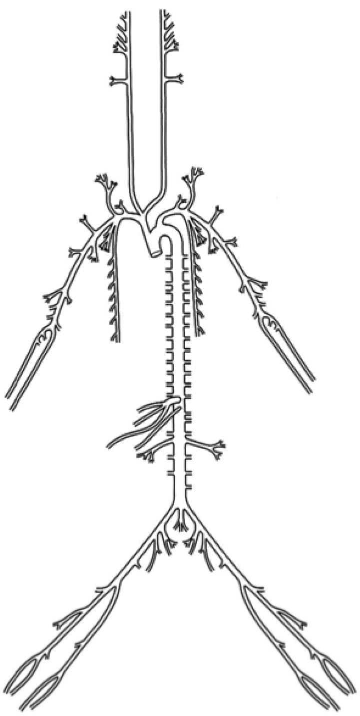
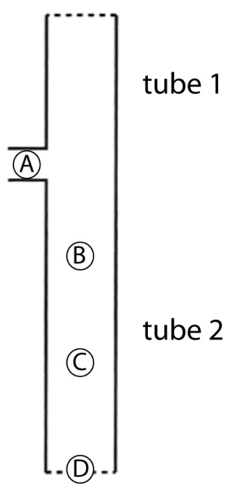
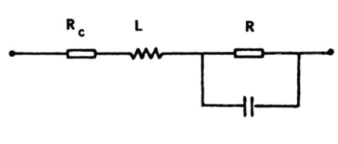
Term Project (if you are registering this course at 6000 level)

Lumped parameter models (*e.g.* windkessel model) are very easy and simple models that can characterize the behavior of cardiovascular system. However, because all the variables are lumped together, it lacks lots of actual features such as vessel wall properties, blood viscosity, wave propagations as well as anatomical information. Distributed models can take these into account. However, considering the complexity of the cardiovascular system, it sometimes involves too many vessel segment and too many parameters.

If we look at the main features of the vascular system as it goes from major vessels into capillaries, and considering the important driven forces at each sites, we can propose a relatively simple hybrid distributed-lumped parameter model. Shown in the figure, the model consists of two viscoelastic tubes of differing lengths, each terminated in a modified windkessel with inductance as well as resistance and compliance. In this way, the influence of vessel wall, wave reflection, Womersley's effect and multiple reflection sites, features that are important in major arteries, can be modeled. When go to the peripheral circulation, these effects become less important, therefore, they can be modeled by lumped parameter model with good accuracy. Combining important individual features of systemic and peripheral circulation, this model predicts a more realistic impedance modulus and phase than lumped parameter models.







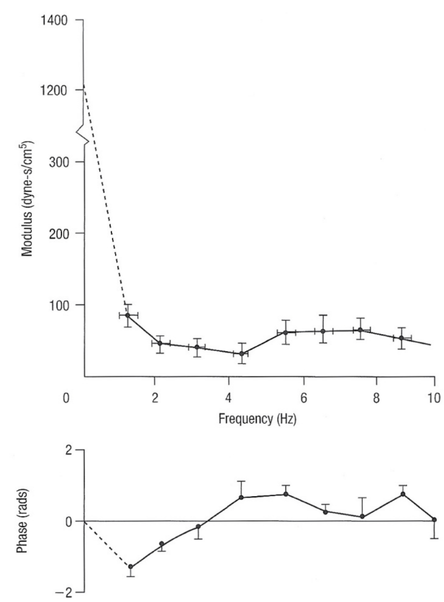


Figure 12.33 *Modulus and phase of ascending aortic impedance obtained in five apparently normal adult humans (n = 5). Reproduced from Nichols* et al. *(1977a)*

For each tube, the input impedance is given by the following relation as shown in class, $Z_{in} = Z_C \dfrac{(1+\Gamma \cdot e^{-2\gamma l})}{(1-\Gamma \cdot e^{-2\gamma l})}$

where Z_C is the characteristic impedance of the tube, Γ is the terminal reflection coefficient, γ is the propagation coefficient, l is the length of the tube.

Terminal reflection coefficient is given by, $\Gamma = \dfrac{Z_T - Z_C}{Z_T + Z_C}$

Z_T is the terminal impedance of the tube. In this model, the terminal is modeled by a lumped parameter model,

$$Z_T = R_C + j\omega L + \frac{R}{1 + j\omega RC}$$

The viscoelastic properties of the wall can be expressed in terms of dynamic elastic modulus as,

$$E^* = E + j\omega\eta$$

where E and η are the Young's modulus and viscosity of the tube.

Dynamic elastic modulus can also be expressed in terms of modulus and phase,

$$E^* = \left|E^*\right| e^{j\phi}$$

where ϕ can be expressed in the alternative form,

$$\phi = \phi_0 (1 - e^{-k\omega})$$

ϕ is a parameter that describes the wall viscosity, the larger the ϕ, the higher the viscosity. ϕ=0 corresponds to the case of pure elastic tube. In addition, we can see that the viscous part of the viscoelastic model is frequency dependent.

The characteristic impedance and propagation coefficient of the tube have been worked out by Womersley,

$$Z_C = \frac{\rho C_0}{\pi r^2 \sqrt{1-\sigma^2}} (1 - F_{10})^{-1/2} e^{j\phi/2}$$

$$\gamma = \frac{j\omega}{C_0} \left(\frac{1-\sigma^2}{1-F_{10}} \right)^{1/2} e^{-j\phi/2}$$

where $j = \sqrt{-1}$, $\omega = 2\pi f$, f is frequency, ρ is blood density, r is the internal radius of the tube, σ is the Poisson ratio, C_0 is the pulse-wave velocity given by Moens-Korteweg formula, $C_0 = \sqrt{\dfrac{Eh}{2\rho r}}$, where E is the Young's modulus, h is the wall thickness.

