Quantum Structuring in Planetary Systems

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1 Introduction

Currently unknown to astronomers is an accurate formation model for celestial systems. How and in what structure planetary systems form has until now been thought the result of the solar nebular disk model first developed by Immanuel Kant in *Universal Natural History and Theory of the Heavens* published in 1755. Recently this model has encountered criticism do to unexplained problems and though much theoretical work has been done to determine which formation model best explains how planetary systems are formed, little experimental work has been done [6] [2] [7]. With the exoplanet database only recently having grown so large we have been left with the opportunity to question formation models in an experimental manner.

The basis of our project is to see if Scale Relativity, a newly derived form of quantum mechanics, can properly explain the structuring of planetary solar systems [3]. At the quantum scale, as for example with electrons, orbital structures form along discrete intervals defined by the expectation of the Schrödinger equation with some potential energy such as angular momentum. Scale Relativity offers the same quantised structuring for planetary systems by using the expectation value of a Schrödinger equation whose potential is the Keplerian potential. It is our goal to see if this discrete structuring is present within all confirmed planets within the Exoplanet Database [1]. To do so we first see if the expectation value for the Kepler-Schrödinger equation for each planet, denoted as the planetary *rank*, tends to be near integer value. As we find planets do in fact seem to cluster around integer values. This is a promising indication that the theoretical background of Scale-Relativity is sound, so far. We show this observation by studying the Kullback-Leibler divergence between the calculated ranks subtracted from their nearest integer value and the uniform distributions from 0 to 0.5, a divergence which implies the calculated ranks tend near integer values rather than all real numbers. Secondly we compare to the calculated ranks to the natural numbers, a similarity which implies the ranks also tend towards integer values.

Following the analysis of discrete planetary orbital numbers as predicted by the Schrödinger Keplerian wave equation we investigate the discretisation of eccentricity. Using KL divergence in a similar manner we examine if experimentally observed values of eccentricity have high set similarity to the predictions of Scale Relativity. We find short comings do exist in the predictions of SR due to a fairly large divergence between predicted and observed probability distributions.

Lastly, we evaluate the results in section 3.1 and 3.2 by clustering planetary attributes through normalised K-means clustering as well as spectral clustering.

2 Background

Scale Relativity assumes space as continuous but non-differentiable which leads to geodesics characterised as being (*i*) infinite in number, (*ii*) fractal, and (*iii*) infinitesimally irreversible. These conditions can then be combined with a covariant derivative. After integrating one finds a geodesic equation in the form of a quantum mechanical equation. In order to find real world examples to test this theory we must look to systems whose structuring results from the three fore mentioned conditions. A perfect candidate is systems developed from ergodic chaos. One such example is planetary and solar system whose formations which are the product of clouds of dust particles and gases subject to Brownian motion. For this reason **the formation of planetary systems serves as an example to the quantum based geodesic equations predicted by Scale Relativity.**

Scale Relativity's derived geodesic equation can be seen as a macro scale version quantum mechanic's Schrödinger wave equation which defines the "kinematic" behaviours of quanta. Our theoretical hypothesis is then that we are able to use the Kepler potential in the Schrödinger equation which would imply the expectation values, the square of the modulus, of the Schrödinger equation define the probability density distribution of each planet as well as be discrete- meaning structuring forms along discrete intervals. **One positive sign would then be if celestial bodies can be identified as forming along discrete probability distributions** whose quantisation is defined by an integer value equivalent to atomic quantum numbers. Our macroscopic constant, defined by the *rank equation*, can be found from the Schrödinger Kepler equation as dependent on period T, some reference period T_0, planet mass M, and some reference mass M_0.

$$n = n_o \left(\frac{T/T_o}{M/M_o} \right)^{1/3} \tag{1}$$

As mentioned, the rank value assigned to a planet can be seen as the planetesimals quantum number but in the classical setting [4]. The rank plays the same role as quantum numbers in the quantum setting for atoms by defining orbital structuring due to being a possible solution to the Schrödinger wave equation.

