Scrambling Encoded Information

Physics Research Project

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Introduction

Our Question

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This process is known as **Quantum Scrambling**. We aim to investigate this process in many-body quantum systems.

Quantum Scrambling

The set-up:

- System with many particles, e.g spin chain or qubit system
- Information is encoded as a string of operators e.g $\mathcal{O} = X \otimes I \otimes Z \otimes Y \otimes I$
- System evolves via unitary conjugation $\to \mathcal{O}(t) = U\mathcal{O}U^\dagger$

How can we tell if information has been scrambled? We should observe:

- An increase in the support of a quantum operator - known as *operator spreading*.
  - Local operator: $IIIX_iIII$
  - Global operator: $X_1X_2X_3X_4X_5X_6X_7$
- A highly entangled system, signified by a growth in *entanglement entropy*

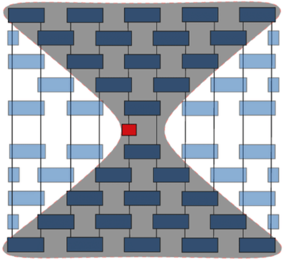
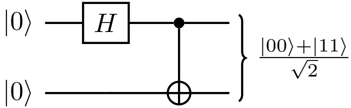


Figure: Emergent light-cone structure

Entanglement Entropy

Entanglement in state description



Entanglement in operator space

$$T^\dagger X T = \frac{X - Y}{\sqrt{2}}, \quad T^\dagger Y T = \frac{X + Y}{\sqrt{2}}$$

- Quantify entanglement using entanglement entropy.
- Initial operator string (e.g a product of Pauli Operators) becomes a product of superpositions of operators $\implies$ entanglement entropy grows
- We specifically look at the (bipartite) Von Neumann Entropy,

$$S_A = -\mathrm{tr}(\rho_A \log_2(\rho_A))$$

Entanglement entropy grows linearly in time, then saturates at the Page value [1]: $S_{\mathrm{max}} \equiv \ln|A|$.

Why?

Some motivation:

- Dates back to study of black holes and the black hole information paradox.
- Recently seen great attention in condensed matter, quantum information and quantum gravity.
- Lack of understanding of evolution in quantum chaotic systems.
- Reveals an irreversibilty of generic unitary dynamics - can no longer recover information with local probes.
- Proposed to characterise dynamics in ultra-cold atom systems.

Now back to our proposition...

Simulating a Quantum System

A Proposition

For sufficiently generic unitary evolution, we expect information to become *scrambled* amongst the degrees of freedom of our system.

To prove this proposition, we simulate our system as a quantum circuit. More specifically, we utilise two circuit models:

- Clifford circuits.
- Non-interacting Fermion circuits.

Both models are classically simulable, such that they can be simulated with polynomial effort.

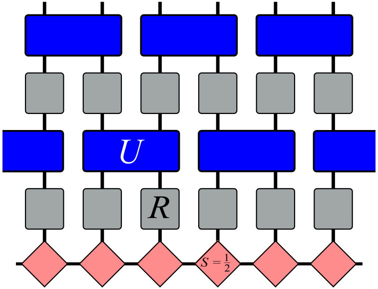
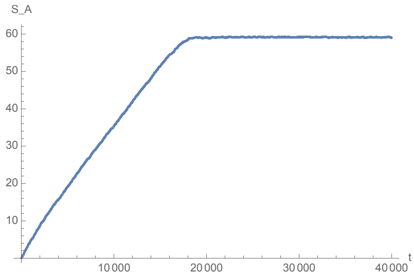


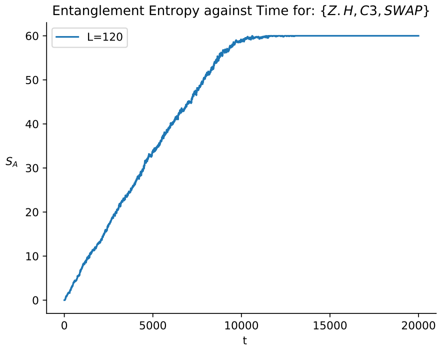
Figure: 'Brickwork' quantum circuit [2]

Clifford Circuits

Circuits comprised of gates from the Clifford Group: $\{H, CNOT, S\}$

- Not sufficient to generate operator complexity: $HXH^\dagger = Z$
- Can recover operator complexity with a non-local description of Pauli strings [3].
- This sacrifices any notion of operator spreading. Initial operator has support over the entire system.





