Chaotic Motion

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Phys 457W Experimental Physics – Spring 2017

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Abstract:

In this paper, for the purpose of better understanding chaotic motion and resonance, we investigate the linear and non-linear torsional pendulum undergoing natural, damped, and driven oscillations. We allowed our linear pendulum, without driving and with only frictional damping, to oscillate until it came to rest and found the natural frequency of our linear system to be 0.713 ± 0.004 Hz. When testing the damped, driven oscillations of the linear pendulum we found a resonant frequency of 0.73 ± 0.01 Hz. While measuring the driving frequency response of the damped, non-linear oscillator we saw the amplitude drop sharply just beyond resonance while decreasing driving frequency, but there was no such drastic change in amplitude observed during our run of increasing driving frequency. We were able to achieve sustained chaotic motion with our nonlinear pendulum at a driving frequency of 0.85 ± 0.01 Hz and a driving arm length of 6.0 ± 0.2 cm.

I. Introduction

In this experiment we explored the linear and non-linear motion of a torsional pendulum. Our pendulum without a point mass has an angular velocity which is linearly dependent on theta, but we also investigated these oscillations with the addition of a point mass which provides the non-linear component. Under the right initial conditions, this nonlinear motion can give rise to chaotic motion. Chaotic systems are defined by a sensitivity to initial conditions, topological mixing, and dense periodic orbits. The system evolves over time such that any given region of its phase space will eventually overlap with any other given region, and every point in the space is approached arbitrarily closely by periodic orbits.[2] True harmonic oscillators have what is called "steady-state" motion that, if disturbed, will eventually return to simple harmonic motion. However, in a chaotic system, if the system is perturbed you can potentially recover a completely different type of motion. Since chaotic systems are completely deterministic we are able to explicitly derive the equations which govern its motion, despite the system being entirely unpredictable. Nevertheless, we will be unable to find any analytic solutions or closed form expressions for $\theta(t)$, which is measured clockwise from the top of the wheel on our pendulum.[1]

Our goal was firstly to find the natural frequency of the linear system by measuring and converting the average period of oscillation as the system comes back to rest after some initial displacement. Using this frequency and other measurements of our initial conditions we are able to map the potential energy of the system as a function of theta. With this knowledge, we can better identify the chaotic attractors of the system which will help us in achieving chaotic motion. A chaotic attractor is a point, or set of points, towards which the chaotic system tends to evolve for a variety of starting conditions. Next, as an additional tool in mapping the potential energy of the system, we aimed to find the resonant frequency of the damped and driven linear system by varying the driving frequency and measuring the amplitude of oscillations. The driving frequency is defined as the frequency with which the arm of our driving motor, situated at the left base of our pendulum, drove the string in order to oscillate the pendulum. The driving frequency which produces the greatest response amplitude is identified as the resonant frequency. We performed a similar procedure with the non-linear system, but this time identifying the resonant frequency qualitatively from a single run of varied driving frequency, as opposed to multiple runs of a single driving frequency. We observed the change in amplitude as we gradually increased driving frequency and also while gradually decreasing.

The equilibrium points of our non-linear system were identified qualitatively on a plot of angle versus time which was recorded while oscillating the point mass about each stable oscillation point without driving. Lastly, beginning with our point mass positioned at the left of the two equilibrium points and by adjusting the driving frequency and driving arm length, we achieved sustained chaotic motion.

II. Theory

In order to derive the equations of motion for our system we may begin by analyzing the torque τ. Our total torque will have four components: a gravitational component τ_g, a component accounting for the driven spring τ_s, a component accounting for the change in the length of the pendulum τ_ℓ, and non-conservative damping torques due to the eddy currents of the magnet and axle friction which I shall collectively refer to as τ_d. That last component is going to depend on our identified resonant frequency of the system. Below I derive these components and arrive at an expression for total torque.

$$\tau_g = \vec{\ell} \times \vec{F} = |\vec{\ell}||\vec{F}| \sin\theta = (\ell)(mg)\sin\theta \tag{1}$$

Where the magnitude of the moment arm $\vec{\ell}$ is the length of the pendulum, m is the mass of the magnetic drag wheel, and the magnitude of force is standard gravitational force. For the next component we begin by applying an angular form of Hooke's law.

$$\tau_s = k\theta = k(r \cdot d) = kr(A\cos(\Omega_D t)) \tag{2}$$

Where θ is measured from the top center of the magnetic drag wheel and equal to the radius of the wheel r multiplied by the displacement of the driven end of the spring d; Ω_D represents our driven angular velocity. For the next component τ_ℓ the magnitude of the moment arm is the radius of the magnetic drag wheel and the spring force is entirely perpendicular to the moment arm, therefore

$$\tau_\ell = rF = r(kx) = rk(r\theta) = kr^2\theta \;\rightarrow\; \tau_\ell = -2kr^2\theta \tag{3}$$

We multiply the result for τ_ℓ by two in order to account for both springs and we change the sign because it is acting against the previous two components. Lastly, the torque due to induced eddy-currents increases with angular velocity and opposes motion, and thus can be written as (-bω). The friction on the axle bearings causes a constant damping torque b' that is always opposite the angular velocity. Thus, we can write the final component as

$$\tau_d = -b'(\sin\omega) + (-b\omega) \tag{4}$$

Therefor,

$$\tau = mgl\sin\theta + krA\cos(\Omega_D t) - 2kr^2\theta - b'\sin\omega - b\omega \tag{5}$$

To derive the potential energy of the system without driving as a function of theta we may begin by breaking it down into two components. The first is spring potential energy which is doubled to account for both springs, and the second is the gravitational potential energy. Combining these, we find

$$V = 2\left(\frac{1}{2}kx^2\right) + mgh = k(r\theta)^2 + mg(\ell\cos\theta) \tag{6}$$

$$V(\theta) = kr^2\theta^2 + mg\ell\cos\theta \tag{7}$$

Where m is the point mass. Note that to map the potential of our system we must find the total energy of the system and subtract the measured rotational kinetic energy. The total energy of the system is equal to the maximum possible potential energy, which occurs at a full π radians.