The written code used in this analysis can be found online [5].

3 Results

3.1 Quantization of the Schrödinger-Keplerian Orbital Quantum Number

Our goal is then to see if the rank of all known planets tend to cluster around integer values. To see if the rank of each planet in the Exoplanet database truly clusters around discrete values we must calculate a reference rank n_o for which we use our solar system and Earth's mass M_0 and period T_0. We calculate the reference rank n_o by varying the value of n_i from 0 to 10 in search for a minimum of the sum of the squared differences between the calculated rank n and the closest integer for all planets (including Pluto!).

$$n_o = min \left(\sum_0^{10} \left(n_i \left(\frac{T/T_o}{M/M_o} \right)^{1/3} - \; int \left(n_i \left(\frac{T/T_o}{M/M_o} \right)^{1/3} \right) \right)^2 \right)$$

The rank assigned to each planet in our system seems to differ depending on which planet defines n_o. As a result our solar system can be seen as composed of two parts, the inner solar system consisting of planets from Mercury to Mars, and the outer solar system consisting of Jupiter to Pluto. Figure 3.1 shows the use of Earth (blue) in one instance as the reference planet and Jupiter in the other (red). We can see that the discretization from Mercury to Jupiter is consistent between both, however following Jupiter the ranks grow dramatically for Earth referenced planets while those of Jupiter remain less than 5. One possible explanation for this is that the discretization of solar systems behaves with an *inner* and *outer* ranking structure. This is consistent with the predictions of Scale Relativity and for this reason we hope to identify celestial systems in our data with more than three planets and use Earth as the reference rank for the inner system and Jupiter as reference for the outer.

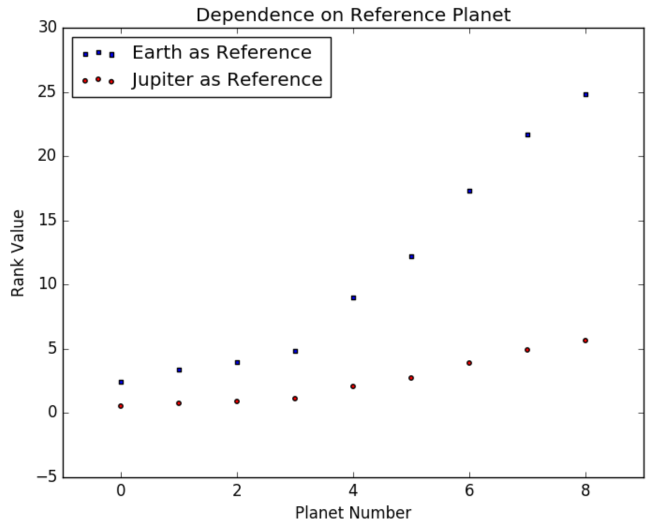

Figure 1: Comparison of rank calculations with differing reference planets.

The orbital rank of each planet in the Exoplanet database was then identified to the integer closest to the rank n calculated with the optimal n_0 value. Figure 2 shows which rank values are more common.

