Development of a regularization term in a TLES code in OpenFOAM

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TLES Introduction

- Re^3 scaling prohibits DNS for high Re
- LES is a commonly used class of methods
  - Only large features resolved, allowing coarser mesh
  - Effect of small features must be modelled → requires closure
- Filtering is used to separate (un)resolved scales

- Conventionally done in spatial domain, but temporal filtering may offer certain advantages
- Implicit assumption that removal of high frequency components will also remove high wavenumber features

TLES Motivation

- Potential Advantages of TLES over LES [1]:

1. Natural linkage to RANS (also uses time filtering)
2. Experimental Data often acquired in time domain
3. Commutation error is problematic for spatial filtering on finite domains or highly stretched grids
4. Filter width should be significantly larger than grid spacing for LES, often not possible in practice
5. More amenable to time-dependent point sources

TLES Theory: Filtering

- Require a causal filter

$$\bar{f}(t,T) = \int_{-\infty}^{t} G(\tau - t; T) f(\tau)\,\mathrm{d}\tau$$

- We use the exponential filter

$$G(t;T) = \frac{\exp(t/T)}{T}$$

$$\bar{f}(t,T) = \frac{1}{T}\int_{-\infty}^{t} \exp\left(\frac{\tau - t}{T}\right) f(\tau)\,\mathrm{d}\tau$$

TLES Theory: Filtering

- Second-order, with the transfer function

$$H(\omega, T) = \frac{1}{1 + iT\omega}$$

- Integral form needs storage of all previous values

$$\frac{\partial \bar{f}}{\partial t} = \frac{f - \bar{f}}{T}$$

- Differential form only subject to time integration scheme

TLES Theory: Closure Problem

- Filtering Navier-Stokes equations gives

$$\frac{\partial \bar{u}_j}{\partial x_j} = 0$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial(\bar{u}_i \bar{u}_j)}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}$$

- Where τ_{ij} is the *temporal stress tensor*

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$$

TLES Theory: Exact Deconvolution

- Stolz and Adams [2] developed an approximate deconvolution model (ADM) for LES
  - Uses linear combination of multiply filtered quantities
  - Extended to TLES by Pruett [1] as temporal ADM (TADM)
- Prof. Jenny [3] proposed an exact deconvolution for the exponential filter (TEDM)

$$\frac{\partial \bar{f}}{\partial t} = \frac{f - \bar{f}}{T} \quad \rightarrow \quad u_i = \bar{u}_i + T\frac{\partial \bar{u}_i}{\partial t}$$

TLES Theory: Exact Deconvolution

- Apply differential filter to $u_i u_j$ and substitute

$$
\begin{aligned}
\frac{\partial \overline{u_i u_j}}{\partial t} &= \frac{u_i u_j - \overline{u_i u_j}}{T} \\
&= \frac{\left(\bar{u}_i + T\frac{\partial \bar{u}_i}{\partial t}\right)\left(\bar{u}_j + T\frac{\partial \bar{u}_j}{\partial t}\right) - \overline{u_i u_j}}{T} \\
&= \frac{\bar{u}_i \bar{u}_j - \overline{u_i u_j}}{T} + \bar{u}_i \frac{\partial \bar{u}_j}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial t} + T\frac{\partial \bar{u}_i}{\partial t}\frac{\partial \bar{u}_j}{\partial t}
\end{aligned}
$$

Remember: $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$

TLES Theory: Exact Deconvolution

- Thus giving an equation for evolution of τ_{ij}

$$\frac{\partial \tau_{ij}}{\partial t} = -\frac{\tau_{ij}}{T} + T\frac{\partial \bar{u}_i}{\partial t}\frac{\partial \bar{u}_j}{\partial t}$$

- Can be solved in alternation with the evolution equation for $\bar{u}$ to advance the system

Regularization

- Large filter widths are highly non-dissipative → unstable
- Use of regularization term from Åkervik et al. [4]

$$-\chi(\bar{u} - \tilde{u})$$

- Addition of linear forcing term to momentum equation
- $\tilde{u}$ filtered with filter width $\tilde{T} > T$
- Forces $\bar{u}$ towards slower varying solution, damps high-frequency changes
- χ is a free control parameter

