation algorithm. This method combines projection, perturbation and stochastic simulation in an accurate (handling ill-conditioning via approximation), numerically stable (handling multicollinearity via regularization) and simple to program algorithm. The main benefit is that the solution is computed only in the ergodic set (equilibrium) and not on domains that are exogenous to the model. The model itself, prior to the estimation step, is expressed as an intertemporal utility maximization problem in the following form:

$$max_{\{k_{t+1},c_t\}_{t=0,\ldots,\infty}} E_0 \Sigma_{t=0}^{\infty} \beta^t u(c_t)$$

subject to $$k_{t+1} + c_t = (1-\delta)\, k_t + a_t f(k_t)$$

$$ln a_{t+1} = \rho ln a_t + \epsilon_{t+1}$$

It is worth noting that $f(k_t)$ = k_t^{α} (Cobb-Douglas form). The algorithm solves the polynomial coefficients by taking initial guesses of capital k_0 and productivity level a_0 and solving for a capital policy function $k_{t+1} = K(k_t, a_t)$ and iterating until convergence. The outputs are values for capital, productivity level and consumption based on the initial conditions and the time interval chosen.

The algorithm implementing the model requires the following input parameters:

- **γ**: utility function parameter
- ***α***: capital share in output
- **B:** discount factor
- ***δ*:** depreciation rate
- **ρ:** persistence of the log of the productivity level
- **Σ:** standard deviation of shocks to the log of the productivity level

The value for each can be determined from real-world data. The utility function parameter is set equal to moderate risk aversion and unit intertemporal elasticity of substitution (CRRA). The capital share in output is obtained using the previous axis of analysis where the sectors of interest can be narrowed down to those within a specific range for capital or labor share of production. The discount factor is obtained using the inflation-adjusted interest rates on 10-year Treasury securities. The values for persistence and standard deviation of shocks to the productivity level (log) are taken from the literature.

The initialization step continues by defining a simulation length T which should preferably be greater than 100 for a smooth output, and the steady state of capital is derived by taking the first-order condition for the representative agent problem (Euler equation).

$$u'(\mathrm{c_t}) = \beta\ \mathrm{E}[u'(\mathrm{c_{t+1}}) * (\alpha * \mathrm{k_{t+1}}^{\alpha-1} + 1 - \delta]$$

$$1 = \beta * [\alpha * \mathrm{k_{t+1}}^{\alpha-1} + 1 - \delta]$$

$$\mathrm{k_{t+1}} = \mathrm{k_t} = \mathrm{k}^*$$

$k^* = (\frac{\alpha*\beta}{1-\beta*(1-\delta)})^{\frac{1}{1-\alpha}}$

The initial condition is used in the context of a first-degree (linear) polynomial as the initial guess, often with manually chosen coefficients (0.95 and 0.05 × steady state capital). This policy function is then used to simulate forward a path of capital and consumption. The right-hand side of the Euler equation is computed at each point using either Monte Carlo or Gauss-Hermite quadrature to approximate conditional expectations.

At each iteration, a time series is simulated using the current guess for the capital policy function (here $y_t = k_{t+1}$) and consumption is computed from the resource constraint.

$c_t = a_t * (k_t)^\alpha + (1-\delta) * (k_t) - k_{t+1}$

$y_t = \beta * \frac{c_{t+1}{}^{-\gamma}}{c_t{}^{-\gamma}} * [1 - \delta + \alpha * (k_t)^\alpha * a_t] * k_t$

The Euler equation residuals are used to construct a regression target for updating the policy. A new estimate of the coefficients is obtained via OLS. To enhance numerical stability, the authors propose using regularized least squares (RLS) via truncated singular value decomposition (SVD). This avoids overfitting when regressors are highly collinear (a common issue when using high-degree polynomials on simulated data). The data is normalized before regression to address scaling issues. A damping factor $\zeta \sim 0.01$ is applied in updating the policy coefficients.

$\hat{\beta} = (X'X)^{-1}X'Y$ where X = [k_t, a_t] and Y = y_t

$\beta^{p+1} = 0.09 * \beta^p + 0.01 * \hat{\beta}$

Once the first-degree policy converges, the polynomial degree is increased up to 5. For each degree, A polynomial basis (Hermite or ordinary) is formed before Monte Carlo or Gauss-Hermite quadrature integration with 1 to 10 nodes is applied. The total number of basis terms increases with degree (including cross-product terms).

