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Section 2

Homework 5

1. Toothbrush Wars
   a) **Response variable:** The change in gingivitis levels based on the Modified Gingival Index (MGI) after 12 weeks.

      Factor: Type of toothbrush (Oral-B Triumph with Smart Guide, Sonicare DiamondClean).

      Experimental Unit: Subject whose teeth were being measured for gingivitis levels.

   b) This study is an experiment because the subjects were randomly assigned to different treatments (toothbrush type), and the examiner was able to measure the results based on the two different treatments.

   c) The data is delimited by a space. There is a header with three different columns identifying the subjects being treated, what toothbrush they used, and the change in MGI for each subject.

   d)
```
data toothbrush;
    infile datalines dlm=" " firstobs=2;
    input id treatment $ changeMGI;
    datalines;
    id treatment changeMGI
    1 OralB 0.441
    2 OralB 0.326
    3 OralB 0.381
    4 OralB 0.299
    5 OralB 0.429
    ...
    128 Sonicare 0.263
    129 Sonicare 0.277
    130 Sonicare 0.303
    ;
    run;
```

e)

Summary for Change in MGI by Toothbrush

Treatment	Mean	St Dev
OralB	0.418	0.073
Sonicare	0.316	0.073

Figure.5.e

It appears in Figure.5.e that the mean change in MGI when the Oral-B toothbrush was used was higher (0.418) compared to when the Sonicare toothbrush was used (0.316), meaning during the experiment, the Oral-B toothbrush led to higher reduced levels of inflammation in subjects than when they started. The standard deviation for both treatments was the same at 0.073 change in MGI scores.

f) Here is the code to help provide a 95% confidence interval and our two-sample t-test results for parts 5j and 5l:

```
proc ttest data=toothbrush;
    class treatment;
    var changeMGI;
run;
```

95% Confidence Interval for Mean Change in MGI

Treatment	Mean	95% CL Mean	
OralB	0.4180	0.3998	0.4362
Sonicare	0.3160	0.2979	0.3342

Figure.5.f

On average, the change in the Modified Gingival Index (MGI) after using the Oral-B toothbrush is 0.4180 (95% CI: 0.3998 to 0.4362), while the average change in MGI after using the Sonicare toothbrush is 0.3160 (95% CI: 0.2979 to 0.3342), as shown in Figure.5.f.

g)

Box Plots for Oral-B vs. Sonicare
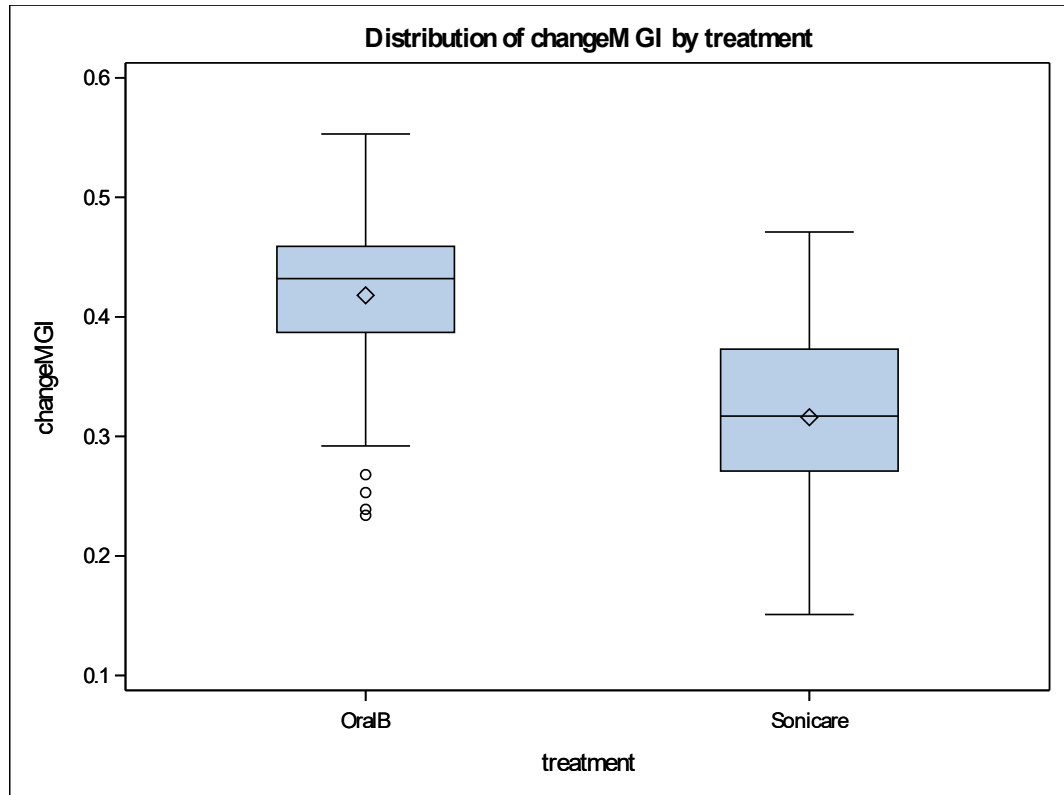
Figure.5.g. The generated boxplots show that, in terms of change in MGI, the subjects who used Sonicare had a larger spread than those who used the Oral-B toothbrush. Oral-B also is left skewed with a few outliers on the lower tail of the data while Sonicare is nearly symmetrical in shape.

