MOCK BOARD EXAMINATION IN ENGINEERING MATHEMATICS

(April 24, 2009)

1) Suppose A = {2, 4, 6, 8, 10, 12}, B = {1, 4, 9, 16} and C = {2, 10}
   a) $A \cup B$ = { 1, 2, 4, 6, 8, 9, 10, 12, 16}
   b) $A \cup B$ = {4}
   c) $A \cup B$ = { 1, 2, 6, 8, 9, 10, 12, 16)
   d) $A \cup B$ = { 1, 4, 9, 16}
2) The sum of two numbers is 21, and one number is twice the other. Find the numbers
   a) 6 and 15
   b) 2 and 12
   c) 7 and 14
   d) 8 and 13
3) If (x + 3) : 10 = (3x – 2) : 8, find ((2x –1).
   a) 1
   b) 4
   c) 2
   d) 3
4) In the expansion of $(x + 4y)^{12}$, the numerical coefficient of the 5^{th} term is
   a) 63 360
   b) 126 720
   c) 506 880
   d) 22 280
5) Determine x, so that: x, 2x + 4, 10x – 4 will be a geometric progression.
   a) 4
   b) 6
   c) 2
   d) 5
6) If angle ϕ = 2, then angle (180° - ϕ) = ___.
   a) 65.4° or 1.1416 radian
   b) 64.5° or 1.1614 radian
   c) 45.6° or 1.6141 radian
   d) 54.6° or 1.4161 radian
7) Suppose A = {2, 4, 6, 8, 10, 12}, B = {1, 4, 9, 16} and C = {2, 10}
   a) $B \cap C$ = { 1, 2, 4, 9, 10, 16}
   b) $B \cap C$ = {0}
   c) $B \cap C = \varnothing$
   d) $B \cap C$ = {2, 10}
8) The hypotenuse of a right triangle is 34 cm. Find the length of the two legs, if one leg is 14 cm longer than the other.
   a) 18 and 32 cm
   b) 15 and 29 cm
   c) 17 and 31 cm
   d) 16 and 30 cm
9) Find the value of x in the equation: csc x + cot x = 3.
   a) $\pi/4$
   b) $\pi/2$
   c) $\pi/3$
   d) $\pi/5$
10) Solve for A in the equation: $\cos^2 A = 1 - \cos^2 A$
    a) 15°, 125°, 225°, 335°
    b) 45°, 125°, 225°, 315°
    c) 45°, 135°, 225°, 315°
    d) 45°, 150°, 220°, 315°
11) a < b if and only if b – a is ___.
    a) negative
    b) positive
    c) zero
    d) none of these
12) A circle with radius 6 has half its area removed by cutting off a border of uniform width. Find the width of the border
    a) 2.2
    b) 1.35
    c) 3.75
    d) 1.76
13) If the radius of the circle is decreased by 20%, by how much is its area decreased?
    a) 46%
    b) 36%
    c) 56%
    d) 26%
14) Exact angle of the dodecagon is equal to ___ deg.
    a) 135
    b) 100
    c) 125
    d) 150
15) A 50-meter cable is divided into two parts and formed into two squares. If the sum of the areas is 100 sq. meters, find the difference in length?
    a) 21.5
    b) 20.5
    c) 24.5
    d) 0
16) a > b if and only if ___
    a) b is more than 1
    b) a is more than 1
    c) b is zero
    d) b is less than a
17) The volume of a cube is reduced to ___ if all the sides are halved.
    a) ½
    b) ¼
    c) 1/8
    d) 1/16
18) A reservoir is shaped like a square prism. If the area of its base is 225 sq. cm., how many liters of water will it hold if its length is 1.5 meters?

a) 337.5
b) 33.75
c) 3375
d) 3.375

19) Find the volume of the sphere whose circumference of a great circle is 18 π.
a) 3984.43
b) 3053.63
c) 3291.68
d) 3643.03

20) When the radius of a sphere is increased by 16%, what percent is the increase in the volume of the sphere?
a) 16%
b) 32%
c) 64%
d) 56%

21) a $\leq$ b if and only if either a < b or
a) a = 0
b) b = 1
c) a = b
d) none of these