$$\bar{V}(\omega) = V(\pi) - \frac{1}{2}I\omega^2 = V(\pi) - \frac{1}{2}\left(\frac{1}{2}Mr^2\right)\omega^2 = kr^2\pi^2 - mg\ell - \frac{1}{4}Mr^2\omega^2 \tag{8}$$

Where M is the mass of the magnetic drag wheel. To find the minima of the system we derive (7) and set it equal to zero.

$$dV = 2kr^2\theta - mg\ell\sin\theta = 0 \;\rightarrow\; \frac{\sin\theta}{\theta} = \frac{2kr^2}{mgl} \tag{9}$$

Notice that if $2kr^2 < mgl$ then if follows that $\sin\theta < \theta$, for which there can be two possible values of theta of equal magnitude and opposite sign, and therefor the system will have two equilibria. Conversely, if $2kr^2 \geq mgl$ then $\sin\theta \geq \theta$ for which there can be only one solution and thus only one equilibrium.

We expected that the natural frequency of our non-linear system with only frictional damping and without driving would be just slightly greater than the resonant frequency found for the same system with driving. This is because in an ideal system, where the damping factor is exactly zero, the two values should be one and the same. However, our system will always have some damping factor even without the magnetic drag applied, primarily due to the friction on the axle. As the damping factor b increases we expected the resonant frequency ω_r to decrease due to the following relationship:[3]

$$\omega_r = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \tag{10}$$

This value of ω can then be used for the calculations of $\bar{V}(\omega)$ and τ_d which will in turn give us a complete set of equations of motion for the system.

III. Materials and Methods

For each of our experiments we analyzed the motion of a torsional pendulum which was assembled according to Appendix 1 of the lab manual and is depicted in Figure 1.[1] The apparatus was connected to a PASCO 850 Universal Interface which collected and relayed the data to the PASCO Capstone software. Using the same interface, we were able to control the driving frequency of our apparatus. The mechanical driver, whose arm length was one of two conditions altered in order to achieve chaotic motion, is attached with string to the left spring. String attached to the other side of that same spring extends up to and over the magnetic drag wheel, which holds the detachable point mass necessary for non-linear motion. The second spring attached to that string is in turn secured at the right base with more string, the length of which is second condition altered in order to achieve chaotic motion.

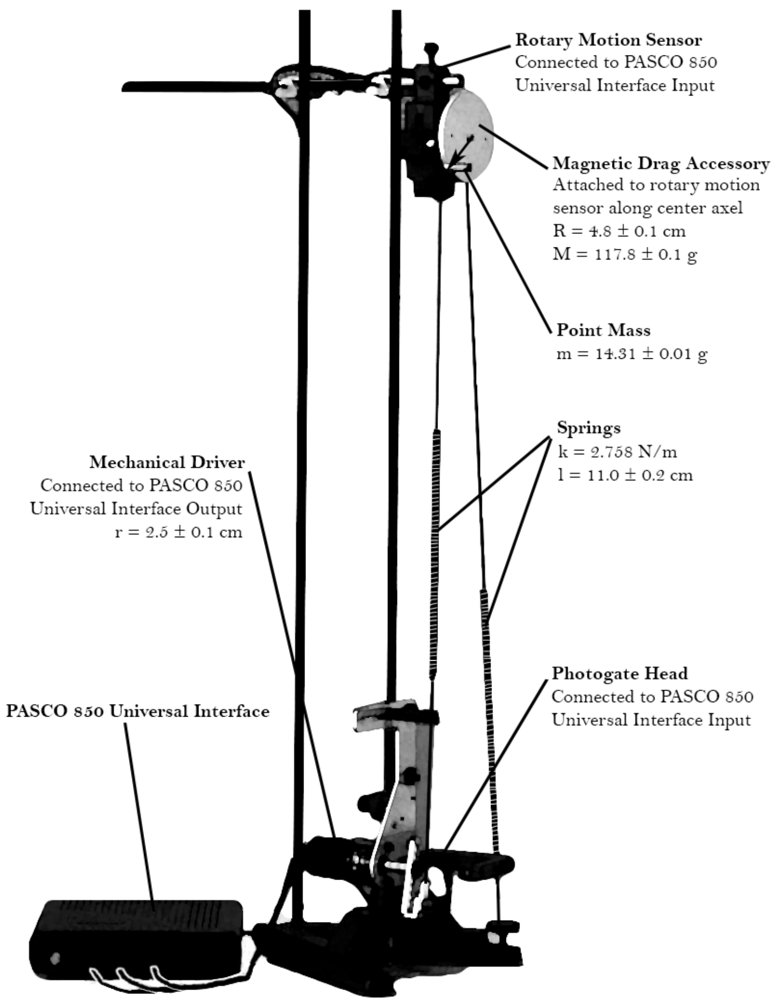


Figure 1 - Diagram of Torsional Pendulum Apparatus

In order to find the natural frequency of our system we performed repeated trials wherein we displaced the drag wheel and allowed the system to oscillate to rest. On the resulting plot of angle versus time we measure the average period, the inversion of which is the natural frequency.

Next, after adding some additional damping factor by moving the damping magnet closer to the drag wheel, we turned on the mechanical driver and varied the driving frequency. For each frequency we measured the average amplitude on the angle versus time plot. The resonant frequency of the system is the driving frequency which was found to produce the greatest amplitude.

Then we adjusted our pendulum such that the hole for the point mass rests at the top and following the attachment of the point mass we then had two stable equilibrium points at which the point mass could stay positioned at rest. Turning off the driving motor, we oscillate the point mass around each stable point consecutively and record angle versus time as well as angular velocity versus angle for analysis.

Next, turning the driving motor on again, we record two separate runs of angle vs time. For the first run we begin at a driving frequency well above resonance and decrement until sufficiently past resonance. For the second run we begin below resonance and increment until sufficiently past resonance. Amplitude at each driving frequency is averaged from a single crest and a single troph and the results are added to a plot of amplitude versus driving frequency.