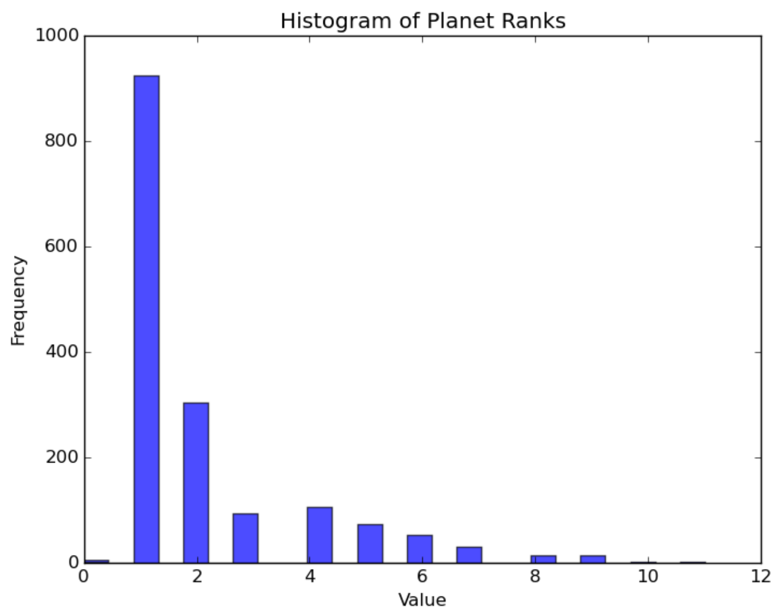

Figure 2: The discrete rank value and its frequency.

Figure 3 shows a histogram of the frequencies of the the calculated rank for each planet subtracted from the integer nearest to that rank. It is promising to see that the ranks calculated for each planet tend to be less than 0.25 away from an integer value.

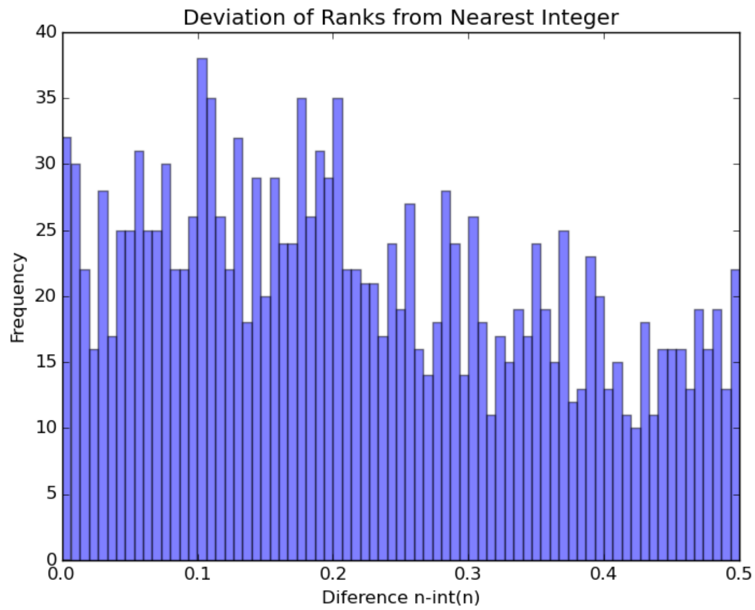

Figure 3: Histogram of $n - int(n)$

Our goal is now to see with what likelihood our data is actually approaching integer values as to apposed to by coincidence. To accomplish this we exploit data mining techniques to measure the set similarities between two probability distributions. To see if our data truly clusters around discrete values we must see if the difference between the calculated rank and its nearest integer value, $rank - round(rank)$, has a higher probability of clustering around integer values than that of a random uniform distribution between 0 and 0.5. Showing this would imply the discretization of our results were not by chance but are exhibiting properties within the system.

Using a random and uniform distribution between 0 and 0.5 and our calculated ranks we then calculate the Kullback-Leibler divergence. The KL-divergence between these two sets demonstrates the relative entropy, or the structural dissimilarity between these two distributions, and was found to be 0.572. This value being mid range between 0 and 1 implies there is some overlapping similarity in the probability distribution of our calculated ranks and the uniform distribution of all real numbers. However, not little enough similarity to show that the rank values do not have arbitrary decimal values between integers but do tend to be structured more towards integer values.

We then compare the true rank values to that of the nearest integers to which they were assigned and find a KL-divergence between of 0.0112281644846. This low value implies a low relative entropy meaning they are in fact structurally similar. This is a promising sign that the planetary systems are accurately modeled along discrete values. In short, by using KL-divergence for Scale Relativity's discrete approximation for Keplerian systems we have been able to test the new model's inefficiency.