Divergence Cleaning

- Divergence of discretization is non-zero in general
- $\nabla \cdot \bar{u} = 0$ condition is not explicitly enforced
- Accrued divergence can be a source of instability

- Zeroing of divergence with Projection Scheme [5] using Helmholtz-Hodge decomposition of filtered velocity

$$\bar{u}^* = \nabla \times A + \nabla \phi$$

Divergence Cleaning

- Take the divergence of both sides

$$\nabla^2 \phi = \nabla \cdot \bar{u}^*$$

- Solve the Poisson equation for ϕ and correct $\bar{u}$

$$\bar{u} = \bar{u}^* - \nabla \phi$$

- Can be performed efficiently with iterative solvers

Velocity Profile Comparisons

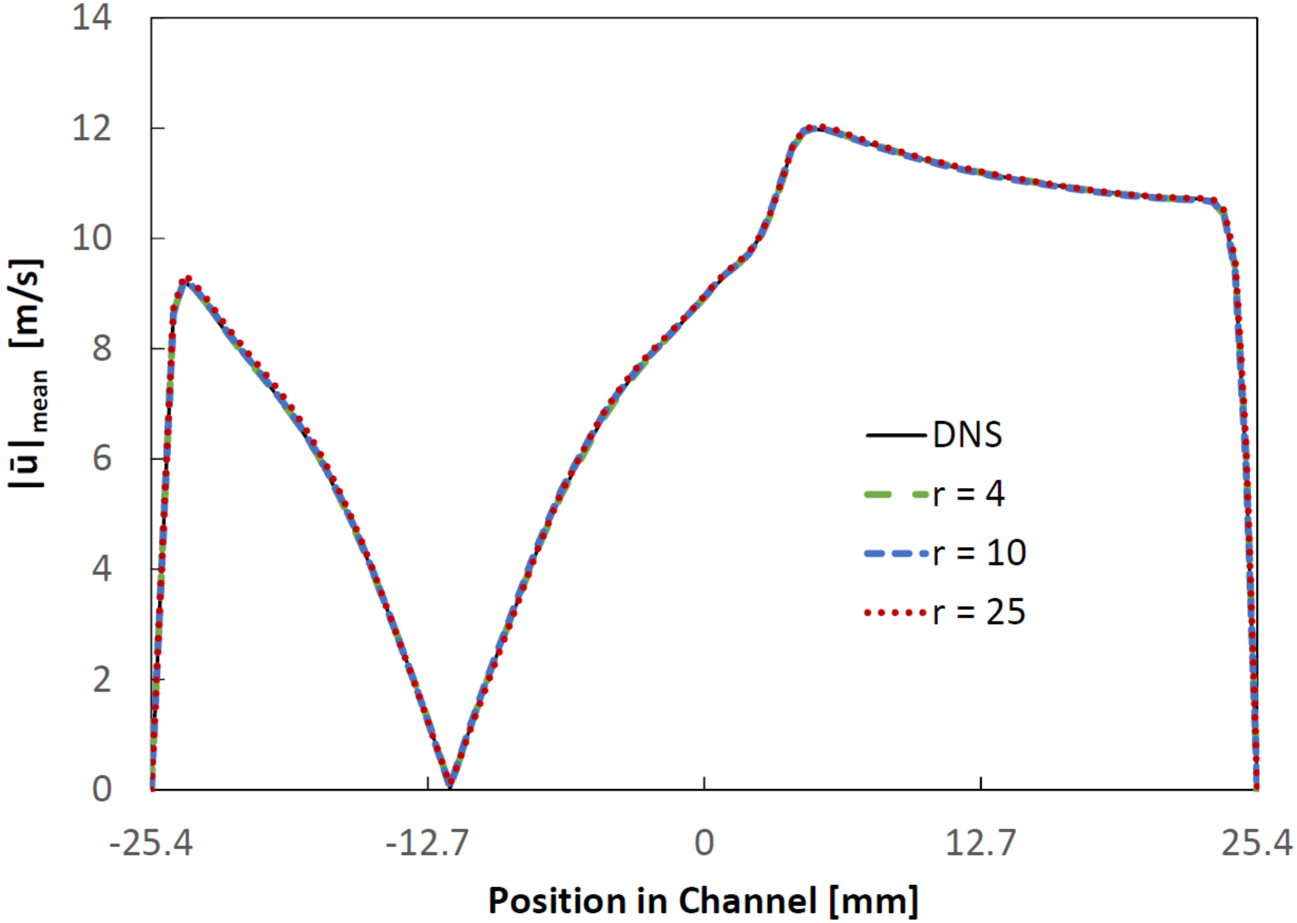
- Line profiles of the $|\bar{u}|_{\mathrm{mean}}$ field used for comparison
- A baseline “DNS” simulation was run, using the same grid
- Location of profile was chosen as the minimum of the recirculation vortex in the baseline simulation