Lastly, we modified the initial conditions of our system in order to produce chaotic motion. The conditions which were modified are the driving arm length and the length of string from the bottom of the right spring to the base of the apparatus. The resulting motion is identified as chaotic if it produces a phase space with considerable topological mixing without coming to rest at some well-defined sinusoidal motion.

IV. Results, Analysis and Data

For the first of our experiments we identified the natural frequency of the linear system with only frictional damping and without driving to be 0.713 ± 0.004 Hz. The data from which this value was extracted from can be found on Table 1 in the appendix.

The potential energy of our system in this same configuration can be mapped using (8) and plugging in the measurements of our apparatus. We arrive at

$$\bar{V}(\omega) = 0.0627157 - 0.0000679\omega^2 \tag{11}$$

With the addition of magnetic damping and driving to the linear system we recorded average amplitude per driving frequency on Table 2 in the appendix. The values are plotted in Figure 2, where a noticeable spike can be seen at resonance.

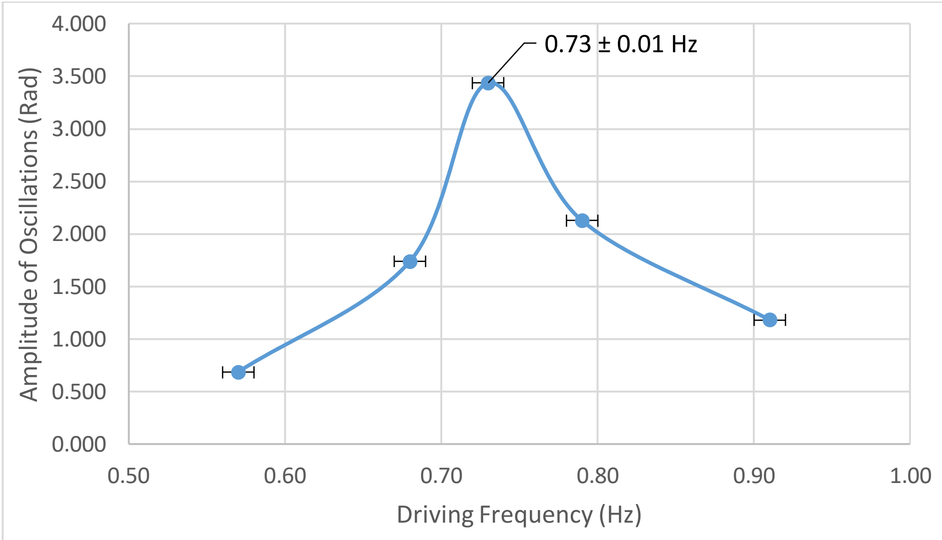


Figure 2 - Amplitude vs Driving Frequency for the Damped, Driven, Linear System

From this we concluded the resonance of our system is around the point 0.73 ± 0.01 Hz, but due to the low resolution of this plot it is still possible that the maximum amplitude occurs somewhere between this point and the two neighboring points. Therefor we should increase our error bar to encompass this entire range, such that resonance is equal to 0.73 ± 0.06 Hz. Within this new extended error bar our measurement aligns with expectations for the resonant frequency to be near to the natural frequency of the system without driving. This value is higher than expected, as the resonant frequency should theoretically be no higher than the natural frequency. However, with this increased error bar there is still a small range within which our predictions are valid if resonance were to be found there. If resonance was greater than our natural frequency than it is safe to assume that the apparatus had been tampered with between the measurement of natural frequency and that of resonant frequency, which occurred on separate days.

The equilibrium points of the non-linear system were experimentally determined to be at ± (2.04 ± 0.04) radians. This agrees, at least within the bounds of the error bars, with the relationship derived in (9) where we see the left half of the equation come to 0.4372 ± 0.0174, while the right half comes to 0.4728 ± 0.0284.

While decrementing driving frequency in 0.05 Hz steps we recorded amplitude versus time. The average amplitude versus driving frequency was compiled into Figure 3, where we observe a significant drop in amplitude just below resonance.

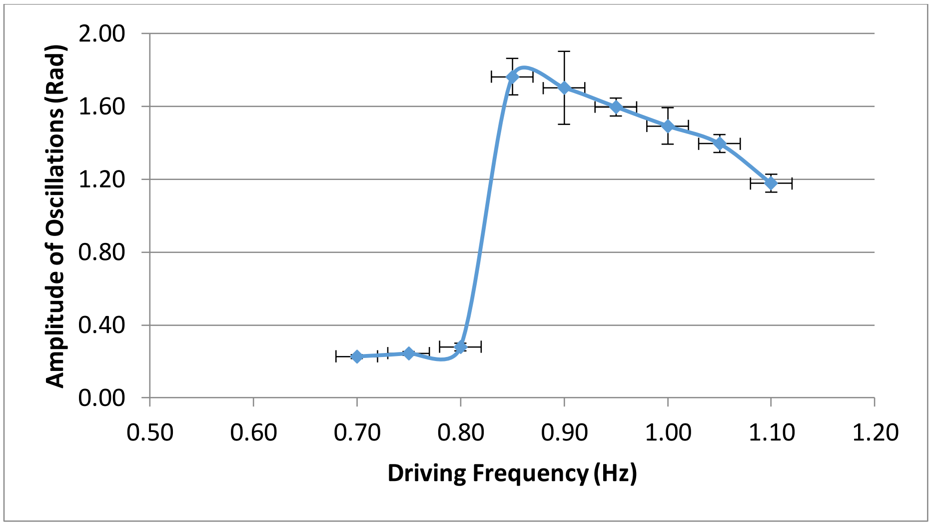


Figure 3 - Amplitude vs Driving Frequency while decrementing

While incrementing driving frequency in 0.05 Hz steps we recorded amplitude versus time. The average amplitude versus driving frequency was compiled into Figure 4, where we saw a more gradual change in amplitude as compared to Figure 3. This is because there are two types of motion which can be exhibited about each equilibrium point, and by changing the direction of the change in amplitude we were able to observe both.