22) Find the equation of the directrix of the parabola $y^2 = 16x$.
a) x = - 4
b) x = - 8
c) x = 4
d) x = 8

23) The diameter of a circle described by $9x^2 + 9y^2 + 2 = 16$ is ___.
a) 4/3
b) 16/9
c) 8/3
d) 4

24) If the points (-2, 3), (x, y) and (-3, 5) lie on a straight line, then the equation of the line is ___.
a) x – 2y – 1 = 0
b) 2x + y – 1 = 0
c) x + 2y – 1 = 0
d) 2x + y + 1 = 0

25) Find the location of the vertex of the parabola defined by the equation: $y = x^2 – 4x + 1$
a) (2, 3)
b) (-2, 3)
c) (2, -3)
d) (-2, -3)

26) a > 0 if and only if ___
I. a is positive
II. a is negative
III. –a < 0
IV. –a > 0
a) I & III only
b) II & IV only
c) I & II only
d) III & IV only

27) Evaluate: $M = \lim_{x \to 2} (x^2 – 4) / (x – 2)$
a) 3
b) 4
c) 2
d) 5

28) The derivative of ln cos x is:
a) sec x
b) –tan x
c) –sec x
d) tan x

29) Find the radius of curvature at any point of the curve y + ln (cosx) = 0.
a) 1
b) 1.5707
c) cos x
d) sec x

30) Find the equation of the normal to $x^2 + y^2 = 1$ at the point (2, 1)
a) X = 3Y
b) X = 2Y
c) X = Y
d) X = 4Y

31) If a < b & b < c, then ___
a) a < c
b) c < a
c) a > c
d) c > a

32) What is the integral of $(3t – 1)^3$ dt?
a) $(1/12)(3t – 1)^4 + c$
b) $(1/12)(3t – 4)^4 + c$
c) $(1/4)(3t – 1)^4 + c$
d) $(1/4)(3t – 1)^3 + c$

33) Find the value of $(1 + I)^5$, where I is an imaginary number.
a) 1 – i
b) 1 + i
c) –4 (1 + i)
d) 4 (1 + i)

34) If a < b, then a + c < b + c, and a – c < b – c if c is
a) subtracted from a only
b) added to b only
c) subtracted from b only
d) any real number

35) If a < b & c < d, then
a) a + c < b – d
b) a + b < b + d
c) a + d < b + c
d) none of these

36) If a < b & if c is any positive number, then __
a) ac < bc
b) ac > bc
c) ac < bd
d) none of these

37) If a < b & if c is any negative number, then ___

a) ac < bc
b) ac > bc
c) ac < bd
d) none of these

38) If 0 < a < b and 0 < c < d, then ___
a) ac < bd
b) ac > bd
c) ab > cd
d) ab < cd

39) If a > b & b > c, then ___
a) a > c
b) c > a
c) a > b is positive
d) a > b is negative

40) If a > b, then a + c > b + c, and a – c > b – c if c is
a) subtracted from a only
b) any real number
c) added to b only
d) subtracted form b only

41) If a > b and c > d, then ___
a) a + d > b + c
b) a + c > b + d
c) a + b > c + d
d) none of these

42) If a > b & if c is any positive number, then
a) ac < bc
b) ac = bc
c) ab > ac
d) ac > bc

43) If a > b & c is any negative number, then ___
a) ac < bc
b) ac = bc
c) ab > ac
d) ac > bc

44) In mathematical logic, there are three traditional laws of thought to exemplify something fundamental on the way, we think. If we say that something cannot be TRUE and FALSE all at the same time, this law is called the Law of ____________.
a) Contradiction
b) Excluded Middle
c) Identity
d) Subaltern

45) Felicito draws three balls in succession (without replacement), from a box containing five (5) Red Balls, Six (6) Yellow Balls, Seven (7) Green Balls. The probability of drawing the balls in the order Red, Yellow and Green is _____
a) 0.2894
b) 0.3894
c) 0.4289
d) 0.3489

46) A family of curves whose equations are the solutions of a given differential equation, i.e. the family of circles: $x^2 + y^2 = c^2$, which is the solution of the differential equation x + y (dy / dx) = 0.
a) Integral Curves
b) Differential Curves
c) Double Points
d) Orthogonals

