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Comparative study of Roe, RHLL and Rusanov fluxes for shock-capturing schemes

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Abstract. Computational Fluid Dynamics provides approximations through iterative calculation to simulate and predict the solution and has been extensively used in the investigation of shockwave. The paper aims to compare the capability of Roe, RHLL and Rusanov flux function in capturing shock phenomena. Two cases have been presented in the paper; one-dimensional case and two-dimensional cases. In general, all the considered flux function capable to capture general pattern of the shockwave phenomena. However, the results shows that it is important to pay great attention to the diffusive component of the fluxes. Strong diffusive component of the flux will helps to bring stability in the solution but at the same time prevent the fluxes to capture small changes of the fluid properties.

1. Introduction

Shockwave is one of propagating disturbance in fluid medium. The disturbance occurs when a wave or the fluid medium travels at higher than a local speed of sound. Similar to the ordinary wave, shockwave carries energy while propagating through the medium. However, the shockwave is uniquely characterized by an abrupt change in the fluid medium properties (pressure, temperature and density) which also known as variable discontinuity. The unique characteristics of the shockwave which trigger discontinuity in flow variables attracts a lot of attention from the world of academics and research. The increasing attention demand for better understanding of shock behavior which now has becomes specialize focus areas in flow investigation. The shockwave investigation can be done through empirical and analytical approach. Although shockwave can be examined empirically, there are very limited cases can be covered by this approach due to sophisticated experimental setup requirement particularly in producing high speed flow. Among the established experimental setup in shockwave investigation is the shock tube. The experimental limitation has set-off extensive analytical work on shock wave investigation [1]. Computational Fluid Dynamics (CFD) in which fluid flow problems are resolved numerically using model equations that representing the governing laws of conservation has now become common in the shockwave investigation. The CFD provides approximations through iterative calculation to simulate and predict the solution as closely to real-world results as possible.

In general, these numerical methods can be classified into the major categories of finite difference, finite volume, and finite element methods [2]. Among these three, finite volume methods has been widely used in the shockwave investigation. In finite volume method, the control volume of the



interest area is define and also known as flow domain. The control volume is divided into a discrete number of cells that form a grid (structured, unstructured or hybrid). Information on fluid properties passes through the domain based on the designated boundaries and initial conditions. Changes that occur across a cell will influence the properties of other cells in the flow direction. All the changes is governed by the governing equation that will ensure any changes is coherent in terms of its physics.

The information on fluid properties that travels from cell to cell along the domain are track in terms of its changes from the domain inlet to its outlet. The transferred data between the cells is recovered in the same manner as if solving a series of Riemann problems. Thus, the set of solution equations that perform such calculations are often known as Riemann solvers [3]. The equations themselves are called flux functions. A lot of flux function has been introduce for the purpose of simulating shockwave. The present paper aims to compare between Roe [4], RHLL [5] and Rusanov [6] flux function for the one-dimensional and two-dimensional cases.

2. Methodology

The present papers considered two cases involving one-dimensional and two-dimensional. All three considered flux functions will be apply for both cases to allow comparison between results. The one-dimensional case make use of shock tube case while the two-dimensional make use of shock diffraction case.

2.1. One Dimensional

Figure 1 shows the overview of one dimensional flow domain of the Sod's shocktube. Total of 80 cells have been generated within $-5.0 \leq X \leq 5$ and $0 \leq Y \leq 1.0$ flow domain. The domain made of ideal gas with $\gamma = 1.4$. The domain is divided into two regions separated by a thin diaphragm. The diaphragm represent the disturbance in the flow (shockwave) separating the pre-shock region and post-shock region. The initial conditions of these regions are defined in Table 1. The results are taken at $t = 1.7$ with CFL = 0.8.

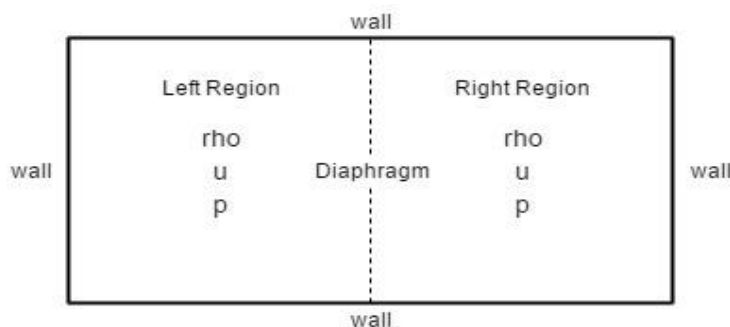


Figure 1. Computational domain of Sod's Shockwave tube considered for one dimensional case.

