Name ____________________ Student Number ____________________

Numerical Methods

Retake Exam 2 – Ordinary differential equations

December 15, 2017

Instructions: You have 120 minutes (13:45-15:45) to complete the exam. You can use the script and the notes you made during the course. You are not allowed to use the internet except for accessing the MATLAB documentation on `www.mathworks.nl/help`.
Create a new folder `exam2` on your laptop. In this folder, create a MATLAB script `exam2.m`; write the solutions to this exam into this file. All solutions to the exam should be created when you execute `exam2.m`. Use sections (starting with `%% Question X`) to separate individual code segments. Indent the code in loops and if statements. Use sensible variable names. You will not be graded for commenting your code.
Once you have finished, compress the folder `exam2` (containing all images and m-files) and email it to both `t.weinhart@utwente.nl`. Write your name and student number into the subject line of the email and onto this sheet. Good luck!

Background: A sinking object

The sinking velocity v of an object in a fluid is described by to following differential equation,

$$\frac{dv}{dt} = -\gamma v^2 + g',, \tag{1}$$

where $\gamma = \frac{3\rho_f C_d}{8\rho_s r}$ denotes the drag constant, and $g' = \frac{\rho_s - \rho_f}{\rho_s} g$ the buoyant gravitational acceleration. Suppose you have an iron cannonball of radius 4 cm sinking in water. The density of iron is $\rho_s = 7860\,\mathrm{kg/m^3}$, the density and drag coefficient of water is $\rho_f = 1000\,\mathrm{kg/m^3}$ and $C_d = 0.44$, respectively. You can assume that the flow around the object is turbulent.

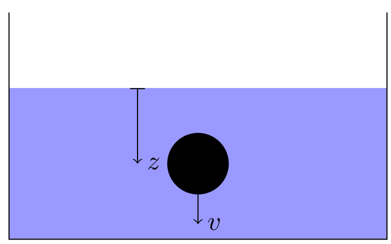

Figure 1: Cannon ball sinking in a water basin; z denotes the sinking depth, v the downwards velocity.

Continue on other side

Questions

1. We want to find the sinking velocity of the cannonball. State the initial value problem (IVP) you need to solve, assuming the cannon ball is initially at rest. [10pt]

State the initial conditions and differential equation.

(2)

2. Use MATLAB's `ode45` solver to find the solution to the initial value problem (2). Solve for $t \in [0, 2]$. Set the relative accuracy of the ODE solver to 10^{-8} to achieve 8 digits of accuracy. Use `fprintf` to display the final velocity to the command window, using a full sentence. [15pt]

3. Instead of fixing the final time, use an `event` function to stop the solution when the velocity reaches $5\,\text{m/s}$. [15pt]

4. Plot both the numerical and the analytical solution for the velocity as a function of time into the same plot. Use markers for the numerical solution and a line plot for the analytical one. You should see the velocity of the projectile decrease towards a constant terminal velocity. Add labels and save the plot as `velocity.pdf`. [15pt]

5. Now solve the IVP (2) using the forward Euler scheme for $t \in [0, 2]$, using 100 time steps. Display the final velocity to the command window. [15pt]

6. To get the trajectory of the projectile, $z(t)$, integrate the velocity, $z(t) = \int_0^t v(t')dt'$ using `cumtrapz` or `trapz`. Plot the resulting trajectory $z(t)$, and save the plot as `position.pdf`. [10pt]