47) Find a, b, c which satisfies the hypothesis of Rolle's theorem for $f(x) = x^2 - 1$.
a) a = 0; b = 1; c = 1/2
b) a = -1; b = 1; c = 1/2
c) a = -1; b = 0;c = 1/2
d) a = -1; b = 1; c = 0

48) A rectangle is inscribed in a square so that each vertex of the rectangle is at the trisection point of different sides of the square. The ratio of the area of the rectangle to that of the square is ___.
a) 7:72
b) 2:7
c) 4:9
d) 5:9

49) An integer greater than one that has no integral factors except itself and one is called ___ number.
a) Prime
b) Irrational
c) Transcendental
d) Differential

50) Find a number "c" which satisfies the conclusion of the "Mean Value Theorem" for f(x) = 1/x, a=2 and b = 4.
a) $\sqrt{5}$
b) $2\sqrt{3}$
c) $7\sqrt{2}$
d) $2\sqrt{2}$

MOCK BOARD EXAM ANSWERS (Mathematics)

1. A $A \cup B = \{1, 2, 4, 6, 8, 9, 10, 12, 16\}$
2. C 7 and 14

Sol'n: Let: x and y be the numbers

Condition 1: $x + y \rightarrow$ Equation 1

Condition 2: $x = 2y \rightarrow$ Sub to Eq. 1

$2y + y = 21$; $3y = 21$; $y = 7$

But $x = 2y = 2 (7) = 14$

The numbers are 1 and 14

3. D 3

Sol'n: $(x + 3) / 10 = (3x – 2) / 8$

$8x + 24 = 30x – 20$

$22x = 44$; $x = 2$

$2x – 1 = 2(2) – 1$

$2x – 1 = 3$

4. B 126 720

Sol'n: in the binomial formula, for the 5^{th} term, r = 4

$(r + 1)$th term $= {}_nC_r\, a^{n-r}\, b^r$

5^{th} term $= {}_{12}c_4\, x^{12-4}\, (4y)^4$

5^{th} term $= \{[12 (11) (10)(9)] / 4!\} (x)^8(4y)^4$

5^{th} term $= 126\ 720\ x^8y^4$

5. A 4

Sol'n: Geometric progression – series of numbers the ratio of any two consecutive terms is constant

That is: $(10x – 4) / (2x + 4) = (2x + 4) / x$

$10x^2 – 4x = 4x^2 + 16x + 16$

$6x^2 – 20x – 16 = 0 \rightarrow$ divide by 2

$3x^2 – 10x – 8 = 0$

$x = 4$

6. A 65.4° or 1.1416 radian

Sol'n: $180° = \pi$ radian

Thus: $(180°- \phi) = \pi r^2 = 1.1416$

7. C $B \cap C = 0$
8. D 16 and 30

Sol'n: By Pythagorean theorem:

