Live Session 3 - Exercises

Questions

Question 1

Suppose you're taking a statistics class and that in each week you are either caught up or behind on the readings.

If you are caught up during the previous week, the probability that you will be caught up the following week is 0.7. If you are behind the previous week, the probability that you will be up to date the following week is 0.4. If we assume that you are caught up during week 1, what is the probability that you are caught up in week 3?

Question 2

A test for certain disease is assumed to be correct 95% of the time: if a person has a disease the test will give a positive result with probability 0.95. If a person does not have disease the test will give a negative result with probability 0.95. A random person drawn from a certain population has a probability 0.001 of having the disease. Given that a person drawn at random just tested positive, what is the probability that they have the disease?

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Solutions

Question 1

Suppose you're taking a statistics class and that in each week you are either caught up or behind on the readings.
If you are caught up during the previous week, the probability that you will be caught up the following week is 0.7.
If you are behind the previous week, the probability that you will be up to date the following week is 0.4.

If we assume that you are caught up during week 1, what is the probability that you are caught up in week 3?

Solution to Question 1

Let C_i be the event that you are up to date during week *i* and B_i be the event that you are behind during week *i*.

We want to find:

$$P(C_3) = P(C_2)P(C_3|C_2) + P(B_2)P(C_3|B_2) = P(C_2)(0.7) + P(B_2)(0.4)$$

Solution to Question 1

Let C_i be the event that you are up to date and Bi be the event that you are behind during week *i*.

We want to find:

$$P(C_3) = P(C_2)P(C_3|C_2) + P(B_2)P(C_3|B_2) = P(C_2)(0.7) + P(B_2)(0.4)$$

We can calculate

$$P(C_2) = P(C_1)P(C_2|C_1) + P(B_1)P(C_2|B_1) = P(C_1)\bullet 0.7+P(B_1)\bullet 0.4$$

$$P(B_2) = P(C_1)P(B_2|C_1) + P(B_1)P(B_2|B_1) = P(C_1)\bullet 0.3+P(B_1)\bullet 0.6$$

Solution to Question 1

Let C_i be the event that you are up to date and Bi be the event that you are behind during week *i*. We want to find:

$$P(C_3) = P(C_2)P(C_3|C_2) + P(B_2)P(C_3|B_2) = P(C_2)(0.7) + P(B_2)(0.4)$$

We can calculate

$$P(C_2) = P(C_1)P(C_2|C_1) + P(B_1)P(C_2|B_1) = P(C_1)\bullet 0.7+P(B_1)\bullet 0.4$$

$$P(B_2) = P(C_1)P(B_2|C_1) + P(B_1)P(B_2|B_1) = P(C_1)\bullet 0.3+P(B_1)\bullet 0.6$$

Since $P(C_1) = 0.7$ and $P(B_1) = 0.3$,

$P(C_2) = 0.7\bullet 0.7+0.3\bullet 0.4 = 0.61$ and $P(B_2) = 0.7\bullet 0.3+0.3\bullet 0.6 = 0.39$

$\Rightarrow P(C_3) = 0.61\bullet 0.7+0.39\bullet 0.4= 0.583.$

Question 2

A test for certain disease is assumed to be correct 95% of the time. If a person has a disease the test will give a positive result with probability 0.95. If a person does not have disease the test will give a negative result with probability 0.95.

A random person drawn from a certain population has a probability 0.001 of having the disease.

Given that a person drawn at random just tested positive, what is the probability that they have the disease?

Solution to Question 2

Let D be the event that the person had the disease (and D' is the complement of D) and T be the event that the test result is positive.

$$P(D|T) = \frac{P(D)P(T|D)}{P(D)P(T|D) + P(D')P(T|D')}$$

$$P(D|T) = \frac{0.001 \cdot 0.95}{0.001 \cdot 0.95 + 0.999 \cdot 0.05}$$

$$P(D|T) = 0.0187$$