Define filter width ratios: $r = \dfrac{T}{\Delta t}$ $\qquad \tilde{r} = \dfrac{\tilde{T}}{T}$

$x = 28.6\ \mathrm{mm}$

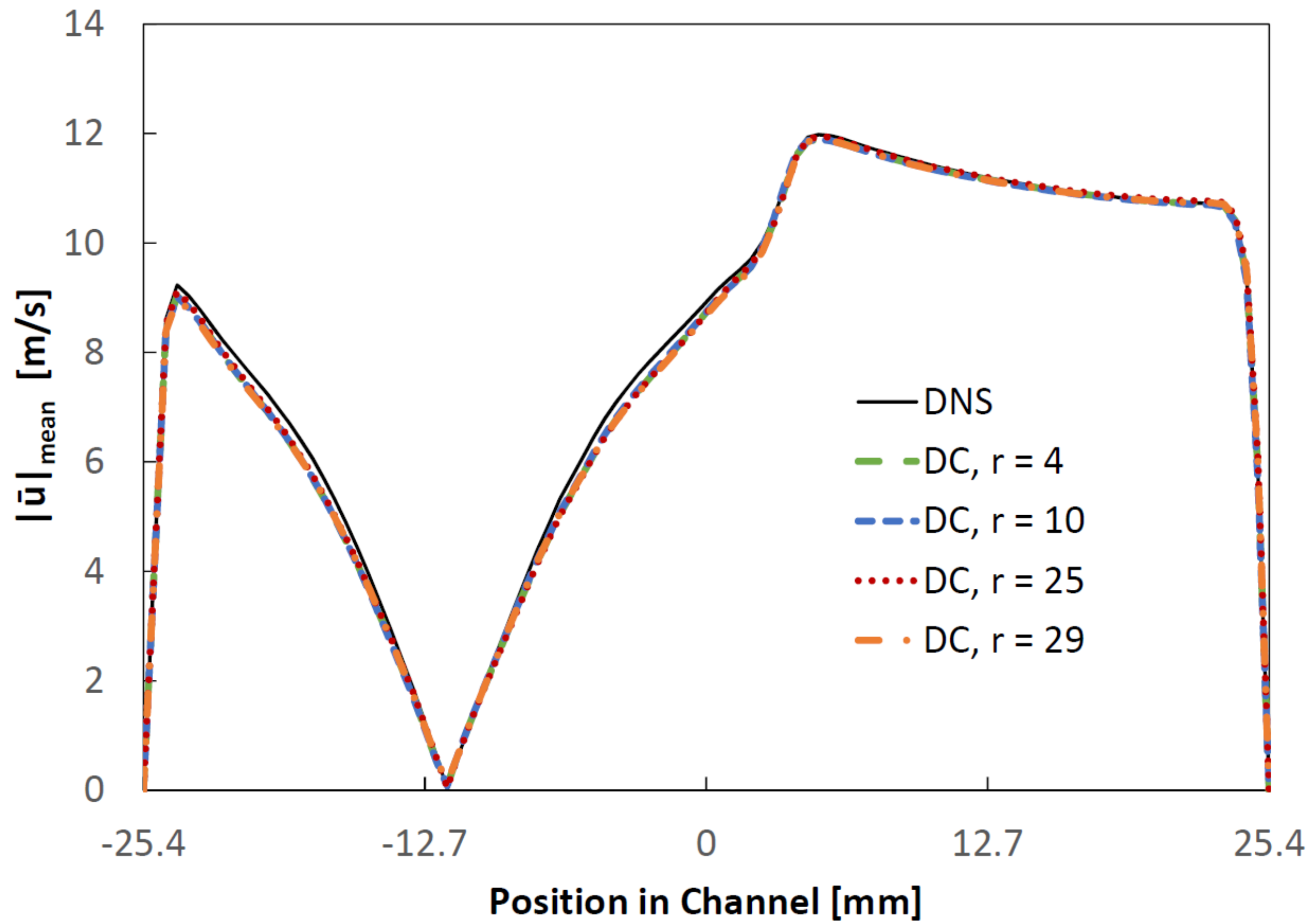
Base TLES Method

$\Delta t = 10^{-6}, t_{\text{final}} = 0.1$



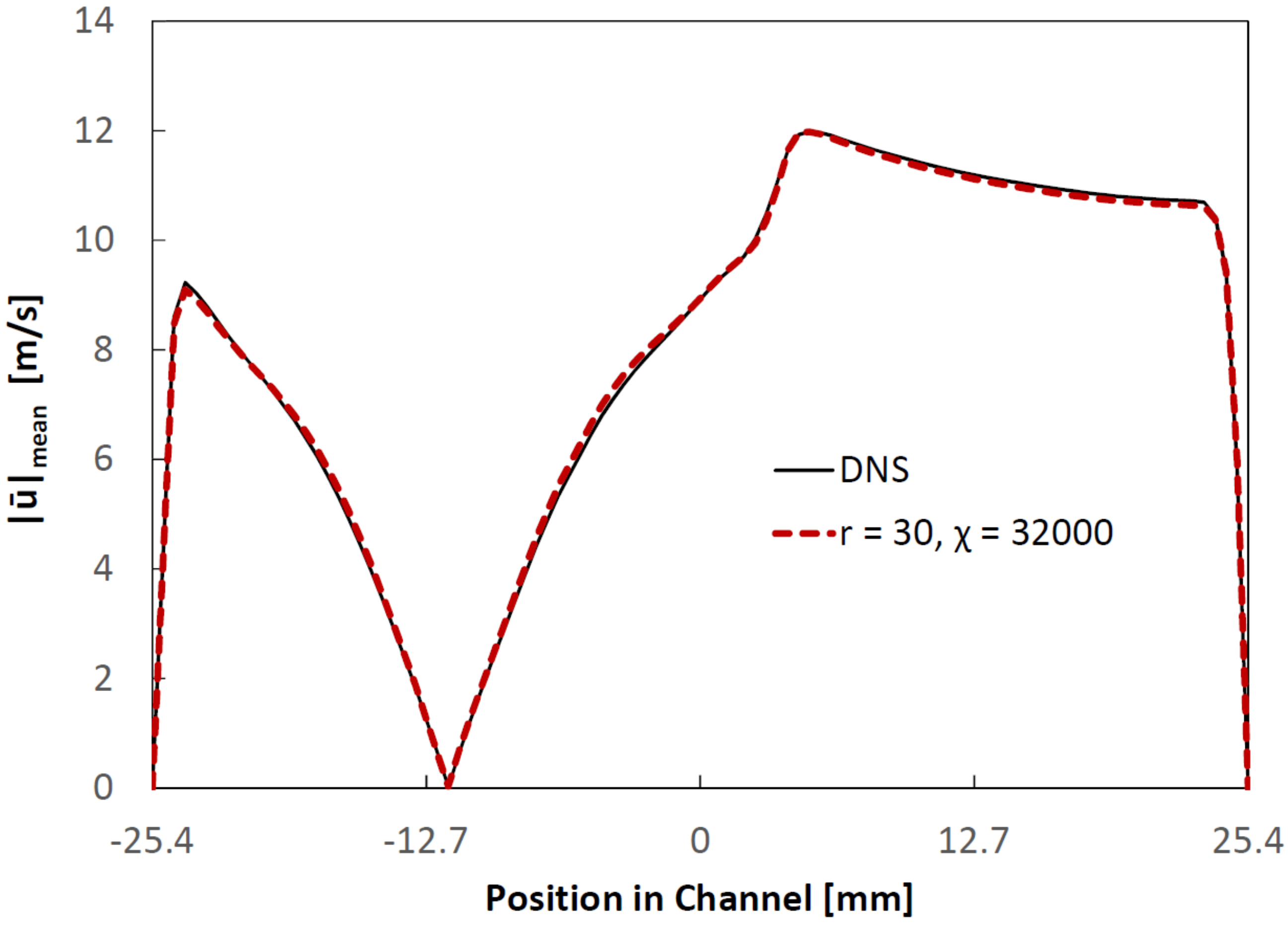
TLES with Divergence Cleaning

