Assessment: Jacobians and Hessians

Sunday, May 22, 2022 10:00 PM

Calculate the Jacobian of the function $f(x, y, z) = x^2cos(y) + e^z sin(y)$ and evaluate at the point $(x, y, z) = (\pi, \pi, 1)$.

$$J = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right]$$

$$J = \left[2x\cos(y), -x^2\sin(y) + e^z\cos(y), e^z\sin(y)\right]$$

$$J(\pi, \pi, 1) = \left[-2\pi, e, 0\right]$$

$u(x, y) = x^2y - cos(x)sin(y)$ and $v(x, y) = e^{x+y}$ and evaluate at the point $(0, \pi)$.

$$J = \begin{bmatrix} 2xy + \sin(x)\sin(y) & x^2 - \cos x\cos(y) \\ e^{x+y} & e^{x+y} \end{bmatrix}$$

$$J = \begin{bmatrix} 0 & 1 \\ e & e^{\pi} \end{bmatrix}$$

Calculate the Hessian for the function $f(x, y) = x^3cos(y) - xsin(y)$.

$$J = \left[3x^2\cos(y) - \sin(y), -x^3\sin(y) - x\cos(y)\right]$$

$$H = \begin{bmatrix} 6x\cos y & -3x^2\sin(y) - \cos(y) \\ -3x^2\sin(y) - \cos(y) & -x^3\cos(y) + x\sin(y) \end{bmatrix}$$

4. Calculate the Hessian for the function $f(x, y, z) = xy + sin(y)sin(z) + z^3e^x$.

$$J = \left[y + z^3e^x, x + \cos(y)\sin(z), \sin(y)\cos(z) + 3z^2e^x\right]$$

$$H = \begin{bmatrix} z^3e^x & 1 & 3z^2e^x \\ 1 & -\sin(y)\sin(z) & \cos(y)\cos(z) \\ 3z^2e^x & \cos(y)\cos(z) & -\sin(y)\sin(z) + 6ze^x \end{bmatrix}$$

$$\Big[\, 3z^2e^x \quad \cos(y)\cos(z) \quad -\sin(y)\sin(z)+6ze^x \,\Big]$$

Calculate the Hessian for the function $f(x,y,z) = xycos(z) - sin(x)e^y z^3$ and evaluate at the point $(x,y,z) = (0,0,0)$

$$J = \Big[\, y\cos(z) - \cos(x)e^y z^3,\ x\cos(z) - \sin(x)e^y z^3,\ -xy\sin(z) - 3z^2\sin(x)e^y$$