$x^2 + (x + 14)^2 = 34^2$

$x^2 + x^2 + 28x + 196 = 1156$

$2x^2 + 28 x – 960 = 0 \rightarrow$ divide by 2

$x^2 + 14x – 480 = 0$

$x = [-14 \pm \sqrt{14^2 – 4(1) (-480)}] / 2(1)$

$x = 16$ and 30

9. D $\pi / 5$

Sol'n: $\csc x + \cot x = 3$

$(1/\sin x) + (\cos x / \sin x) = 3 \rightarrow$ multiply by sin x

$1 + \cos x = 3 \sin x$

By trial and error: $x = \pi / 5$

10. C 45°, 135°, 225°, 315°

Sol'n: $\cos^2 A = 1 – \cos^2 A$

$2 \cos^2 A = 1$

$\cos^2 A = ½$; $\cos A = \pm \sqrt{½}$

$A = \text{inv} \cos \pm \sqrt{½} = 45°, 135°, 225°, 315°$

11. B positive
12. D 1.76

Sol'n: $A_{remaining} = ½\, A_{original}$

$\pi r^2 = ½\, \pi (6)^2$; $\pi (6 – z)^2 = \pi (6)^2 / 2$

$6 – z = \sqrt{18}$; $z = 1.757$

13. B 36%

Sol'n: A_1 = area of original circle $= \pi R^2$

r = reduced radius of circle = 0.8 R

$A_2 = \pi (0.8)^2 = 0.64\pi R^2 = 0.64 A_1$ or $64\%A_1$

% reduction = 100% - 64% = 36%

14. D 150

Sol'n: dodecagon = 12 sided polygon

$\gamma = [(n – 2) 180°] / n = [(12 – 2) (180°)] / 12 = 150°$

15. C 24.5

Sol'n: $4x + 4y = 50 \leftarrow$ sum of perimeters of the two squares. Divide by 4.

$x + y = 12.5 \rightarrow$ Eq. 1

$x^2 + y^2 = 100 \leftarrow$ sum of area. Substitute Eq. 1

$(12.5 – y)^2 + y^2 = 100$

$156.25 – 25 + y^2 + y^2 = 100$

$2y^2 –25y + 56.25 = 0$

$y = [25 \pm \sqrt{25^2 – 4 (2)(556.25)}] / 2 (2)$

$y = 2.94 \rightarrow$ Substitute to Eq. 1

$x = 12.5 – y = 12.5 – 2.94 = 9.06$

$4x – 4y = 4 (9.06) + 4 (2.94) = 24.48$

16. D b is less than a
17. C 1/8

Sol'n: Let: V_1 = original volume of cube $= x^3$

$V_2 = (x/2)^3 = (1/8) x^3$

18. B 33.75

Sol'n: $A = 225\ cm^2 (1m/100\ cm)^2 = 0.0225m^2$

$V = A \times L = 0.0225 (1.5)$

$V = 0.03375\ m^3 (1000\ liters / 1m^3) = 33.75$ liters

19. B 3053.63

Sol'n: A great circle is one whose radius is the same as that of the sphere: $C = 2 \pi R$; $18\pi = 2\pi R$; $R = 9$

Volume $= (4/3) \pi R^3 = (4/3) \pi (9)^3 = 3053.63\ m^3$

20. D 56%

Sol'n: Volume of 1^{st} sphere $= (4/3) \pi R_1^3$

Volume of 2^{nd} sphere $= (4/3) \pi (1.16R_1)^3$

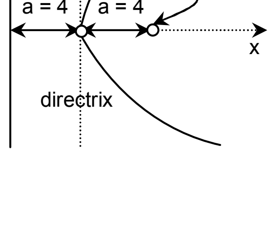
Volume of 2^{nd} sphere $= (1.56) (4/3) \pi R_1^3$

Volume of 2^{nd} sphere = 1.56 x Volume of 1^{st} sphere

Thus, increase in volume is 56%

21. C a = b
22. A x = -4

Sol'n:



$y^2 = 16x$; $(y - 0) = 16 (x - 0)$
By inspection: Vertex is at V (0, 0)
$4a = 16$; $a = 4$
By inspection, the equation of the directrix is at:
$x = -4$

23. C 8/3
Sol'n: $9x^2 + 9y^2 = 16 \rightarrow$ divide by 9
$x^2 + y^2 = 16/9 = (4/3)^2 \rightarrow$ circle whose r = 4/3
d = diameter = 2r = 2 (4/3) = 8/3

24. D $2x + y + 1 = 0$
Sol'n: By definition of slope:
$m = (y_2 - y_1) / (x_2 - x_1) = (y - y_1) / (x - x_1)$
$m = (5 - 3) / (-3 + 2) = (y - 3) / (x + 2)$
$m = 2 / -1 = (y - 3) / (x + 3)$
$-2x - 4 = y - 3$; $2x + y + 1 = 0$

25. C (2, -3)
Sol'n: Principle: At the vertex, slope of the tangent line, dy/dt = 0; $y = x^2 - 4x + 1 \rightarrow$ take d/dt of both sides
$dy/dt = 2x - 4$; $0 = 2x - 4$
$x = 2 \rightarrow$ substitute to the equation of the parabola
$y = 2^2 - 4(2) + 1$
Thus, the vertex is a point (2, -3)

