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MATH 448 Final Exam
4/25/19

1.

(a)

```
% Golden Search minimization algorithm
function ret=goldsearch(f, a, b)
n=0;
phi = (1+sqrt(5))/2;
tol = 1e-10;

c=b-(b-a)/phi;
d=a+(b-a)/phi;
while (abs(a-b)>tol && n<50000)

   fc = f(c);
   fd = f(d);
   if fc<fd
      b=d; d=c;
      c = b-(b-a)/phi;
      fd=fc;
   elseif fc>fd
      a=c; c=d;
      d=a+(b-a)/phi;
   end
   %fprintf("%10.15f,  %10.15f,  %10.15f \n", a, b, abs(a-b))
   n=n+1;
end

ret=d;
truemin = 2.387425886722793;

fprintf('\nthe approximate value of the minimum is %0.10f with true value %0.10f\n', d, truemin)
fprintf('error %.10f\n', abs(d-truemin))
```

(b)

Matlab code gives

```
>> goldsearch(g, 1, 4)

the approximate value of the minimum is 2.3874258866 with true value 2.3874258867
error 0.0000000001
```

Calculus gives
2.387425886722793

2.

(a)

```
>> x1=(0.4+0.9i)^0;
```

Durand Method

```
>> x1=(0.4+0.9i)^0;
>> x2=(0.4+0.9i)^1;
>> x3=(0.4+0.9i)^2;
>> x4=(0.4+0.9i)^3;

>> a=0;b=0;c=-1;d=-1;
```

First iteration

```
>> durand
  1.086422045817177 - 0.251514793202897i
 -0.265722915469425 + 0.048908509685217i
 -0.128345843587953 - 0.312879109981729i
 -0.692353286759799 + 0.515485393499408i
```

After running durand 10 times the approximate roots are:

```
>> durand
```

1.220744084605760 - 0.000000000000000i
-0.724491959000516 + 0.000000000000000i
-0.248126062802622 - 1.033982060975968i
-0.248126062802622 + 1.033982060975968i

Aberth's Method

First iteration

```
>> z1=1i; z2=3i; z3=5i; z4=7i;
>> z2=z2-(df(z2)/f(z2)-1/(z2-z1)-1/(z2-z3)-1/(z2-z4))^-1;
>> z3=z3-(df(z3)/f(z3)-1/(z3-z1)-1/(z3-z2)-1/(z3-z4))^-1;
>> z4=z4-(df(z4)/f(z4)-1/(z4-z1)-1/(z4-z2)-1/(z4-z3))^-1;
>> z1=z1-(df(z1)/f(z1)-1/(z1-z4)-1/(z1-z2)-1/(z1-z3))^-1;
>> disp([z1 z2 z3 z4]')

 -0.192673350273651 - 0.894396677727069i
 -0.014889794922061 - 2.374796428585050i
 -0.005913350896374 - 3.508227265037532i
 -0.007313998314218 -17.246445778349297i
```

10th iteration

```
>> z2=z2-(df(z2)/f(z2)-1/(z2-z1)-1/(z2-z3)-1/(z2-z4))^-1;
>> z3=z3-(df(z3)/f(z3)-1/(z3-z1)-1/(z3-z2)-1/(z3-z4))^-1;
>> z4=z4-(df(z4)/f(z4)-1/(z4-z1)-1/(z4-z2)-1/(z4-z3))^-1;
>> z1=z1-(df(z1)/f(z1)-1/(z1-z4)-1/(z1-z2)-1/(z1-z3))^-1;
>> disp([z1 z2 z3 z4]')
```

-0.248126062802622 - 1.033982060975968i
1.220744084605760 - 0.000000000000000i
-0.248126062802622 + 1.033982060975968i
-0.724491959000516 + 0.000000000000000i

Plugging in the previous roots into the function will give the y values as well…

Matlab's roots() command gives

```
>> roots([a, b, c, d, e])
ans =
```

1.220744084605760 + 0.000000000000000i
-0.248126062802622 + 1.033982060975968i

-0.248126062802622 - 1.033982060975968i
-0.724491959000515 + 0.000000000000000i

So yes they match up correctly.

