Quantum Scrambling Review*

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I. INTRODUCTION

II. THEORY

A. Quantum Information and Circuits

Quantum Information and Quantum Computation is built upon the concept of quantum bits (qubits for short), represented as a computational basis state of either $|0\rangle$ or $|1\rangle$. With this, the quantum state of a system can be completely specified by writing the state as a linear combination, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where α, β are complex numbers. This is easily extended to multi-party quantum systems of n qubits. Then we may write computational basis states as strings of qubits using the tensor product;

$$|x_1\rangle \otimes |x_2\rangle \otimes ... \otimes |x_n\rangle \equiv |x_1 x_2 ... x_n\rangle.$$

The dynamics of such systems can be represented via quantum circuits, made up of quantum logic gates acting upon the qubits of a system. Analogous to a classical computer which comprised of logic gates that act upon bit-strings of information. In contrast however, quantum logic gates are formed of linear operators, often written in matrix form. An important set of such gates is the Pauli gates, equivalent to the set of Pauli matrices, $P \equiv \{X, Y, Z\}$ for which X, Y and Z are defined as;

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad Y = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

These gates are all one-qubit gates, as they only act upon a single qubit. Another important one-qubit gate is known as the Hadamard, H, which maps computational basis states to a superposition of computational basis states, written explicitly in it's action;

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}},$$
$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}},$$

or in matrix form;

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

B. Operator Scrambling and Spreading