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Section 2

Homework 10

1. Microwave Popcorn
   a) Response variable: percentage of popped kernels for each bag.

   b) Whole-plot factor: Popcorn Brand (Expensive, Generic)
   Experimental unit for the whole-plot factor: Individual box of six microwavable bags of popcorn

   c) Split-plot factor: Temperature (Frig, Room)
   Experimental unit for the split-plot factor: Individual bag from box of microwavable popcorn

   d) Box is nested within Brand since one box cannot be both expensive and generic, but there are boxes for both expensive and generic brands of popcorn.

   e) Bag is nested within Box because you will always have a unique bag number with each box, but across boxes you could, for example, have the first bag from multiple boxes. The same bag can't come from multiple boxes.

   f) Temperature is crossed with Brand because both temperatures (Frig, Room) occur across both Brands (Expensive, Generic).

   g) We are using an SP[1;1] model with $y_{ijk} = \mu + \alpha_i + \beta_{j(i)} + \gamma_k + \delta_{ik} + \varepsilon_{ijk}$ where y_{ijk} is the percentage of popped kernals from Popcorn Brand level i (Expensive, Generic) with Box j (nested in i) and temperature (Frig, Room) level k, μ is the grand mean of all the kernals popped, α_i is the treatment effect for the i^{th} Brand level, $\beta_{j(i)}$ is the block effect for the j^{th} Box (nested in i), The variable γ_k is the treatment effect for the k^{th} Temperature level, δ_{ik} is the interaction effect for the i^{th} Brand level and the k^{th} Temperature level, and ε_{ijk} is the error for the i^{th} Brand level on Bag j (nested in i) and the k^{th} Temperature level.

   In this model, i = 1, 2 indexes levels of the Brand (Expensive, Generic), j = 1, …, n indexes Bags nested in the i^{th} level of Brand, k = 1, … , K indexes levels of the Temperature (Frig, Room).

   h) $H_{NullBrand}$: $\alpha_1 = \alpha_2 = 0$
   $H_{AltBrand}$: At least one $\alpha_i \neq 0$

i) $H_{NullTemperature}$: $\gamma_1 = \gamma_2 = 0$

$H_{AltTemperature}$: At least one $\gamma_k \neq 0$

j) $H_{NullBrandxTemperature}$: $\delta_{11} = \delta_{12} = \ldots = \delta_{ik} = 0$

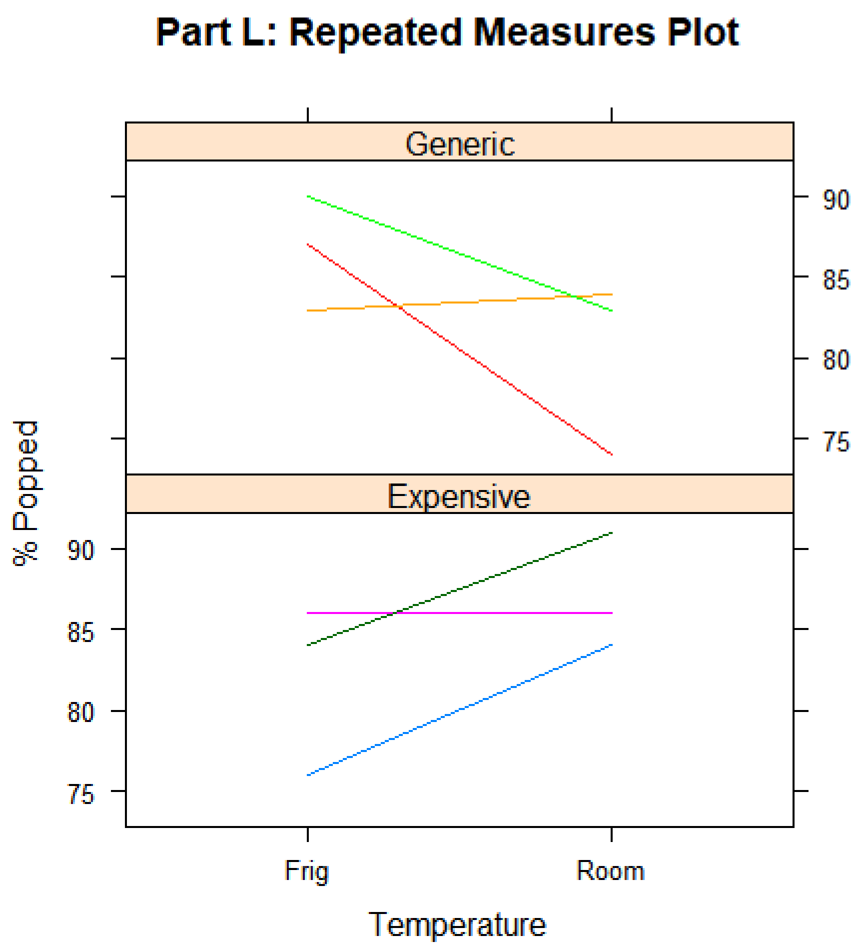
$H_{AltBrandxTemperature}$: At least one $\delta_{ik} \neq 0$