(b)

%Higher Dimension Newton Minimization

```
>> x=[1;2];
>> y=JM(x)\(-FM(x))
y =
   1.0e-10 *
  -0.161018297136683
  -0.128691347485762
>> x=x+y
x =
   1.220744084605760
   1.490216120099954

>> x=[-2;-4];
>> y=JM(x)\(-FM(x))
y =
   1.0e-06 *
   0.164011640443312
   0.155864980652063
>> x=x+y
x =
  -0.724491959000536
   0.524888598656408

>> x=[2; 3i];
>> y=JM(x)\(-FM(x))
y =
   1.0e-16 *
 -0.329299250518749 + 0.486602944217349i
 -0.842861977247513 + 0.187766243773091i
>> x=x+y
x =
 -0.248126062802622 + 1.033982060975968i
 -1.007552359378179 - 0.513115795597015i

>> x=[2;-3i]
>> y=JM(x)\(-FM(x))
y =
   1.0e-15 *
 -0.058590373495930 - 0.003641105572739i
  0.021545921698697 + 0.122969696661371i
x=x+y
x =
 -0.248126062802622 - 1.033982060975968i
 -1.007552359378179 + 0.513115795597015i
```

(c)

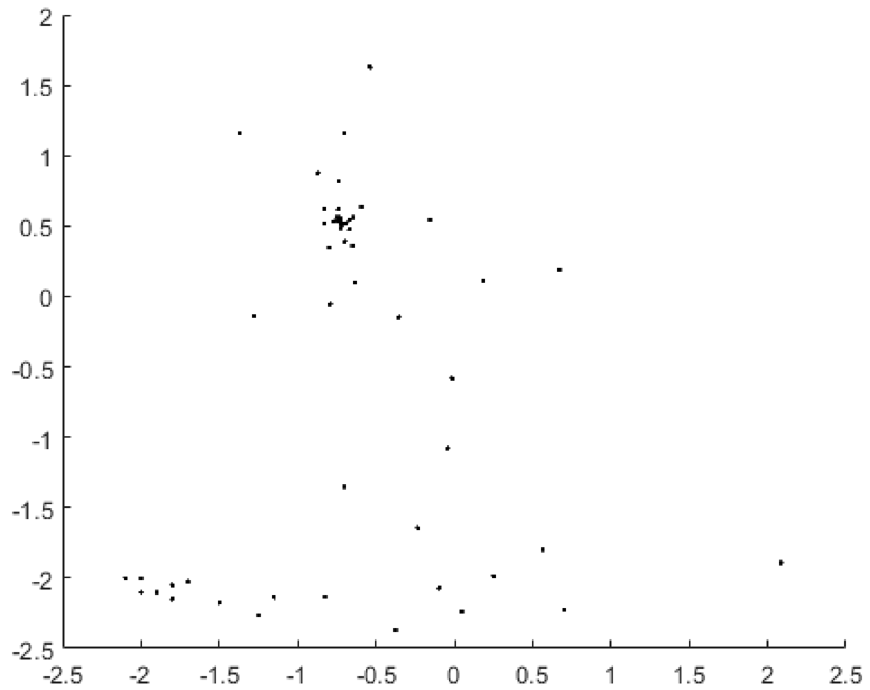
Because at a minimum both of the equations must be zero since both parts of the function are squared and therefore are never negative in R.

