

**COSC 320 – 001**  
**Analysis of Algorithms**  
**2022/2023 Winter Term 2**

**Project Topic Number: 2 Milestone 3**  
**KML and LCSS**

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### **Problem Formulation.**

#### **LSCC Algorithm**

Input:

- A corpus of  $n$  documents  $D = \{d_1, d_2, \dots, d_n\}$  in plain text format
- A potentially plagiarized document  $P$  in plain text format
- An integer parameter  $k$  representing the length of substrings to compare
- A similarity threshold  $s$

Output:

The set of documents  $D$  that  $P$  was plagiarized from

1. For each document  $d_i$  in  $D$ :
  - 1.1 Create a list  $L_i$  of all substrings of length  $k$  in  $d_i$
2. Create a list  $L_p$  of all substrings of length  $k$  in  $P$
3. For each substring  $s_p$  in  $L_p$ :
  - 3.1 For each document  $d_i$  in  $D$ :
    - 3.1.1 For each substring  $s_i$  in  $L_i$ :
      - 3.1.1.1 Use the LSCC algorithm to find the length  $l$  of the longest common subsequence between  $s_p$  and  $s_i$
      - 3.1.1.2 If  $l/k \geq s$ , add  $d_i$  to the set of matched documents
4. Return the set of matched documents

Such that for each document  $d_i$  present, there exists a pattern of 3 or more consecutive words in  $A$  that match a pattern of 3 or more consecutive words in document  $d_i$  with at least 24% consecutive words matched of the word length of  $d_i$

#### **KMP Algorithm**

Input:

- A corpus of  $n$  documents  $D = \{d_1, d_2, \dots, d_n\}$  in plain text format
- A potentially plagiarized document  $P$  in plain text format

Output:

The set of documents from which  $P$  was plagiarized, represented as a set  $S$

1. Define a function  $KMP(\text{pattern}, \text{text})$  that implements the KMP algorithm to find all occurrences of a pattern in a given text. The function should return a list of indices where the pattern occurs in the text.
2. Initialize an empty set  $S$
3. For each document  $d_i$  in the corpus:
  - a. Find all occurrences of  $p$  in  $d_i$  using the KMP algorithm.
  - b. If any occurrences are found, add  $d_i$  to the set  $S$ .
4. Return  $S$

Such that for each document  $d_i$  present, there exists a pattern of 3 or more consecutive words in  $A$  that match a pattern of 3 or more consecutive words in document  $d_i$  with at least 24% consecutive words matched of the word length of  $d_i$

#### **Pseudo-code.**

**for both**

```
import documents(corpus_file_directory), ppdoc(potentially_plagarized_document)
number_of_docs_in_corpus = 0
```

```

for i in range (length(documents)):
    for j in range(length(documents[ i ].comment)):
        number_of_docs_in_corpus += 1           //this is to count the number of separate
                                                //comments so that we can count each //comment as a
                                                //separate document as //checking plagiarism between
                                                //2 separate //comment fragments makes no sense

corpus = [String s = "" for i in range(length(documents))] // blank string for every comment
n = 0
for i in range (length(documents)):
    for j in range(length(documents[ i ].comment)):
        for k in range(length(document[ i ].comment[ j ].letters)):
            letter = letter.lowercase
            if letter == a,b,c,...x,y,z + " ":
                corpus[n] = corpus[n] + document[ i ].comment[ j ].letters[ k ]
            n += 1                                //counter to keep track of which document we
                                                //are on
String ppdw = ""                                //ppdw potentially plagiarized document words
wc = 0                                           //wc = word count in ppdw
for i in range(length(ppdoc)):
    if(ppdoc[ i ].lowercase() == a,b,c, ..., x,y,z, " "): //this gets rid of the potential punctuation or
        ppdw = ppdw+ppdoc[ i ].lowercase()              //capitalization problems that could occur
        if((ppdoc[ i ] == " " and ppdoc[ i +1 ] != " ") or ppdoc[ i+1 ] == None):
            wc = wc +1
count = 0                                       //number of plagiarized words found
tc = 0                                        //tc for temporary count, series of words are only
                                                //counted if >=3 words in a row are potentially
                                                //plagarized
perc = 0                                     //%of words plagiarized
pseudo for KMP
for i in range (length(corpus)):
    KMP-MATCHER(corpus, ppdw)
perc = (count/wc)*100
if(perc >= 24):
    print("the document in question has a high probability of having been plagiarized showing " + perc + "% of the
    document which is potentially plagiarized")
else:
    print("this document is not likely plagiarized from the corpus provided, it shows a potential" + perc + "%
    plagiarized material which is within tolerance")
KMP-MATCHER(comment, ppdw):
    n = comment.length
    m = ppdw.length
    e = PREFIX-FUNCTION(ppdw)
    q = 0
    for i in range(1,n):
        while(q>0 and ppdw[q+1] != comment[i]):
            temp = q
            q = e[q][0]
            prefix_spaces = e[temp][1]

