Quantum Scrambling

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Abstract goes here, state your claim!

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I. REPORT PLAN

A. Introduction

- Introduce quantum scrambling with a short description
- Where and why is this being studied
- What are the uses and conclusions we can draw - state examples
- What are our aims
- Outline the structure of the report

B. Background Theory

1. Qubit Systems

- What are qubits?
- Why do we use qubits?
- Quantum operators + Quantum Circuits
- Entanglement In qubit systems

2. *Fermionic Systems*

- What are fermions? - Introduce Building blocks
- Fock space and states; creation and annihilation operators; anticommutation relations
- Correlation Functions + Wicks theorem
- Majorana Fermions
- Entanglement in Fermionic Systems

3. *Encoding Information*

- How do we encode information? Introduce Pauli strings and why they're a convinient way to encode information.
- Ask stephen about this.

C. Quantum Scrambling

1. *Operator Spreading*

- Detailed explanations
- Introduce the picture of Spreading
- How we measure spreading in full generality
- Here we can focus on literature on spreading (since it is abundant)

2. *Entanglement Entropy*

- Explain how a system generates entanglement and how this looks in terms of Pauli strings
- Start with shannon entropy as a measure for loss of information
- introduce von neumann entropy and the conditions for an entropy we want
- also introduce renyi but state not used.
-

D. Appendix

- Density Matrices
- Wicks theorem
- Schur Decomposition
- Group Theory (Generators)
- Clifford Group

II. INTRODUCTION

Many-body systems and their dynamics play a central role in our understanding of modern physics. The dynamics of quantum many-body systems applies to a wide variety of fields in contemporary physics. Fields such as, quantum computation and information, modern condensed matter theory, quantum gravity.

III. QUBIT SYSTEMS

A. Quantum Bits

An intuitive example of a quantum many-body system, is that of a quantum computer. In a similar fashion to how a 'bit' is the basic unit of information within classical computation and logic, the 'qubit' (quantum-bit) forms the basic unit of information within quantum computation. Drawing the analogy from classical computation, where a bit occupies a binary state of 0 or 1, a quantum bit is a quantum state, denoted $|0\rangle$ or $|1\rangle$. These states form an orthonormal basis in $\mathbb{C}^2$ and are known as computational basis states. However, the analogy with classical computing ends here, as qubits can be in a linear superposition of states, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where α, β are complex probability amplitudes.

To describe a system of many qubits, the use of the tensor product is required. The Hilbert space of an n qubit system is constructed via the tensor product of subsystem Hilbert spaces for each qubit,

$$\mathcal{H} = \mathcal{H}^{\otimes n} \equiv \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \cdots \otimes \mathcal{H}_n. \tag{1}$$

The state of an n qubit system is constructed identically, and is often expressed as a binary string,

$$|x_1\rangle \otimes |x_2\rangle \otimes \cdots \otimes |x_n\rangle \equiv |x_1 x_2 \ldots x_n\rangle. \tag{2}$$

B. Circuits

To evolve a many-body state, such as an n qubit state, the time-evolution operator, U is used in the following way,

The evolution and dynamics of many-body systems can be represented via quantum circuits, constructed from a set of quantum logic gates acting upon the qubits of a system. Analogous to a classical computer which is comprised of logic gates that act upon bit-strings of information. In contrast, quantum logic gates are linear operators acting on qubits. This allows for the decomposition of a unitary evolution into a sequence of linear transformations, represented as matrices[1]. Such evolutions are often represented as a circuit diagrams, with each time step in the unitary evolution corresponding to a gate action upon a set of qubits. Analogous to circuit diagrams in classical computation, each quantum logic gate has a specified gate symbol, allowing the creation of complicated quantum circuitry that can be directly mapped to a sequence of linear transformations acting on a finite set of qubits. Some example gate symbols can be seen in Fig. 1.

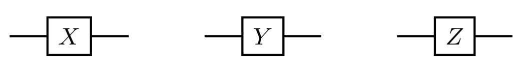

FIG. 1: Gate symbols for the Pauli operators in quantum circuits.

[1] Any linear map between two finite dimensional vector spaces, in this case finite dimensional Hilbert spaces, may be represented as a matrix.

C. Encoding Information

IV. FERMIONIC SYSTEMS

V. EFFICIENT QUANTUM CIRCUITS

VI. QUANTUM SCRAMBLING

VII. RESULTS