Sam Tenney

Section 2

Homework 8

1. Caffeine-free Beverages
   a) Response variable: the sum of 20 beverage taste judges, where each judge scores on a 1-10 scale (meaning higher Taste Scores reflect a better beverage).

      Factors: Sweetener (Sugar, Corn Syrup, Aspartame, Ace-K) and Carbonation (Yes, No)

      Treatments: There are eight treatment combinations of the factors and factor levels. They are as follows: Sugar-Yes, Sugar-No, Corn Syrup-Yes, Corn Syrup-No, Aspartame-Yes, Aspartame-No, Ace-K - Yes, Ace-K – No.

      Experimental Unit: Taste judges who receive the drink combination and give it a Taste Score

   b) This study is an experiment because the chemist randomly assigns experimental combinations to the judges and measures the response from the judges. A treatment is applied to an experimental unit which is not done in an observational study, but in an experiment.

   c)
```r
# Read in the data from
https://blades.byu.edu/stat230data/caffeine.txt
caffeine <- read.table(text = "run sweetener carbonation taste
 1  CornSyrup No          189
 2  Aspertame No          187
 3  CornSyrup Yes         191
 4  AceK      Yes         173
 5  Aspertame Yes         171
 6  AceK      No          180
 7  Sugar     No          187
 8  CornSyrup Yes         184
 9  Aspertame No          187
10  AceK      No          185
11  Sugar     No          190
12  AceK      Yes         163
13  Sugar     Yes         198
14  Sugar     Yes         199
15  CornSyrup No          182
16  Aspertame Yes         178", header = TRUE, sep = '')
```

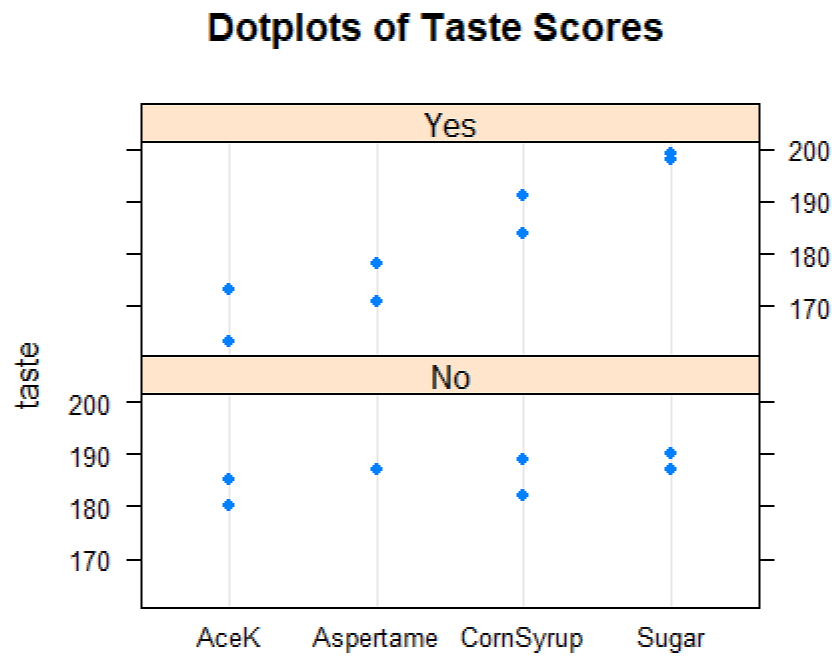
d)
```r
# Calculate the summary statistics for each treatment
aggregate(taste~sweetener+carbonation, data = caffeine, FUN = mean)
aggregate(taste~sweetener+carbonation, data = caffeine, FUN = sd)
```

Summary Statistics for Caffeinated Beverage Data

Sweetener	Carbonation	Mean Taste Score	Standard Deviation
Ace-K	No	182.5	3.54
Aspartame	No	187.0	0.00
Corn Syrup	No	185.5	4.95
Sugar	No	188.5	2.12
Ace-K	Yes	168.0	7.07
Aspartame	Yes	174.5	4.95
Corn Syrup	Yes	187.5	4.95
Sugar	Yes	198.5	0.71