(d)

>> fminsearch(@f, [-2, -2], opt)

ans =

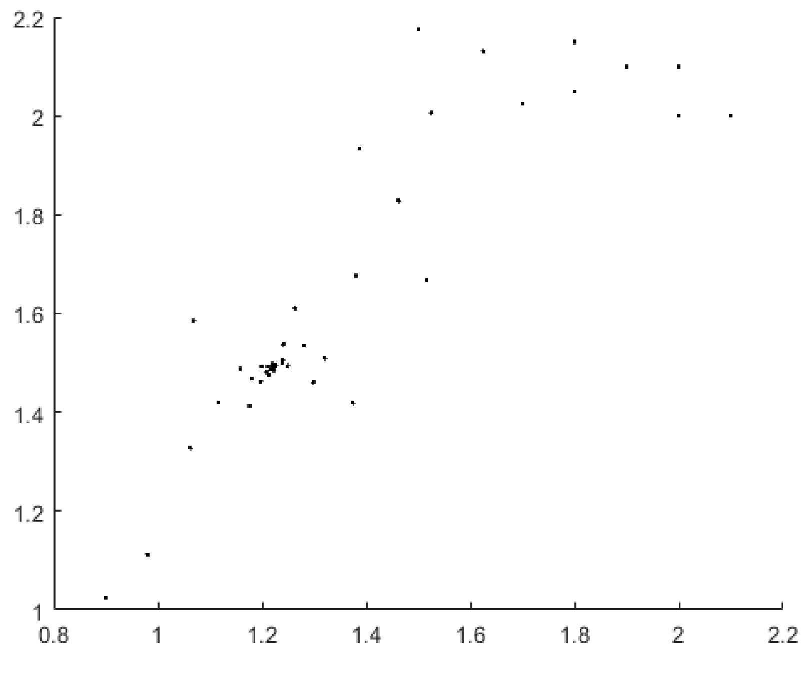
-0.724491959045917   0.524888598633890



>> fminsearch(@f, [2, 2], opt)