k)
```
# Read in the data
popcorn <- read.table(text = "Brand,Temp,Box,Bag,% Popped
Expensive,Room,1,1,84
Expensive,Frig,1,2,76
Expensive,Room,2,3,86
Expensive,Frig,2,4,86
Expensive,Room,3,5,91
Expensive,Frig,3,6,84
Generic,Room,4,7,74
Generic,Frig,4,8,87
Generic,Room,5,9,84
Generic,Frig,5,10,83
Generic,Room,6,11,83
Generic,Frig,6,12,90", header = TRUE, sep = ",")
```

```
# Look at the data
str(popcorn)

## 'data.frame':    12 obs. of  5 variables:
##  $ Brand    : Factor w/ 2 levels "Expensive","Generic": 1 1 1 1 1
1 2 2 2 2 ...
##  $ Temp     : Factor w/ 2 levels "Frig","Room": 2 1 2 1 2 1 2 1 2
1 ...
##  $ Box      : int  1 1 2 2 3 3 4 4 5 5 ...
##  $ Bag      : int  1 2 3 4 5 6 7 8 9 10 ...
##  $ X..Popped: int  84 76 86 86 91 84 74 87 84 83 ...
```

l)



m)

ANOVA Table

Error: Box

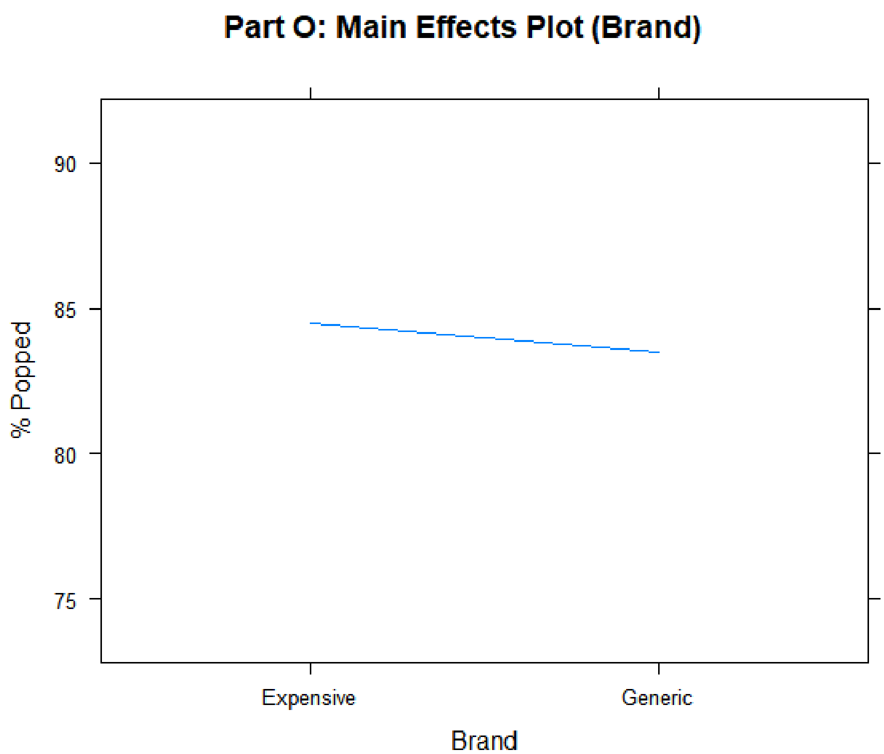
Source	Df	Sum Sq	Mean Sq	F value	P value
Brand	1	9.26	9.26	--	--