For the sweeteners Ace-K and Aspartame, the mean Taste Score seems to drop when carbonation was present, whereas for Corn Syrup and Sugar, the mean taste scores increased when carbonation was present. The spread for Corn Syrup taste scores remained the same with or without carbonation, but for Ace-K and Aspartame, the results were much more spread out from the mean when carbonation was present. Sugar's Taste Score spread decreased when carbonation was present.

e)



The spreads are similar whether carbonation is present or not. When carbonation is not in the drinks, the results seem to be more consistent across the different sweeteners, whereas when carbonation is in the drinks, Ace-K and Aspartame generally scored lower, while Corn Syrup and Sugar scored higher. It appears that Ace-K has the largest change in score when going from no carbonation to having carbonation present in the beverages. There are no unusual observations in our graph.

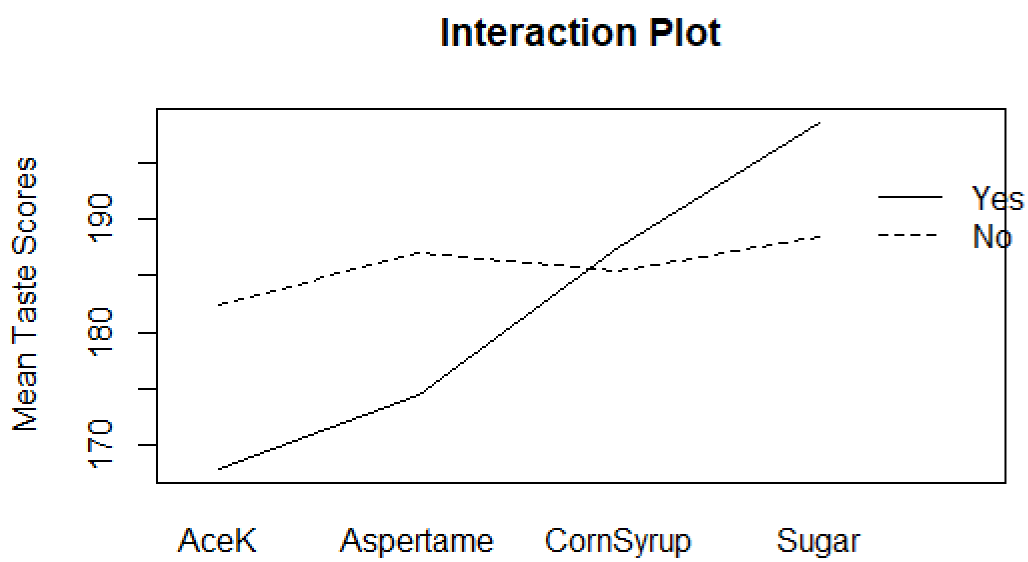
f) The ANOVA model for the caffeinated beverage data is $y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \varepsilon_{ijk}$. y_{ijk} is the taste score where each judge scores on a 1-10 scale (meaning higher Taste Scores reflect a better beverage) for the k^{th} replicate of the i^{th} sweetener level (Sugar, Corn Syrup, Aspartame, Ace-K) and the j^{th} carbonation level (Yes or No). The variable μ is the grand mean of all the taste scores given by the judges on a scale from 1-10. The variable α_i is the treatment effect for the i^{th} sweetener level. The variable β_j is the treatment effect for the j^{th} carbonation level. The variable γ_{ij} is the interaction effect for the i^{th} sweetener level and the j^{th} carbonation level. The error for the k^{th} replicate with i^{th} the sweetener level and the j^{th} carbonation level is represented by ε_{ijk}.

