Problem A: It's Hip to be Square

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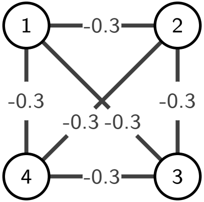
Abstract

We develop a model for tracking the behavior of two groups, conformists (squares) and anti-conformists (hipsters), as they try to reach their respective style preferences through fad adoption. Fads are linked by a weighted graph, where the edge connecting fads i and j is a measure of how compatible i and j are. We develop a system of differential equations to generate the behavior of the fad popularity in the two groups over time.

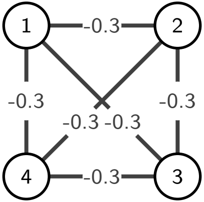
Video 0

A Fad Interaction Graph Example



G_1

A Fad Interaction Graph Example



G_1

$$A_1 = \begin{bmatrix} 1 & -0.3 & -0.3 & -0.3 \\ -0.3 & 1 & -0.3 & -0.3 \\ -0.3 & -0.3 & 1 & -0.3 \\ -0.3 & -0.3 & -0.3 & 1 \end{bmatrix}$$

The Model

$$
\begin{cases}
\frac{d\tilde{s}_i}{dt} = \tilde{s}_i(1-\tilde{s}_i)\Big[\sum_{k=1}^{N} a_{ik}(\tilde{s}_k + \alpha\tilde{h}_k)\Big], \tilde{s}_i(0) = \tilde{s}_{i0} \\
\frac{d\tilde{h}_i}{dt} = \tilde{h}_i(1-\tilde{h}_i)\Big[\Big(\sum_{k=1}^{N} a_{ik}\tilde{h}_k\Big) - \beta\tilde{s}_i + \frac{\gamma}{\tilde{s}_i}\Big], \tilde{h}_i(0) = h_{i0} \\
\alpha, \beta > 1, \gamma > 0.
\end{cases}
$$

The Model

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\frac{d\tilde{s}_i}{dt} = \tilde{s}_i(1-\tilde{s}_i)\Big[\sum_{k=1}^{N} a_{ik}(\tilde{s}_k + \alpha\tilde{h}_k)\Big], \tilde{s}_i(0) = \tilde{s}_{i0} \\
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\alpha, \beta > 1, \gamma > 0.
\end{cases}
$$

$\tilde{s}_i$: proportion of squares following trend i
$\tilde{h}_i$: proportion of hipsters following trend i
α: parameter governing the weight squares place on hipsters' tastes (3.0)
β: parameter governing the aversion of hipsters to squares (3.0)
γ: parameter governing the desire of the hipsters to be contrarian (0.5)
a_{ik}: interaction coefficient between fad i and k

Analysis of the Model

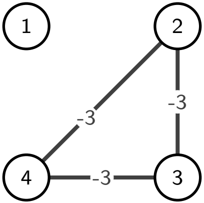
It's clear from the terms out in front that there are EQ points of a population whenever a particular h_i or s_i is equal to 0 or 1. However, even when a particular population is approaching 0 or 1, it is possible for the influence of other fads to shock the group out of this long-term behavior.

Analysis of the Model

It's clear from the terms out in front that there are EQ points of a population whenever a particular h_i or s_i is equal to 0 or 1. However, even when a particular population is approaching 0 or 1, it is possible for the influence of other fads to shock the group out of this long-term behavior.

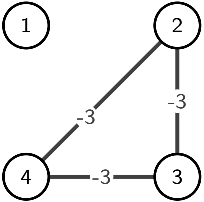
Another thing to note is the absence of any real negative terms in the equation for $\frac{d\tilde{s}_i}{dt}$ apart from potential negative values for A_{ik}. Thus we expect the comformists to come to dominate fads which have very weak interactions with other fads. In the same vein, we expect the non-comformists to dominate fads that interact very negatively with other fads.

An Example with High Negativity



G

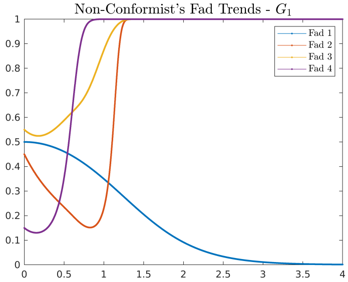
An Example with High Negativity

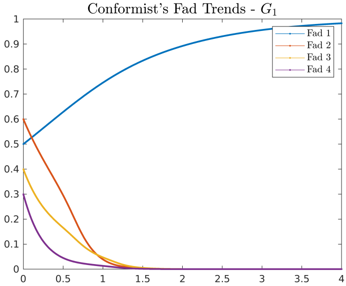


G

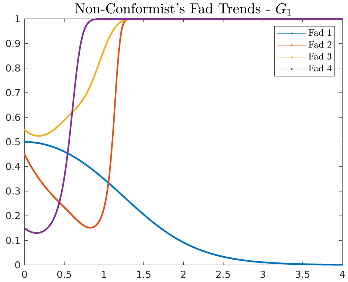
$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & -3 \\ 0 & -3 & 1 & -3 \\ 0 & -3 & -3 & 1 \end{bmatrix}$$