Table 1. Initial condition for Sod's shockwave.

Parameters	Left region	Right region
Velocity, u	0	0
Density, rho	1.0	0.12
Pressure, P	1.0	0.1

2.2. Two Dimensional

Figure 2 shows the overview of shock diffraction case for two-dimensional case of the present study. The case has been widely use to evaluate the stability of flux functions in simulating the shock. The flow domain is modelled as a square domain; $-1.0 \leq X \leq 1.0$ and $0.0 \leq Y \leq 1.0$. The upper left boundary is designated as an inlet while the lower left and top boundary are set as wall. The other two sides; the right and bottom are set as an outlet as shown. The domain is builds by quadrilateral grid consists of 160,801 nodes. The inlet is set to have incoming normal flow to the boundary at Mach number of 5.09. Table 2 shows the initial conditions of the flow domain. The results for this case are taken at $t = 0.18$ and $CFL = 0.95$.

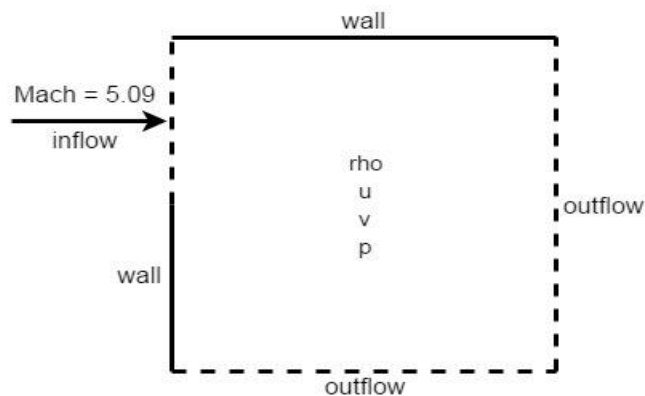


Figure 2. Computational domain for shock diffraction considered for one dimensional case.

Table 2. Initial conditions of shock diffraction problem

Parameters	Inlet Condition
Velocity, u	0
Velocity, v	0
Density, rho	1.0
Pressure, P	$1/\gamma$

3. Results and discussions

3.1. Sod's shocktube

Figure 3 shows the density variation of a Sod's shock tube case for Roe, RHLL and Rusanov flux functions. The exact solution of the case represented in solid line distinguish three phenomena; expansion wave ($-2.5 \leq X \leq -0.5$), contact discontinuity wave ($X = 1.5$) and shockwave ($X = 3.0$) as shown in the figure. The formation of these three non-linear waves represent the tendency of the flow to equalize the pressure inside the tube. The expansion wave propagates to the left region, $X < 0$ causes the expansion of the high pressure gas while the shockwave propagates to the right region, $X > 0$ causes the compression of the low pressure gas. The expansion and compression of gas are separated by contact discontinuity. Comparison between the solution of Roe, RHLL and Rusanov flux functions shows good solution stability with the absent of oscillation in the fluid properties profiles.

However, all solutions produce discrepancy in comparison with the exact solution at the aforementioned wave formation. Based on Figure 3, Roe solution produces the smallest discrepancy at the endpoint of expansion wave among all, while Rusanov solution produce the largest discrepancy. RHLL solution in the other hand produces nearly identical profile in comparison with the Roe solution. The larger discrepancy produces by Rusanov solution indicated by rounded corner at both

endpoint of the expansion wave. Similar pattern can also be observed in the velocity profile presented in Figure 4. From the comparison, Roe scheme gives the best performance in solving the shock tube problem.