g)

```
# Create ANOVA table
caffeineFacMod <- aov(taste~sweetener+carbonation+sweetener:carbonation, data
= caffeine)
anova(caffeineFacMod)

## Analysis of Variance Table
##
## Response: taste
##                       Df Sum Sq Mean Sq F value   Pr(>F)
## sweetener              3 734.50 244.833 13.8913 0.001545 **
## carbonation            1  56.25  56.250  3.1915 0.111840
## sweetener:carbonation  3 414.25 138.083  7.8345 0.009132 **
## Residuals              8 141.00  17.625
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

# Create table of 95% confidence intervals for all pair-wise comparisons of f
actor and interaction levels
TukeyHSD(caffeineFacMod, which="sweetener:carbonation")

##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = taste ~ sweetener + carbonation + sweetener:carbonation
, data = caffeine)
##
## $`sweetener:carbonation`
##                              diff         lwr        upr     p adj
## Aspertame:No-AceK:No          4.5 -12.1127398 21.1127398 0.9465629
## CornSyrup:No-AceK:No          3.0 -13.6127398 19.6127398 0.9937505
## Sugar:No-AceK:No              6.0 -10.6127398 22.6127398 0.8227965
## AceK:Yes-AceK:No            -14.5 -31.1127398  2.1127398 0.0950689
## Aspertame:Yes-AceK:No        -8.0 -24.6127398  8.6127398 0.5810259
## CornSyrup:Yes-AceK:No         5.0 -11.6127398 21.6127398 0.9140444
## Sugar:Yes-AceK:No            16.0  -0.6127398 32.6127398 0.0601896
## CornSyrup:No-Aspertame:No    -1.5 -18.1127398 15.1127398 0.9999214
## Sugar:No-Aspertame:No         1.5 -15.1127398 18.1127398 0.9999214
## AceK:Yes-Aspertame:No       -19.0 -35.6127398 -2.3872602 0.0246064
## Aspertame:Yes-Aspertame:No  -12.5 -29.1127398  4.1127398 0.1743365
## CornSyrup:Yes-Aspertame:No    0.5 -16.1127398 17.1127398 1.0000000
## Sugar:Yes-Aspertame:No       11.5  -5.1127398 28.1127398 0.2342184
## Sugar:No-CornSyrup:No         3.0 -13.6127398 19.6127398 0.9937505
## AceK:Yes-CornSyrup:No       -17.5 -34.1127398 -0.8872602 0.0383078
## Aspertame:Yes-CornSyrup:No  -11.0 -27.6127398  5.6127398 0.2704638
## CornSyrup:Yes-CornSyrup:No    2.0 -14.6127398 18.6127398 0.9994824
## Sugar:Yes-CornSyrup:No       13.0  -3.6127398 29.6127398 0.1500109
## AceK:Yes-Sugar:No           -20.5 -37.1127398 -3.8872602 0.0159894
## Aspertame:Yes-Sugar:No      -14.0 -30.6127398  2.6127398 0.1107351
## CornSyrup:Yes-Sugar:No       -1.0 -17.6127398 15.6127398 0.9999949
```

```
## Sugar:Yes-Sugar:No              10.0  -6.6127398 26.6127398 0.3567804
## Aspertame:Yes-AceK:Yes           6.5 -10.1127398 23.1127398 0.7667317
## CornSyrup:Yes-AceK:Yes          19.5   2.8872602 36.1127398 0.0212838
## Sugar:Yes-AceK:Yes              30.5  13.8872602 47.1127398 0.0012663
## CornSyrup:Yes-Aspertame:Yes     13.0  -3.6127398 29.6127398 0.1500109
## Sugar:Yes-Aspertame:Yes         24.0   7.3872602 40.6127398 0.0061540
## Sugar:Yes-CornSyrup:Yes         11.0  -5.6127398 27.6127398 0.2704638
```