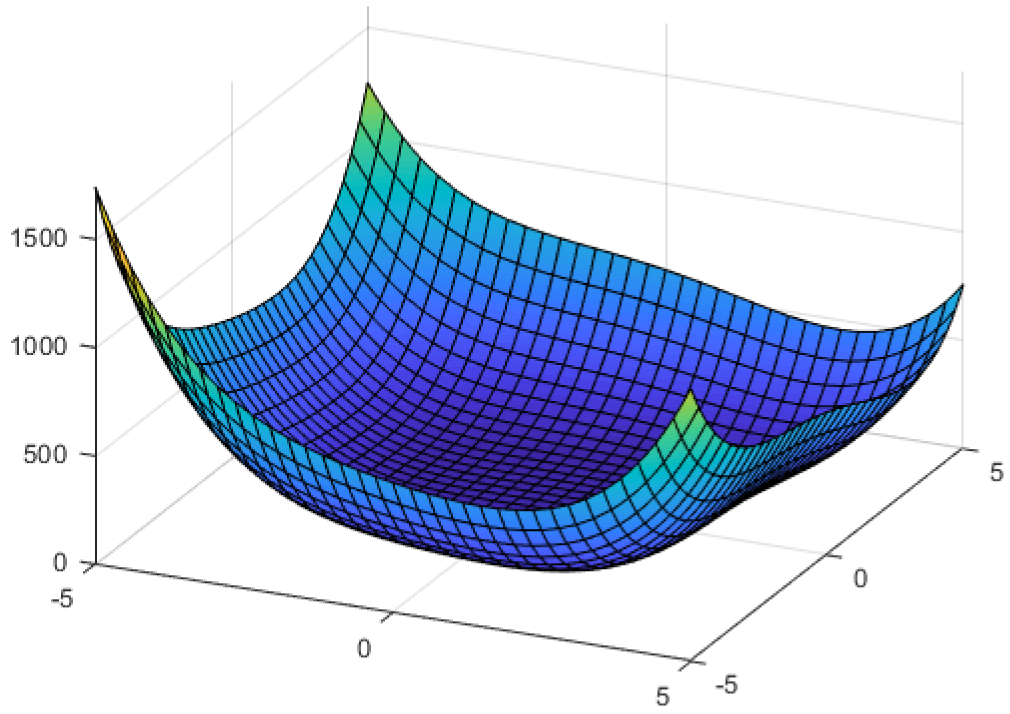
ans =

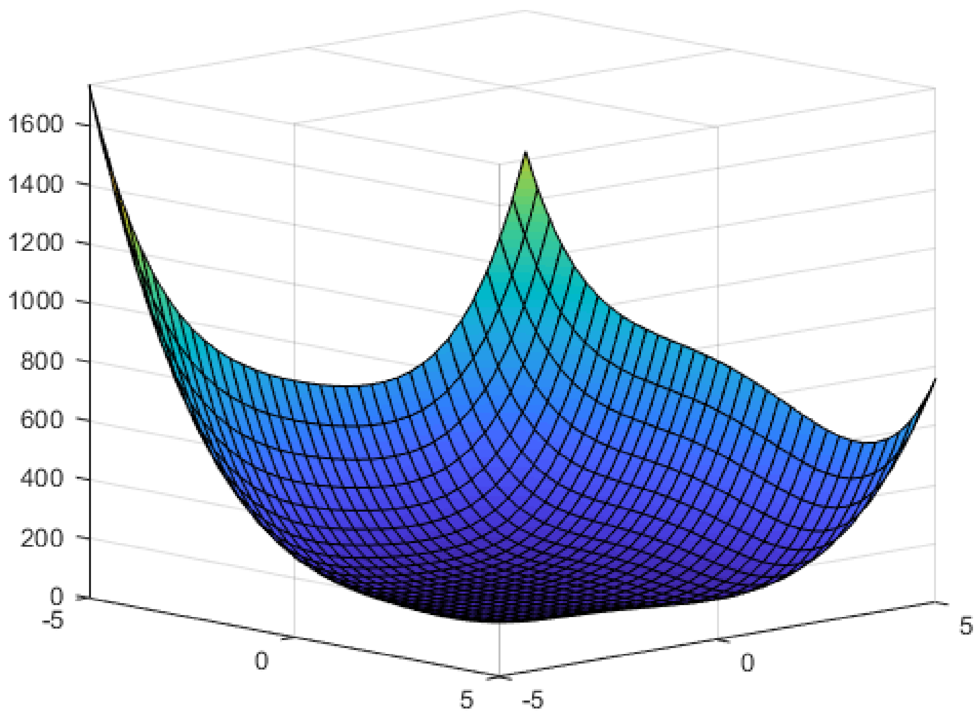
1.220744084620451   1.490216120151177



The two real solutions came out using fminsearch as expected. The points are converging to the minimum. Method is good since there is a long flat valley.

>> fsurf(f2)





(e)

% looking for the 1st root using Steepest Descent

```
>> x=[3,3];
>> steepDescent
a =
  0.062500000000000
x =
  1.375000000000000                 0
>> steepDescent
a =
  0.186509957852831
x =
 -0.480628369731098   0.705240778131018
>> steepDescent
a =
```

```
  0.303357691656985
x =
 -0.632270146828918   0.436346598390595
>> steepDescent
a =
  0.250000000000000
x =
 -0.732500452482564   0.495434064963469
>> steepDescent
a =
  0.247199937904922
x =
```

-0.728506754597176   0.526564533556525

% now looking for the the next root

```
>> x=[1,1];
>> steepDescent
a =
  0.062500000000000
x =
  0.875000000000000   1.250000000000000
>> steepDescent
a =
  0.181661484165570
x =
  0.915448064833740   1.357861506223307
>> steepDescent
a =
  0.087680117595272
x =
  0.986329491533330   1.300833111479657
>> steepDescent
a =
  0.073275439859802
x =
  0.990629409471990   1.364923611954040
>> steepDescent
a =
  0.201004008981486
x =
  1.092084052032483   1.350767644581195
>> steepDescent
a =
  0.048667475087922
x =
  1.082853738026191   1.405720023680523
>> steepDescent
a =
  0.467994011420385
x =
  1.219189772050151   1.468550949557925
>> steepDescent
a =
  0.039645925042302
x =
```

```
  1.212502443670108   1.484534779074316
>> steepDescent
a =
  0.249229839560836
x =
  1.216872879854062   1.490185503604595
>> steepDescent
a =
  0.045150306798016
x =
  1.218247773278352   1.488318848414849
>> steepDescent
a =
  0.111386967258575
x =
  1.218682439781968   1.489477144106549
>> steepDescent
a =
  0.062370609640723
x =
```
1.219317145752969   1.488994099152432