1st degree = 3 coefficients

2nd degree = 6 coefficients

3rd degree = 10 coefficients

4th degree = 15 coefficients

5th degree = 21 coefficients

Rather than relying on random draws, deterministic nodes and weights from Gauss-Hermite quadrature are used to compute expectations (conditional) accurately. This significantly improves accuracy over Monte Carlo integration and avoids sampling noise. This integration relies on the number of nodes (1 through 10), the nodes themselves (error term value of the regression in the subsequent time step) and the weights associated with each node. It is used to compute the next-period productivity levels using:

$\ln a_{t+1} = \rho \ln a_t + \epsilon_{t+1}$ where $\epsilon_{t+1} \sim N(0,\sigma^2)$

To proceed with computing the capital series, we must select a regression method and choose whether to normalize the data as well as the degree of regularization. In our case, the regularized LS using truncated SVD (proposed by the literature) is used because as opposed to a standard SVD where the X decomposition only yields orthogonal matrices U, V (rotation of space) and a diagonal matrix of singular values (principal component magnitudes), the truncated version removes small singular values (which otherwise result in large β coefficients) and their corresponding directions (little variance). The aim here is to make the solution less sensitive to noise, avoid overfitting and address numerical instability. For each polynomial degree, we construct the matrix of state variables and compute an initial guess for the coefficients on each using the chosen regression. These coefficients are updated at each step until convergence to produce the time series of capital.

The final step consists of evaluating the accuracy of the results by generating a separate random draw of productivity shocks (ϵ_t) from which we construct a “test” series of productivity levels and capital (Monte Carlo performs poorly according to the literature). The Euler equation residual is evaluated at each test point using quadrature, with the mean and maximum reported to assess accuracy.

Appendix III shows the sensitivity of the model output’s initial conditions to the fixed parameters. Although, the simulation occurs in the ergodic set only it is worth noting the predictive nature of the algorithm if the fixed parameters are chosen to approximate the real economic “environment”. We thus show the discrepancies in capital steady state (k*) for different values of capital share , as well as different discount rates (analogous to different interest rate environments).

Appendix IV displays the outputs plotted alongside their respective real-world data. We notice that the simulated model approximates Total Factor Productivity but not capital and consumption as provided by the Penn Table (kpriv) and the Fed (pce) respectively.

III. Conclusion

Even though the model provides a resulting range that is within the same order of magnitude as the starting value for either dataset, it does not capture growth because it is stationary by construction. Hence, we will need a follow-up to this work that makes use of a time-variant stochastic autoregressive model to describe the simultaneous relationships between the 3 variables (productivity, capital and consumption). Primiceri 2003-which addresses the dynamics between inflation, unemployment and interest rates-provides a robust model that can be retrofitted for this application.

Sources

1. U.S. Bureau of Economic Analysis, Personal Consumption Expenditures [PCE], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/PCE

2. Board of Governors of the Federal Reserve System (US), Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity, Quoted on an Investment Basis, Inflation-Indexed [DFII10], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/DFII10