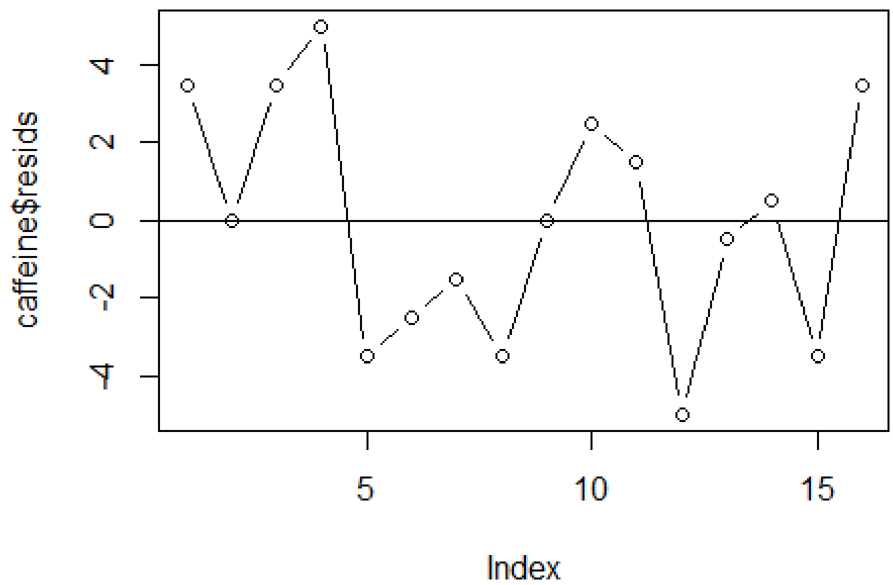
h)



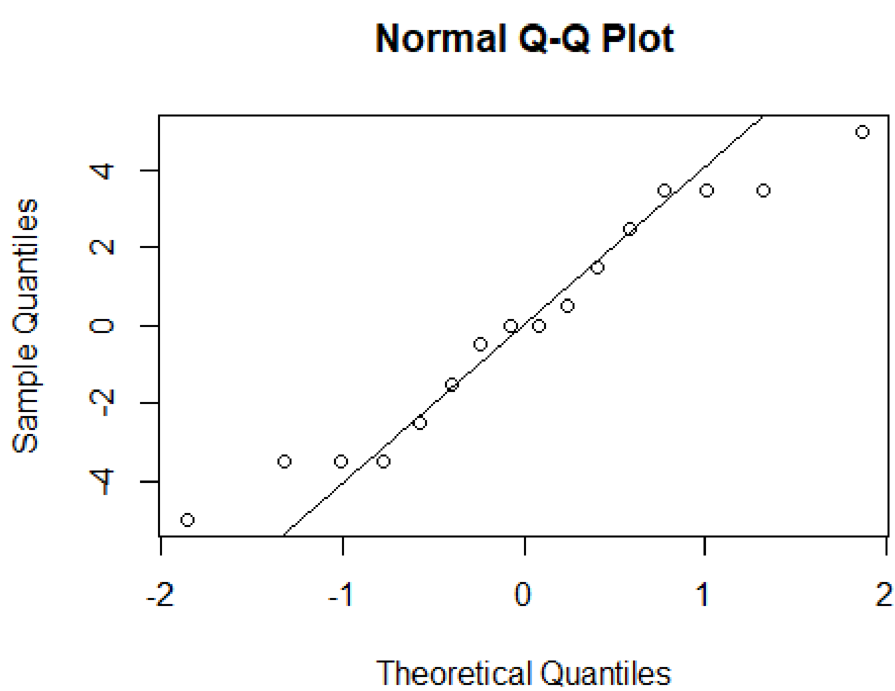
The most significant interaction difference is between Sugar:Yes-AceK:Yes with a p-value of about 0.001. The least significant interaction difference is between CornSyrup:Yes-Aspartame:No with a p-value of 1.00.

i) Check Assumptions

```
# Calculate Residuals
caffeine$resids <- resid(caffeineFacMod)
mean(caffeine$resids)

## [1] 1.249001e-16

# Index Plot: Check Independence
plot(caffeine$resids, type="b")
abline(h=0)
```




```r
# Normal qq plot: check normality
qqnorm(caffeine$resids)
qqline(caffeine$resids)
```



```r
# Ratio of sds: check equal variance
aggregate(taste~sweetener+carbonation, data = caffeine, FUN = sd)

##   sweetener carbonation     taste
## 1      AceK          No 3.5355339
## 2 Aspertame          No 0.0000000
## 3 CornSyrup          No 4.9497475
## 4     Sugar          No 2.1213203
## 5      AceK         Yes 7.0710678
## 6 Aspertame         Yes 4.9497475
## 7 CornSyrup         Yes 4.9497475
## 8     Sugar         Yes 0.7071068
```