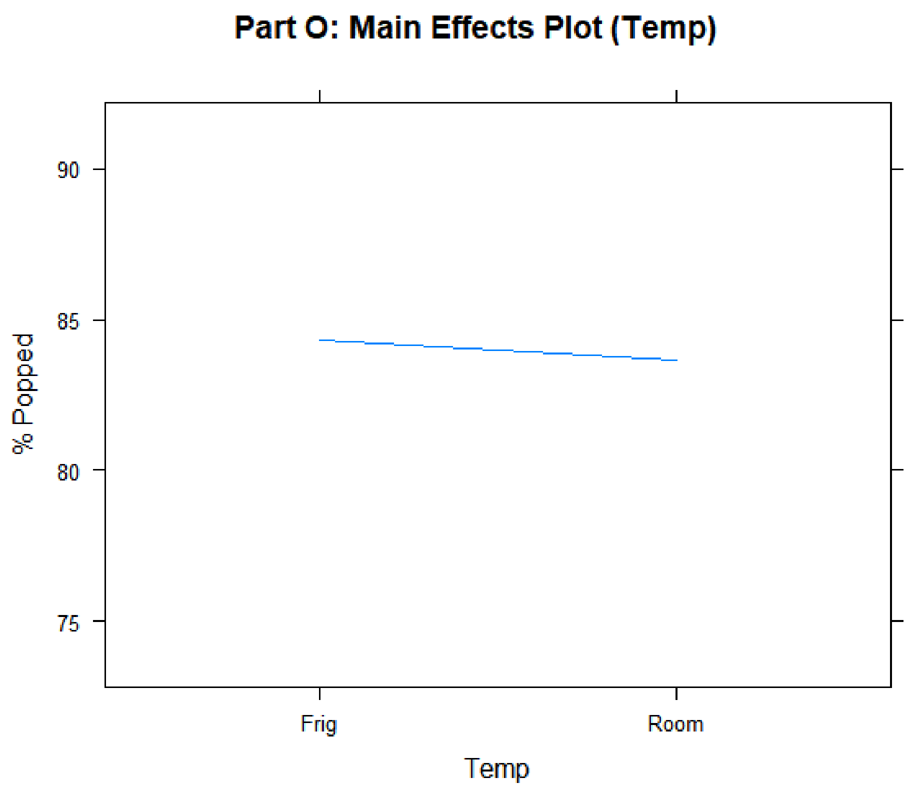
Error: Within

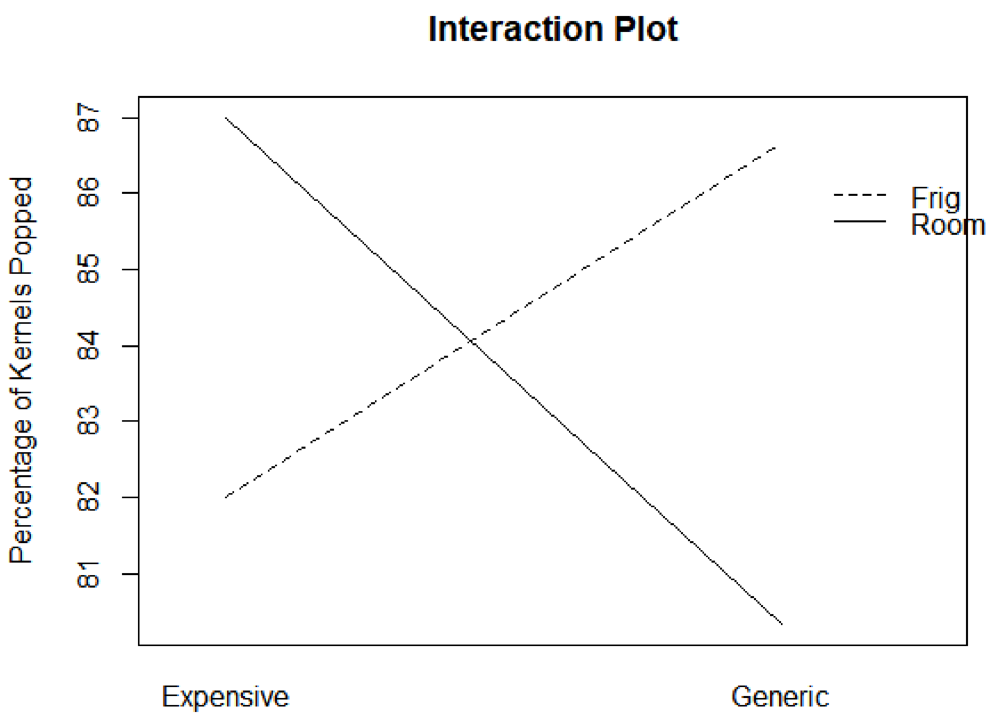
Source	Df	Sum Sq	Mean Sq	F value	P value
Brand	1	84.87	84.87	7.80	0.03
Temp	1	1.33	1.33	0.12	0.74
Brand:Temp	1	96.33	96.33	8.85	0.02
Residuals	7	76.21	10.89	--	--