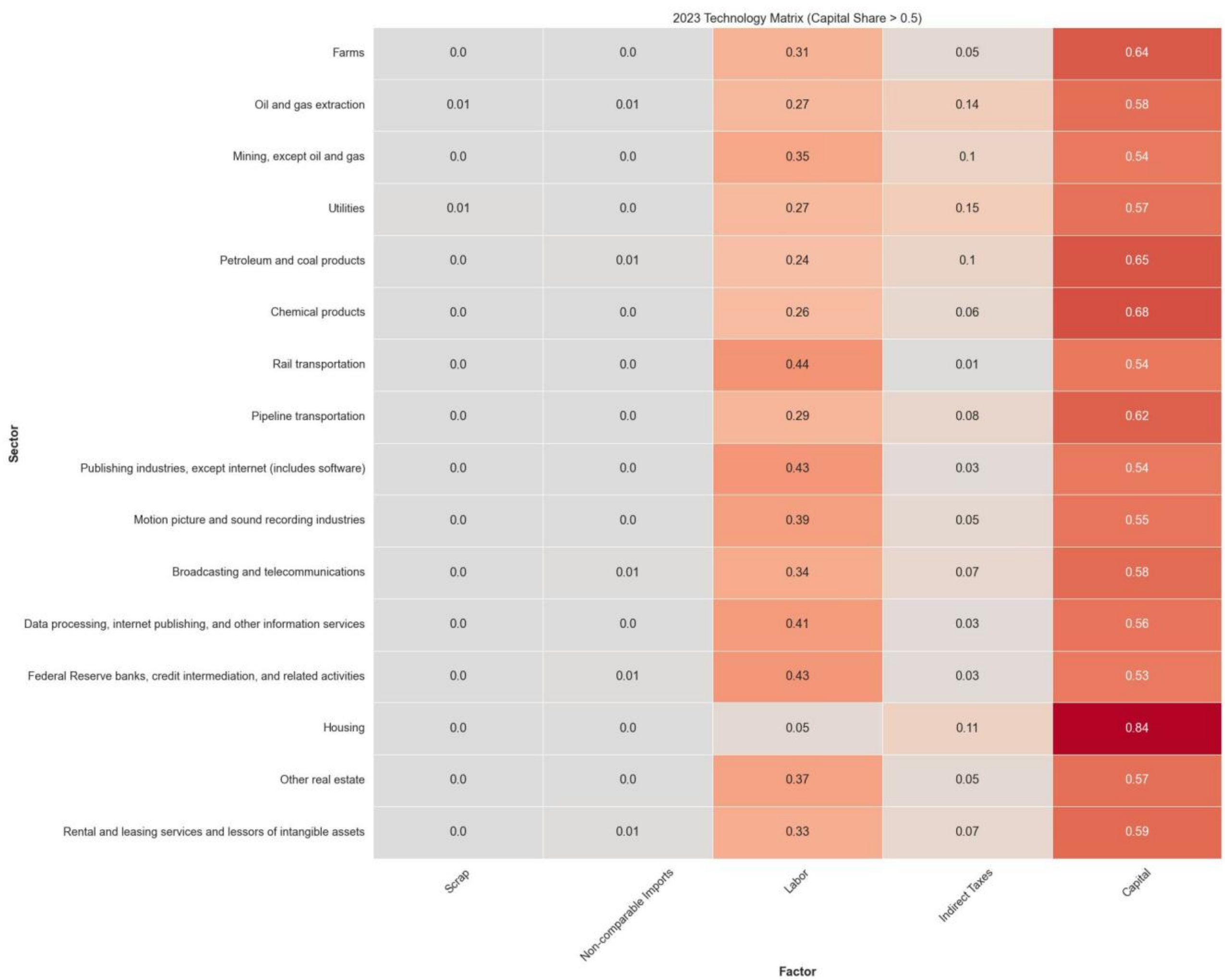
3. U.S. Bureau of Economic Analysis. Industry Economic Accounts: Input-Output Data. https://www.bea.gov/industry/input-output-accounts-data.

4. U.S. Bureau of Labor Statistics. Labor Productivity vs. Total Factor Productivity: A Comparison. https://www.bls.gov/productivity/educational-material/laborproductivity-total-factor-productivity-comparison.htm.

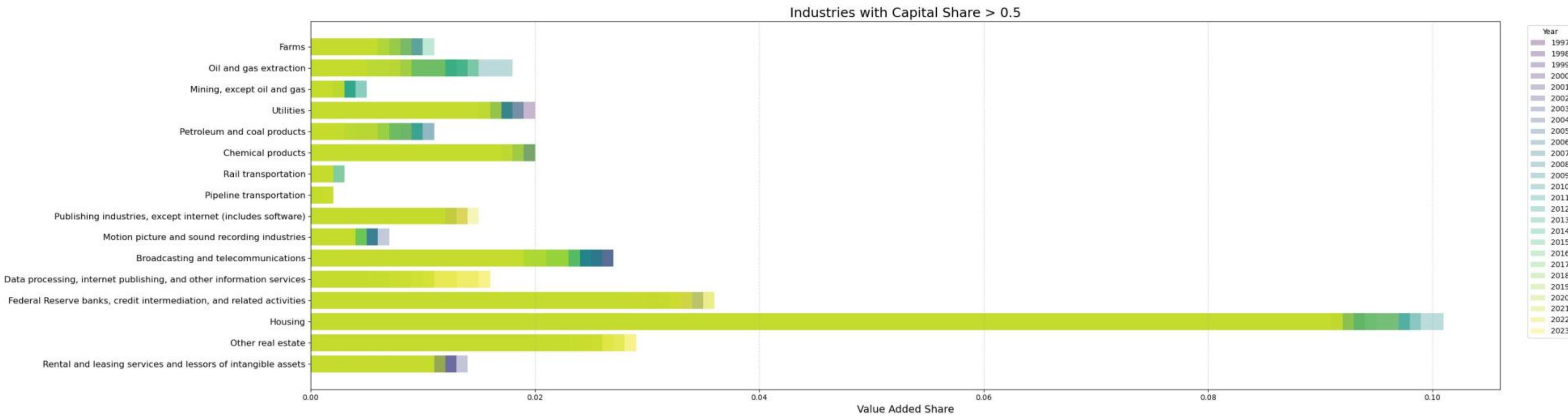
5. Fisher, Eric O'N. The Singular Value Decomposition of a Technology Matrix. CESifo Working Paper No. 4566. Munich: CESifo Group, January 2014. https://ssrn.com/abstract=2385908.