h) The following is the model for our two-sample t-test to test for significance in our toothbrush data. The variable y_{ij} is the change in MGI for the ith toothbrush (Oral-B, Sonicare) and the jth replicate of that toothbrush. The mean change in MGI for the toothbrush i is represented by μ_i. The error for the ith toothbrush type and the jth replicate, ε_{ij}, is equal to $y_{ij} - \mu_i$. The model ends up being $y_{ij} = \mu_i + \varepsilon_{ij}$.

i) Our null hypothesis, H_o, is that there isn't a difference in the mean change in MGI for individuals who brush using the Oral-B or Sonicare toothbrushes. The alternative hypothesis, H_a, is there is a difference between the mean change in MGI for individuals who brush using the Oral-B or Sonicare toothbrushes.
Notation:
μ_{OB} = Mean change in MGI for Oral-B users
μ_{SC} = Mean change in MGI for Sonicare users

H_o: $\mu_{OB} = \mu_{SC}$ or $\mu_{OB} - \mu_{SC} = 0$
H_a: $\mu_{OB} \neq \mu_{SC}$ or $\mu_{OB} - \mu_{SC} \neq 0$

j) See code in 5f for how the two-sample t-test results were calculated.

Two-Sample t-test Statistics

Method	Variances	DF	t-Value	p-Value
Pooled	Equal	128	7.92	<.0001

Figure.5.j

k) Based on our small p-value ($< .0001$) in Figure.5.j we can reject the null hypothesis and conclude that there is a difference in mean MGI for individuals brushing with the Oral-B and Sonicare toothbrushes.

l) See code in 5f for how the 95% confidence interval for the difference in mean MGI was calculated.

95% Confidence Interval for Difference in
Mean Change in MGI

Type of CI	Mean	Lower	Upper
Difference in means	0.1020	0.0765	0.1275

Figure.5.l

On average, the difference between MGI change for individuals who used the Oral-B toothbrush and those who used the Sonicare toothbrush is 0.1020 (95% CI: 0.0765 to 0.1275). Our confidence interval does not contain zero, so the interval confirms what we observed in our t-test (H_a: $\mu_{OB} \neq \mu_{SC}$ or $\mu_{OB} - \mu_{SC} \neq 0$).

m) The ANOVA model is $y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$. The variable y_{ij} is the change in MGI for the ith toothbrush (Oral-B, Sonicare) and the jth replicate of that toothbrush. The variable μ is the grand mean of all the data. The variable α_i is the treatment effect for the ith toothbrush. The error for the ith toothbrush type and the jth replicate is represented by ε_{ij}.

n) The null hypothesis is our treatment effect for the Oral-B and Sonicare toothbrushes are equal to zero. The alternative hypothesis is that at least one of the toothbrushes treatment effect is not equal to zero.