3.2 Quantization of the Runge-Lenz Vector

The Runge-Lenz (RL) vector serves to describe the orientation and shape of two orbiting bodies. Classical mechanics defines the RL vector as $\mathbf{A} = \mathbf{p} \times \mathbf{L} - \mathbf{r}/r$ and is oriented on the orbitals major axis towards the perihelion. Though we will not show the derivation here Scale Relativity predicts that the permitted values of the z component of the RL vector in the quantum regime will be discrete multiples of the quantum number, or rank, derived from the Schrödinger-Kepler wave equation and defined by equation 1. What is more, the eccentricity e is predicted equal to $\mathbf{A}_z$. It is then predicted that the exoplanet eccentricity distribution should show peaks around quantized values defined by the formula $e = k/n$ where n is the rank as before and k is an integer varying from 0 to $n - 1$ [3]. Our goal is then to determine with what certainty can is the theoretical prediction true that

$$\mathbf{A}_z = e = \frac{k}{n} \tag{2}$$

In order to determine the KL divergence between the eccentricities observed and cataloged in the Exoplanet database to the eccentricity values predicted by the Scale Relativity's hypothesis we begin calculating the rank for each planet followed with calculating each planets set of eccentricities as defined by equation 2. We then choose the eccentricity value predicted by Scale Relativity that is closest to the experimentally observed value. To determine if Scale Relativity's predictions reflect the observed eccentricities we take the KL divergence between the probability distributions of true eccentricities to the set of predicted eccentricities. The resulting KL divergence is a value of 2.091945. The resulting large KL divergence is indication that the predictions of Scale Relativity do not reflect the true eccentricity values well. The short comings of Scale Relativity's predictions can be seen in the histogram plot shown in Figure 4. Both observed and predicted eccentricities show a strong clustering around eccentricity values neat 0 (meaning circular orbits) however Scale Relativity predicts many more circular orbits than actually exist. What is more, Scale Relativity seems to overlook many eccentricity values between 0 and 0.1.

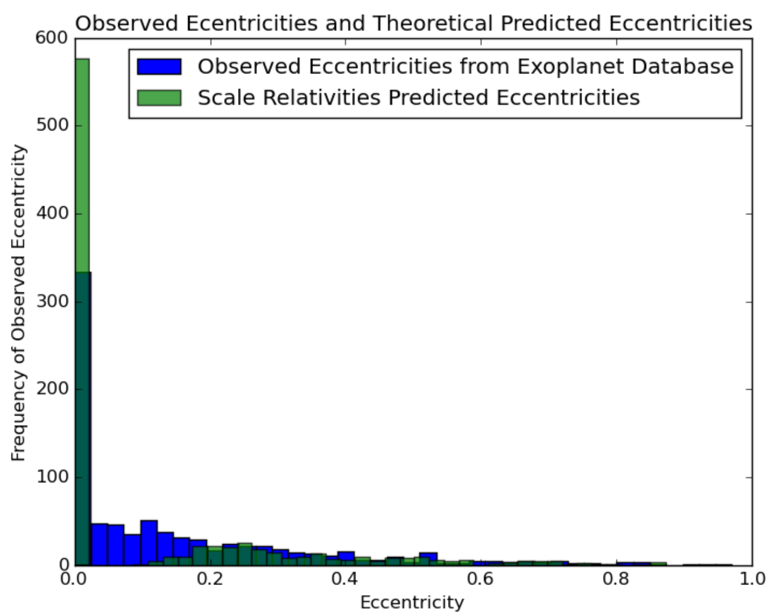

Figure 4: Comparison of theoreticaly predicted experimentaly observed eccentricities

The large divergence between predicted and observed values does not however discourage the potential of Scale Relativity. Due to the experimental difficulty of observing eccentricity it is one of the most limited of observed quantities in the Exoplanet database. For this reason the smaller subset gave us a limited probability distribution to both theoretically predict and compare with. With the two probability distributions being smaller than those seen in section 3.1 could be reason as to why more divergence is observed.

