stuff that needs math formatting

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June 19, 2023

1 number of layers needed

Goal: determine the number of layers N needed to cover at least the proportion k of the unit disk with a $(2, p, q)$ regular hyperbolic tiling.

$$f(\mathbf{x}) := \ln\left(\frac{1 + ||\mathbf{x}||}{1 - ||\mathbf{x}||}\right)$$

Equivalently, $f(\mathbf{x}) = 2\,\text{arctanh}(||\mathbf{x}||)$.
If $\mathbf{x}$ is a number in the open unit disk in $\mathbb{C}$, $f(\mathbf{x})$ gives the hyperbolic distance of $\mathbf{x}$ to the origin in the Poincaré disk model. If $\mathbf{x}$ is a real number, $f(\mathbf{x})$ gives the hyperbolic distance to a point of Euclidean norm $\mathbf{x}$.

Suppose the tiling is generated from a rotationally symmetric regular hyperbolic polygon centered at the origin. Then the distance d of each vertex of this polygon is uniquely determined by p and q due to congruence of all similar triangles in hyperbolic space. N can be computed by taking the following ratio:

$$\frac{f(k)}{f(d)}$$