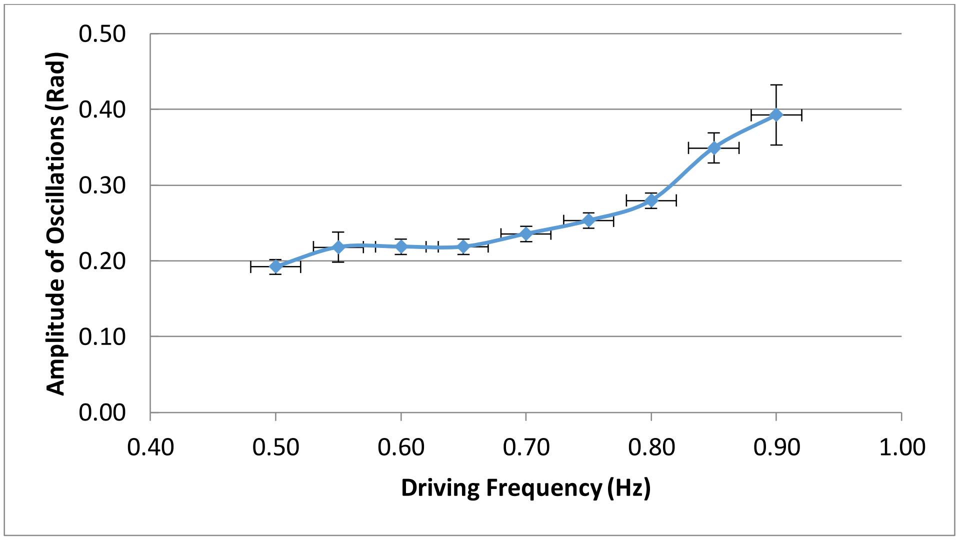


Figure 4 - Amplitude vs Driving Frequency while incrementing

As a means of comparison, Figures 3 and 4 are combined into a single Figure 5.

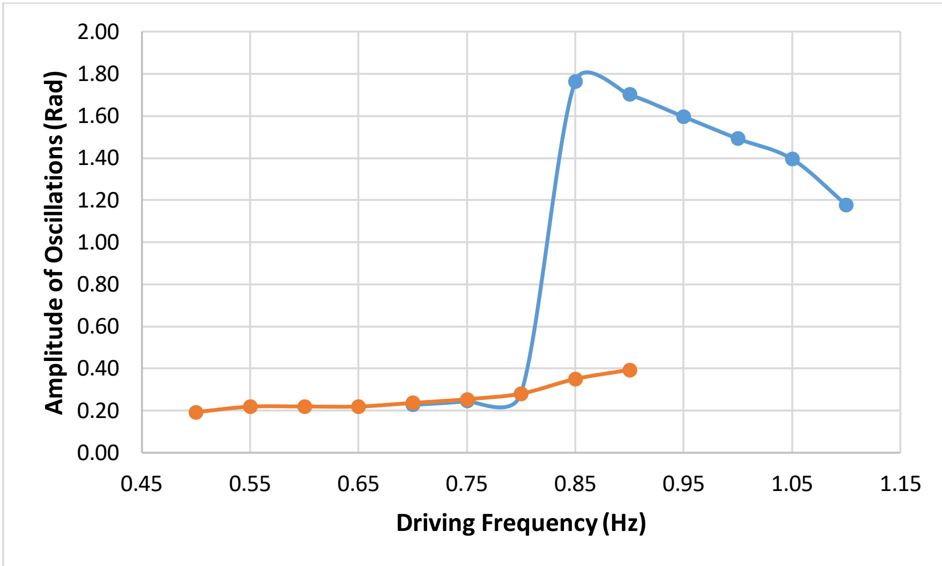


Figure 5 - Amplitude vs Driving Frequency, blue indicates decremented data, orange indicates incremented data

Lastly, we were able to achieve chaotic motion with our non-linear pendulum using a driving arm length of 6.0 ± 0.2 cm and driving frequency of 0.85 ± 0.01 Hz. The resulting Poincare plot forms the peculiar pattern shown in Figure 6.

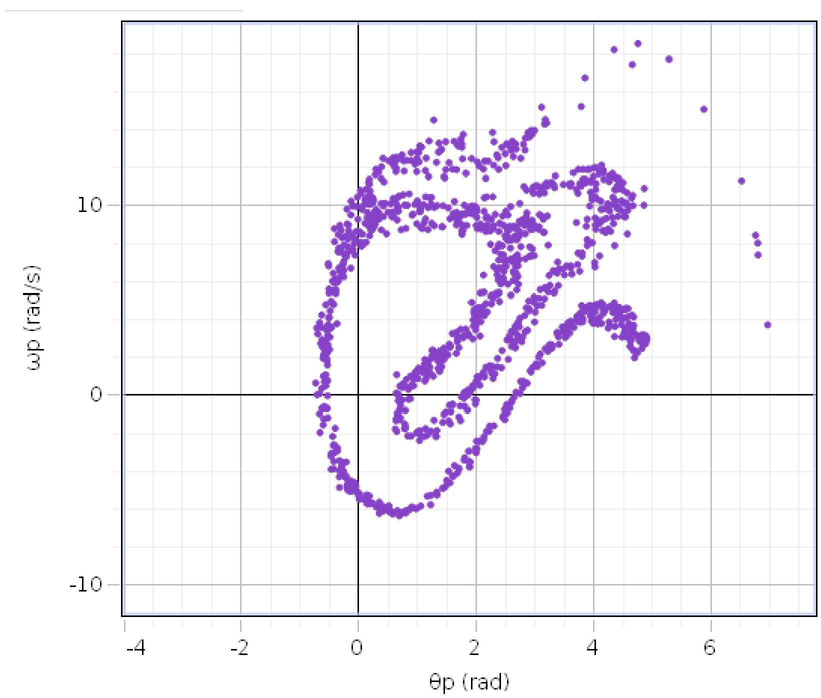


Figure 6 - Angular Velocity vs Angle of chaotic non-linear pendulum

The potential energy vs angle plot seen in Figure 7 displays the well-defined shape of our potential well. In this case it slopes to the left, but theoretically an ideal chaotic system would balance the well on either side. This would mean that probabilities of any given potential energy would be balanced on the left and right sides of the wheel. That said, our system sustained chaotic motion for the entire duration of our recording with no signs of settling to any discernable oscillatory pattern.

