Numerical Methods

Exam 3 – PDE

December 15, 2017

Heat flow through a wall

A house wall consists of a 5 cm layer of insulation and a 30 cm layer of brick. The heating of the house is set such that the temperature inside the house is kept at a constant $T_\mathrm{i} = 25°\mathrm{C}$.

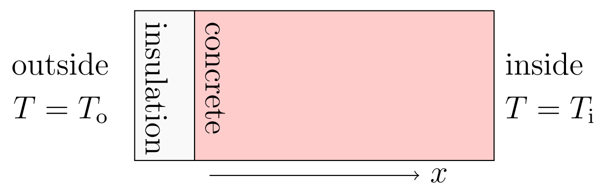

Figure 1: Sketch of the wall geometry.

Material properties (density, thermal capacity, heat transfer coefficient):
brick: ρ_b=1500 $\frac{\mathrm{kg}}{\mathrm{m}^3}$, C_b=1000 $\frac{\mathrm{J}}{\mathrm{kgK}}$, λ_b=1.2 $\frac{\mathrm{W}}{\mathrm{mK}}$; insulation: ρ_i=500 $\frac{\mathrm{kg}}{\mathrm{m}^3}$, C_i=2000 $\frac{\mathrm{J}}{\mathrm{kgK}}$, λ_i=0.5 $\frac{\mathrm{W}}{\mathrm{mK}}$.
Assume that the outside temperature oscillates between 10°C in the night and 20°C during the day, and can be described by the formula (with $t_o = 1\,\mathrm{day} = 86400\,\mathrm{s}$)

$$T_\mathrm{o} = 15°\mathrm{C} + \sin(2\pi t/t_o) \cdot 10°\mathrm{C}.$$

Calculate the temperature $T(t, x)$ inside the wall for the first five days. Assume that the wall has an initially constant temperature of 25°C. Plot the temperature profile at 12:00 and 24:00 on the last day. Use appropriate labels.

The heat flux at the inside wall, $\phi(t) = \lambda \frac{\partial T}{\partial x}|_{x=30\,\mathrm{cm}}$, equals the power consumption per m^2 of wall that the heating has to generate. Plot $\phi(t)$ for the first five days (it should show an oscillating pattern). Add appropriate axis labels.

A simple rheometer

Consider two concentric rings of radii $R_1 = 9\,\mathrm{mm}$ and $R_2 = 10\,\mathrm{mm}$, with the space in between filled with water (viscosity $0.001\,\mathrm{Pa\,s}$, density $1000\,\mathrm{kg/m^3}$). The outer ring is driven at a constant angular velocity $\Omega = 1\,\mathrm{rad/s}$, while the inner ring is stationary. The angular velocity of the water ω satisfies the equation

$$\rho \frac{\partial \omega}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\eta r \frac{\partial \omega}{\partial r}\right).$$

Find the steady-state velocity profile $v_\theta(r) = r\omega(r)$ of the fluid between the two rings (you can use either `pdepe` or `bvp4c`). Plot it using appropriate axis labels.

The shear stress of the liquid is given by the flux $\tau = \eta r \frac{\partial \omega}{\partial r}$. Measure the shear stress on the inner and the outer ring, and check that they are nearly the same. Display both values in a full sentence to the command window, using `fprintf`.

Note, because there is a simple relationship between the shear stress and the viscosity, the so-called 'Couette cell' is often used to measure the viscosity of a liquid.

Cooling of a spherical object

A copper ball of radius 6 cm and with an initial temperature of 100°C is dropped into a large water basin of temperature 20°C. Assume that the water basin is large enough and well-convected such that you can assume a constant temperature of 20°C at the surface of the copper ball.

Properties of copper: thermal conductivity $\lambda_\mathrm{c} = 400\,\mathrm{W/m/K}$, specific heat $c_\mathrm{c} = 385\,\mathrm{J/kg/K}$, density $8960\,\mathrm{kg/m^3}$.

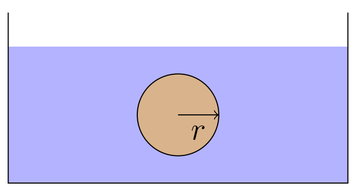

Figure 2: Copper ball in a water basin.

The temperature of the copper ball is governed by the heat equation. Carefully think about which boundary and initial conditions you should apply.
Plot the temperature of the center of the sphere as a function of time. Use an appropriate timescale to observe the temporal decay. Add appropriate axis labels.

When does the center of the sphere reach a temperature of $T = 30$°C? Find the exact time (using e.g. `interp1`) and use `fprintf` to display your answer in the command window.