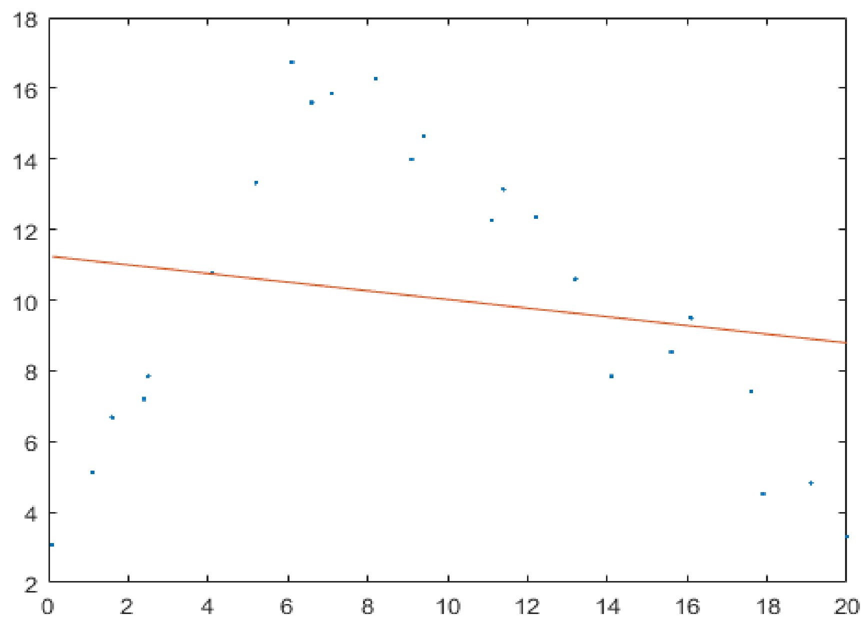
3.

% Linear

```
>> A=[length(x), sum(x);
  sum(x), sum(x.^2)];
>> b=[sum(y);
  sum(x.*y)]
>> c=A\b
c =
```
11.244642801208141
-0.122699427217409

```
>> sum((y-(x*c(2)+c(1))).^2)
S =
```
4.172865272092502e+02