The mean of our residuals is so small, we can say it's zero, so we can assume the means of the different combinations are constant. The index plot above shows no real pattern, so we can assume our residuals are independent. The normal QQ plot is pretty straight, so we can assume that our residuals are normally distributed. Our standard deviation ratio, 7.07 / 2.12 (not counting dividing by zero) is greater than 2 so we can't assume that our variances are constant. Not all the assumptions are met.

2. Choir Heights
   a) Response variable: Height of singers in inches.

      Factors: Sex (male, female) and singing part (low, high)

      Conditions: There are four different conditions. Male-low, male-high, female-low, female-high.

      Experimental Units: Each individual singer.

   b) This is an observational study since no treatment is being applied by an external force. The researcher is simply observing data that has been collected about singers.

   c)

```sas
data singers;
   infile datalines dlm=",";
   input height sex $ part $;
   datalines;
height,sex,part
64,f,high
62,f,high
66,f,high
65,f,high
```

```
60,f,high
....
low
72,m,low
66,m,low
72,m,low
70,m,low
69,m,low
;
run;
```

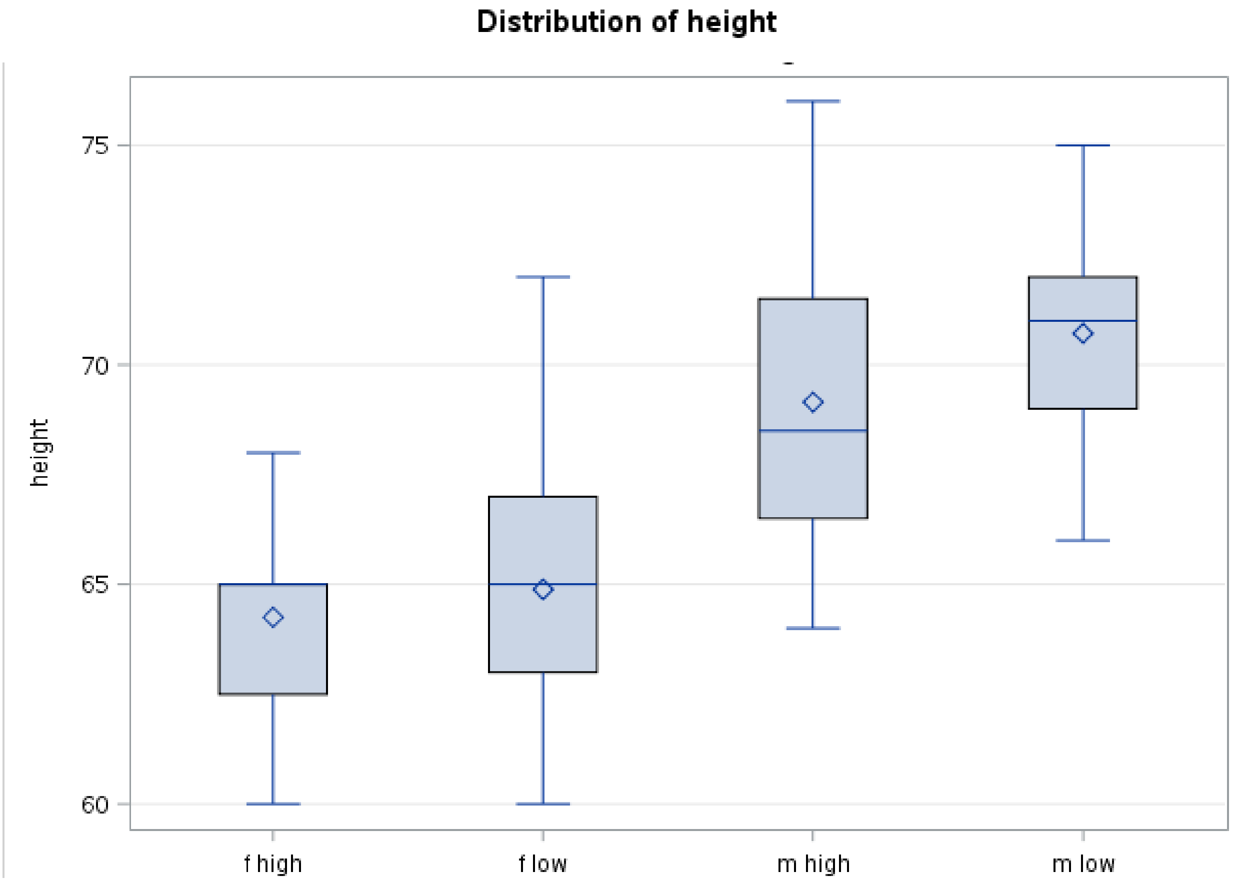
d)
```
proc glm data=singers;
   class sex part;
   model height = sex|part;
   means sex*part / tukey;
run;
```

