

# Quantum Scrambling Review\*

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Abstract Goes Here...

## CONTENTS

I. INTRODUCTION	1	$ x_1\rangle \otimes  x_2\rangle \otimes \dots \otimes  x_n\rangle \equiv  x_1x_2\dots x_n\rangle.$
II. Background Theory	1	
A. Quantum Information and Circuits	1	
B. Clifford Circuits and Stabilizer Formalism	2	
References	3	

## I. INTRODUCTION

## II. BACKGROUND THEORY

### A. Quantum Information and Circuits

Quantum Information and Quantum Computation is built upon the concept of quantum bits (qubits for short), represented as a computational basis state of either  $|0\rangle$  or  $|1\rangle$ . With this, the quantum state of a system can be completely specified by writing the state as a linear combination,  $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ , where  $\alpha, \beta$  are complex numbers. This is easily extended to multi-party quantum systems of  $n$  qubits. Then we may write computational basis states as strings of qubits using the tensor product;

The evolution and dynamics of such systems can be represented via quantum circuits, made up of quantum logic gates acting upon the qubits of a system. Analogous to a classical computer which is comprised of logic gates that act upon bit-strings of information. In contrast, quantum logic gates are linear operators acting on qubits, often represented in matrix form. This allows us to decompose a complicated unitary evolution into a sequence of linear transformations on one or more qubits. A common practice is to create diagrams of such evolutions, with each quantum gate having their own symbol, analogous to circuit diagrams in classical computation, allowing the creation of complicated quantum circuitry that can be directly mapped to a string of linear operators acting on a set of qubits. Some example gate symbols can be seen in Fig. 1.

The gates shown in Fig. 1 are known as the Pauli operators, equivalent to the set of Pauli matrices,  $P \equiv \{X, Y, Z\}$  for which  $X, Y$  and  $Z$  are defined in

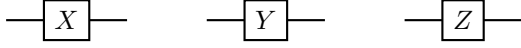


FIG. 1: Gate symbols for the Pauli operators in quantum circuits.

their matrix representation as;

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad Y = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}, \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

These gates are all one-qubit gates, as they only act upon a single qubit. Another important one-qubit gate is known as the Hadamard,  $H$ , which maps computational basis states to a superposition of computational basis states, written explicitly in it's action;

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}},$$

$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}},$$

or in matrix form;

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

The Hadamard, along with the CNOT (1) gate is used to generate entanglement in circuits to produce states that cannot be written in product form. For example, if we prepare the state  $|\psi_1\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$ . We cannot write this as

$$|\psi_1\rangle = [\alpha_0|0\rangle + \beta_0|1\rangle] \otimes [\alpha_1|0\rangle + \beta_1|1\rangle]$$

$$= \alpha_0\beta_0|00\rangle + \alpha_0\beta_1|01\rangle + \alpha_1\beta_0|10\rangle + \alpha_1\beta_1|11\rangle$$

since the  $\alpha_0$  or  $\beta_1$  must be zero in order to ensure the  $|01\rangle$ ,  $|10\rangle$  vanish. However, this would make the

coefficients of the  $|00\rangle$  or  $|11\rangle$  terms zero, breaking the equality. Thus,  $|\psi_1\rangle$  cannot be written in product form and is said to be entangled. This defines a general condition for a arbitrary state to be entangled.

## B. Clifford Circuits and Stabilizer Formalism

A notable family of circuits are known as *Clifford circuits*. These are circuits comprised only of unitary operators from the Clifford group, namely the Hadamard, CNOT (Controlled-Not) and the S (phase) gate. In matrix form, these gates are defined as,

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad S = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{bmatrix} \quad (1)$$

Quantum circuits made only of Clifford gates are said to be *classically simulable*, that is, circuits of this construction can be efficiently simulated on a classical computer with polynomial effort via the Gottesman-Knill theorem [1]. Clifford gates do not form a set of universal set of quantum gates.

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- [1] D. Gottesman, “The heisenberg representation of quantum computers,” 1998.