26. A a is positive and –a is negative

27. B 4
Sol'n: $M = \lim_{x\to 2} \dfrac{(x + 2)(x - 2)}{x - 2}$
$M = \lim_{x \to 2} (x + 2)$
$M = 2 + 2 = 4$

28. B - tan x
Sol'n: $y = \ln \cos x$
$dy / dt = 1 / \cos x (-\sin x) = -\tan x$

29. D sec x
Sol'n: $d / dx (y + \ln (\cos x) = 0)$
$y' + (-\sin x / \cos x) = 0$
$y' = \tan x$
$d/dy (y' = \tan x)$
$\rho = [1 + (\tan x)^2]^{3/2} / y'' = [1 + (\tan x)^2]^{3/2} / \sec^2 x$
$\rho = (\sec^2 x)^{3/2} / \sec^2 x = (\sec^3 x) / \sec^2 x = \sec x$

30. B $x = 2y$
Sol'n: The normal to a circle passes thru the center. By inspection, the circle $x^2 + y^2 = 1$ has its center at (0, 0). Thus the line passes thru (0, 0) and (2, 1). By the two (2) point from: $m = (y_2 - y_1) / (x_2 - x_1) = (y - y_1) / (x - x_1)$
$(1 - 0) / (2 - 0) = (y - 0) / (x - 0)$
$x = 2y$

31. A a < c

32. A $(1/12) (3t - 1)^4 + c$
Sol'n: Let: $x = \int (3t - 1)^3 dt$
$x = 1/3 \int (3t - 1)^3 3\, dt$
$x = 1/3 [(3t - 1)^4 / 4] + c = [(3t - 1)^4 / 12] + c$

33. C -4 (1 + i)
Sol'n: Change (1 + i) to polar form
$(1 + i) = \sqrt{2} \angle 45°$
recall $(r \angle \theta)^n = r^n \angle \theta$
$(1 + i)^5 = (\sqrt{2} \angle 45°)^5 = (\sqrt{2})^2 \angle 5 (45°)$
$(1 + i)^5 = 5.657\angle 225° = 5.657(\cos 225° + i\sin 225°)$
$(1 + i)^5 = -4 - 4i = -4 (1 + i)$

34. D any real number
35. B a + b < b + d
36. A ac < bc
37. B ac > bc
38. A ac < bd
39. A a > c
40. B any real number
41. B a + c > b + d
42. D ac > bc
43. A ac < bc
44. A Contradiction
45. C 0.4289
46. A Integral Curves
47. D a = -1; b = 1; c = 0

To satisfy the hypothesis of Rolle's Theorem:
f(a) = f(b) = 0, f is continuous on (a, b).
Qf $(x) = x^2 - 1 = (x+1)(x - 1)$;
Therefore: f(-1) = f(1) = 0; a =-1; b=1
f' (x) 2x; f'(c) = 0, c ε (-1, 1); 2c = 0 and finally c = 0

48. C 4:9
Let x = length of each side of the square;
W = Width of the inscribed rectangle and
L = Length of the inscribed rectangle.
The area of the square is As $(x)^2$.
By Pythagorean Theorem, the rectangle's length and width are:
$W = \sqrt{(x/3)^2 + (x/3)^2} = (x / 3) \sqrt{2}$ and
$L = \sqrt{[(2/3)(x)]^2 + [(2/3)(x)]^2} = (2x /3) \sqrt{2}$
The area of the inscribed rectangle is
$Ar = (L)(W) = [(2x/3) \sqrt{2}\ ] [(x / 3) \sqrt{2}\ ] = (4/9)(x)^2$
*The ratio of the areas = A_r / A_s
$= [(4/9)(x)^2] / (x)^2 = 4 / 9$

49. A Prime

50. D $2 \sqrt{2}$
$y = f(x) = 1/x$ and $y' = -1 / x^2$ f(b) = 1/4 and f(a) = 1/2. By the "Mean Value Theorem":
f'(c)(b -a) = f(b) - f(a) or f'(c) (4-2) = 1/4 -1/2;
f'(c)(2) = -1/4 or f'(c) = -1/8 = $1/ (c)^2$
Since$(c)^2 = 8$; $c = 2 \sqrt{2}$