6. Judd, Kenneth L., Lilia Maliar, and Serguei Maliar. "Numerically Stable and Accurate Stochastic Simulation Approaches for Solving Dynamic Economic Models." Quantitative Economics 2 (2011): 173–210. https://doi.org/10.3982/QE14.

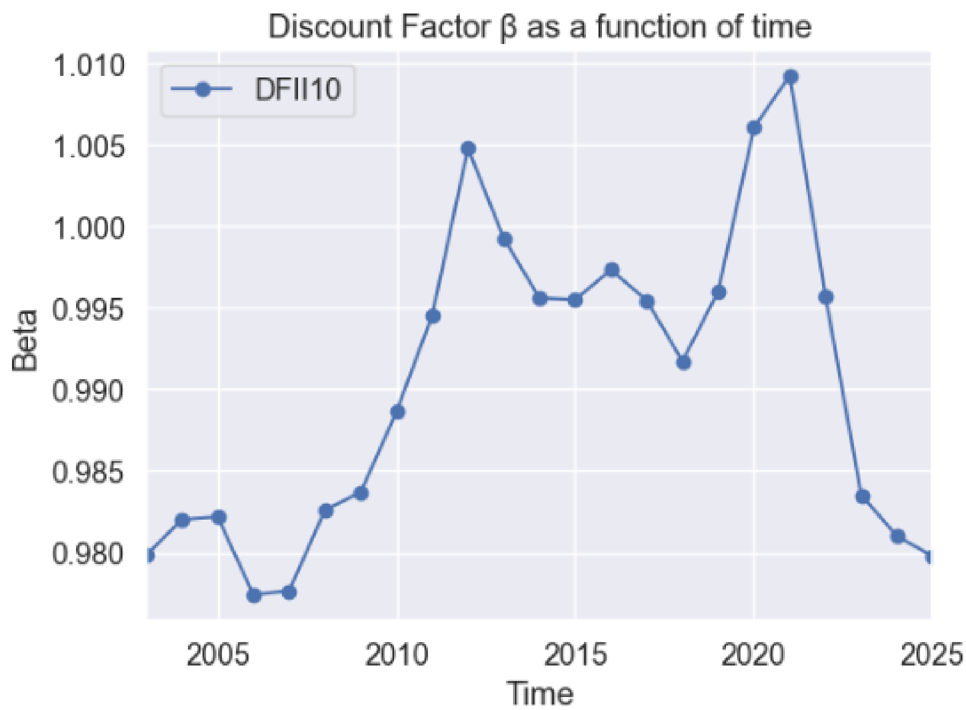
Appendix I

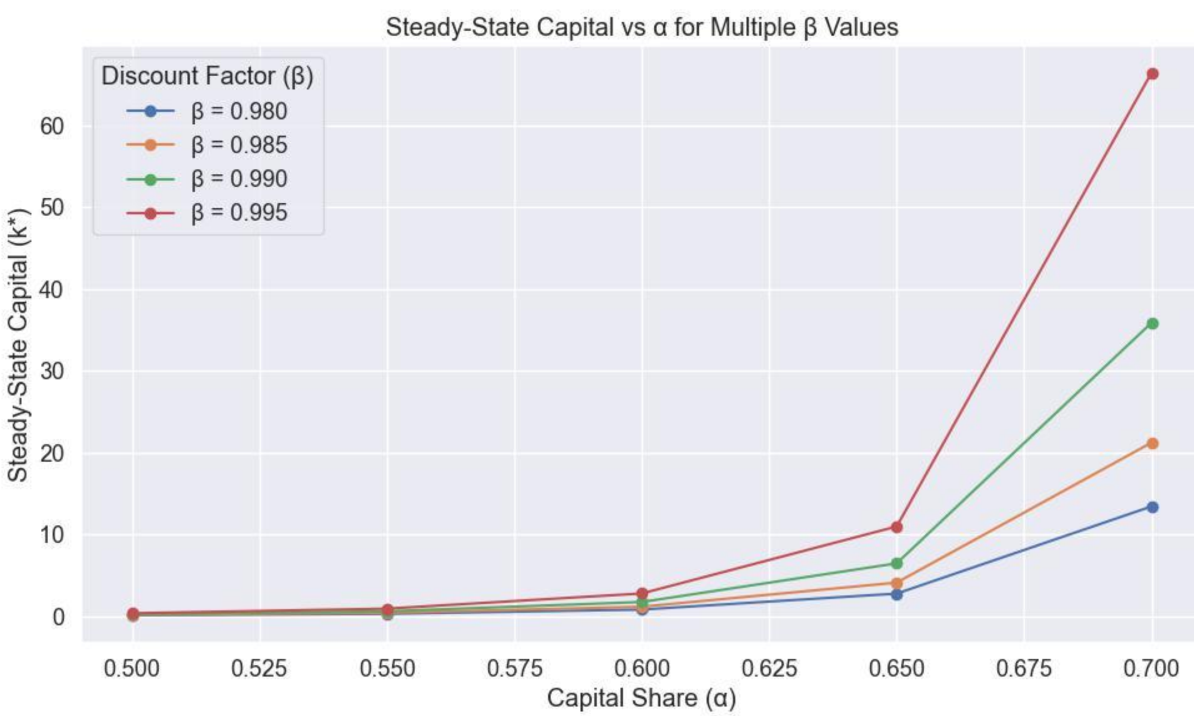

2023 Technology Matrix (Capital Share > 0.5)
Sector
Factor
Scrap
Non-comparable Imports
Labor
Indirect Taxes
Capital
Farms 0.0 0.0 0.31 0.05 0.64
Oil and gas extraction 0.01 0.01 0.27 0.14 0.58
Mining, except oil and gas 0.0 0.0 0.35 0.1 0.54
Utilities 0.01 0.0 0.27 0.15 0.57
Petroleum and coal products 0.0 0.01 0.24 0.1 0.65
Chemical products 0.0 0.0 0.26 0.06 0.68
Rail transportation 0.0 0.0 0.44 0.01 0.54
Pipeline transportation 0.0 0.0 0.29 0.08 0.62
Publishing industries, except internet (includes software) 0.0 0.0 0.43 0.03 0.54
Motion picture and sound recording industries 0.0 0.0 0.39 0.05 0.55
Broadcasting and telecommunications 0.0 0.01 0.34 0.07 0.58
Data processing, internet publishing, and other information services 0.0 0.0 0.41 0.03 0.56
Federal Reserve banks, credit intermediation, and related activities 0.0 0.01 0.43 0.03 0.53
Housing 0.0 0.0 0.05 0.11 0.84
Other real estate 0.0 0.0 0.37 0.05 0.57
Rental and leasing services and lessors of intangible assets 0.0 0.01 0.33 0.07 0.59

Appendix II


Industries with Capital Share > 0.5
Farms
Oil and gas extraction
Mining, except oil and gas
Utilities
Petroleum and coal products
Chemical products
Rail transportation
Pipeline transportation
Publishing industries, except internet (includes software)
Motion picture and sound recording industries
Broadcasting and telecommunications
Data processing, internet publishing, and other information services
Federal Reserve banks, credit intermediation, and related activities
Housing
Other real estate
Rental and leasing services and lessors of intangible assets
0.00
0.02
0.04
0.06
0.08
0.10
Value Added Share
Year
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019
2020
2021
2022
2023

