Prescribed strain rate tensor:

$$v_{ij} = \begin{bmatrix} a & b \\ b & d \end{bmatrix} \qquad (\text{symmetric part of } \nabla v)$$

$$v_{ij} \equiv \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$$

$a = v_{11}$
$b = v_{12} = v_{21}$
$d = v_{22}$

$a = \dfrac{\partial v_x}{\partial x} \qquad b = v_{12} = \dfrac{1}{2}\left(\dfrac{\partial v_1}{\partial x_2} + \dfrac{\partial v_2}{\partial x_1}\right) = \dfrac{1}{2}\left(\dfrac{\partial v_x}{\partial y} + \dfrac{\partial v_y}{\partial x}\right)$

$d = v_{22} = \dfrac{\partial v_y}{\partial y}$

Deviatoric & volumetric parts :-

$$v_{ij} = v_{ij}^{D} + v_{ij}^{V}$$

$$v_{ij}^{V} = \frac{1}{2} \operatorname{tr}[v_{ij}]\, I = \frac{1}{2}(a+d)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{a+d}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$v_{ij}^{D} = v_{ij} - v_{ij}^{V} = \begin{bmatrix} a & b \\ b & d \end{bmatrix} - \frac{a+d}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{a-d}{2} & b \\ b & \frac{d-a}{2} \end{bmatrix} \qquad \{\text{traceless}\}$$

$$v_s = \sqrt{v_{ij}^{*} v_{ij}^{*}} = \left\{ \sqrt{v_{11}^{*} v_{11}^{*} + v_{12}^{*} v_{12}^{*} + v_{21}^{*} v_{21}^{*} + v_{22}^{*} v_{22}^{*}} \right\}$$

$$= \sqrt{\left(\frac{a-d}{2}\right)^2 + b^2 + b^2 + \left(\frac{d-a}{2}\right)^2} = \sqrt{2b^2 + \frac{(a-d)^2}{2}}$$

GSM Equations

(I) $\qquad \partial_t \rho + \nabla_i (\rho v_i) = 0$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_x}{\partial x} + \frac{\partial \rho v_y}{\partial y} = 0$$

Taking spatially constant ρ

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial v_x}{\partial x} + \rho \frac{\partial v_y}{\partial y} = 0 \implies \frac{\partial \rho}{\partial t} + \rho\left(\underbrace{\frac{\partial v_x}{\partial x}}_{v_{11}} + \underbrace{\frac{\partial v_y}{\partial y}}_{v_{22}}\right) = 0$$

(II) $\partial_t(\rho v_i) + \nabla_i(\sigma_{ij} + \rho v_i v_j) = \rho g_i$

No idea about the velocity fields

(III) $$\frac{1}{R_T}\frac{\partial T_g}{\partial t} = -T_g\left[1 - \xi_T^2 \nabla_i^2\right]T_g + (f v_s)^2$$

$$\frac{1}{R_T}\frac{\partial T_g}{\partial t} = -T_g^2 + T_g \xi_T^2 \nabla_i^2 T_g + (f v_s)^2$$

Again assuming spatially homogeneous T_g. $\{\nabla_i^2 T_g = 0\}$

$$\frac{1}{R_T}\frac{\partial T_g}{\partial t} = -T_g^2 + f^2 v_s^2$$

$$v_s = \sqrt{v_{ij}^* v_{ij}^*} = \left\{ v_{ij}^* = v_{ij}^D = \begin{bmatrix} \frac{a-d}{2} & b \\ b & \frac{d-a}{2} \end{bmatrix} \right\} = \sqrt{2\left(\frac{a-d}{2}\right)^2 + 2b^2}$$

$$\| \quad (v_{11}^* v_{11}^* + v_{12}^* v_{12}^* + v_{13}^* v_{13}^* + \dots)^{1/2}$$

(IV) $\partial_t u_{ij}^* = v_{ij}^* - \lambda T_g u_{ij}^*$

Clearly, now there will be evolution of shear strains

$\rightarrow \partial_t u_{11}^* = v_{11}^* - \lambda T_g u_{11}^*$ $\qquad \rightarrow \partial_t u_{12}^* = v_{12}^* - \lambda T_g u_{12}^*$

$\partial_t u_{21}^* = v_{21}^* - \lambda T_g u_{21}^*$ $\qquad \rightarrow \partial_t u_{22}^* = v_{22}^* - \lambda T_g u_{22}^*$

(V) $\partial_t \Delta + v_{ll} = \alpha_1 u_{ij}^* v_{ij}^* - \lambda_1 T_g \Delta$

$$\underline{\underline{u}}^* = \begin{bmatrix} u_{11}^* & u_{12}^* \\ u_{21}^* & u_{22}^* \end{bmatrix} \qquad \underline{\underline{v}}^* = \begin{bmatrix} v_{11}^* & v_{12}^* \\ v_{21}^* & v_{22}^* \end{bmatrix}$$

$\Delta = -u_{11} - u_{22} - u_{33}$

$v_{u} = v_{11} + v_{22} + v_{33}$

$u_{ij}^* v_{ij}^* =$ einstein summation $= u_{11}^* v_{11}^* + u_{12}^* v_{12}^* + u_{21}^* v_{21}^* + u_{22}^* v_{22}^*$

$\rho, T_g, u_{ij}^*, \Delta$
$\hookrightarrow$ 4 components
3 distinct "

ρ, T_g, Δ
u_{ij}^*

Stresses, $\sigma_{ij} = \pi_{ij} + P_T S_{ij} - \eta_1 T_g v_{ij}^*$ $\rightarrow 57\times 1$ $\rightarrow u_{ij}^*$

again, $\pi_{ij} = \pi_{ij}^v + \pi_{ij}^D$
(or $\pi_{ij}^* \Rightarrow$ traceless)

$\pi_{ij}^v = \underbrace{\frac{1}{D} tr[\pi_{ij}]}_{P_\Delta} I$

$\pi_{ij}^* = -2A\sqrt{\Delta}\, u_{ij}^*$

$\sqrt{\Delta}$: 57×1, u_{ij}^*: 57×4

$\pi_{11}^* = -2A\underbrace{\sqrt{\Delta}}_{57\times 1}\, \underbrace{u_{11}^*}_{57\times 1}$

$P_\Delta = \sqrt{\Delta}\left(B\Delta + Au_s^2/2\Delta\right)$

$u_s = \sqrt{u_{ij}^* u_{ij}^*} = \sqrt{\underbrace{u_{11}^{*2} + u_{12}^{*2} + u_{21}^{*2} + u_{22}^{*2}}_{57\times 1}}$

t1 : $1^2 + 2^2 + 3^2 + 4^2$

$$u^* = \begin{array}{cccc} u_{11} & u_{12} & u_{21} & u_{22} \\ \vdots & \vdots & \vdots & \vdots \end{array}$$

at a time t

$\pi_{ij}^v = P_\Delta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\qquad \pi_{ij}^* = \begin{bmatrix} - & - \\ - & - \end{bmatrix}$ {traceless}

$\pi_{ij} = \pi_{ij}^v + \pi_{ij}^*$

$\boxed{\text{reshape}(A', 1, 4)}$

$P_\Delta = 57\times 1$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{array}{cccc} 11 & 21 & 12 & 22 \\ 1 & 0 & 0 & 1 \end{array}$

$57\times 1 \qquad 1\times 4$

57 $\quad$ 1 0 0 1

57×4

$\begin{bmatrix} - \\ - \\ - \\ - \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} - & 0 & 0 & - \\ \vdots & & & \end{bmatrix}$