Problem A: It's Hip to be Square

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1 Introduction

Our task is to develop a model to describe interactions within a given community between the non-conformists (hipsters) and conformists (squares) of that community. To this end, we construct a network of 'trends' or 'fads' connected by weighted edges, which measures the correlation between two given trends. We call this the "trend interaction graph". In practice, trends of interest might be wearing scarves with argyle prints, or going to thrash metal concerts. A researcher using this model could find these empirically given micro-level data on the members of the subculture.

The remainder of this paper will proceed as follows: we will first explain the hypothesized behavior of the hipsters and squares in our model, second we will present our model, and last we will discuss the results of a simulation of a baseline model with fairly neutral interactions between fads.

2 Model

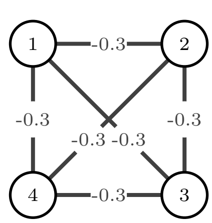
By hipster we mean individualistic members of the population who set trends within the community, this is the "authentic in-group" of the culture. By squares we mean individuals who have the sole desire to appear as members of the community, whatever that may entail.

There are three main features of the hipsters behavior: i) hipsters follow trends according the interaction of the trend interaction graph and the prevalence of trend involvement of other hipsters; ii) hipsters avoid trends that squares have joined, iii) hipsters are attracted to trends that squares have vacated. Again, squares have one defining behavior: they are attracted to what is popular.

We now present our model with N fads, where the variables of the system measure the popularity of trends among hipsters and squares.

$$\left\{ \begin{array}{l} \frac{d\tilde{s}_i}{dt} = \tilde{s}_i(1-\tilde{s}_i)\left[\sum_{k=1}^{N} A_{ik}(\tilde{s}_k + \alpha\tilde{h}_k)\right], \tilde{s}_i(0) = \tilde{s}_{i0} \\ \frac{d\tilde{h}_i}{dt} = \tilde{h}_i(1-\tilde{h}_i)\left[\sum_{k=1}^{N} A_{ik}\tilde{h}_k - \beta\tilde{s}_i + \frac{\gamma}{\tilde{s}_i}\right], \tilde{h}_i(0) = \tilde{h}_{i0} \end{array} \right. \quad \alpha, \beta > 1, \gamma > 0.$$

We use s_i and h_i to denote the respective number of squares and hipsters following trend i. $\tilde{h}_i$ and $\tilde{s}_i$ are analogously defined in terms of proportion of the total population of each subgroup. If there are, for example, four fads in a culture, the system will have an eight dimensional phase space. The matrix A is the adjacency matrix of the trend interaction graph. For instance, for the following graph G_1, we get the corresponding matrix A.



$$A = \begin{bmatrix} 1 & -0.3 & -0.3 & -0.3 \\ -0.3 & 1 & -0.3 & -0.3 \\ -0.3 & -0.3 & 1 & -0.3 \\ -0.3 & -0.3 & -0.3 & 1 \end{bmatrix}$$

We keep the total population of hipsters and squares constant, but the total involvement in each fad is not constant. In particular, each person may be involved in any number of fads between 0 and N. Because of the logistic terms in both equations, $\tilde{h}_i$ and $\tilde{s}_i$ are bounded by 0 and 1. Hence $\sum_i \tilde{h}_i$ and $\sum_i \tilde{s}_i$ are both in the interval $[0, N]$.

Of note, when $\tilde{h}_i \approx 1$, we can deduce that almost the entirety of the hipster population is following trend i. Thus by observing only the popularity of a given fad, we can track the behavior of hipsters and squares themselves.

3 Some Notes

It's clear from the terms out in front that there are EQ points of a population whenever a particular h_i or s_i is equal to 0 or 1. However, even when a particular population is approaching 0 or 1, it is possible for the influence of other fads to shock the group out of this long-term behavior.