```

```

tc = tc - prefix_spaces
if(tc >= 3):
    count = count + tc
    tc = 0

tc = tc + prefix_spaces
//adds back words that were temporarily moved out
//of tc

if ppdw[q+1] == comment[i]:
    if(comment[i] == "" or None):
        tc = tc + 1
        q=q+1
    if(q>0 and ppdw[q+1] != comment[i]):
        if(tc >= 3):
            count = count + tc
            tc = 0

if(tc >= 3):
    count = count + tc
    tc = 0

```

**PREFIX-FUNCTION(ppdw):**

```

m = length(ppdw):
e = [[0 for i in 2] in range m]
k = 0
spaces = 0
for q in range(2,m)
    while(k > 0 and ppdw[k+1] != ppdw[q])
        k = e[0][k]
    if(ppdw[k+1] == ppdw[q])
        k = k + 1
    if((ppdw[k+1] == "" and ppdw[k+2] != None) or (ppdw[k+1] == "" and ppdw[k+2] != "")):
        spaces = spaces + 1
    //counts the number of spaces in the longest prefix that is also a suffix
    e[q][0] = k
    e[q][1] = spaces
    spaces = 0
return e

```

**pseudo for LCSs**

```

max_similarity = 0

for i in range(length(corpus)):
    j = 0
    i = 0
    lcs = LCS(ppwd, corpus[i])
    if(lcs/min(corpus[i], ppwd) > max_similarity):

```

```

    max_similarity = lcs/min(corpus[i], ppwd) > max_similarity
max_similarity = max_similarity *100 //convert from decimal to percent
if(max_similarity > 24):
    print("this document shows signs of potentially being plagiarized and has a lcs that contains" max_similarity "%
        similarity with at least one tested document")
else:
    print("this document has similarities that are within tolerance and has not likely been plagiarized")

LCS(ppwd, corp)
lcs = [[0 for i in range length(corp)] for j in range(ppwd)]
running == True
while(running):
    if(i == ppwd.length-1 or j == corp.length-1):
        running == false
    elseif(ppwd[i] == corp[j]):
        lcs[i, j] = 1 + LCS[i+1, j+1]
    else:
        lcs[i,j] = max(lcs[i-1, j-1], lcs[i, j-1])
return lcs[length(corp)-1, length(ppwd)]

```

**Algorithm Analysis.** Include the analysis of the algorithm and proof of correctness and running time of your algorithm.

**Proof of correctness for kml:**

Proof by loop invariance:

for this method to be correct the count and tc (temporary count) must accurately display the number of words in groups of 3 or greater and words that could potentially be in groups of 3 or greater respectively that are present in both the document in question and the corpus of documents. At the end of the loops all the tc = 0 and count must = all matched strings of length  $\geq 3$ . Also, without loss of generality I will do a proof that is

Initialization: start of a loop

Before a loop has run no letters have been checked and so the count is correctly placed at zero and the temporary count is also correctly placed at 0

maintenance: if it is true at the start is it true later

Case 1: character between copus document and document being checked is the same

In both cases, i is set to i + 1 at the end so that the next loop is ready to go and we are not comparing the same element repetitively.

Case 1.1: End of word:

then tc = tc + 1 and count is unchanged, count is then correct by assumption and tc is also correct as a new potentially in a group of 3 matched wor has been added to the temporary count and since the temporary count was correct before this word was added by assumption, it must also be correct not

Case 1.2 Middle of word

then by assumption, tc and count are both correct already and we are not at the end of a word so no word can be added to either the count or tc which means these values remain unchanged which is correct.