$\Delta t = 10^{-6}, t_{\text{final}} = 0.1$



TLES with Regularization

$\Delta t = 10^{-6}, t_{\mathrm{final,DNS}} = 0.1, t_{\mathrm{final,r=30}} = 8.4$



Minimum χ Required for Stabilization

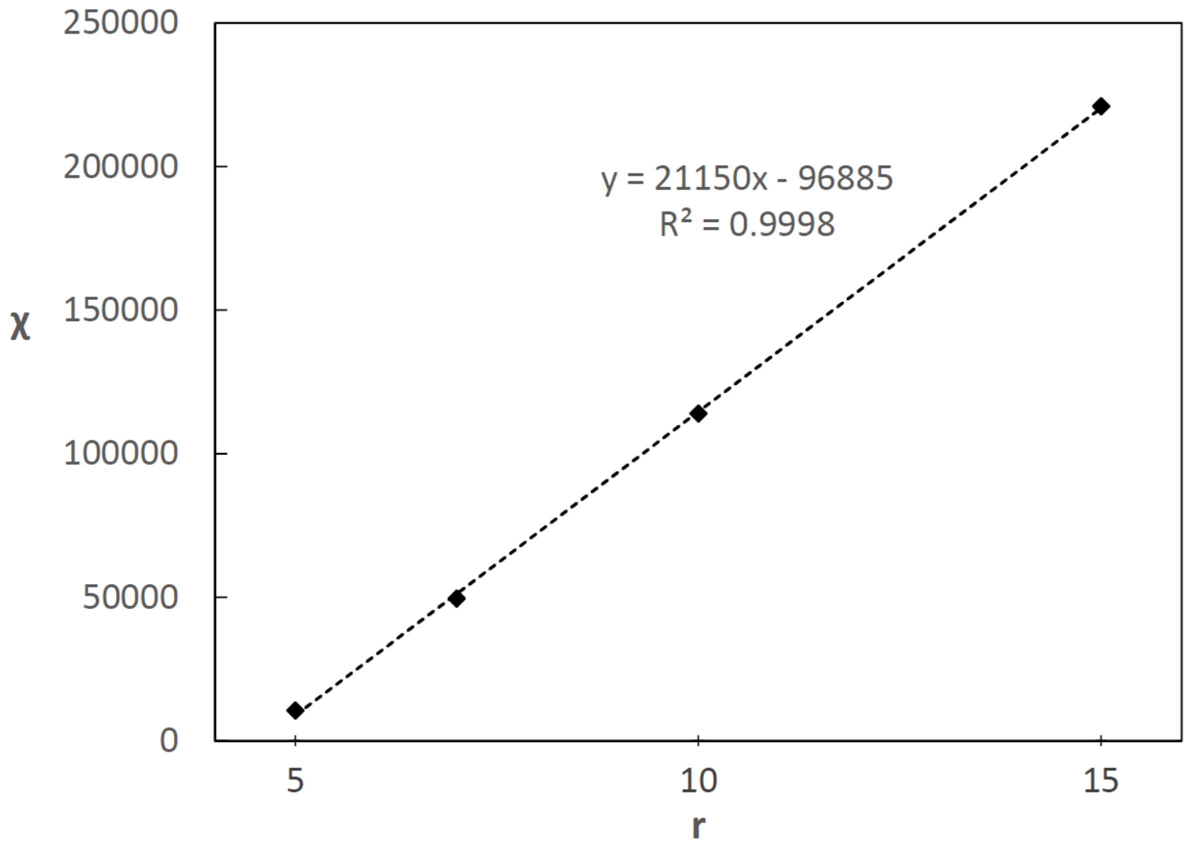
$$-\chi(\bar{u} - \tilde{u})$$

- Use of filtered instead of de-convoluted field
- Large enough χ could stabilize any tested filter width
- Can also stabilize larger time steps
- However, time evolution becomes *very* slow!
- Desirable to use minimum χ which achieves stability
  - Is there an observable relationship to other parameters $r, \tilde{r}, \Delta t$

