W2023 Hw7 Shan He

1. (a) Yes; I'd think that it will be approximately a normal distribution with most Americans consuming around 2 lb of ground beef per month. And as the monthly ground beef consumption deviates from 2lb, in both directions, there will be fewer and fewer amount of people.

(b) Yes; invoking Central Limit Theorem, the sample size here is larger than 30. Considering that the population distribution won't be heavily skewed, I would expect the sample mean to be approximately normal.

(c) Assuming a t distribution, we can use the following equation:

$$95\%\ CI = \left(\bar{x} - t_{\alpha/2,99}\cdot\frac{s}{\sqrt{n}},\ \bar{x} + t_{\alpha/2,99}\cdot\frac{s}{\sqrt{n}}\right) \qquad t_{0.025,99} = 1.984$$

$$= \left(2.45 - 1.984\cdot\frac{2}{\sqrt{100}},\ 2.45 + 1.984\cdot\frac{2}{\sqrt{100}}\right)$$

$$= (2.0532,\ 2.8468)$$

2. Referring to $\rightarrow$ table of critical values for t distribution,

① $n=10$, $t_{\alpha/2,9} = 1.96$, using R: "a ← (1-pt(1.96,9))·2", a = 0.08
So the CI is actually $100(1-0.08)\% = 92\%$ CI

② $n=200$, $t_{\alpha/2,199} = 1.96$, using R: " a ← (1-pt(1.96,199))·2", a=0.05
So the CI is actually $100(1-0.05)\% = 95\%$ CI

3. a. $L(\lambda) = \lambda e^{-\lambda x_1} \cdot \lambda e^{-\lambda x_2} \cdot \ldots \cdot \lambda e^{-\lambda x_n}$

b. $\log(L(\lambda)) = \log(\lambda e^{-\lambda x_1}) + \log(\lambda e^{-\lambda x_2}) \ldots + \log(\lambda e^{-\lambda x_n})$

$= \log(\lambda) + \log(e^{-\lambda x_1}) + \log(\lambda) + \log(e^{-\lambda x_2}) \ldots + \log(\lambda) + \log(e^{-\lambda x_n})$

$= n \cdot \log(\lambda) - \lambda x_1 - \lambda x_2 - \ldots - \lambda x_n$ or $\sum_{i=1}^{n} (\log(\lambda) - \lambda x_i)$

c. $d \log(L(\lambda)) / d\lambda = n \cdot \frac{1}{\lambda} - x_1 - x_2 - \ldots x_n = 0$

$n \cdot \frac{1}{\lambda} - (x_1 + x_2 + \ldots + x_n) = 0 \Rightarrow n \cdot \frac{1}{\lambda} - n \cdot E(x) = 0$

$\frac{1}{\lambda} = E(x) = \mu. \Rightarrow \lambda = \frac{1}{\mu}$

d. see next page