Multiple Model Adaptive Estimation Kalman Filtering

AERO 553 / Winter 2023

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LEARN BY DOING

CAL POLY

1. Introduction

- Goal: develop a practical application of a Multiple Model Adaptive Estimation Kalman Filter (MMAEKF) to estimate the state of a system (linear carriage position)
- Different position estimation methods using acoustics and accelerometer data are compared to the encoder position.
- Application: tracking an object that is difficult to measure in real-life scenarios - position estimation to be used in a controller

2. System Model

System Model

- Combined measurement model (using both the accelerometer and acoustic sensor data) is used to describe the system
- The system uses a state-space representation of the equations of motion to model the platform as a particle:

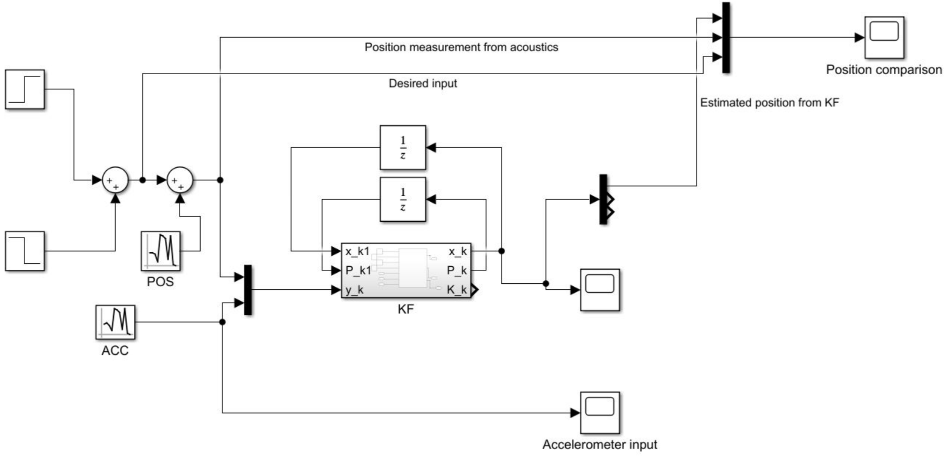
$$\begin{bmatrix} \mathrm{x} \\ \dot{x} \\ \ddot{x} \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & \mathrm{dt} & \frac{1}{2}dt^2 \\ 0 & 1 & \mathrm{dt} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{x} \\ \dot{x} \\ \ddot{x} \end{bmatrix}_{k} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \sim N(0, Q)$$

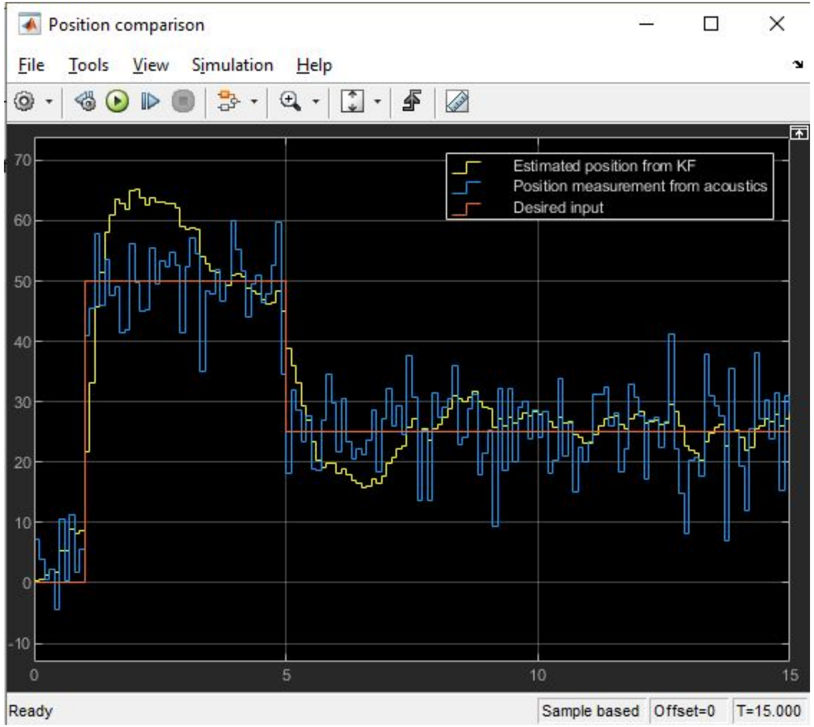
$$\begin{bmatrix} x \\ \ddot{x} \end{bmatrix}_{k} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{x} \\ \dot{x} \\ \ddot{x} \end{bmatrix}_{k} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sim N(0, \sigma^2_{acoustic}) & 0 \\ 0 & \sim N(0, \sigma^2_{IMU}) \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot 0.1$$

Where x is position measured via acoustic triangulation and $\ddot{x}$ is acceleration measured via IMU.

