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w203 - Sec 1

# Week 4 Pre-Class Warm-up

*w203 Instructional Team*

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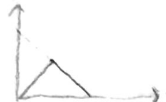
### The 'Pyramid' Distribution

Suppose that $X$ is a continuous random variable with the following PDF.




$$f_X(x) = \begin{cases} x, & 0 \le x < 1 \\ 2 - x, & 1 \le x < 2 \\ 0, & \text{otherwise} \end{cases}$$

a. Find the cumulative density function of $X$, $F_X$, and plot it.

b. Compute $E(X)$

c. Compute $var(X)$

d. Suppose $Y(X) = X^2$. Explain why $Y$ is also a random variable.

e. Compute $E(Y)$

a. for $x < 0$:

$F(x) = 0$

for $0 \le x < 1$

$F(x) = \int_0^x x \, dx = \frac{1}{2}x^2$

for $1 \le x \le 2$

$F(x) = \frac{1}{2}x^2 \Big|_0^1 + \int_1^x (2-x)\,dx = 0.5 + \left(2x - \frac{1}{2}x^2\right)\Big|_1^x = 0.5 + 2x - \frac{1}{2}x^2 - 2 + 0.5$

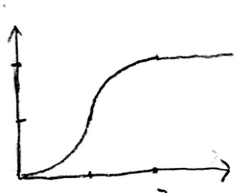
$= 2x - \frac{1}{2}x^2 - 1$

for $x > 2$

$F(x) = 1$

$\Rightarrow F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{2}x^2, & 0 \le x < 1 \\ 2x - \frac{1}{2}x^2 - 1, & 1 \le x \le 2 \\ 1, & x > 2 \end{cases}$

Plot.




b. $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$

$= \int_0^1 x \cdot x \, dx + \int_1^2 x \cdot (2-x)\,dx$

$= \frac{1}{3}x^3 \Big|_0^1 + \left(x^2 - \frac{1}{3}x^3\right)\Big|_1^2$

$= \frac{1}{3} + \left(4 - \frac{8}{3} - 1 + \frac{1}{3}\right) = \boxed{1}$

c. $Var(X) = E(X^2) - (E(X))^2$

$E(x^2) = \int_0^1 x^3\,dx + \int_1^2 x^2(2-x)\,dx$

$= \frac{1}{4}x^4 \Big|_0^1 + \left(\frac{2}{3}x^3 - \frac{1}{4}x^4\right)\Big|_1^2$

$= \frac{1}{4} + \left(\frac{16}{3} - 4 - \frac{2}{3} + \frac{1}{4}\right) = \frac{3}{12} + \left(\frac{11}{12}\right) = \frac{14}{12}$

$= \frac{7}{6}$

$Var(X) = \frac{7}{6} - 1^2 = \boxed{\frac{1}{6}}$

d. $\boxed{S \; \omega} \xrightarrow{X} \mathbb{R} \xrightarrow{Y(x)=x^2} \mathbb{R}$

Since $Y(x)$ is a function of random variable $X$, its domain is a sample space and its range is the set of real numbers

e. $E(Y) = \int_{-\infty}^{\infty} Y(x) f(x)dx$

$= \int_{-\infty}^{\infty} x^2 f(x)dx.$

Same as computed in c, we get $\boxed{7/6}$