- Various parameter sets were checked for stability over the first 100 time steps

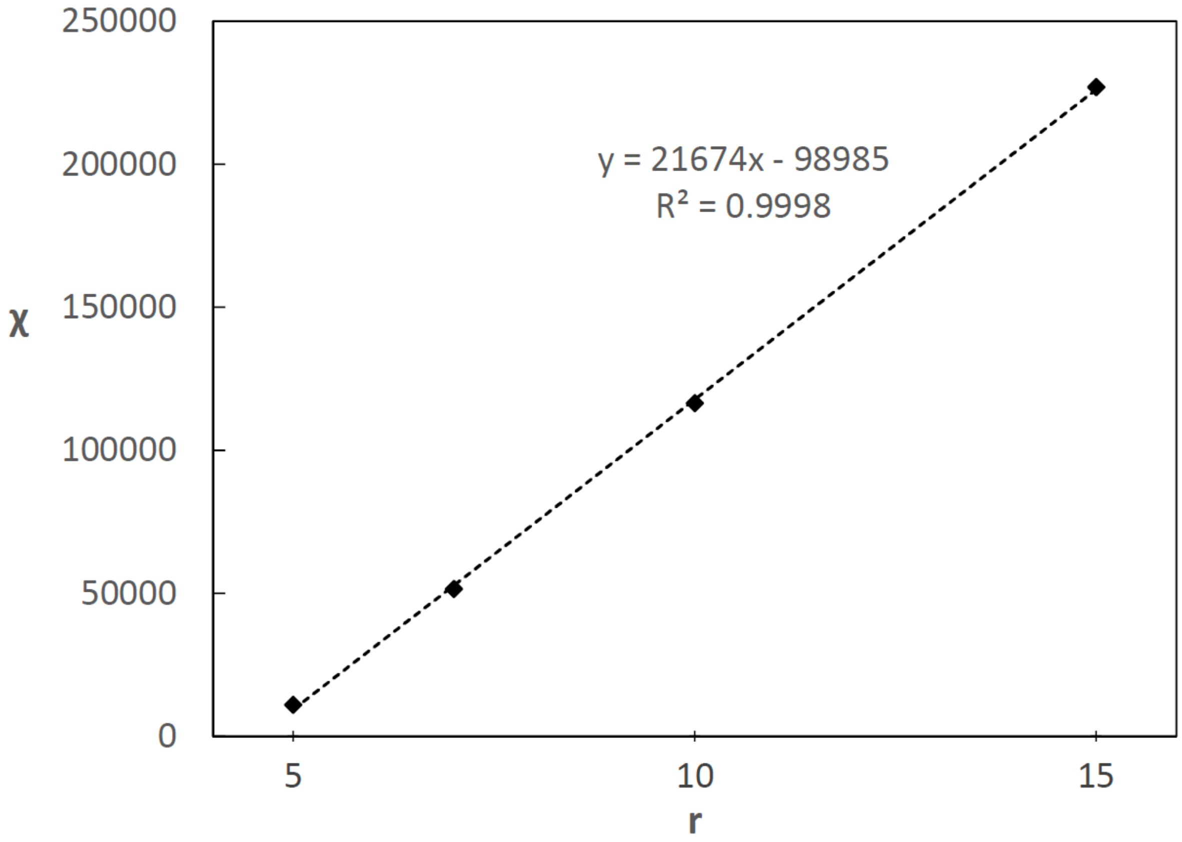
Minimum χ Required for Stabilization

$$\Delta t = 10^{-5}, \tilde{r} = 100$$



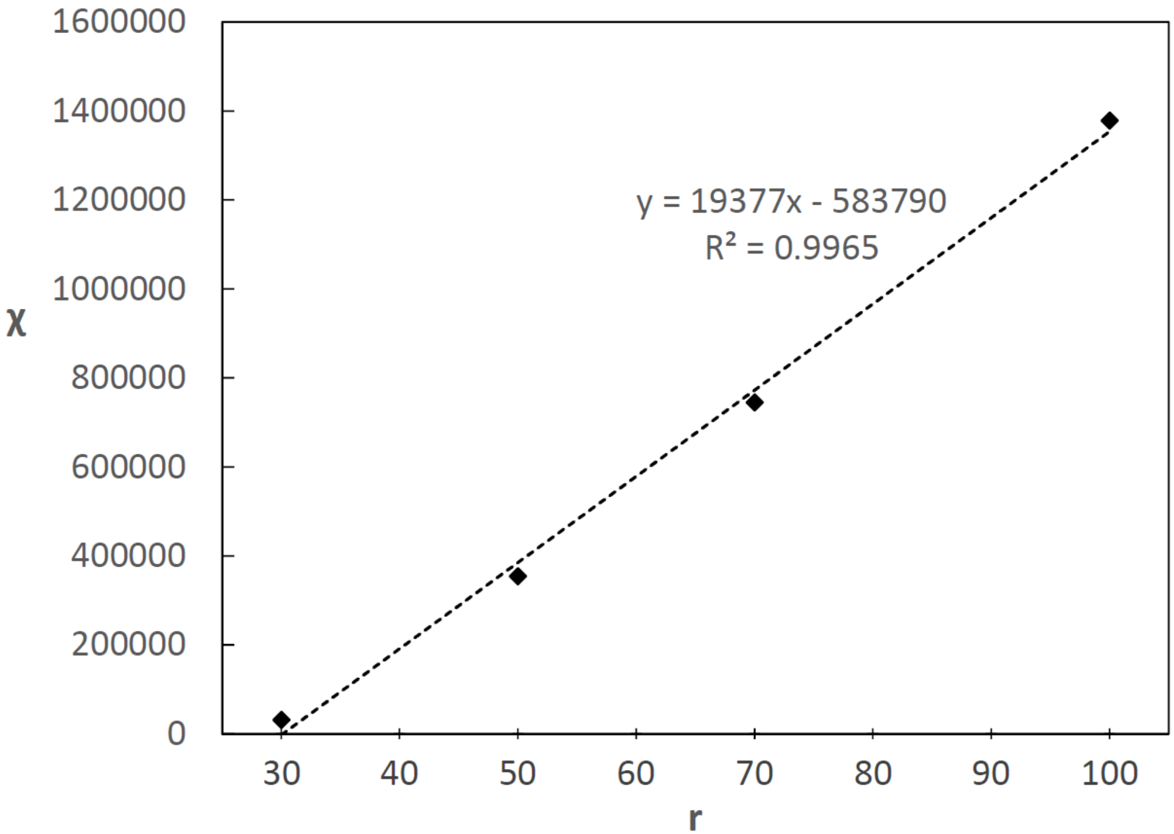
Minimum χ Required for Stabilization

$\Delta t = 10^{-5}, \tilde{r} = 10$



Minimum χ Required for Stabilization

$\Delta t = 10^{-6}, \tilde{r} = 100$



Final Thoughts

- Divergence cleaning helps, but does not solve instability
  - Suggests non-zero divergence is caused by, rather than cause of

- Regularization is an effective stabilizer
  - Renders the system evolution very slow
  - Desire minimal (and possibly dynamic) χ
  - Investigate potential effects of DC on minimum χ relation

Bibliography

(1) C. Pruett, “Temporal large-eddy simulation: theory and implementation,” Theor. Comput. Fluid Dyn. 22, 275 (2008).

(2) S. Stolz, N.A. Adams, and L. Kleiser, “An approximate deconvolution model for large eddy simulation with application to incompressible wall-bounded flows,” Phys. Fluids 13, 997 (2001).

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(4) A. Åkervik, L Brandt, D. S. Henningson, J. Hoepffner, O. Marxen, and P. Schlatter, “Steady solutions of the Navier-Stokes equations by selective frequency damping,” Phys. Fluids 18, 068102 (2006).

(5) G. Tóth, “The $\nabla \cdot \boldsymbol{B} = 0$ Constraint in Shock-Capturing Magnetohydrodynamics Codes,” J. Comput. Fluids 161, 605 (2000).

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Questions?