Case 2: the characters are not the same

2.1: q > 0

If the characters being compared are not the same then we must look at the e matrix. e[0][q] where q = the index of the character in the algorithm that is being compared with the qth character in the corpus document, e[0] = matrix storing longest prefix that is also a suffix for the qth character in the document being checked for plagiarism, and e[1] which simply stores the number of spaces in the current longest prefix that is also a suffix in the document that is being looked at. First, a temporary variable is set to q then

the  $q$ th character is set to the  $e[0][q]$ th character. This character is the best character that can be used to compare the least amount of characters and the proof for the  $e[0]$  matrix working can be found in Cormen, Thomas H. et al. *Introduction to Algorithms*, MIT Press, 2009. pages 1007 - 1008 (it is presented as the  $\pi$  matrix). After  $q$  is set to the  $e[0][q]$ th character, to stop words from being counted multiple times,  $e[1][temp]$  is accessed which records the number a spaces in the longest prefix that is also a suffix, this number is then placed in the variable `prefix_spaces`. Prefix spaces is then temporarily subtracted from `tc` to prevent words from being counted 2+ times. Then the size of `tc` is checked and if `tc > 3` then `count = tc` which then makes the count accurate and then `tc` is set to `tc = 0` to remove the words that have been added to the count and then `tc = prefix_spaces` which then adds back the words that are now potentially words in groups of greater than 3, this then makes it so that `tc` is also accurate. Thus maintenance is maintained.

Case 2.2:  $q = 0$

If  $q = 0$  then it is a nice simple case,  $q \neq 0$  so the while loop breaks, the if statement for just this eventuality it run, if `tc > 3`, then `count = tc + count`, and `tc = 0`. And because both `tc` and `count` are correct prior to the run, they must also be correct after as they have been correctly added. Then the  $i$ th index in the corpus document is moved up by 1 so that we are ready for the next loop

Thus maintenance holds in all cases

Termination:

Case 1 End of file -- character between corpus document and document being checked is the same:

The final for loop is broken and the `tc` is checked to see if it is greater than 3, if it is then `count = count + tc`, and because `count` and `tc` are by assumption correct before the final/terminating loop, the new count must also be correct. Then `tc` is set to zero which gives exactly what is desired.

Case 2 End of file -- character between corpus document and the document being checked are different:

in this case  $q$  will  $= 0$  and so the while loop will be broken (if started) and case 2.2 will take over, but at the end  $i = \text{length of corpus document}$  and so the for loop is broken and the `count` and `tc` are accurate for the same reason as in cases 2.2. The final for loop will not affect values as `tc = 0` and thus the termination gives the desired result.

Thus, as initialization, maintenance, and termination are valid in this algorithm the algorithm must be correct. This particular proof was done with a corpus of one document; however, but only difference is that the algorithm runs through multiple documents counting all of the words in groups of 3+ in all if the document in the corpus and the document being checked for plagiarism. Thus this algorithm is correct (Note: this proof leans on the correctness of the  $\pi$  function proof from Cormen, Thomas H. et al. *Introduction to Algorithms*, MIT Press, 2009. pages 1007 - 1008 which is an additional 2 pages)

### **Proof of correctness for LCSS:**

Induction can be used to prove the correctness of LCSS.

Base Case: The first step of the algorithm is the first iteration of the loop, where the first character of  $m$  and  $n$  are compared. So the problem becomes finding the longest common substring between two single character strings, for example, "a" and "b". It is obvious if the characters are the same then the character itself would be the longest common substring

Inductive Step: Since step 1 is proved to be true, then step  $i$  of the algorithm would also be true. In that case at step  $i+1$  the next character of the strings will be compared

Case 1: If the characters are the same they would add to the string, they would be added to the substring at step  $i$  and be saved. Since at step  $i$  the algorithm already produces the correct longest common substring, then the added character would obviously create the current longest common substring.