$H_o: \alpha_i = 0$
$H_a: \alpha_i \neq 0$

o)

One-Way ANOVA Table

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	0.33802	0.33802	62.78	<.0001
Error	128	0.68923	0.00538	--	--
Corrected Total	129	1.02726	--	--	--

Figure.5.o

p) Based on the small p-value (<.0001) from the one-way ANOVA we conducted on the previous step, we can reject the null hypothesis and conclude that at least one of the toothbrushes' treatment effect is not equal to zero. This means that there is a difference between the two toothbrushes effect on the subjects' mean change in MGI.

q) The p-value (<.0001) is the same for both our two-sample t-test and our one-way ANOVA.

r) `sqrt(62.78)`

```
## [1] 7.923383
```

The square root of the generated F-statistic (62.78) is 7.92. This looks familiar because it is the same value as our t-value in Figure.5.j.

s)

```
title1 'ANOVA Table';
proc glm data=toothbrush plots=diagnostics;
    class treatment;
    model changeMGI = treatment;
    output out=toothbrushRes residual=res;
run;

data toothbrushRes;
    set toothbrushRes;
    id=_N_;
run;

title1 'Index Plot of Residuals vs. Order';
proc gplot data=toothbrushRes;
    symbol1 color=black interpol=join value=dot;
    plot res*id / vref = 0;
run;
```
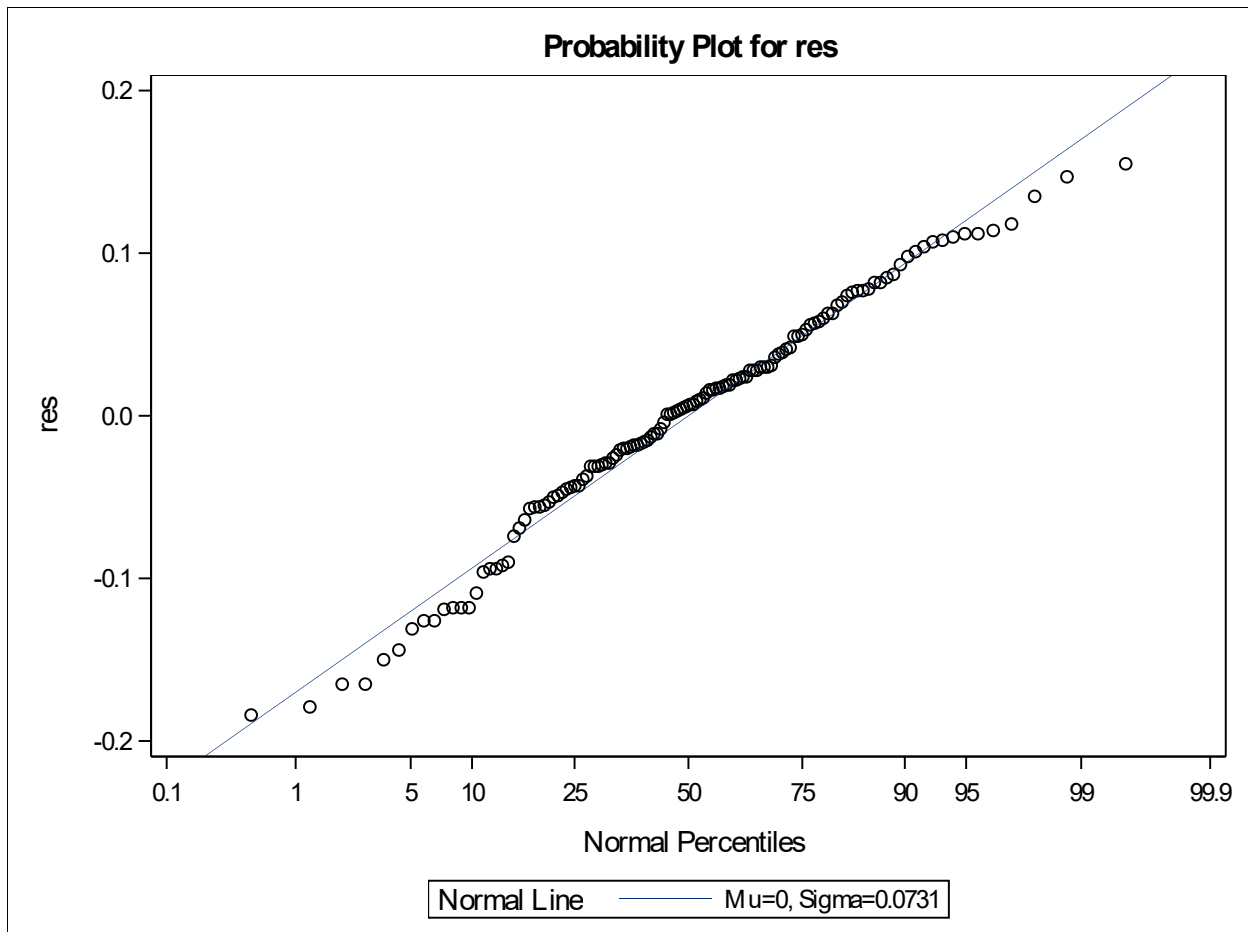