MATLAB Simulation of KF





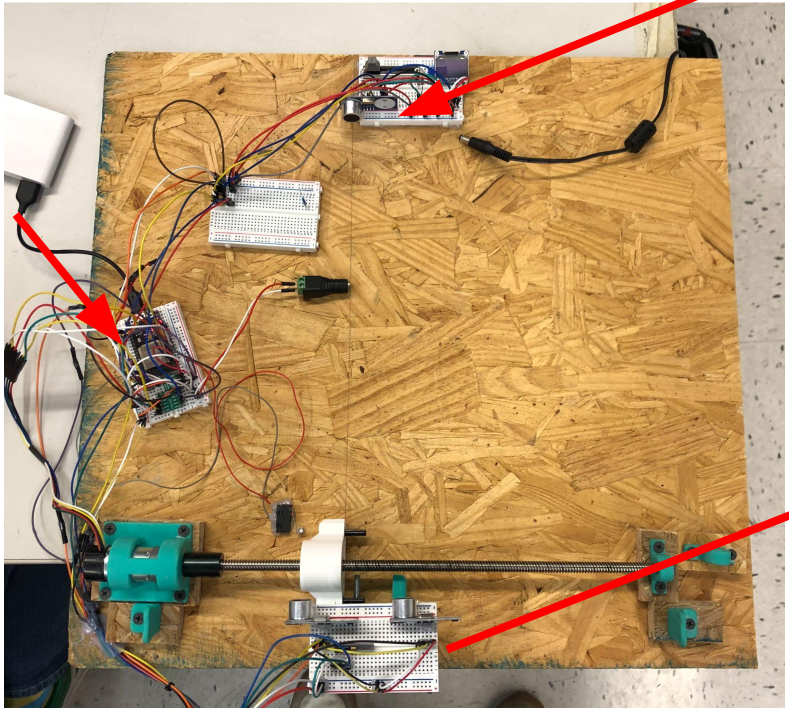
3. Multiple Model Adaptive Estimation

Dynamic Theory

- MMAE is a technique for estimating the state of a dynamic system
- Combines several KFs to improve estimation accuracy
- Each KF model represents a different dynamic model of the system
- MMAE selects the model that produces the most accurate estimate (least residual)
- Useful for systems with changing dynamics or unknown model parameters
- This project: KFs for signal processing, not in a state-space controller:
    - Stability, controllability, and observability of the plant cannot be investigated