n) We reject $H_{NullBrand}$ because the p-value for the Brand main effect is small (0.03). The brand of popcorn is statistically significant when describing the percentage of kernels that popped. We fail to reject $H_{NullTemperature}$ because the p-value for the Temperature main effect is very large (0.74). We reject $H_{NullBrandxTemperature}$ because the p-value for the interaction effect is very small (0.02). The difference in the interaction effects is statistically significant.

o)




Part O: Main Effects Plot (Temp)
90
85
80
75
% Popped
Frig
Room
Temp



There seems to be an interaction between Brand and Temperature. The main effect plots don't show much of a difference between each Brand and each Temperature. The Expensive-Room Temperature popcorn seemed to have the highest percentage of kernels popped with Refrigerated-Generic popcorn close behind.

p)
```
# Calculate grand mean, mean for each Brand, and the Brand effects
mean(popcorn$X..Popped)

## [1] 84

brandMean <- aggregate(X..Popped~Brand, data=popcorn, FUN=mean)
brandMean

##       Brand X..Popped
## 1 Expensive      84.5
## 2   Generic      83.5
```

The Brand effect for expensive is 0.5 (84.5 – 84), while the Brand effect for the generic popcorn is -0.5 (83.5 – 84). You can see that the expensive brand is slightly above 84 on the "Part O: Main Effects Plot (Brand)" plot in part o, while the generic brand is slightly below 84.

q) Temperature

```r
# Calculate the grand mean, the mean for each Temperature, and the T
emperature effects
mean(popcorn$X..Popped)

## [1] 84

tempMean <- aggregate(X..Popped~Temp, data=popcorn, FUN=mean)
tempMean

##   Temp X..Popped
## 1 Frig  84.33333
## 2 Room  83.66667
```

The Temperature effect for the Refrigerator temperature popcorn is 0.33 (84.33 – 84). The Temperature effect for the room temperature popcorn is -0.33 (83.67 – 84). You can see that the Refrigerator temperature is slightly above 84 on the "Part O: Main Effects Plot (Temp)" plot in part o, while the Room temperature is slightly lower.

r) Interaction

```r
# Calculate the grand mean, the mean for each Brand x Temperature, a
nd the interaction effects
mean(popcorn$X..Popped)

## [1] 84

brandTempMean <- aggregate(X..Popped~Brand+Temp, data=popcorn, FUN=m
ean)
brandTempMean

##       Brand Temp X..Popped
## 1 Expensive Frig  82.00000
## 2   Generic Frig  86.66667
## 3 Expensive Room  87.00000
## 4   Generic Room  80.33333
```

The interaction effect for Expensive-Frig is 82 – 84 – 0.5 – 0.33 = -2.83
The interaction effect for Generic-Frig is 86.67 – 84 – (-0.5) – 0.33 = 2.84
The interaction effect for Expensive-Room is 87 – 84 – 0.5 – (-0.33) = 2.83
The interaction effect for Generic-Room is 80.33 – 84 – (-0.5) – (-0.33) = -2.84

The interaction can be seen on the Interaction Plot in part o. For example, the two positive interactions are Generic-Frig and Expensive-Room. On the interaction plot, both of those points are above the grand mean of 84.