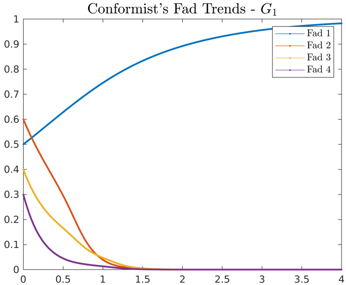
Plots with High Negativity





Plots with High Negativity

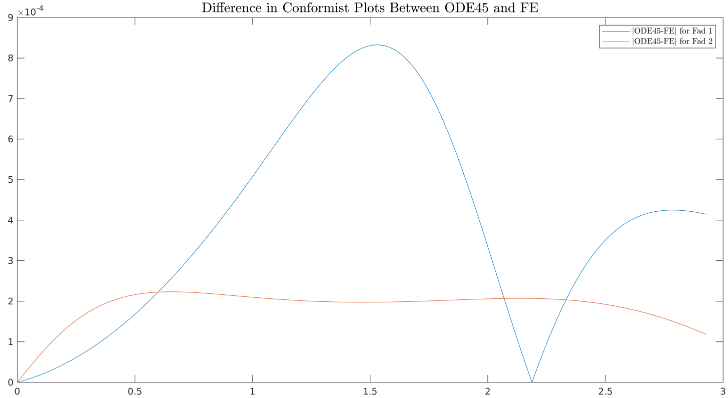




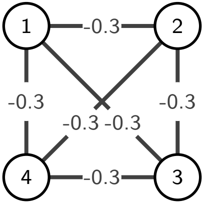
Video 1

Video 2

ODE45 vs. FE

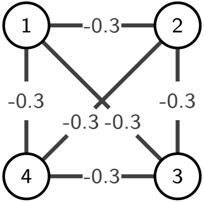

Difference in Conformist Plots Between ODE45 and FE
×10^-4
9
8
7
6
5
4
3
2
1
0
0
0.5
1
1.5
2
2.5
3
|ODE45-FE| for Fad 1
|ODE45-FE| for Fad 2

G_2



G_1

G_2



G_1

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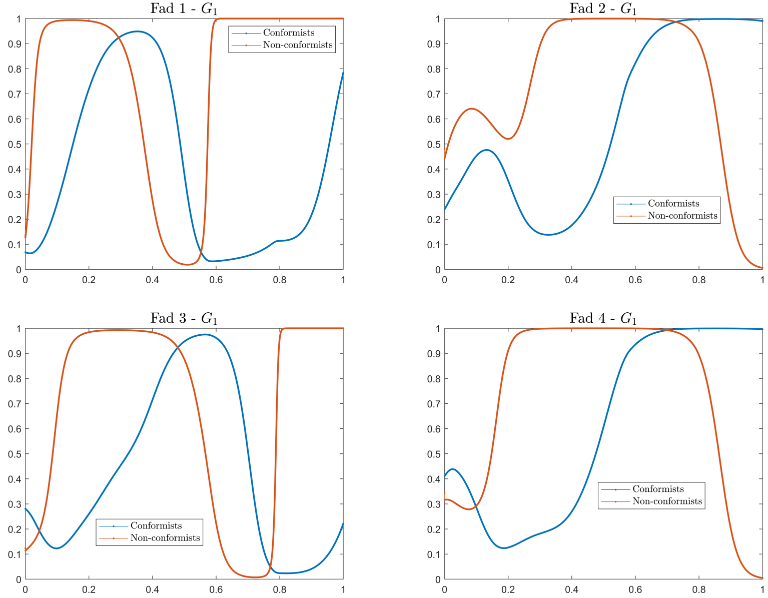
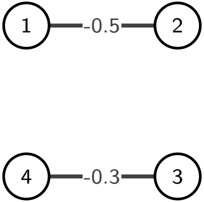
Plots for G_1:

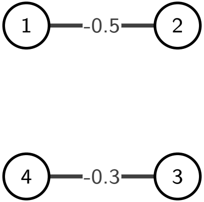

Fad 1 - G_1
Conformists
Non-conformists
Fad 2 - G_1
Conformists
Non-conformists
Fad 3 - G_1
Conformists
Non-conformists
Fad 4 - G_1
Conformists
Non-conformists

G_2:



