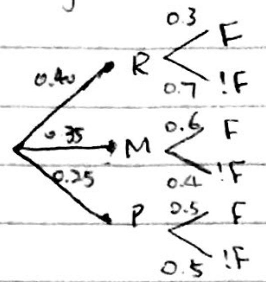
W203 Hw2: Probability Theory

Shan He

1.

Tree Diagram



Known Possibilities:

$P(R) = 0.40$, $P(F|R) = 0.30$

$P(M) = 0.35$, $P(F|M) = 0.60$

$P(P) = 0.25$, $P(F|P) = 0.50$

(a) $P(R \cap F) = P(R) \cdot P(F|R) = 0.4 \cdot 0.3 = \boxed{0.12}$

(b)
$$\begin{aligned} P(F) &= P(R) \cdot P(F|R) + P(M) \cdot P(F|M) + P(P) \cdot P(F|P) \\ &= 0.4 \cdot 0.3 + 0.35 \cdot 0.6 + 0.25 \cdot 0.5 \\ &= 0.12 + 0.21 + 0.125 \\ &= \boxed{0.455} \end{aligned}$$

(c) $P(R|F) = \dfrac{P(R \cap F)}{P(F)} = \dfrac{0.12}{0.455} = \boxed{0.264}$

2. (a) let R be event of red toy
W be event of waterproof toy
C be event of cool toy

Known Probabilities: $P(R) = \frac{1}{2}$, $P(W) = \frac{1}{2}$
$P(C) = \frac{1}{3}$, $P(R \cap W) = \frac{1}{4}$
$P(R \cap C) = \frac{1}{6}$, $P(W \cap C) = \frac{1}{6}$
$P(!R \cap !C \cap !W) = \frac{1}{6}$

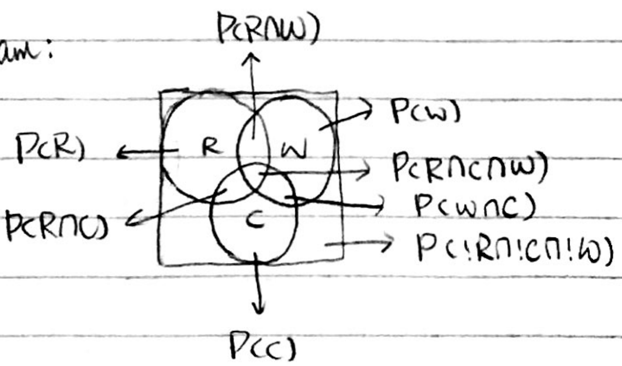
$P(R \cup W) = P(R) + P(W) - P(R \cap W) = 1 - \frac{1}{4} = \frac{3}{4}$

$P(R \cup C) = P(R) + P(C) - P(R \cap C) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$

$P(C \cup W) = P(C) + P(W) - P(C \cap W) = \frac{1}{3} + \frac{1}{2} - \frac{1}{6} = \frac{2}{3}$

$P(R \cup C \cup W) = 1 - P(!R \cap !C \cap !W) = \frac{5}{6}$

Area Diagram:



(b) $P(R \cup W \cup C) = 1 - P(!R \cap !C \cap !W)$

$= P(R) + P(C) + P(W) - P(R \cap C) - P(R \cap W) - P(C \cap W) + P(R \cap W \cap C)$

$P(R \cap W \cap C) = -(P(R) + P(C) + P(W) - P(R \cap C) - P(R \cap W) - P(C \cap W)) + 1 - P(!R \cap !C \cap !W)$

$= -(\frac{1}{2} + \frac{1}{3} + \frac{1}{2} - \frac{1}{6} - \frac{1}{4} - \frac{1}{6}) + 1 - \frac{1}{6}$

$= -(\frac{12}{24} + \frac{8}{24} + \frac{12}{24} - \frac{4}{24} - \frac{6}{24} - \frac{4}{24}) + \frac{24}{24} - \frac{4}{24} = \boxed{\frac{1}{12}}$

(c) $P(!C|R) = \frac{P(!C \cap R)}{P(R)}$

$P(!C \cap R) = P(R) - P(C \cap R) = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$

$P(!C|R) = \frac{1/3}{1/2} = \boxed{\frac{2}{3}}$

(d) $P(C|R \cup W) = \frac{P(C \cap (R \cup W))}{P(R \cup W)}$

$= \frac{P(C) + P(R \cup W) - P(C \cup R \cup W)}{P(R) + P(W) - P(R \cap W)}$

$= \frac{P(C) + P(R) + P(W) - P(R \cap W) - P(C \cup R \cup W)}{P(R) + P(W) - P(R \cap W)}$

$= \frac{\frac{1}{3} + \frac{1}{2} + \frac{1}{2} - \frac{1}{4} - \frac{5}{6}}{\frac{1}{2} + \frac{1}{2} - \frac{1}{4}}$

$= \frac{\frac{1}{4}}{\frac{3}{4}} = \boxed{\frac{1}{3}}$

3. a) $P(A) = \frac{1}{2}$; $P(B) = \frac{2}{3}$

$$
\begin{aligned}
P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\
&= \tfrac{1}{2} + \tfrac{2}{3} - P(A \cup B) \\
&= \tfrac{7}{6} - P(A \cup B)
\end{aligned}
$$

And we know $P(A \cup B) \in [0, 1]$ & $P(A \cap B) \in [0, 1]$ according to the axiom of probability.

But furthermore, we know that $P(A \cup B) \geq (P(A), P(B))_{max} = P(B)$

So $P(A \cap B)|_{max} = \frac{7}{6} - \frac{2}{3} = \boxed{\frac{1}{2}}$, when $P(A \cup B) = P(B)$

$P(A \cap B)|_{min} = \frac{7}{6} - 1 = \boxed{\frac{1}{6}}$, when $P(A \cup B) = 1$

b) $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

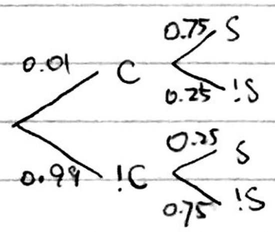
from a), we know that $\dfrac{P(A \cap B)}{P(B)}\Big|_{max} = \dfrac{P(A \cap B)|_{max}}{P(B)} = \boxed{1}$

$\dfrac{P(A \cap B)}{P(B)}\Big|_{min} = \dfrac{P(A \cap B)|_{min}}{P(B)} = \dfrac{1/6}{2/3} = \boxed{1/4}$

4. Tree Diagram :

let C be the event of having completed W203,

S be the event of liking statistics



known probabilities

$P(C) = 0.01$

$P(!C) = 0.99$

$P(S|C) = 0.75$

$P(S|!C) = 0.25$

$$P(C|S) = \frac{P(C \cap S)}{P(S)} = \frac{P(C) \cdot P(S|C)}{P(S)} = \frac{P(C) \cdot P(S|C)}{P(C) \cdot P(S|C) + P(!C) + P(S|!C)}$$

$$= \frac{0.01 \cdot 0.75}{0.01 \cdot 0.75 + 0.99 \cdot 0.25}$$

$$= \boxed{0.029}$$