UNIVERSITY EXAMINATIONS 2024/2025
YEAR 1 SEMESTER II EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN SPECIFY THE DEGREE

SPS 0001: SAMPLE EXAMINATION

DATE: MONDAY 15^{TH} DECEMBER 2024 **TIME: 8.30 am - 10.30 am**

INSTRUCTIONS TO CANDIDATES:

1. *Answer question ONE and any other TWO questions.*
2. *Be neat and show all your workings*

This paper consists of 5 printed pages.

QUESTION ONE: 30 MARKS

1. (a) Let $A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$ find the characteristic polynomial of A. [3 marks]

   (b) Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $F(x, y) = (4x + 5y, 2x - y)$ and the following basis of $\mathbb{R}^2 : E = \{e_1, e_2\} = \{(1, 0), (0, 1)\}$ and $S = \{u_1, u_2\} = \{(1, 2), (2, 3)\}$. Find the matrix A representing F relative to the basis E. [4 marks]

   (c) The center of a group $z(a) = \{X \in G | x * g = g * x, \forall g \in G\}$ prove that $z(a)$ is a subgroup. [6 marks]

   (d) Let X be a topological space, show that the classes of closed subsets of X possess the following properties:

   (i) X and $\emptyset$ are closed sets. [1 mark]

   (ii) The intersection of any closed set is closed. [3 marks]

   (iii) The finite union of closed sets is closed. [3 marks]

   (e) Let $A = \{4, 2, 6, 1\}$ and $B = \{a, 2, c, 5, 4\}$ evaluate

   (i) $B - A$ [1 mark]

   (ii) $A \cap B$ [1 mark]

   (iii) $|A \cup B|$ [1 mark]

   (f) From the following data estimate the value of

$$\int_1^5 \log x dx$$

   Using Simpson's $\frac{1}{3}$ rule [5 marks]

x	1.0	1.5	2.0	2.5	3.0	4.0	4.5	5.0
$Y = \log x$	0.000	0.4055	0.6931	0.9163	1.0986	1.3865	1.1041	1.6099

   (g) Solve the system of equation below by use of Gauss elimination method with partial pivoting. [5 marks]

$$x + y + z = 7$$
$$3x + 3y + 4z = 24$$
$$2x + y + 3z = 16$$

QUESTION TWO: 20 MARKS

2. (a) A solid cylinder of 10cm diameter and 40 cm long consists of two parts made of different materials. The first part of the base is 1.0cm long and of specific gravity = 6.0. The other part of the cylinder is made of the material having a specific gravity of 0.6 as shown in Figure 1. Determine if it can float vertically in water. [10 marks]

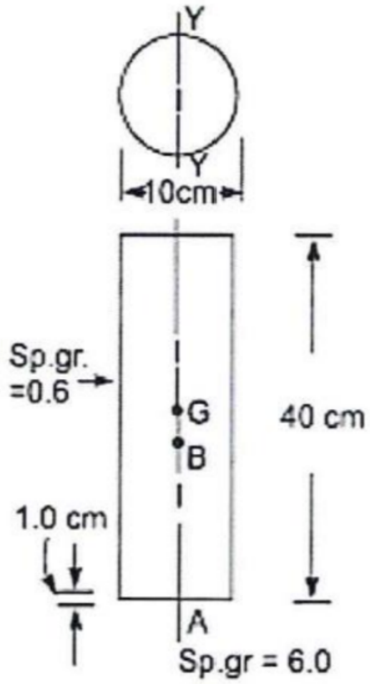

Figure 1: Diagram for use

(b) Evaluate the following limits

(i) $\lim_{x \to 0} \frac{\sin x}{2x}$ [3 marks]

(ii) $\lim_{x \to \infty} \frac{8x^3 - 9x - 1}{-12x^3 + 1}$ [3 marks]

(c) For what values of the constant K is the function continuous for all values of X [3 marks]

$$f(x) = \begin{cases} KX + 5 & X < 1 \\ x^2 - 3x + 4 & X \geq 1 \end{cases}$$

(d) Determine whether the series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ converges or diverges. [3 marks]

QUESTION THREE: 20 MARKS

3. (a) In multiple regression, prove that the variance of $\hat{\beta}_0$ and $\hat{\beta}_1$

$$V(\hat{\beta}_0) = \frac{\uparrow \sigma^2 \sum_{i=1}^{n} x_i^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \quad and \quad V(\hat{\beta}_1) = \frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

[7 marks]

(b) Assuming the factor ANOVA model $y_{ij} = \mu + t_i + e_{ij}$ where $e_{ij} \sim N(0, \sigma^2)$

(i) Obtain the estimate u_1, t_1, t_2, t_3 and t_4 of the factor level means. [5 marks]

(ii) Test the hypothesis that the four types of fertilizers have the same response means against the alternative that the factor levels are not equal and construct the ANOVA. [8 marks]

QUESTION FOUR: 20 MARKS

4. (a) Write an R program that perform the following integrals. [5 marks]

$$\int_0^1 \int_0^2 \int_1^3 \frac{1}{150}(2x_1 + x_2 + x_3^2)dx$$

(b) Write an R Code to give the following output. **NB: Use for loop** [5 marks]

```
1*1 = 1
3*3 = 9
5*5 = 25
7*7 = 49
9*9 = 81
11*11 = 121
13*13 = 169
15*15 = 225
```

(c) Interpret the following multiple linear regression output. [10 marks]

```
Call:
lm(formula = Y ~ X1 + X2)

Residuals:
Min      1Q  Median      3Q     Max
-1.4568 -0.4870 -0.1646  0.8023  1.1672

Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.1123     0.4035  -0.278   0.7887
X1            0.4580     0.5331   0.859   0.4187
X2           -0.4837     0.2475  -1.954   0.0916 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.051 on 7 degrees of freedom
Multiple R-squared:    0.4,Adjusted R-squared:  0.2285
F-statistic: 2.333 on 2 and 7 DF,  p-value: 0.1673
```

QUESTION FIVE: 20 MARKS

5. (a) Use Taylor's series to show that if h is small then

$$\tan^{-1}(x+h) \cong \tan^{-1} x + \frac{h}{1+x^2} - \frac{xh^2}{(1+x^2)^2}$$

[5 marks]

(b) Given that $f(x,y) = tan^{-1}(\dfrac{y}{x})$ show that $\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial^2 f}{\partial y \partial x}$ [6 marks]

(c) If $z = x^2 + y^2$ where $x = r\cos\theta$ and $y = r\sin 2\theta$ find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ [10 marks]

THE END