Numerical Methods

PDE 4 Worksheet – Boundary value problems

December 4, 2017

Matlab solvers for the transient and steady-state heat equation

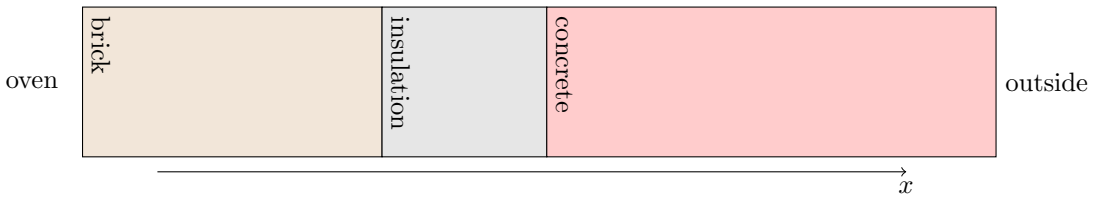

Figure 1: Sketch of the wall geometry.

The wall of an oven consists of a 20 cm layer of fire-proof bricks, a 11 cm layer of insulation, and a 30 cm layer of concrete, as shown in Figure 1. Assume the oven is very large, so the wall is nearly planar. Assume that the oven temperature is initially equal to the ambient temperature $T_o = 20\,^\circ\mathrm{C}$; then the oven is heated to its maximum allowed temperature of $T_i = 1200\,^\circ\mathrm{C}$. The material properties are:

brick: density $\rho = 1500\,\mathrm{kg/m^3}$, thermal capacity $C_\mathrm{p} = 1000\,\mathrm{J/kg/K}$, heat transfer coefficient $\lambda = 1.2\,\mathrm{W/m/K}$;

insulation: density $\rho = 500\,\mathrm{kg/m^3}$, thermal capacity $C_\mathrm{p} = 2000\,\mathrm{J/kg/K}$, heat transfer coeff. $\lambda = 0.1\,\mathrm{W/m/K}$;

concrete: density $\rho = 2000\,\mathrm{kg/m^3}$, thermal capacity $C_\mathrm{p} = 1000\,\mathrm{J/kg/K}$, heat transfer coeff. $\lambda = 0.7\,\mathrm{W/m/K}$.

Questions

1. State the initial-boundary value problem you have to solve to find the temperature profile $T(t, x)$. Solve these equations using `pdepe`, and plot the temperature profile after $t = 0, 1, \ldots, 5$ days. After how many days is the system near steady state?

2. State the boundary value problem you have to solve to find the *steady-state* temperature profile $T(x)$. Solve the equations using `bvp4c`. Plot the temperature profile $T(x)$ and compare to the solution you got from question 1.

3. The amount of heat transferred through the wall is given by the local heat flux, $f = \lambda \frac{dT}{dx}$. Check that the local heat flux is constant in steady state by plotting $f(x)$ for the solutions to question 1 and 2.

4. Assume that the bricks and the insulation are heat-proof, but the concrete will fail if it is heated to temperatures above 350 °C. Find the maximum temperature in the concrete layer, T_{max}, and display the answer in a formatted sentence to the command window.[1] Will the concrete fail?

5. If the oven is very small, the geometry of the oven has to be taken into account. Assume that the oven walls are cylindrical with an inner radius of $r_i = 0.5\,\mathrm{m}$ (consequently, the outer radius is 1.11 m); assume the same wall setup (brick-insulation-concrete) as before. Solve for the temperature profile $T(t, x)$ using `pdepe`. Compare the maximum temperature in the concrete layer, $T_{max}(t)$, for the cylindrical oven and the planar oven.

[1] Because the temperature increases over time, it is sufficient to find T_max for the steady state solution. You should get about 318 °C

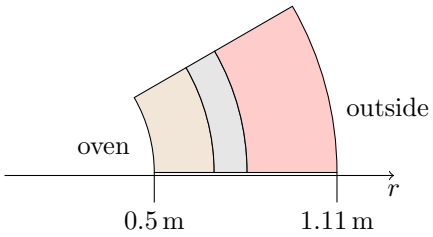

Figure 2: Sketch of the cylindrical wall geometry.