Application

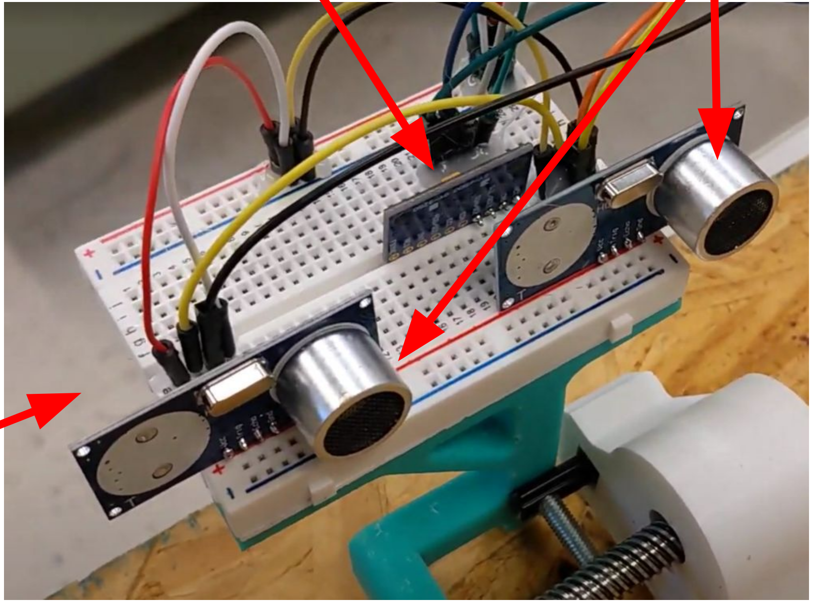
ESP32



Ultrasonic
Transmitter

IMU

Ultrasonic
Receivers

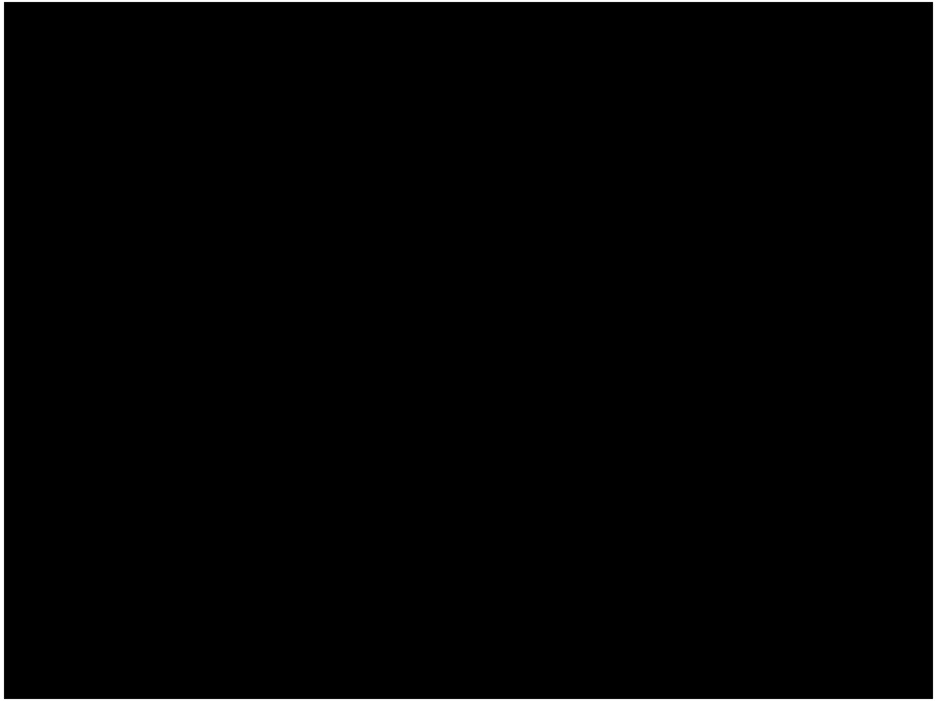


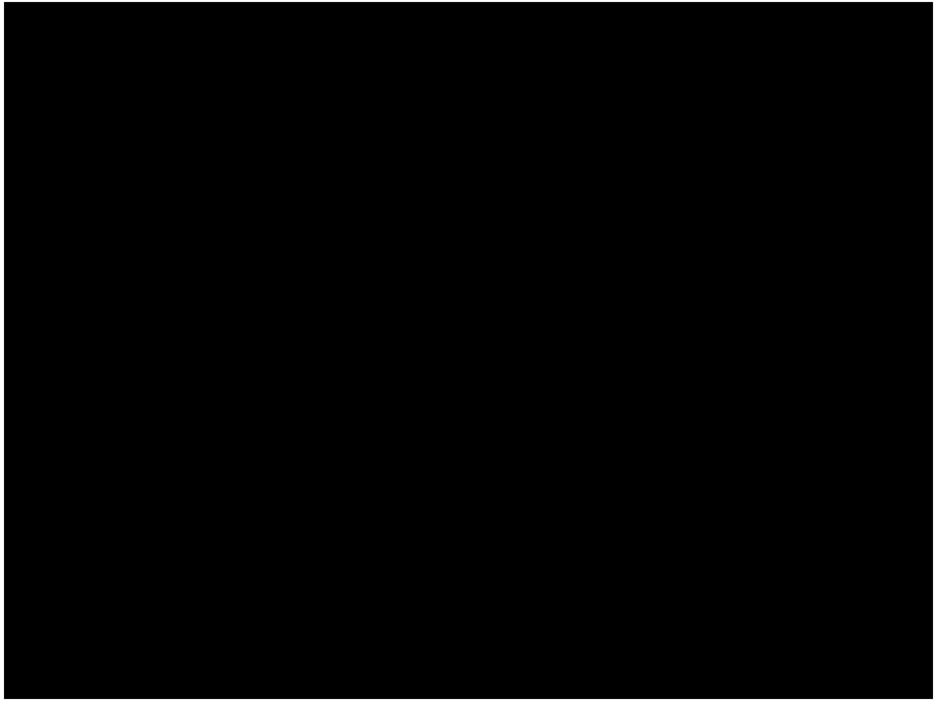
Application

- MMAE is applied to a linear carriage system.
- 3 KFs in our model:
  - Combined model (both acoustics and accelerometer) (3x3)
  - Just acoustic position readings (1x1)
  - Just accelerometer (twice integrated) position readings (1x1)
- Filtered measurements from the KF models are weighted according to their accuracy (residuals)
- Encoder position used to compare position estimation to actual position for percent error, not for filtering

4. System Evaluation

System in Operation





Measured Output

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         Average best model: Acoustic time-of-flight (with KF)