G_2

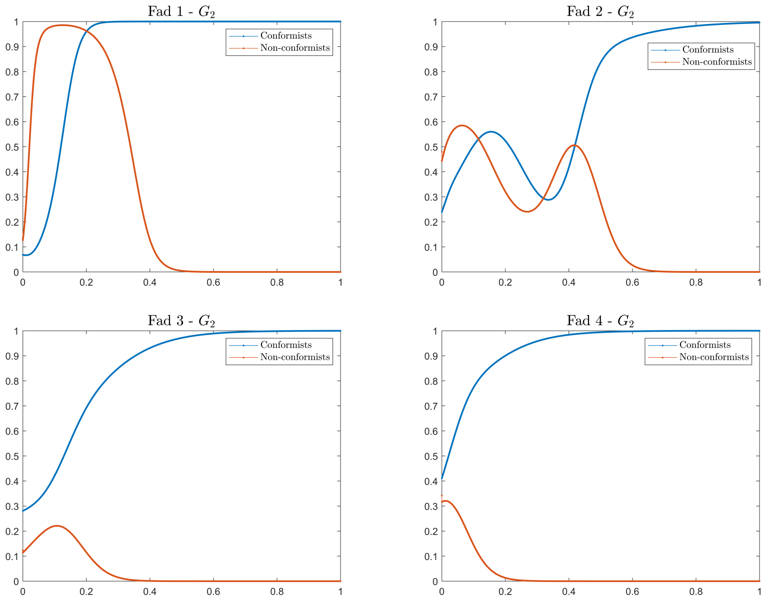
G_2:



G_2

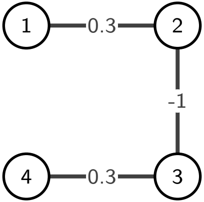
$$A_2 = \begin{bmatrix} 1 & -0.5 & 0 & 0 \\ -0.5 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.3 \\ 0 & 0 & -0.3 & 1 \end{bmatrix}$$

Plots for G_2:



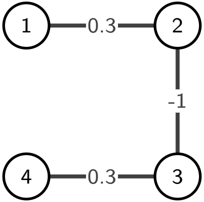
Fad 1 - G_2
Conformists
Non-conformists
Fad 2 - G_2
Conformists
Non-conformists
Fad 3 - G_2
Conformists
Non-conformists
Fad 4 - G_2
Conformists
Non-conformists

G_3:



G_3

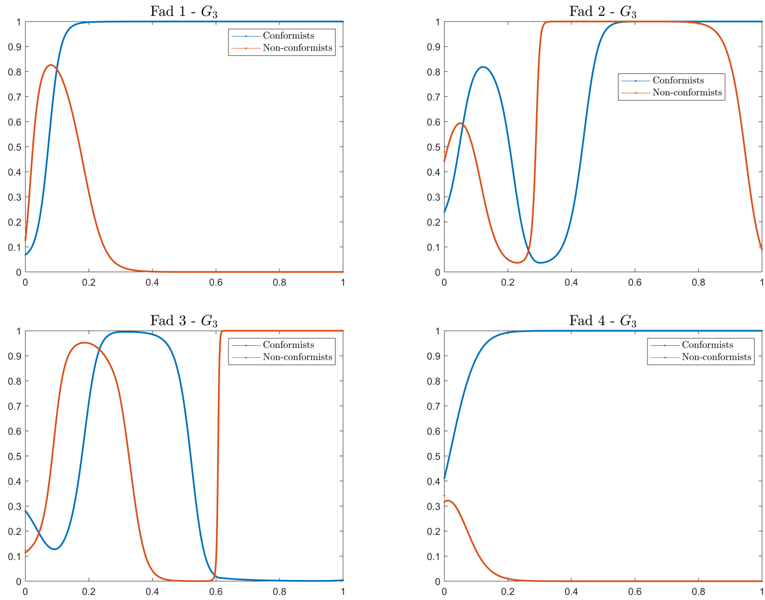
G_3:



G_3

$$A_3 = \begin{bmatrix} 1 & 0.3 & 0 & 0 \\ 0.3 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0.3 \\ 0 & 0 & 0.3 & 1 \end{bmatrix}$$

Plots for G_3:



Fad 1 - G_3
Conformists
Non-conformists
Fad 2 - G_3
Conformists
Non-conformists
Fad 3 - G_3
Conformists
Non-conformists
Fad 4 - G_3
Conformists
Non-conformists

Additional Issue 1

If a company were making an item whose purchase was correlated to some fad within the population, their best strategy would be to market exclusively to a small group of hipsters. Since hipsters respond quite rapidly to fad adoption within their community, our hypothetical company would see a sharp uptick in sales as the hipsters rush to buy their product.

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After the hipsters adopt the given fad, their behavior would drive the squares to adopt it as well, leading to a slower, sustainable level of sales in the immediate future.

Works Referenced

David Smith and Lang Moore, "The SIR Model for Spread of Disease - The Differential Equation Model," Convergence (December 2004)

https://github.com/sam-snarr/scudem-challenge