Index Plot of Residuals vs. Order
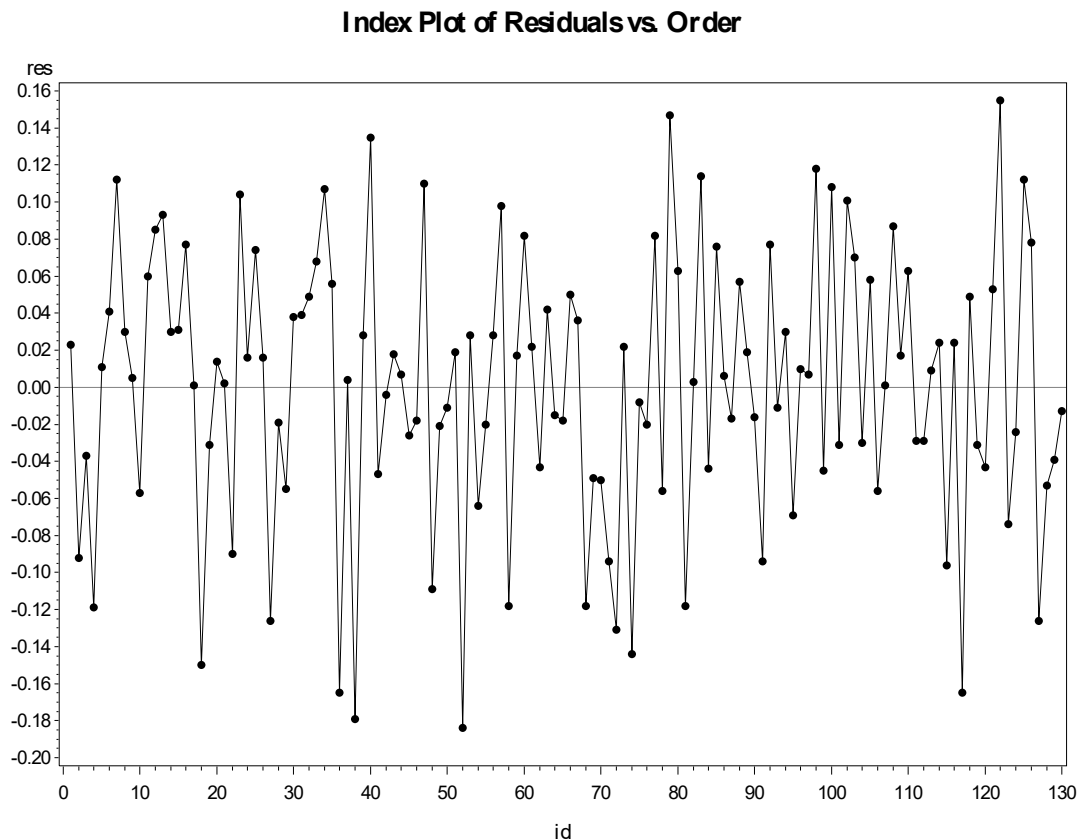
Figure.5.s

There are no obvious patterns in the index plot (Figure.5.s) so we can assume the observations are independent.

t)

```
title1 'QQ Plot of Residuals';
proc univariate data=toothbrushRes;
    var res;
    probplot / normal(mu=est sigma=est color=red);
run;
```


Probability Plot for res

Normal Line ——— Mu=0, Sigma=0.0731

Figure.5.t

The normal QQ plot generated from SAS appears to be a relatively straight line, so
we can assume that our residuals are normally distributed.

u) The standard deviations are equal, so the ratio is 0.0734 / 0.0734 = 1. The ratio is less
than 2 so the equal variance assumption is satisfied.

v) We wanted to know if subjects experienced a difference in mean MGI depending on
the type of toothbrush they used (Oral-B, Sonicare). We used a two-sample t-test and
a one-way ANOVA. We found on average the difference in mean MGI in our sample
to be 0.1020 (95% CI: 0.0765 to 0.1275). Our calculated p-value from the t-test and

ANOVA to be <0.001 which is small enough to reject our null hypothesis that the mean MGI in both toothbrushes are the same and conclude that there must be a difference in the change in mean MGI in subjects who used the toothbrushes.