3.3 Spectral Clustering Graphs of Period and Mass

In further better our understanding of the similarity between planets we perform spectral clustering over the periods of the planets. This was accomplished by defining each period a node and assigning an edge between nodes if the two periods are within $\epsilon = 5\ days$. The location of each node is defined as the second and third columns of the eigenvectors of the Laplacian matrix. Defining D as the degree matrix, A as the adjacency matrix, and I the identity matrix the normalised Laplacian matrix is $L = I - D^{-1/2}AD^{-1/2}$. We then take the eigenvector's of the Laplacian matrix and scale each value in the eigenvector by $1/\sqrt{\lambda_i}$ where λ_i is the eigenvalue for eigenvector i.

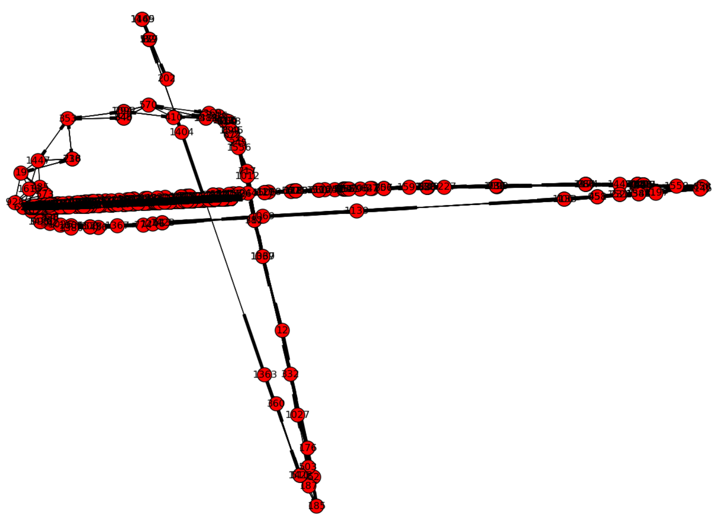

Figure 5: Adjacency matrix of periods Defined as having a connected edge if the periods are within 5 days of each other

Figure 5 shows 3, potentially 4, fairly distinct cuts. Since period is sufficient to perform the analysis done in section 3.1 this strengthens the argument towards Scale Relativity. Namely, despite there being a unique groupings of planets based on period, the calculated probability distributions posed in section 3.1 still had low divergence from those experimentally observed.

In order to compare multiple attributes defining a planets orbital structure it was necessary to adjust to the difference between units of our data. We cluster planets based on both mass and period, defining two planets as connected by an edge if two planet's periods are within $\epsilon = 5$ days of one another *and* their masses are within $\delta = 1/100 \times mass\ Jupiter$ as shown in Figure 6. That is the cut of range separates planets within $1.898 \times 10^{25} kg$ of each other. Our choice of ϵ and δ was based on eyeballing the orders of magnitude in the data in the Exoplanet database. Secondly, with respect to our solar system our choice of ϵ and δ would disconnect Earth from Jupiter, thereby preserving the inner and outer structuring of our solar system which we found in section 3.1.

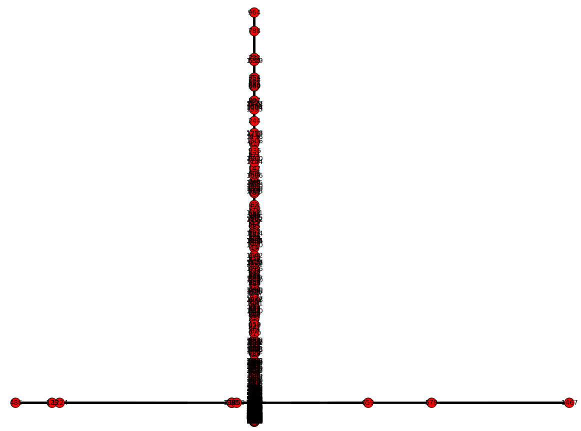

Figure 6: Adjacency matrix of periods Defined as having a connected edge if the periods are within 5 days of each other and mass within 1/100 the mass of Jupiter

The cuts depicted in Figure 6 are interesting in that they show there is a distinct separation into two clusters among planets. The density of the vertical component shows that this clustering is more than just separation by mass and period. The sparsity of points along the horizontal component shows us strong outliers exist in our data.

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