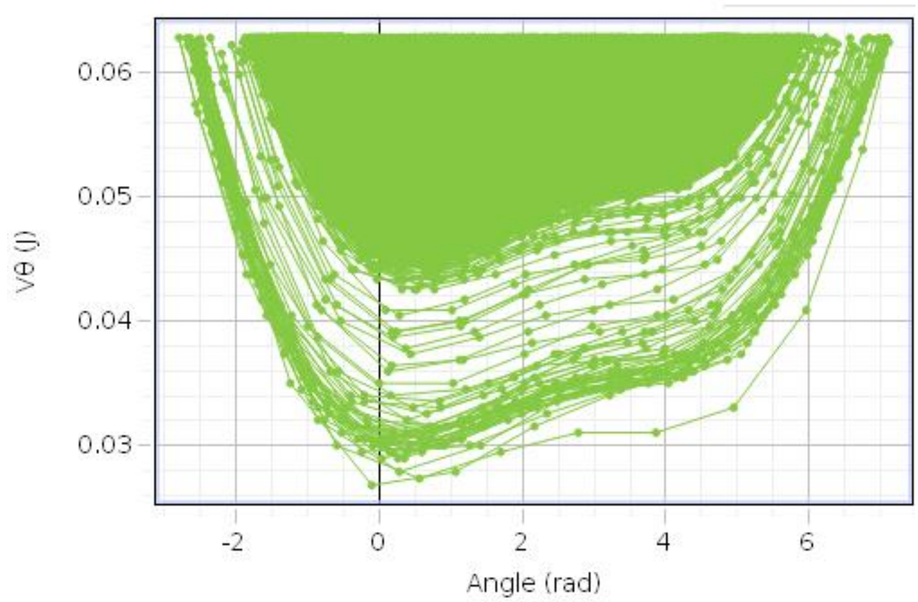


Figure 7 - Potential Energy vs Angle for chaotic non-linear pendulum

V. Conclusion

The results of each experiment panned out quite nicely. The resonant frequency of the damped and driven linear system fell close to the natural frequency of the linear system with only frictional driving and no driving, which is in line with my predictions for the experiment. As for the resonant frequency of the damped and driven nonlinear pendulum, the results defied my prediction which was to see a large jump around resonance regardless of the direction in which amplitude was changed. That said, these results are in line with experimental theory which allows for two types of motion exhibited around resonance. Lastly, our mapping of potential energy helped us to find a sustained chaotic motion for the damped and driven nonlinear system by balancing the chaotic attractors. Our results saw a slight favoring of the left equilibrium point, but the motion was fully chaotic and uninterrupted nonetheless.

VI. References

1. *Chaotic Pendulum* [PDF]. (n.d.). PSU Experimental Physics Labs.
2. Hasselblatt, B., Katok, A. B. (2010). *A First course in dynamics: with a panorama of recent developments*. Cambridge: Cambridge University Press.
3. *Damped Harmonic Oscillator*. (n.d.). Retrieved March 02, 2017, from http://hyperphysics.phy-astr.gsu.edu/hbase/oscda.html

VII. Appendix

Table 1 - Natural frequency data for the linear system with only frictional damping and no driving

Trial Peaks [s] (±0.05)	1	2	3	4	5
	0.90	0.90	1.05	1.35	1.20
	2.30	2.30	2.40	2.75	2.65
	3.65	3.70	3.85	4.20	4.05
	5.05	5.10	5.25	5.60	5.45

Period [s] (±0.07)					
	1.40	1.40	1.35	1.40	1.45
	1.35	1.40	1.45	1.45	1.40
	1.40	1.40	1.40	1.40	1.40

Frequency [Hz]					
	0.714	0.714	0.741	0.714	0.690
	0.741	0.714	0.690	0.690	0.714
	0.714	0.714	0.714	0.714	0.714

Table 2 - Resonance data for the damped and driven linear system

Frequency (Hz)	δ(Frequency)	Amplitude (Rad)	δ(Amplitude)
0.57	0.01	0.684	0.003
0.68	0.01	1.740	0.011
0.73	0.01	3.437	0.005
0.79	0.01	2.128	0.013
0.91	0.01	1.182	0.006