              Model is best: 53.85% of the time

  Best model for this dataset: Acoustic time-of-flight
         Model percent error: 1.79%

       Desired position (mm): 100
        Actual position (mm): 98.82, DC motor controller percent error: 1.18%
        Accel. position (mm): 155.89, percent error: 58.07%
      Acoustic position (mm): 100.59, percent error: 1.79%
     KF accel. position (mm): 156.02, percent error: 57.93%
   KF acoustic position (mm): 94.37, percent error: 5.05%
   KF combined position (mm): 172.67, percent error: 78.74%
          MMAE position (mm): 128.14, percent error: 28.34%
         MMAE KF proportions: 49.91%, 46.26%, 3.84%
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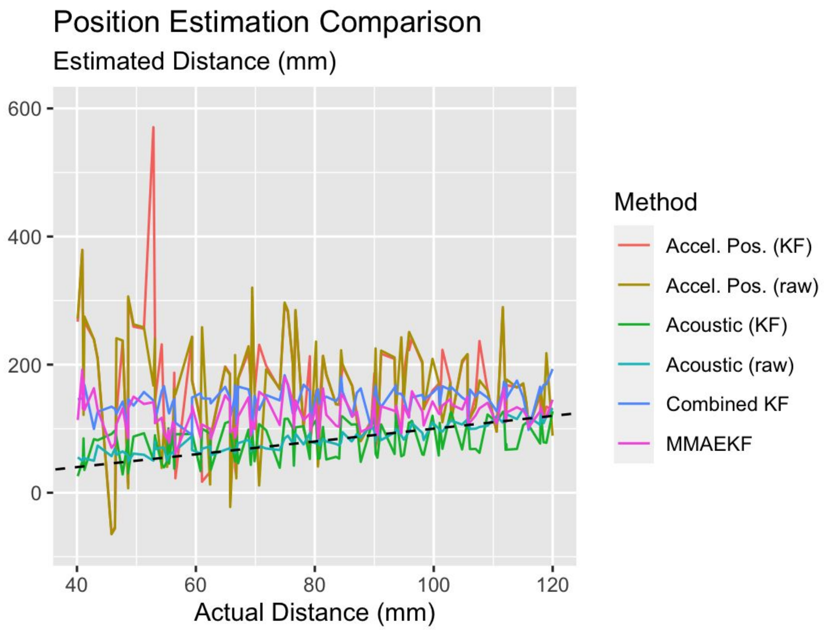
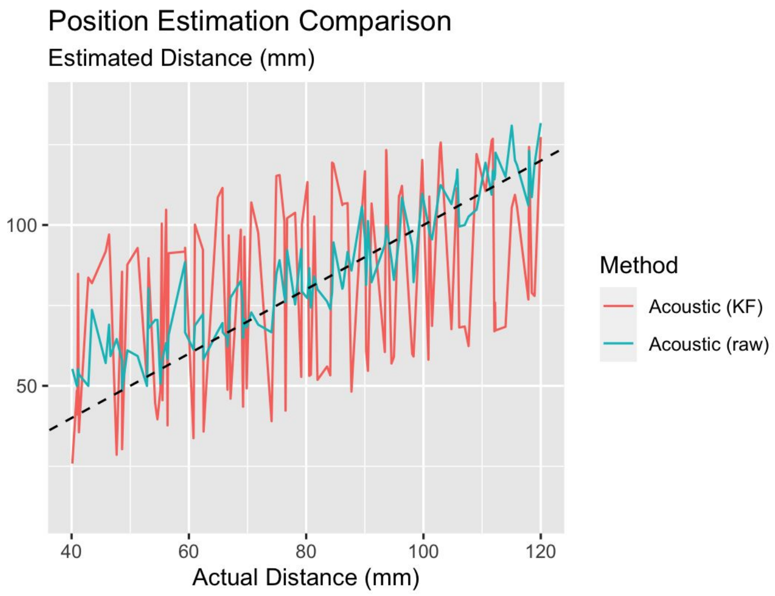
Results