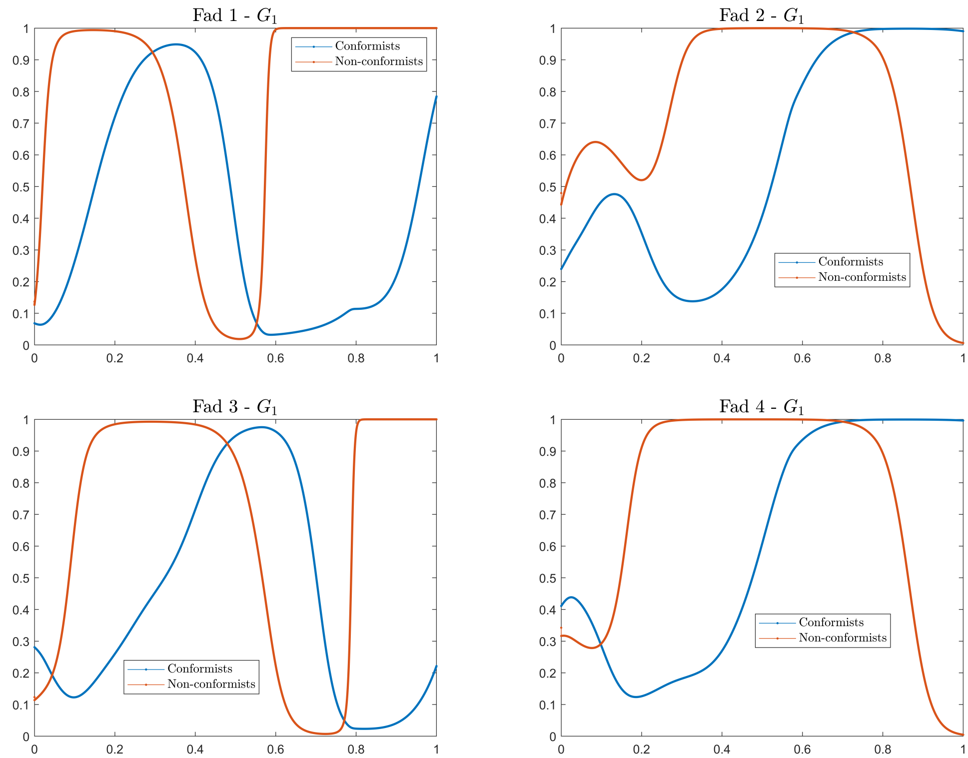
Another thing to note is the absence of any real negative terms in the equation for $\frac{d\tilde{s}_i}{dt}$ apart from potential negative values for A_{ik}. Thus we expect the comformists to come to dominate fads which have very weak interactions with other fads. In the same vein, we expect the non-comformists to dominate fads that interact very negatively with other fads.

4 Results

Using the graph G_1 for the underlying fad interaction we get the following plots of squares and hipsters. These plots were generated using a Forward Euler approximation written in MATLAB to solve the model, with initial conditions $s_1 = 10$, $s_2 = 31$, $s_3 = 40$, $s_4 = 60$, $h_1 = 20$, $h_2 = 70$, $h_3 = 18$, $h_4 = 50$.

Consider fad 1. It is clear that the squares follow the hipsters; as the proportion of hipsters following fad 1 increases, so to does the proportion of squares. However, the squares exhibit more inertia than the hipsters, who are quicker to pick up and drop fads. Note also that the hipsters avoid the squares. The increase in square prevalence induces a turning point hipster involvement. Finally, a small level of square involvement reignites hipster involvement.

A property we wish to emphasize is that hipsters respond quickly to a change in proportion of squares following a fad. This is demonstrated in fad 3 around time 0.8. In fad 4 near 0.6, as the popularity of the fad among squares approaches 1, the hipsters quickly respond by leaving the fad.


Fad 1 - G_1
Conformists
Non-conformists
Fad 2 - G_1
Conformists
Non-conformists
Fad 3 - G_1
Conformists
Non-conformists
Fad 4 - G_1
Conformists
Non-conformists