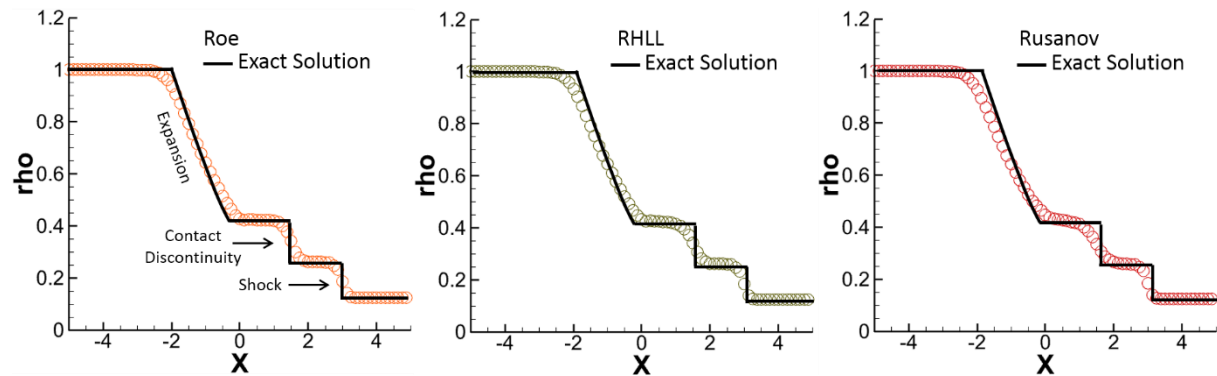


Figure 3. Density distribution for Roe, RHLL and Rusanov for one-dimensional case

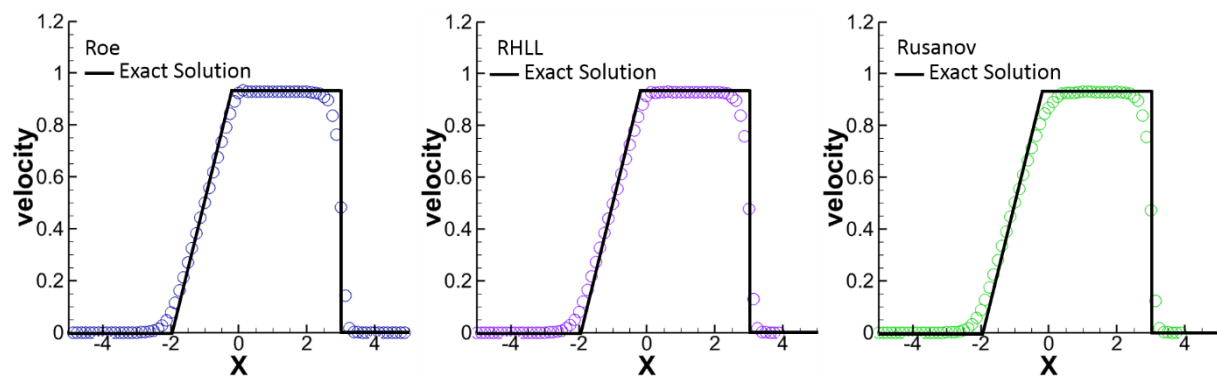


Figure 4. Velocity distribution for Roe, RHLL and Rusanov for one-dimensional case

3.2. Shock diffraction

Figure 5 shows the density contour plot for all considered flux functions for the case of shock diffraction. Several flow phenomena can be distinguished from the results. At the top side of the domain a clear contour separating the region marked as ES is region of reflected expansion wave due to the existence of the upper wall. SS is the secondary shock which occurs after the slipstream (SL). The slipstream created from the flow separation from the lower left side. The separation also created a main vortex marked as V in the figure.

In comparison between Roe, RHLL and Rusanov flux, all flux functions are capable to capture the general pattern of the density distribution throughout the domain. However, Roe flux as shown in Figure 5 (marked as A) indicates to have concern in terms of stability. Such occurrence cannot be observed in the case of RHLL and Rusanov. The instability observed in the case of Roe appears more likely when the shock is aligned with the mesh. This is the reason for the instability to occur only at the tip of the DS.

The stability discussed in the earlier paragraph is closely related to the dissipative nature of the flux. At the presence of pressure difference along a shockwave, the fluid will be pushed to the low-pressure region where the flow speed will be accelerated. Consequently, the pressure in the low-pressure region

gets even lower and causes instability in the solution. To prevent such condition, stronger dissipative component of the flux functions is required. RHLL and Rusanov have a much stronger dissipative component which dampen the instability. Although strong dissipative component provides the required stability, too strong dissipative component also prevent the flux function to capture smaller change of the fluid properties.