Position Estimation Comparison
Estimated Distance (mm)
600
400
200
0
40
60
80
100
120
Actual Distance (mm)
Method
Accel. Pos. (KF)
Accel. Pos. (raw)
Acoustic (KF)
Acoustic (raw)
Combined KF
MMAEKF

Results

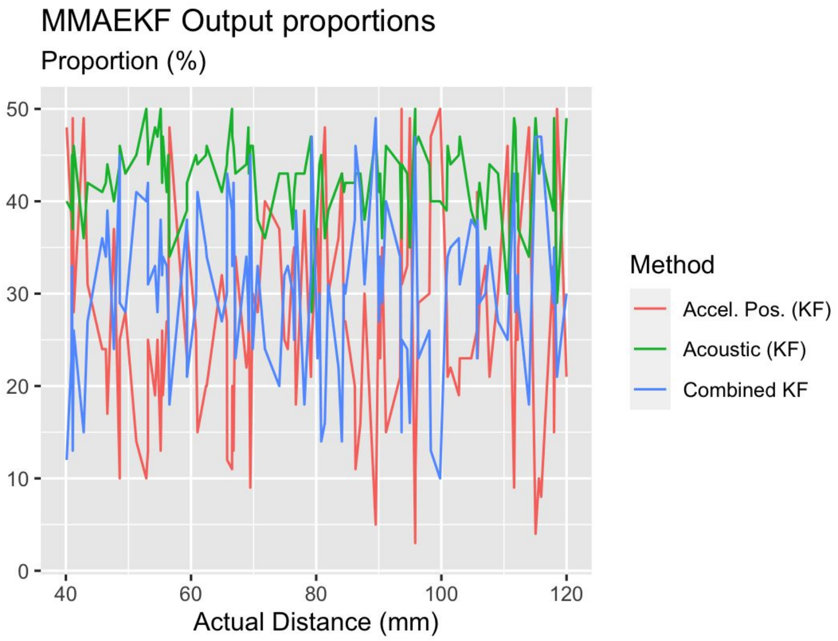


Position Estimation Comparison
Estimated Distance (mm)
100
50
40
60
80
100
120
Actual Distance (mm)
Method
Acoustic (KF)
Acoustic (raw)

Stationary Position Estimation Comparison
Estimated Distance (mm) when held at 83mm
100
90
80
0
25
50
75
100
Sample
Method
Acoustic (KF)
Acoustic (raw)