Sam Lucas
2/6/17

Partner:
Alex Cook

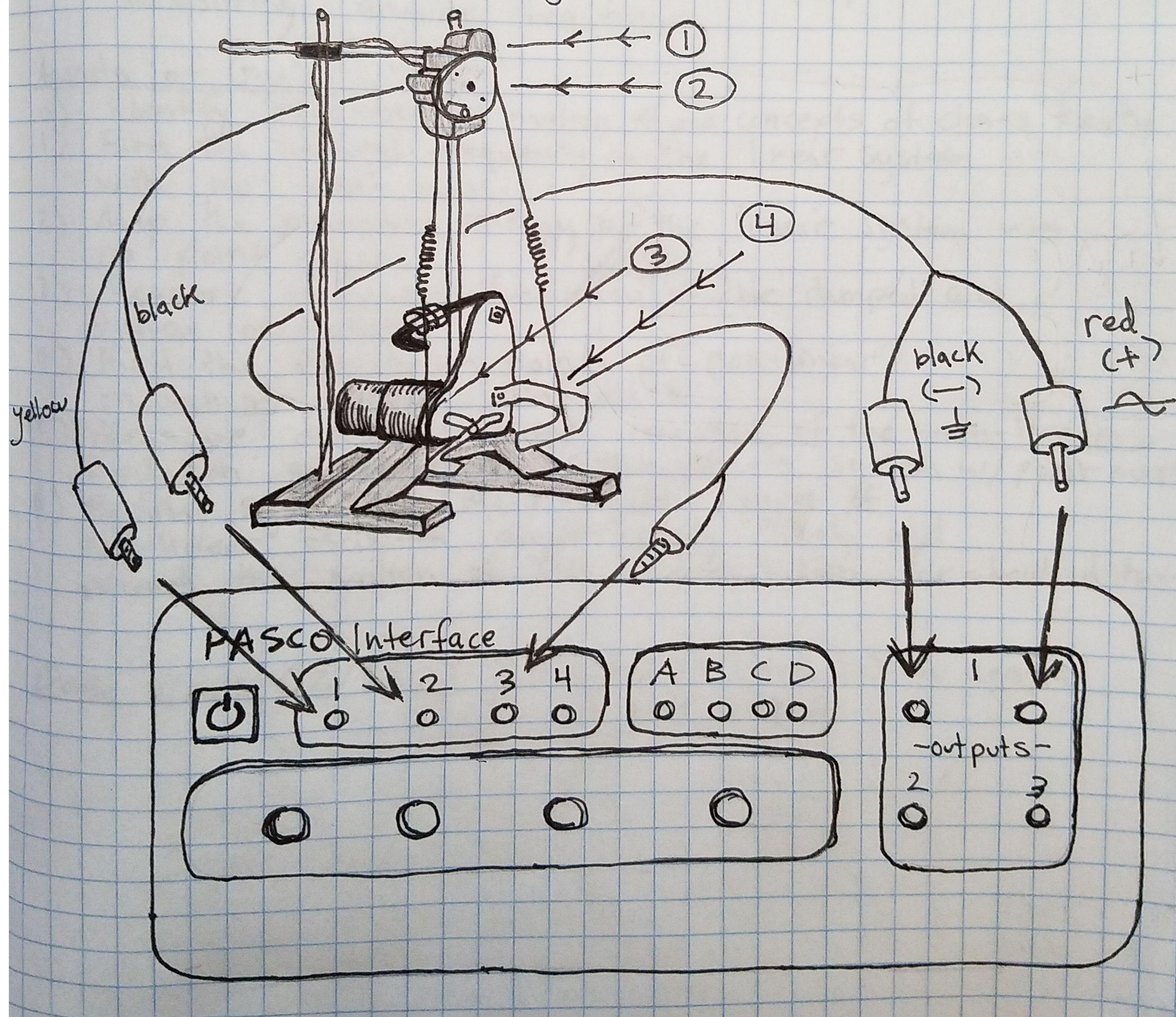
Chaos

Purpose of Experiment: To explore ~~linear~~ the motion of a torsional pendulum undergoing natural oscillations, damped and driven oscillations, and chaotic motion.

References: Chaotic Pendulum, pdf, psu.app.box.com/v/phys457-chaoslab

Daily Goals: To familiarize with the experiment, setup, concepts in harmonic motion and chaotic motion and to lay out an agenda of goals.

Setup: Apparatus assembly procedure found on pg. 8-9 of the experiment guide.



① Rotary motion sensor, attached to ~~slots~~ digital ~~■~~ input slots 1 and 2 on PASCO 850 Universal Interface.

② Magnetic drag accessory attached to rotary motion sensor for the purpose of applying and controlling a damping factor in the simple harmonic motion.

③ Mechanical driver, connected to both number 1 output slots on the PASCO Interface, for the purpose of applying and controlling a driving factor in the oscillatory motion.

④ Photogate head, connected to digital input slot 3 of the PASCO interface, for the purpose of measuring angular frequency

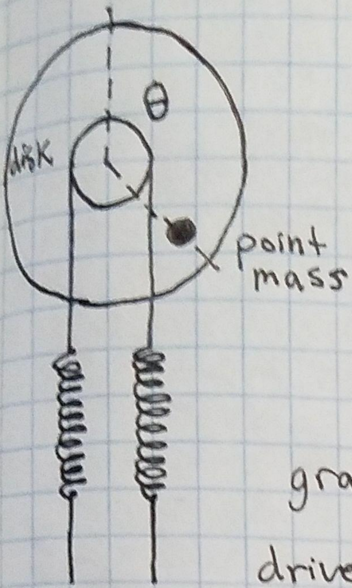
Agenda of Goals:

0) Identify equations of motion ~~■~~ and concepts of chaos theory
1) Find the natural frequency of the linear system with no point mass.
2) Map the potential energy of the linear system with no point mass.
3) Measure resonant frequency of the damped and driven oscillations
4) Find the equilibrium points of non-linear oscillations, using point mass.
5) Measure driving frequency response of the non-linear oscillation amplitude for a non-chaotic system w/ point mass.
6) Modify system to make all four types of non-linear oscillation about equally likely and record the motion of this chaotic system for about an hour.

Daily Goals (2/8/17): • accomplish item (0) on agenda of goals above

Chaos Theory:

initial position – θ measured clockwise from this point



driven angular ~~frequency~~ velocity $\Omega_D = \varphi / t$

φ = angle of drive shaft where $\varphi = 0$ @ $t = 0$

displacement of driven end of spring $d = A\cos(\Omega t)$

torque $\tau = (\text{moment arm}) \times (\text{force}) = \vec{r} \times \vec{F}$
$= lF\sin\theta$

gravity component of $\tau \rightarrow F = mg \rightarrow \tau = mgl\sin\theta$

driven spring component $\rightarrow = Kx$ where x is displacement
$x = d = A\cos(\Omega t) \rightarrow \tau = KrA\cos(\Omega t)$

component accounting for change in $l \rightarrow = Kx = K(rF)$
F here $= r\theta \rightarrow \tau = Kr^2\theta$
also, make it negative since it acts against the other components, and multiply by 2 since this component applies to both springs.

so, all together $\tau = -2Kr^2\theta + KrA\cos(\Omega_D t) + mgl\sin(\theta)$

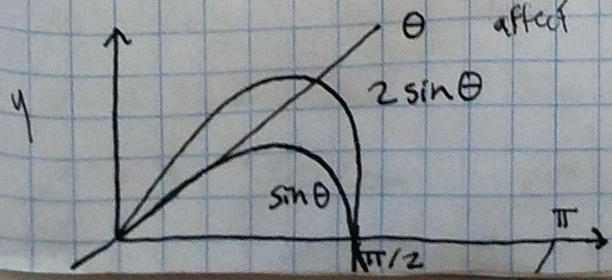
For a system w/o driving $(d=0)$ $V = 2(\frac{1}{2}Kx^2) + mgh$
where the first component is multiplied by 2 to account for both springs. Since x here $= r\theta$ and $h = l\cos(\theta)$ we get
$$V = Kr^2\theta^2 + mgl\cos(\theta)$$

To find minima of this system we derive this eq. and set it equal to zero.

$$dV = 2Kr^2\theta - mgl\sin(\theta) = 0 \rightarrow \frac{\theta}{\sin\theta} = \frac{mgl}{2Kr^2}$$

$$\text{or } \theta = \left(\frac{mgl}{2Kr^2}\right)\sin\theta$$

Using this relation we can see how the applitude of the sin function will affect the number of minima recieved.