Case 2: If the characters are different, then the characters at step  $i+1$  are no longer common, so the longest common substring would be from step  $i$ , which is proven to be correct, would be saved

Because at step 1 the algorithm is proved correct, and so step  $i$  is also correct, then by induction  $i+1$  will also be correct, and the algorithm would produce the correct result all the way until the end of the array, where the element stored would be the longest common substring, and the algorithm would terminate

### Time Complexity/Running time for KMP

The KMP algorithm has a time complexity of  $O(n+m)$ , where  $n$  is the length of the text and  $m$  is the length of the pattern. For each document in the corpus, the KMP algorithm is called once, so the time complexity of the PlagiarismDetector function is  $O(n*(n+m))$ . So the time complexity using the notation would be  $O(n * di + P)$

### running time LCSS (Time complexity is $O(mn)$ I believe rather than $O(m + n)$ in the textbook, I will have to look at a bit)

The first step of the algorithm is to create a table or a 2d array of size  $m*n$ , where  $m$  and  $n$  are the size of the two strings to be compared. The initial values are all zero and take an approximate constant time of  $c$ , so this means the total time complexity would be

$$c * m * n = O(mn)$$

Then the algorithm will loop through each index of the array and compare the corresponding character at the same index in string  $m$  and  $n$ . The two characters are compared in a constant time  $c_1$ , if they are the same they will be added to the biggest neighbor and save in this index of the array. If the characters are different, then only the biggest neighbor is added. Due to the way, the array is iterated there will be only three neighbors, and the time to search for a single neighbor is also a constant time  $c_2$ . These steps are repeated for the entire array which has the size of  $m*n$ , So in total the time complexity is

$$m * n * (c_1 * 3 * c_2) = O(mn)$$

Then only the last index of the array would store the longest common substring. To access it would be a simply constant time  $c_1 = O(1)$ , so the total time complexity of the whole algorithm is

$$O(mn) + O(mn) + O(1)$$

Taking the most significant growth rate results in

$$O(mn)$$

**Unexpected Cases/Difficulties:** If you encountered any issues for design and implementation of your work, include it here. Also, mention the solutions you have considered to alleviate the issue.

General: It took some time to think of how to implement both methods to check over more than one document as both methods were designed to compare one document at a time to fix this problem we, to fix the issue with capitalization and punctuation differences we converted all the used text to lowercase and we also removed all punctuation

LCS: Because LCS does not look for specific strings this presents a problem for what we defined to be plagiarism, to remedy this particular problem we will redefine plagiarism for this method so a plagiarized document will be one that has the longest common subsequence that is equal to the largest value obtained from taking the  $\max\{lcs[i]/(\min\{\text{length}(\text{ppwd}), \text{length}(\text{corpus}[i])\})\}$  ( $\forall i \in \mathbb{N} \in \text{corpus.length}$ ) where  $lcs[i]$  is defined as the longest common subsequence between the document being checked for plagiarism and the  $i^{\text{th}}$  document in the corpus and  $\min\{\text{length}(\text{ppwd}), \text{length}(\text{corpus}[i])\}$  = the length of the shorter of the 2 documents, this method was taken from <https://reader.elsevier.com> which is very interesting document. This presents an issue if a long document is compared with a short document which is addressed in the document mentioned; however, due to time constraints we will not be implementing the fix. The fix used is using an additional measure of local lcs

KMP: One potential issue with the proposed algorithm is that it might produce false positives. For example, if a common phrase or sentence is present in both the corpus of documents and the potentially plagiarized document, it might be identified as plagiarism even if it is not. We can use stop-word removal or stemming to reduce the impact of common phrases.

Another issue is that the time complexity of the algorithm can be quite high, especially for large set of documents.

To address this issue, we can use techniques such as indexing or hashing to speed up the matching process.

Another potential issue is that the algorithm might not detect plagiarism if the copied text is heavily modified or paraphrased. To address this issue, we can use techniques such as cosine similarity or latent semantic analysis (LSA) to compare the semantic similarity between the documents, instead of relying only on exact word matches

**Task Separation and Responsibilities.** Who did what for this milestone? Explicitly mention the name of group members and their responsibilities.

Samuel S: Pseudocode for (LCSS, KMP), part of unexpected cases, Proof of Correctness for KMP

Sam G.: Problem Formulation for both algorithms, part of unexpected cases and difficulties (KMP), Running time and Time Complexity (KMP)

Jimmy J.: Time Complexity and Proof of Correctness (LCSS)