Interactions

Up to this point, each predictor variable has been incorporated into the regression function through an additive term $\beta_1 X_1$. In an interaction model, such a term is called a main effect.

For a main effect, a variable increases the average response byβif for each unit increase in Xi, regardless of the levels of the other variables.

An interaction between two variables, i.e. X_i and X_{ji}, is an additive term of the form $\Upsilon_{ij} X_i X_j$ in the regression function. For example, if there are two variables, the main effects and interactions give the following regression function:

$$E(Y|X) = \alpha + \beta_1 X_1 + \beta_2 X_2 + \gamma_{12} X_1 X_2.$$

With an interaction, the slope of X_1 depends on the level of X_2, and vice versa. For example, by holding X_2 constant (or fixed) the regression function can be written as:

$$E(Y|X) = (\alpha + \beta_2 X_2) + (\beta_1 + \gamma_{12} X_2) X_1,$$

which implies that for a given level of X_2, the effect increases by $\beta_1 + \gamma_{12} X_2$ for each unit increase in X_1.

In statistics, an interaction between independent variables A and B implies that the effect of A depends on the value of B and that the effect of B depends on the value of A. This will be illustrated in the following example.

EXAMPLE

Assume a researcher has two substances and hypothesizes that one substance, the "Excitatory" substance, will increase the response while another, the "Inhibitory" substance, will decrease it. In this study, the researcher could have three groups: Excitatory, Inhibitory, or Control (i.e. neither substance).

<u>Experiment 1: No interaction</u>

Now suppose the research wants to know what happens when both the Excitatory and Inhibitory substance are administered together. In this case, we would add a fourth group makes this a two by two design.

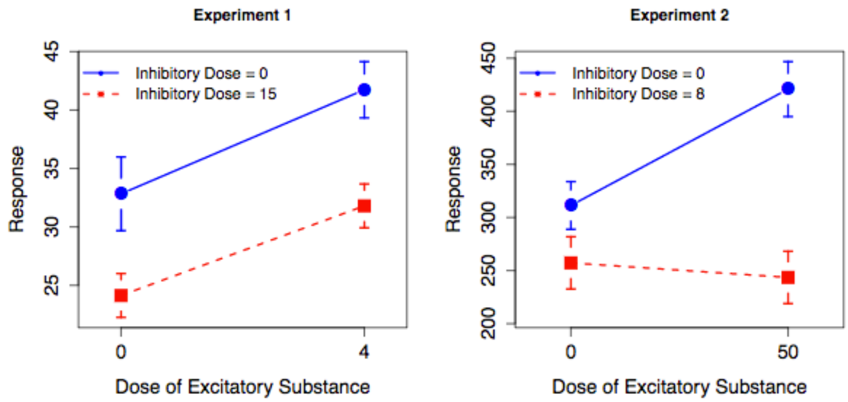
There are two predictors in this study: Excitatory with values of 0 and some amount (we'll say 4 units for this example) and Inhibitory with values of 0, and say, 15. Note that we can also recode the variables as 0 if the substance is administered and 1 otherwise.

A simple model has two predictors, the Excitatory (E) dose and the Inhibitory (I) dose is given by: $\widehat{Response} = \beta_0 + \beta_1 E + \beta_2 I$. This is the main effect model or, or sometimes, the additive model.

An interactive model adds a third predictor variable, i.e. the product of E and I. The model is given by: $\widehat{Response} = \beta_0 + \beta_1 E + \beta_2 I + \beta_3 (E \times I)$.

<u>Experiment 2: Significant Interaction</u>

Our second experiment has the same design as the first but with different Response, Excitatory, and Inhibitory variables. The doses for E are 0 and 50 while those for I are 0 and 8.



Experiment 1

	Parameter Estimates	
Source	Estimate	Std. Error
Intercept	32.839	2.700
Excitatory	2.226	0.955
Inhibitory	-0.580	0.255
Exit. * Inhib.	-0.021	0.090

Experiment 2

	Parameter Estimates	
Source	Estimate	Std. Error
Intercept	311.300	28.260
Excitatory	2.191	0.799
Inhibitory	-6.759	4.996
Exit. * Inhib.	-0.308	0.141

Suppose that the effect of the Excitatory substance is significant (i.e. β_1=2.191, t_{obs}=2.74, p=.01) while that of the Inhibitory substance is not (β_1=−6.759, t_{obs}=−1.35, p=.19). Note that we cannot conclude anything substantive from this because there is a significant interaction (β_1=−0.308, t_{obs}=−2.18, p=.04). In other words, a significant interaction trumps the significance of the variables involved in that interaction.

To understand interaction between independent variables, consider the prediction equation for Experiment 2:

$$\widehat{Response} = 311.3 + 2.191E + 6.759I + 0.308(E \times I)$$

We can factor out the E to get: $\widehat{Response} = 311.3 + 6.759I + (2.191 - 0.308I)E$. Then the slope for E is 2.191-0.308I, which is a function of I so when no inhibitory substance was administered, the slope for E is 2.191 and when the inhibitory substance was administered, the slope for E is (2.191−.308×50)=-0.27, as is shown the in the figure above.

When the Inhibitory substance is absent, then 50 units of E increases the response and when the Inhibitory substance is present, then 50 units of E has no effect. Hence, the effect of E depends on the value of I.

Similarly, consider the prediction equation for Experiment 1:

$$\widehat{Response} = 2.829 + 2.226E - 0.580I - 0.021(E \times I)$$

$$= 2.829 - 0.580I + (2.226 - 0.021I)E$$

Notice that the interaction term in this example was not significantly different from 0, i.e. when the Inhibitory dose is 0, the slope of the prediction line for E is 2.23 and when the dose is 15, the slope is 1.91. Hence, even though the interaction term is included in the prediction equation, the interaction in Experiment 1 is not significant and we can improve the model by dropping the interaction term, i.e. $\widehat{Response} = 33.138 + 2.072E - 0.62I$ (Note that any change in I has no effect on the slope of E, which remains invariant at 2.072).

Now suppose instead of specifying a dose in Experiment 2, E and I are recoded as dummy variables (i.e. Excitatory and Inhibitory substances were coded as "No" and "Yes") and assume that the parameter estimates in Experiment 2 are given by the following table:

Parameter Estimates

Source	Estimate	Std. Error
Intercept	311.30	28.26
ExcitatoryYes	109.55	39.97
InhibitoryYes	-54.08	39.97
ExitYes * InhibYes	-123.24	56.52

Then the predicted value for any observation starts with the constant 311.30 and if the Excitatory substance was administered, then 109.55 is added to that number so $\widehat{Response}$ = 420.85 in cases where no Inhibitory substance was administered. If it was administered, then 54.08 and 123.24 are subtracted and $\widehat{Response}$ = 243.50. In this way, we can arrive at the predicted values for all four groups.