Note how a larger ratio (> 1) allows the sin curve to cross the theta line twice, but a ratio of ≤ 1 would allow only a single cross at 0.

net non-conservative damping torque $\tau_{nc} = -b'(\sin\omega) + (-b\omega)$

or $\tau_{nc} = -b'(\omega/|\omega|) + (-b\omega)$

where $(-b\omega)$ is damping torque created by induced eddy-currents when a magnet is brought near the rotating aluminium disk, and the other compent is the result of friction on the axel ~~bearings~~ bearings.

~~dam~~ Daily Goals (2/13/17): • finish notes on chaos theory and equations of motion
- find the natural frequency of the linear system
- map the potential energy of the linear system
- parts 4 and beyond in the experiment guide if time allows

Equations of motion for damped/driven system:

$$\frac{d\theta}{dt} = \omega \qquad \frac{d\phi}{dt} = \Omega_D$$

$$\alpha = \frac{\tau}{I} = \frac{d\omega}{dt} = -\frac{b}{I}\,\omega\sin(\omega) - \frac{2Kr^2}{I}\theta + \frac{mgl}{I}\sin\theta + \frac{KrA}{I}\cos(\Omega t)$$

equations of motion about minima:

$$\alpha_{near\ \theta_0} = \frac{\tau_{near\ \theta_0}}{I} = \tau_{conserved\ -near\ \theta_0} + \tau_{nonconserved} + \tau_{driving}$$

$$\frac{d^2(\theta-\theta_0)}{dt^2} = -\left(\frac{2Kr^2}{I} + \frac{mgl}{I}\cos\theta_0\right)(\theta-\theta_0)$$

$$-\frac{b}{I}\frac{d(\theta-\theta_0)}{dt} + \frac{KrA}{I}\cos(\Omega_D t)$$

Method/Procedure: Apparatus assembled and setup according to Appendix 1 of experiment guide.

Measuring K of Springs: ~~0.048 ± 0.001m~~ ~~θ_0 = 180 ± 5~~

We hung a mass from the spring and measured displacement.

$l_0 = 11 \pm 0.2$ cm $\qquad F = Kx = mg \qquad m = 1.97\,g = 0.00197\,Kg$

$g = 9.8\ m/s^2$

$l' = 11.7 \pm 0.2$ cm $\qquad K = \frac{mg}{x} = \frac{(0.00197)(9.8)}{(0.007)} = \boxed{2.758\ N/m}$

measuring V & V_0:

$r = 4.8 \pm 0.1$ cm $= 0.048 \pm 0.001$ m
$\theta_0 = 180 \pm 5°$ $k = 2.758$

$$V_0 = kr^2\theta^2 = (2.758)(0.048)^2(\pi)^2 = 0.0627 \text{ J}$$

$$V(\theta) = V_0 - KE_{rotation} = kr^2\theta^2 - \frac{1}{2}I\omega^2$$

$$= kr^2\theta^2 - \frac{1}{2}\left(\frac{1}{2}Mr^2\right)\omega^2 \quad (\theta/t)^2$$

V_0

$$\boxed{V(\theta) = kr^2\theta^2 - \frac{1}{4}Mr^2\omega^2}$$

where M is mass of magnetic drag wheel (#2 on diagram drawn on pg 13)

In our runs for experiment #2 in the packet we measure ω to find $V(\theta)$

$M = (117.8 \pm 0.1)$g
$= (0.1178 \pm 0.0001)$ kg

$$V(\theta) = V_0 - \frac{1}{4}Mr^2\cancel{\left(\frac{\theta}{t}\right)^2}\left(\frac{\theta-\theta_0}{t}\right)^2$$

Finding natural frequency:

We ran 5 trials with ~~a~~ the ~~undamp~~ linear pendulum (no point mass or driving) and allowed the system to oscillate until it came to rest. We then measured period on the Angle vs. Time graph ~~by~~ using the maximum peaks as points of reference. Below we recorded the time of each peak, followed by a table ~~which~~ of the periods calculated from their differences. Then all those measurements for period were averaged. Natural frequency could then be found by $T = 1/f$ or $f = 1/T$.

	run 1	run 2	run 3	run 4	run 5
1st peak	0.9	0.9	1.05	1.35	1.2
2nd peak	2.3	2.3	2.4	2.75	2.65
3rd peak	3.65	3.7	3.85	4.2	4.05
4th peak	5.05	5.1	5.25	5.6	5.45

1→2	1.4	1.4	1.35	1.4	1.45
2→3	1.35	1.4	1.45	1.45	1.4
3→4	1.4	1.4	1.4	1.4	1.4

average $T = 1.404$ sec

error bar $= \frac{\sigma}{\sqrt{n}}$

for $n = 15$ data points

~~$\sigma = 0.0297$~~

~~error bar~~ $= \cancel{\frac{0.0297}{\sqrt{15}}} = \cancel{0.01}$

$f = 1/T$

$\therefore$ ~~natural frequency~~ ~~(1.40 ± 0.01) s~~

$$\boxed{\text{natural frequency } f = 0.713 \pm 0.004 \text{ Hz}}$$

converted to f we get $f_{avg} = 0.713$ Hz
and we get $\sigma = 0.015 \rightarrow$ error bar $= \frac{0.015}{\sqrt{15}} = 0.004$

Daily Goals (2/15/17): • resonant freq measurements for damped, driven oscillations
• non-linear oscillations (using point mass)
• hopefully begin identifying resonance in non-linear oscillator

