Quantum Scrambling Review*

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Abstract Goes Here...

CONTENTS

I. INTRODUCTION

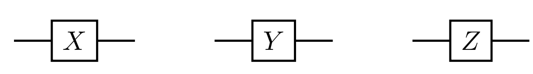
II. THEORY

A. Quantum Information and Circuits

Quantum Information and Quantum Computation is built upon the concept of quantum bits (qubits for short), represented as a computational basis state of either $|0\rangle$ or $|1\rangle$. With this, the quantum state of a system can be completely specified by writing the state as a linear combination, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where α, β are complex numbers. This is easily extended to multi-party quantum systems of n qubits. Then we may write computational basis states as strings of qubits using the tensor product;

$$|x_1\rangle \otimes |x_2\rangle \otimes ... \otimes |x_n\rangle \equiv |x_1 x_2 ... x_n\rangle.$$

The evolution and dynamics of such systems can be represented via quantum circuits, made up of quantum logic gates acting upon the qubits of a system. Analogous to a classical computer which is comprised of logic gates that act upon bit-strings of information. In contrast, quantum logic gates are formed of linear operators acting on qubits, often represented in matrix form. This allows us to decompose a complicated unitary evolution into a sequence of linear operators acting on one or more qubits, to be understood more clearly. A common practice is to create diagrams of such evolutions, with each quantum gate having their own symbol, analogous to circuit diagrams in classical computation, allowing the creation of complicated quantum circuitry that can be directly mapped to a string of linear operators acting on a set of qubits. Some example gate representations can be seen in fig 1



(a) pauliX (b) pauliY (c) pauliZ

An important set of such gates are the Pauli gates, equivalent to the set of Pauli matrices, $P \equiv$

$\{X, Y, Z\}$ for which X, Y and Z are defined as;

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad Y = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

These gates are all one-qubit gates, as they only act upon a single qubit. Another important one-qubit gate is known as the Hadamard, H, which maps computational basis states to a superposition of computational basis states, written explicitly in it's action;

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}},$$
$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}},$$

or in matrix form;

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

The Hadamard is widey utilised to create entanglement in circuits to generate states that cannot be written in product form. For example, if we prepare the state $|\psi_1\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$. We cannot write this as

$$\begin{aligned} |\psi_1\rangle &= [\alpha_0|0\rangle + \beta_0|1\rangle] \otimes [\alpha_1|0\rangle + \beta_1|1\rangle] \\ &= \alpha_0\beta_0|00\rangle + \alpha_0\beta_1|01\rangle + \alpha_1\beta_0|10\rangle + \alpha_1\beta_1|11\rangle \end{aligned}$$

since the α_0 or β_1 must be zero in order to ensure the $|01\rangle$, $|10\rangle$ vanish. However, this would make the coefficients of the $|00\rangle$ or $|11\rangle$ terms zero, breaking the equality. Thus, $|\psi_1\rangle$ is said to be entangled. This defines a general condition for state to be entangled.

B. Clifford Circuits and Stabilizer Formalism

A classical computer with n bits, has 2^n possible ways of arranging those bits into a binary vector, forming a n-dimensional state space. This is different to the case in quantum computation. A n-qubit system is described by a complex vector, with 2^n components, forming a 2^n dimensional Hilbert space due to the presence of entangled states. This makes quantum computation intractable in classical computation with polynomial effort, making it difficult to classically simulate and understand

comprised unitary operators from the Clifford group, namely the Hadamard, CNOT (Controlled-Not) and the S (phase) gate. In matrix form, these gates are defined as,

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \qquad S = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{bmatrix}$$

C. Operator Scrambling and Spreading