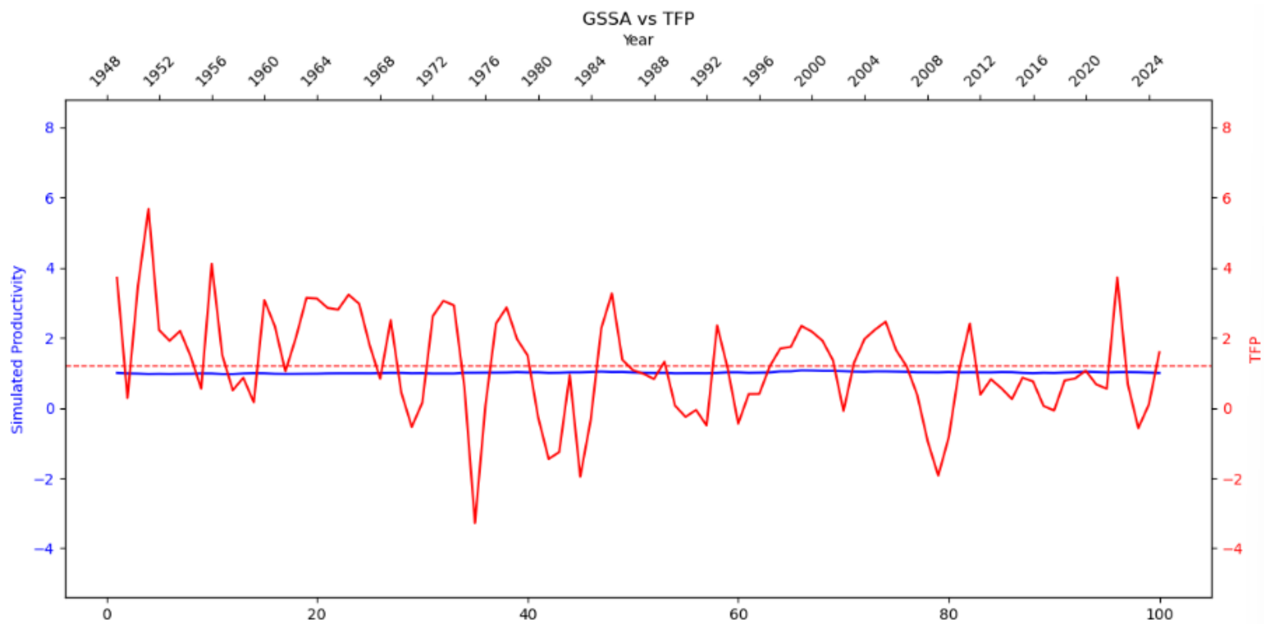
Appendix III

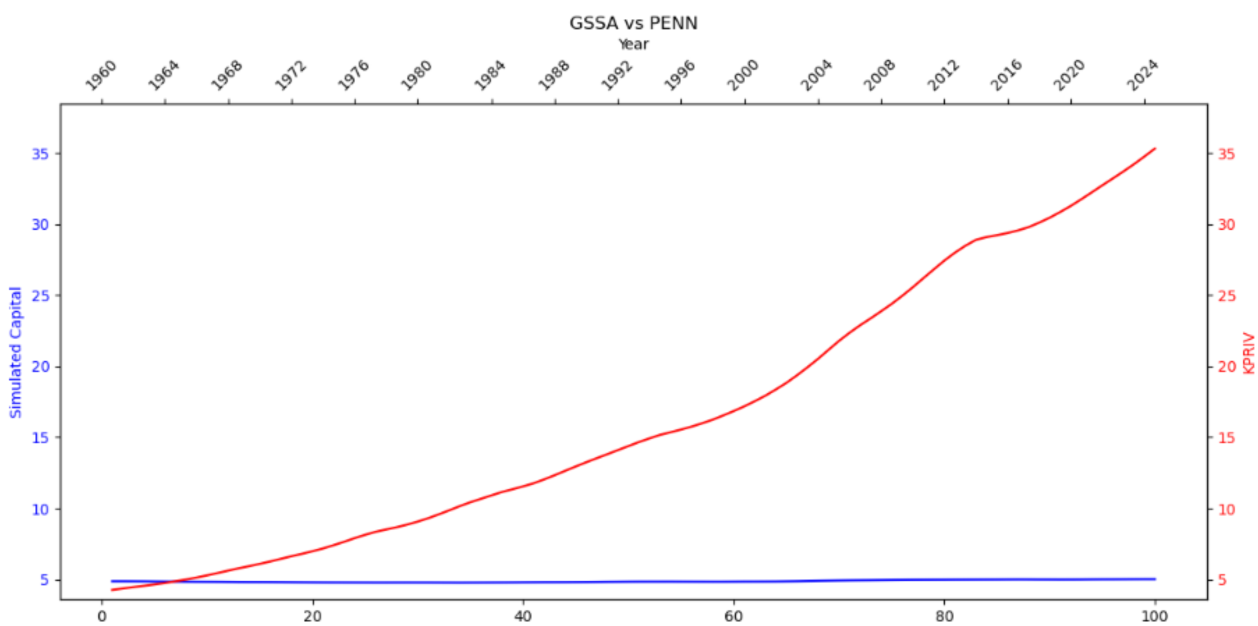


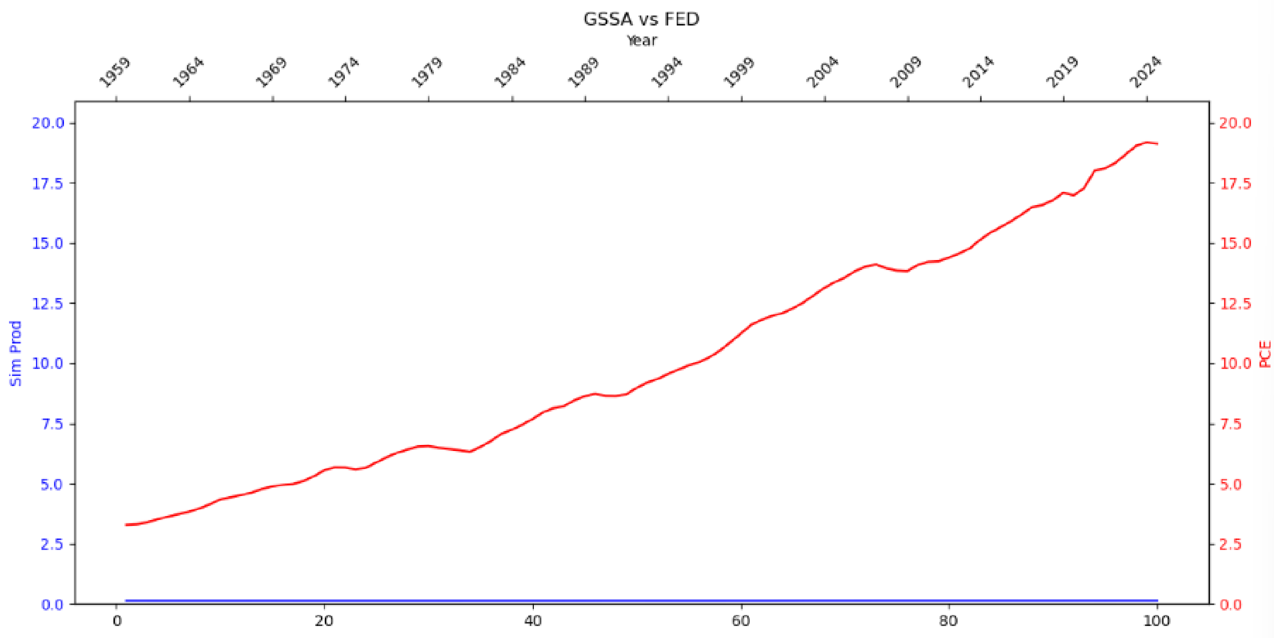



Steady-State Capital vs β for Multiple α Values
Capital Share (α)
alpha = 0.500
alpha = 0.550
alpha = 0.600
alpha = 0.650
alpha = 0.700
Steady-State Capital (k*)
0
10
20
30
40
50
60
0.980
0.982
0.984
0.986
0.988
0.990
0.992
0.994
Discount Factor (beta)

Appendix IV


GSSA vs TFP
Year
1948
1952
1956
1960
1964
1968
1972
1976
1980
1984
1988
1992
1996
2000
2004
2008
2012
2016
2020
2024
Simulated Productivity
TFP
8
6
4
2
0
−2
−4
0
20
40
60
80
100


GSSA vs PENN
Year
1960
1964
1968
1972
1976
1980
1984
1988
1992
1996
2000
2004
2008
2012
2016
2020
2024
Simulated Capital
KPRIV
35
30
25
20
15
10
5
0
20
40
60
80
100


GSSA vs FED
Year
1959
1964
1969
1974
1979
1984
1989
1994
1999
2004
2009
2014
2019
2024
Sim Prod
PCE
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
0
20
40
60
80
100