Below we measure the crests and trophs of the sin wave (Angle vs. Time) and average their magnitudes to find the approximate amplitude for a given driving frequency. The averages of those amplitudes, over 10 runs per frequency, are then plotted (amplitude vs. frequency) ~~against~~ and from that graph a resonant frequency can be identified as the largest peak.

peak	3V → 0.57 H	amp	3.5V → 0.68 Hz
1	(.785 + .576)/2 =	0.681	(1.972 + 1.571)/2 =
2	(.768 + .628)/2 =		(1.955 + 1.571)/2 =
3	(.785 + .576)/2 =	0.681	(1.955 + 1.571)/2 =
4	(.768 + .628)/2 =		(1.955 + 1.571)/2 =
5	(.785 + .576)/2 =	0.681	(1.955 + 1.571)/2 =
6	(.768 + .611)/2 =		(1.955 + 1.571)/2 =
7	(.768 + .576)/2 =	0.672	(1.937 + 1.553)/2 =
8	(.768 + .611)/2 =		(1.902 + 1.518)/2 =
9	(.768 + .576)/2 =	0.672	(1.868 + 1.484)/2 =
10	(.768 + .593)/2 =		(1.868 + 1.501)/2 =

peak	4V → 0.79 Hz	4.5V → 0.91 Hz
1	(1.885 + 2.305)/2 = 2.095	(1.204 + 1.222)/2 =
2	(1.902 + 2.321)/2 =	(1.169 + 1.204)/2 =
3	(1.902 + 2.321)/2 =	(1.169 + 1.204)/2 =
4	(1.920 + 2.321)/2 =	(1.169 + 1.204)/2 =
5	(1.902 + 2.304)/2 =	(1.187 + 1.222)/2 =
6	(1.885 + 2.304)/2 =	(1.187 + 1.204)/2 =
7	(1.902 + 2.321)/2 =	(1.152 + 1.187)/2 =
8	(1.937 + 2.339)/2 =	(1.134 + 1.169)/2 =
9	(1.972 + 2.391)/2 =	(1.134 + 1.187)/2 =
10	(2.007 + 2.426)/2 = 2.217	(1.152 + 1.187)/2 =

format: (crest − troph)/2 = avg. amplitude

Daily Goals (2/20/17): • measure driving-frequency response of the non-linear oscillator amplitude
• Identify the 4 steady state attractors by varying initial conditions
• Balance attractors and achieve chaotic motion

freq (Hz)	time (s)	amplitude (rads)	error (rads)
1.10	15	(1.239 + 1.117) / 2 = 1.178	0.035
1.05	30	(1.553 + 1.239) / 2 = 1.396	0.05
1.00	50	(1.728 + 1.257) / 2 = 1.493	0.10
0.95	70	(1.885 + 1.309) / 2 = 1.597	0.05
0.90	90	(2.059 + 1.344) / 2 = 1.702	0.20
0.85	105	(2.147 + 1.379) / 2 = 1.763	0.10
0.80	120	(−0.017 + 0.576) / 2 = 0.280	0.02
0.75	140	(−0.07 + 0.559) / 2 = 0.245	0.01
0.70	160	(−0.087 + 0.541) / 2 = 0.227	0.01

$\Omega_D = 1.10$ • was achieved by applying 5.4V to driver. We found as we were performing to above experiment that a difference of ~0.3V corresponding with the necessary difference of 0.05 Hz in our driving frequency.

freq (Hz)	time (s)	amplitude (rads)	error (rads)
0.50	10	(0.122 + 0.262) / 2 = 0.192	0.01
0.55	25	(0.157 + 0.279) / 2 = 0.218	0.02
0.60	40	(0.105 + 0.332) / 2 = 0.219	0.01
0.65	60	(0.105 + 0.332) / 2 = 0.219	0.01
0.70	75	(0.122 + 0.349) / 2 = 0.236	0.01
0.75	90	(0.157 + 0.349) / 2 = 0.253	0.01
0.80	110	(0.192 + 0.367) / 2 = 0.280	0.01
0.85	130	(0.262 + 0.436) / 2 = 0.349	0.02
0.90	150	(0.314 + 0.471) / 2 = 0.393	0.04

$A = (6.0 \pm 0.2)\,cm$

For part 7 of the experiment, we found an example of chaotic motion with A = 6 cm and starting with the point mass at equilibrium, hanging to the right side. A map of potential wells show the system favors the right half, but chaotic motion persists indefinitely.

Poincare plot does seem to form some vague pattern eventually, but is overall very sporadic.

$\Omega_D = (0.85 \pm 0.01)\,Hz$

peak	3.75 V → 0.73 Hz
1	(3.526 + 3.281) / 2 =
2	(3.543 + 3.316) / 2 =
3	(3.560 + 3.316) / 2 =
4	(3.560 + 3.316) / 2 =
5	(3.578 + 3.334) / 2 =
6	(3.543 + 3.316) / 2 =
7	(3.578 + 3.316) / 2 =
8	(3.543 + 3.316) / 2 =
9	(3.578 + 3.316) / 2 =
10	(3.578 + 3.334) / 2 =

Below we record data for part 6b of experiment guide: *

time (s)	freq (Hz)	amplitude (rads)
0	1.10	(0.925 + 0.401) / 2 =
30	1.05	(1.100 + 0.541) / 2 =
50	1.00	(1.396 + 0.698) / 2 =
80	0.95	(1.658 + 0.803) / 2 =
100	0.90	(1.920 + 0.890) / 2 =
120	0.85	(2.059 + 0.925) / 2 =
140	0.80	(2.251 + 0.970) / 2 =
180	0.75	(2.496 + 1.030) / 2 =
200	0.70	(0.611 + 0.244) / 2 =

Data Void

replaced by first table on next page

↑ the time at which the

It was at this point we observed a nonlinear spike in amplitude before returning to a much smaller stable amplitude.

* this data may need to be retaken next class to ensure the setup is the same for both our trials of inreasing freq as well as dec freq.

we repeated part 6 of the experiment guide on 2/20/17 since the setup varied slightly

our new measurement from the bottom of the right spring ~~and~~ to the base is 10 cm.