PML HW1

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a)

Assume $P(B|A) \geq P(B)$
We have

$$\begin{aligned} P(B) &= P(B|A)P(A) + P(B|\neg A)(1 - P(A)) \quad \text{(By partition law)} \\ &\geq P(B)P(A) + P(B|\neg A)[1 - P(A)] \end{aligned}$$

$$\implies P(B)[1 - P(A)] \geq P(B|\neg A)[1 - P(A)] \implies P(B) \geq P(B|\neg A)$$

b)

$$P(A|B) = \frac{P(A,B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)} \geq \frac{P(B)P(A)}{P(B)} = P(A)$$

c)

We have

$$\begin{aligned} P(A) &= P(A|B)P(B) + P(A|\neg B)(1 - P(B)) \\ &\geq P(A)P(B) + P(A|\neg B)(1 - P(B)) \quad \text{(from (b))} \end{aligned}$$

$$\implies P(A)[1 - P(B)] \geq P(A|\neg B)[1 - P(B)] \implies P(A) \geq P(A|\neg B)$$

d)

Now we assume that $P(B|A) = 1$:
We first observe that this immediately implies that $P(\neg B|A) = 0$. Then we have:

$$P(\neg A|\neg B) = \frac{P(\neg A, \neg B)}{P(\neg B)} = \frac{P(\neg A, \neg B)}{P(\neg B, A) + P(\neg B, \neg A)}$$

But we observe that now $P(\neg B, A) = P(\neg B|A)P(A) = 0$. Therefore our expression reduces to 1

e)

This is just a special case of part (a) (since $1 = P(B|A) \geq P(B)$ with $P(B) \in [0, 1]$ by definition of probability)

f)

This is just a special case of part (c) (since $1 = P(A|B) \geq P(A)$ necessarily)