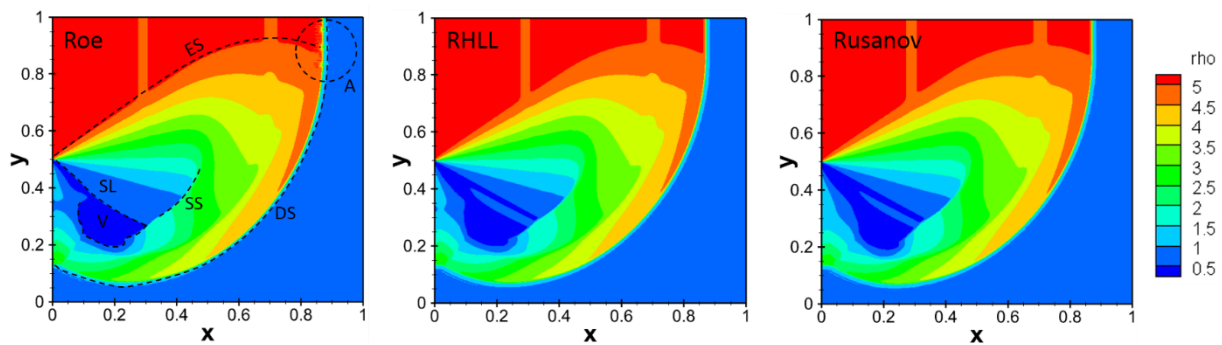


Figure 5. Density distribution for Roe, RHLL and Rusanov for two-dimensional case

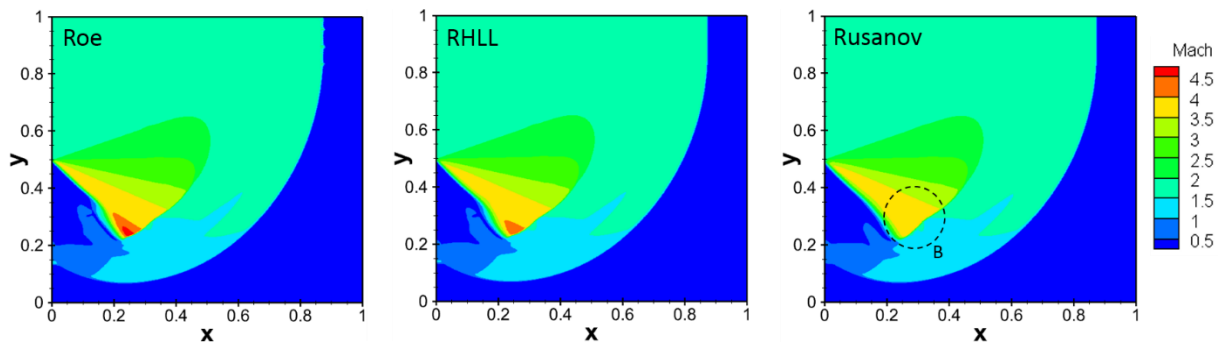


Figure 6. Mach number distribution for Roe, RHLL and Rusanov for two-dimensional case

Figure 6 shows the Mach number distribution for Roe, RHLL and Rusanov flux function. All flux function generally produce similar pattern of Mach number distribution. However, the figure obviously indicates the strength of the dissipative component of Rusanov flux function. The existence of high Mach number at the secondary shock region (marked as B) in the case of Roe become insignificant in the case of RHLL and non-exist for the case of Rusanov. Although, dissipative nature of the flux is important, too strong dissipative component will make the solution incapable to capture details of the flow phenomenon.

4. Conclusion

A comparative study of shock capturing schemes is carried out by using Roe, RHLL and Rusanov flux functions. Based on the results obtain, the effect of each scheme properties can be conclude as follow:

- Roe scheme has the property of good capturing contact discontinuity. However, this property causes the Roe scheme to suffer shock instability in multidimensional problem.
- Rusanov scheme is a dissipative scheme. This property causes the Rusanov scheme to produce very diffusive solution, thus unable to capture contact discontinuity.
- RHLL scheme has both properties of Roe and Rusanov scheme. These properties make RHLL scheme capable in suppress the shock instability and at the same time avoid diffusive solution.

Acknowledgment

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