Summary Statistics for Singers

Sex	Singing Part	Obs	Mean height (inches)	Std Dev (inches)
Female	High	36	64.25	1.87
Female	Low	35	64.89	2.79
Male	High	20	69.15	3.22
Male	Low	39	70.72	2.36

Naturally, the men are taller than women on average. It seems the singing part doesn't make much of a difference in height for either sex as the only difference is bass singers were taller than tenors on average by about 1.5 inches. The height of tenors had a larger spread than any of the other singing parts while sopranos had the smallest spread on average.

e)



Females were shorter than the men on average. Tenors and altos had the largest spreads while sopranos and basses had the smallest spreads. It is difficult to tell what direction sopranos are skewed. Altos are nearly symmetric, tenors are slightly skewed right, and basses are slightly skewed left. There are no outliers according to the boxplots.

f) The ANOVA model for the singers data is $y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \varepsilon_{ijk}$. y_{ijk} is the height in inches for the k^{th} (where $k = 1, \ldots, n_{ij}$) replicate of the i^{th} sex level (male, female) and the j^{th} singing part level (low, high). The variable μ is the grand mean of all the singers' heights in inches. The variable α_i is the treatment effect for the i^{th} sex level. The variable β_j is the treatment effect for the j^{th} singing part level. The variable γ_{ij} is the interaction effect for the i^{th} sex level and the j^{th} singing part level. The error for the k^{th} replicate with i^{th} the sex level and the j^{th} singing part level is represented by ε_{ijk}.

g)

```
proc glm data=singers;
     class sex part;
     model height = sex|part;
     means sex*part / tukey;
run;
```

ANOVA Table using Type I SS

Source	DF	Type I SS	Mean Square	F Value	p-Value
Sex	1	1018.86	1018.86	161.13	<.0001
Part	1	33.09	33.09	5.23	0.02
Sex * Part	1	6.58	6.58	1.04	0.31

The main effects, or rather, when sex and singing part are accounted for separately, the heights of the singers are significantly different when they are the next term in the model, since both of their p-values are lower than 0.05. The difference in heights is not significant, however, when looking at the interaction effect between sex and singing part when they are the next term in the model.

h)

ANOVA Table using Type III SS

Source	DF	Type III SS	Mean Square	F Value	p-Value
Sex	1	872.65	872.65	138.00	<.0001
Part	1	36.79	36.79	5.82	0.02
Sex * Part	1	6.58	6.58	1.04	0.31

The main effects, or rather, when sex and singing part are accounted for separately, the heights of the singers are significantly different when they are the last term in the model, since both of their p-values are lower than 0.05. The difference in heights is not significant, however, when looking at the interaction effect between sex and singing part when they are the last term in the model.

i) The SS for sex is smaller in the Type III model because the Type I model takes into account the order the data is read in. The previous data has an effect on the variance, and since we read in the data by sex with females first, this had an effect on our variance since their heights are different than males. The Type III model makes its calculations as if each data entry is the last one, so previous data entries don’t have as large of an effect on the variance.