The function F_{10} is defined by,

$$F_{10} = \frac{2J_1(\alpha j^{3/2})}{\alpha j^{3/2} J_0(\alpha j^{3/2})}$$

J_0, J_1 are zero and first-order Bessel functions, and the Womersley number $\alpha = r\sqrt{\frac{\rho\omega}{\mu}}$

Table. Values of parameters of tubes and distal terminations representing nominal normal human values (subscript 1 and 2 indicate the value for tube 1 and 2, respectively)

Property	**Value**	**Unit**
Blood density	$\rho = 1.05$	g/cm^3
Blood viscosity	$\mu = 0.04$	poise
Young's modulus	$E_1 = 7\times10^6,\ E_2 = 6\times10^6$	dyn/cm^2
Wall thickness	$h_1 = 0.015,\ h_2 = 0.020$	cm
Tube radius	$r_1 = 0.35,\ r_2 = 0.5$	cm
Length	$l_1 = 20,\ l_2 = 38$	cm
Poisson's ratio	$\sigma_1 = 0.5,\ \sigma_2 = 0.5$	
Wall viscosity	$\phi_{01} = \phi_{02} = 12°,\ k_1 = k_2 = 2$	
Peripheral resistance	$R_1 = R_2 = 12\times10^3$	dyn.s.cm^{-5}
Peripheral inductance	$L_1 = L_2 = 450$	dyn.s^2.cm^{-5}
Peripheral compliance	$C_1 = 5\times10^{-6},\ C_2 = 8\times10^{-6}$	cm^5/dyn
Peripheral characteristic impedance	$R_{c1} = 260,\ R_{c2} = 240$	dyn.s.cm^{-5}

Part 1

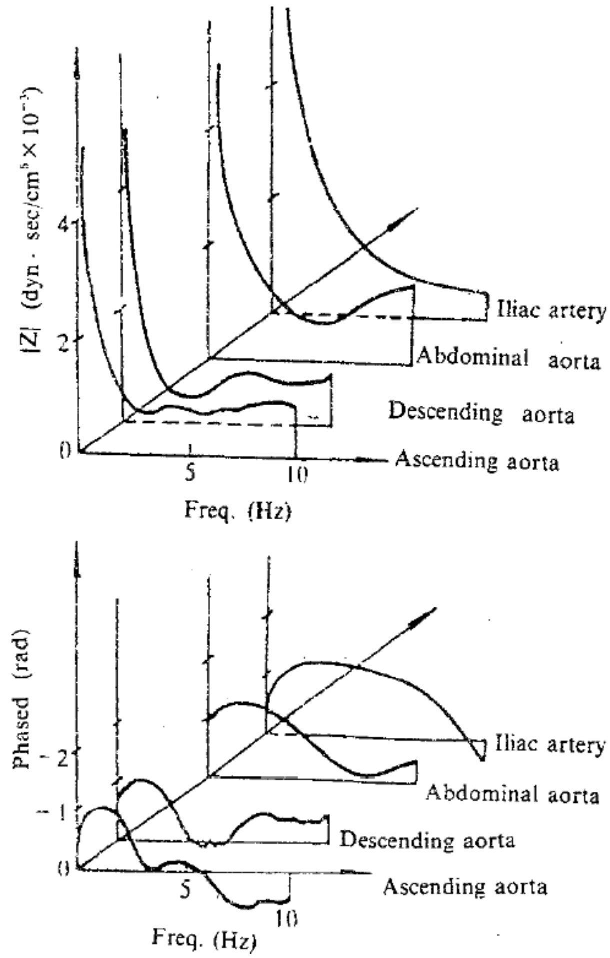
1. Using the values given in the table, calculate the input impedance at the entrance, and plot the modulus and phase separately (like those shown in the class) as a function of frequency f, where $\omega = 2\pi f$, and compare it to actual human input impedance. Discuss the model. (Hint: calculate the input impedance for each tube and combine them together at the entrance, be sure to use the individual's value which are different for each tube and terminal. In Matlab, set all the variables as complex number type (e.g., $j = \sqrt{-1}$), in this way, all the calculation will come out right)

2. Vary the Elastic modulus of each tube, individually or together, and plot the results, be sure the label each curve and used value clearly. Discuss the results.

3. Vary the viscous component (ϕ_0) of the viscoelastic model, considering the condition where it is pure elastic, and the condition where it is very viscous, compare the results to previous one. Discuss the results.

4. In hypertensive patients, the Young's modulus and peripheral resistance increase. Using this model, predict the input impedance as a result of these effects, and discuss the changes of input impedance in hypertensive cases.

5. Plot the input impedance at the anatomical location A(entrance), B(1/3 of the lower tube), C(2/3 of the lower tube), and D(terminal). Discuss the changes of impedance as it moves from A to D, compare it to actual measured input impedance (shown below).


|Z| (dyn · sec/cm⁵ × 10⁻³)
4
2
0
5
10
Freq. (Hz)
Iliac artery
Abdominal aorta
Descending aorta
Ascending aorta
Phased (rad)
- 2
- 1
0
5
10
Freq. (Hz)
Iliac artery
Abdominal aorta
Descending aorta
Ascending aorta