So $y = 11.2446 - 0.1227x$



% quadratic

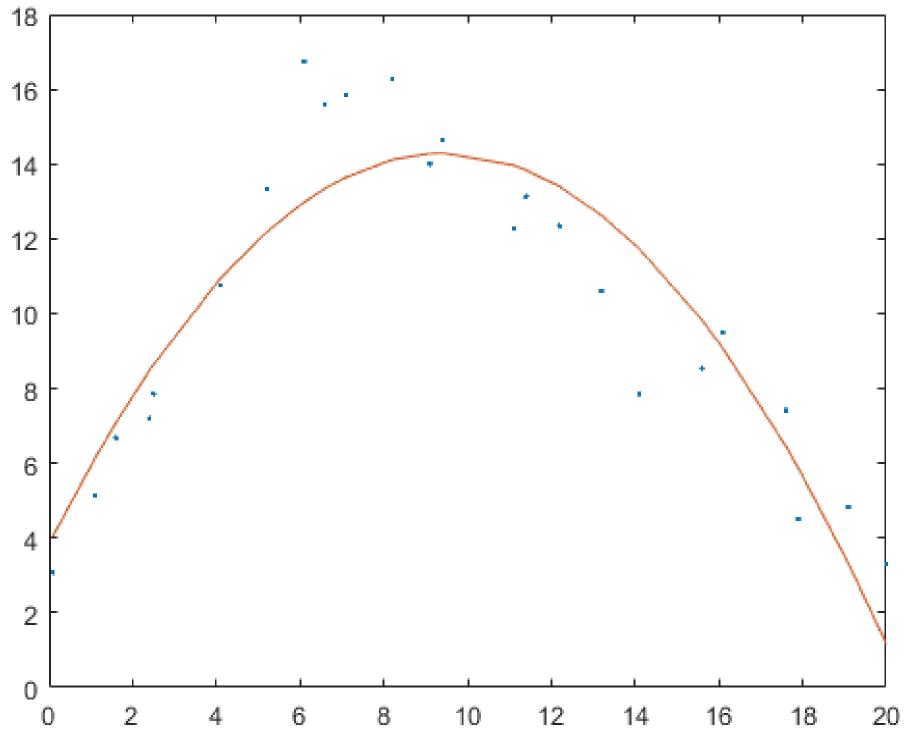
```
>> A= [length(x), sum(x), sum(x.^2); sum(x), sum(x.^2), sum(x.^3); sum(x.^2), sum(x.^3), sum(x.^4)]
>> b=[sum(y);
  sum(x.*y);
  sum(y.*x.^2)];
>> c =A\b
c =
```

3.830554635074556
2.217397324655864
-0.117509707495235

```
>> sum((y-(x.^2*c(3)+x*c(2)+c(1))).^2)
S =
```

69.606048786689456

So $y = 3.83055 + 2.2173x - 0.1175x^2$



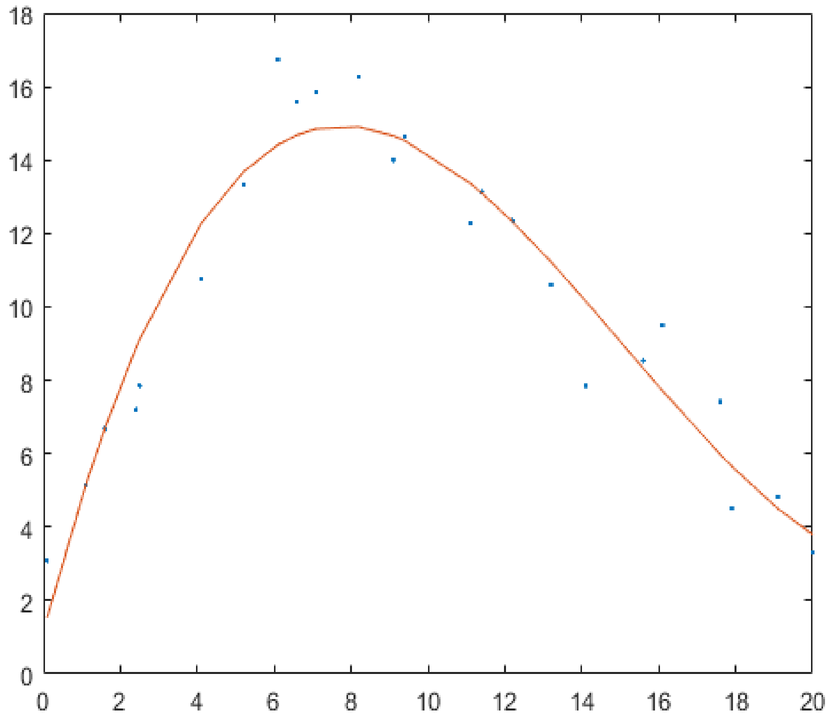
%cubic

```
>> A= [length(x), sum(x), sum(x.^2), sum(x.^3); sum(x), sum(x.^2), sum(x.^3), sum(x.^4); sum(x.^2), sum(x.^3), sum(x.^4), sum(x.^5); sum(x.^3), sum(x.^4), sum(x.^5), sum(x.^6)];

>> b=[sum(y);
  sum(x.*y);
  sum(y.*x.^2);
  sum(y.*x.^3)];

>> c =A\b
c =
```

1.135817111223791
4.007623065260419
-0.346696821555106
0.007648732850660

```
>> sum((y-(x.^3*c(4)+x.^2*c(3)+x*c(2)+c(1))).^2)
S =
```
32.475350445810221

So $y = 1.135 + 4.007x - 0.3466x^2 + 0.0076x^3$



These regressions are not that great. The corner at the top is pretty sharp. The curve does not capture this very well.

