# Development of a regularization term in a TLES code in OpenFOAM

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# Abstract

Text

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# 1 Introduction

DNS is computationally expensive, so go TLES!

An brief overview of some of the basic theory underpinning TLES is presented next, adapting from the work presented by Pruett [1] and Jenny [2].

## 1.1 TLES

Since for TLES filtering is done during the course of the numerical experiments, the filtering operation must be *causal*, ie. it must depend only upon the current and previous values of the quantity being filtered and not on any future values. Letting an overbar denote a time-filtered quantity and $T$ denote the characteristic filter width, one obtains the following causal time-filtering operation for some filter kernal $G(t,T)$:

$$\bar{f}(t,T) = \int_{-\infty}^{t} G(\tau - t; T) f(\tau) d\tau \tag{1.1}$$

In this work only the exponential filter is discussed, namely:

$$G(t;T) = \frac{\exp(t/T)}{T} \longrightarrow \bar{f}(t,T) = \frac{1}{T}\int_{-\infty}^{t} \exp\left(\frac{\tau - t}{T}\right) f(\tau) d\tau \tag{1.2}$$

which is a second order low-pass filter and has the transfer function:

$$H(\omega, T) = \frac{1}{1 + iT\omega} \tag{1.3}$$

Since the integral formulation would require storage of the quanitity at all previous time points in the simulation, the following equivalent differential form is used for implementation, as it can be integrated using standard time-marching schemes to update the filtered quantities at each step (where the explicit time-dependence of the quanitites is now dropped for convenience):

$$\frac{\partial \bar{f}}{\partial t} = \frac{f - \bar{f}}{T} \tag{1.4}$$

Applying such a filtering operation to the incompressible Navier-Stokes equations gives the following system for the evolution of temporally-filtered velocity fields, using the same initial condition as for the unfiltered fields $\bar{u}(0,x;T) = u(0,x)$:

$$\frac{\partial \bar{u}_j}{\partial x_j} = 0 \tag{1.5}$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial(\bar{u}_i\bar{u}_j)}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \nu\frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} \tag{1.6}$$

where $u$ is the fluid velocity, $p$ is the pressure, $\nu$ is the kinematic pressure, and subscripts $i, j \in \{1, 2, 3\}$ indicate the 3D cartesian direction. The element $\tau$ in the final term is referred to as the *temporal stress tensor* and is defined as:

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i\bar{u}_j \tag{1.7}$$

Solving the temporally-filtered Navier-Stokes equations requires a residual-stress model to handle the unkown value $\overline{u_i u_j}$ and close the system. Stolz and Adams [3] dveloped an approximate deconvolution model (ADM) for for spatial LES which was then adapted for TLES by Pruett [1]. For this project, however, a new method termed the temporal exact deconvolution model TEDM created by Jenny [2] is used to provide the closure.

For the TEDM method, it is observed that by inserting the velocity field into the differential form of the filter given in Eq. (1.4) the unfiltered field can be recovered as follows:

$$u_i = \bar{u}_i + T\frac{\partial \bar{u}_i}{\partial t} \tag{1.8}$$

The same Eq. (1.4) can then be applied to the unknown quantity $\overline{u_i u_j}$ and the just obtained relation for the unfiltered field in Eq. (1.8) can be inserted in the resulting expression to obtain the following:

$$\begin{aligned}
\frac{\partial \overline{u_i u_j}}{\partial t} &= \frac{u_i u_j - \overline{u_i u_j}}{T} \\
&= \frac{\left(\bar{u}_i + T\frac{\partial \bar{u}_i}{\partial t}\right)\left(\bar{u}_j + T\frac{\partial \bar{u}_j}{\partial t}\right) - \overline{u_i u_j}}{T} \\
&= \frac{\bar{u}_i\bar{u}_j - \overline{u_i u_j}}{T} + \bar{u}_i\frac{\partial \bar{u}_j}{\partial t} + \bar{u}_j\frac{\partial \bar{u}_i}{\partial t} + T\frac{\partial \bar{u}_i}{\partial t}\frac{\partial \bar{u}_j}{\partial t}
\end{aligned} \tag{1.9}$$

This can then be rearranged to produce an equation for the time evolution of the temporal stress tensor $\tau$ containing only the known filtered velocities:

$$\frac{\partial \tau_{ij}}{\partial t} = -\frac{\tau_{ij}}{T} + T\frac{\partial \bar{u}_i}{\partial t}\frac{\partial \bar{u}_j}{\partial t} \tag{1.10}$$

Suitable time integration schemes can then be used to solve Eqs. (1.6) and (1.10) in alternation to evolve the system in time.

## 1.2 Regularization

A regularization term based on work by Åkervik et al. [4] was investigated as a potential means to improve the stability of the method, and is outlined here.

## 1.3 Divergence Cleaning

An implicit condition of the incompressible Navier-Stokes equations is that the divergence of the velocity field must be zero. However, numerical discretization schemes generally lead to non-zero values of the divergence which can be a potential source of isntability during simulations. To this end, explicit divergence cleaning (DC) using the projection scheme, following the procedure outlined in section 5 of the work by Tóth [5] for DC of magnetic fields, was also considered for possible stabilizing effects.

# 2 Implementation

Implementation of the regularization term and divergence cleaning scheme was carried out using the open source OpenFOAM CFD code. For this seminar, a solver implementing the fundamental TLES methodology, which had been develped during a previous project, was used as a starting code.

# 3 Results

All simulations were carried out on the standard Pitz-Daily backward facing step geometry provided with OpenFOAM. The numerical viscosity of $\nu = 10^{-5}m^2/s$ and the inlet velocity of $u = 10m/s$ were kept constant for all runs. Using this inlet velocity and the channel width $L = 50.8mm$, the Reynold's number of the flow can be approximated as:

$$Re = \frac{uL}{\nu} = 50800$$

Since most cases of instability in the simulation manifested within the first few dozen time steps (approx. 30-40 or less), and given the large number of parameter sets that needed to be tested, for the purposes of this resport a simulation was considered 'stable' if it ran for 100 timesteps without failing. It is noted that a small number of the runs performed crashed even after 80+ timesteps, so this 100 timestep rule is certainly not an ironclad guarantee of stability over a much longer run. However, as in all of these cases it was the transition from stable to unstable that was under investigation, it suffices to warn that the reported minimal $\chi$ values required for stability should be regarded as providing only marginal stability, and slightly larger values might be safer in a real simulation to provide some margin of safety.

# 4 Conclusion

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