Discussion

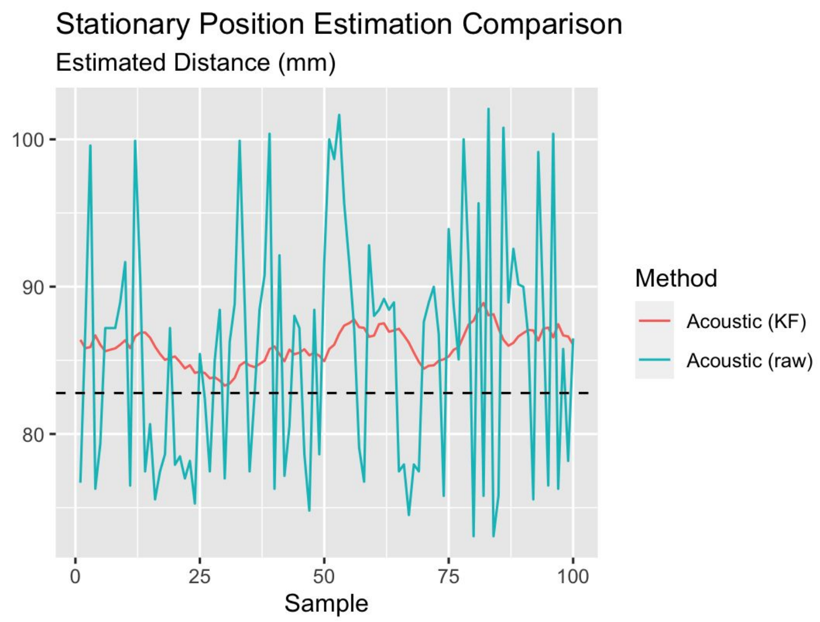
- MMAEKF successfully adjusted the proportion of each measurement used
- Ex: Airplane Wing Accelerometer



5. Comparison to Classical Methods

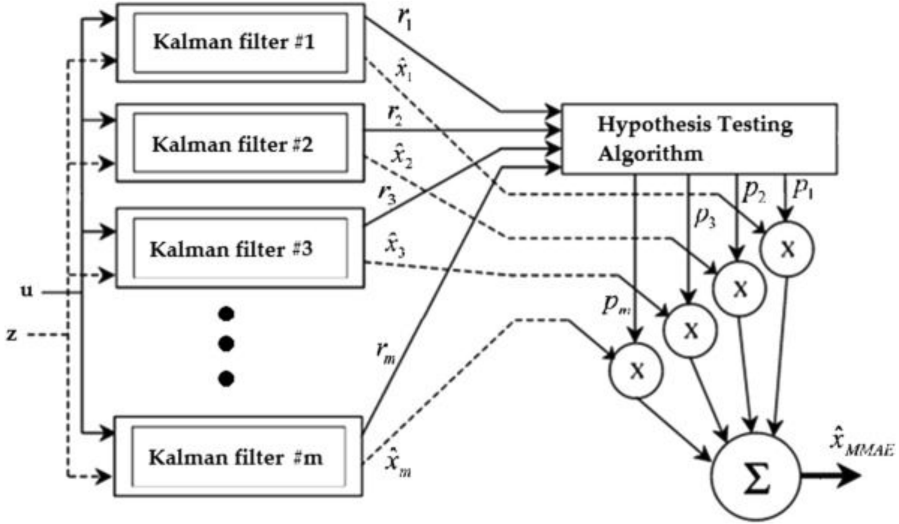
Comparison

- Classical filtering methods include low-pass or high-pass filtering
  - N/A in this system - noise is not isolated in a particular frequency range
- MMAEKF is superior due to system adaptation, providing more accurate readings each time step.
- Allows for multiple noise state models (Ex: Aircraft Wing Accelerometer)



6. Conclusion

- MMAEKF is effective in combining multiple measurements in variable systems
- MMAEKF can be implemented in real-time on embedded systems with limited computational resources.
- Applications in GPS, navigation, Missile guidance [3]
- MMAEKF is an important advance in signal processing and estimation theory with large potential



7. References

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- [2] Frabosilio, J., & Lyons, J. (2023, March 20). Linear Position Controller. EE 513 - Winter 2023. Retrieved from https://cpslo-my.sharepoint.com/:w:/g/personal/jlyons06_calpoly_edu/EVHZh_uWLDNAoM7PYSkW0x4BvMAR2L0xoFjLaxz6DwgfRA?e=YE741o
- [3] Weicun Zhang, Sufang Wang, Yuzhen Zhang, "Multiple-Model Adaptive Estimation with A New Weighting Algorithm", Complexity, vol. 2018, Article ID 4789142, 11 pages, 2018. https://doi.org/10.1155/2018/4789142
- [4] Y. WANG, S. FAN, J. WANG, and G. WU, “Quick identification of guidance law for an incoming missile using multiple-model mechanism,” Chinese Journal of Aeronautics, vol. 35, no. 9, pp. 282–292, Sep. 2022, doi: https://doi.org/10.1016/j.cja.2021.10.032.