4.

See attached code that I wrote to solve this.

Here is a plot of the best fit. See last page for proof/derivation of piecewise linear fit.

\>> piecewisefit

The Best Fit is row 10:

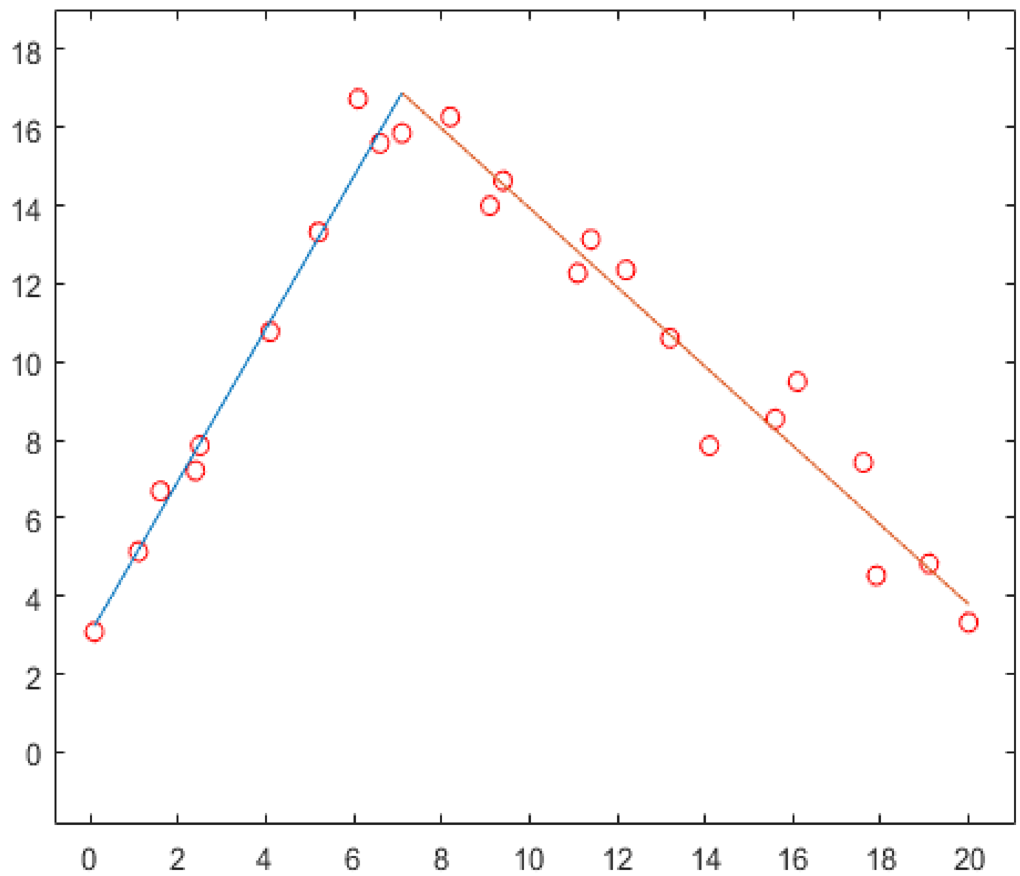
Sq.Error = 17.654541

Slope 1 = 1.951240

Intercept 1 = 3.034009

Slope 2 = -1.016165

Intercept 2 = 24.102580



See next page for derivation #4