Fermionic Systems

We can describe quantum circuits using operators from second quantization, namely the creation, $a_i^\dagger$ and annihilation, a_i operators.

- System of N qubits: $|n_1\rangle \otimes |n_2\rangle \otimes \cdots \otimes |n_L\rangle$
- System of N Local Fermionic Modes: $|n_1, n_2, \ldots, n_L\rangle \equiv (a_1^\dagger)^{n_1}(a_2^\dagger)^{n_2} \ldots (a_L^\dagger)^{n_L}|\underline{\mathbf{0}}\rangle$.

Obey crucial anti-commutation relations:

$$\{a_i, a_j\} = \{a_i^\dagger, a_j^\dagger\} \equiv 0 \qquad\qquad \{a_i, a_j^\dagger\} = \delta_{ij} I$$

To enforce anti-commutation relations, fermionic operators are defined using global operators. Implies inherent non-locality to fermionic description.

Non-interacting Fermion Circuits

- Use fermionic operators to construct unitary gates: $U = \exp(i\frac{\pi}{4}(a_0^\dagger a_1 + a_1^\dagger a_0))$
- Build a circuit using unitary gates. For non-interacting Fermions, no terms higher than quadratic order in fermionic operators.
- Shown to be classically simulable by Terhal and DiVincenzo [4].
- Also has been shown to exhibit Operator Spreading and a growth in entanglement entropy. Good candidate for studying quantum scrambling.

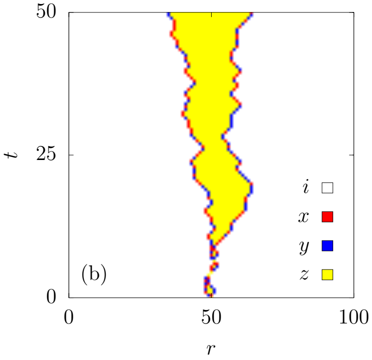


Figure: Operator spreading in Free Fermion Circuit [5]

Non-Interacting Fermion Circuits - Results

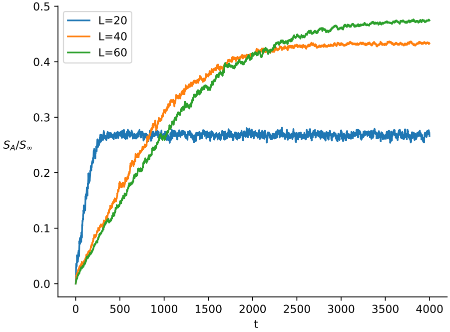


Figure: Numerical results for a free fermion circuit with system size, $L =$20, 40, and 60.

- Free fermion circuits exhibit quantum scrambling.
- Free Fermion circuits weakly scramble.
- Smaller system sizes saturate earlier.
- Entropy saturation value is dependent on system size.

Comparison

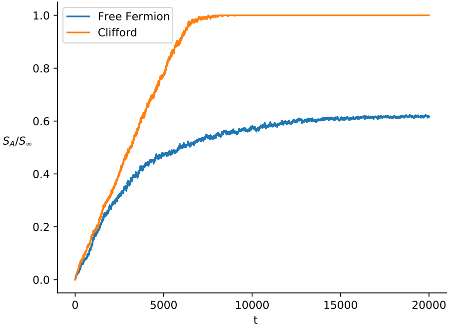


Figure: Numerical results for Clifford circuit and free fermion circuit. System size: $L = 100$

- Free Fermion weakly entangle compared to Clifford circuits.
- Entropy saturation occurs later in free fermion circuit
- Clifford circuits: Exhibit entanglement entropy growth but no operator spreading.
- Free Fermion circuits: Exhibit both entanglement entropy growth and operator spreading, but weaker.

Further Directions

- Changing geometry of free-fermion circuits to boost entanglement growth.
- Finding new, less constrained sets of gates in the Clifford group to scramble operators.
- Recent work on dual-unitary circuits has shown them to be good candidates for the study of scrambling using tensor network methods.
- Including higher-order terms in fermion circuits (interaction terms).

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