Signal and System

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def. **Signal**
Everything that carry information can be says as a signal, e.g., R, C, R^R, N^N.
def. **System**
x is a System $\iff x \in A^A$ where A is sets.
anno. $x \in A^A$ *is another presentation of "*x *is a function mapping from* A *to* A*". In this article, the* A^A *presentation will be used*
def. **continuous time signal**
x is continuous time signal $\iff x \in A^R$ where A is a set.
At this article, it will be restricted to C^R or R^R *def.* **discrete time signal**
x is discrete time signal $\iff x \in A^Z$ where A is a set.
At this article, it will be restricted to C^Z or R^Z *oper.* **addition** of **continuous time signal**
The addtition of the two continuous time signal is $f+g$ generated by the equation $(\forall x \in R)[(f+g)(x) = f(x)+g(x)]$.
oper. **scaling** of **continuous time signal**
The scaling of the continuous signal f with factor $a \in R$ is af generated by the equation $(\forall x \in R)[(af)(x) = a(f(x))]$.
oper. **addition** of **discrete time signal**
The addtition of the two discrete time signal is $f+g$ generated by the equation $(\forall x \in R)[(f+g)(x) = f(x)+g(x)]$.
oper. **scaling** of **discrete time signal**
The scaling of the discrete signal f with factor $a \in R$ is af generated by the equation $(\forall x \in R)[(af)(x) = a(f(x))]$.
oper. **addition** of **system**
The addtition of the two systems is $f+g$ generated by the equation $(\forall x \in A)[(f+g)(x) = f(x)+g(x)]$. A is the signal field.
oper. **scaling** of **system**
The scaling of the system f with factor a is af generated by the equation $(\forall x \in A)[(af)(x) = a(f(x))]$. A is the signal field.
oper. **composition** of **system**
The composition of the two system f, g is $f \circ g$ generated by the equation $(\forall x \in A)[(f \circ g)(x) = f(g(x))]$. A is the signal field.
prop. **linearity** of **system**
A system f is linear $\iff \left(\forall (x,y) \in A^2\right)[f(x+y) = f(x)+f(y)] \wedge (\forall (a,x) \in R \times A)[f(ax) = af(x)]$. A is the signal field.
prop. **time-invariant** of **system**
A system f is time-invariant
$\iff \left(\forall (x,y) \in A^2\right)[(\exists t_0 \in B)[(\forall t \in B)[x(t) = y(t+t_0)]]$
$\to (\exists t_0 \in B)[(\forall t \in B)[x(t) = y(t+t_0)] \wedge (\forall t \in B)[(f(x))(t) = (f(y))(t+t_0)]]]$. A is the signal field, and B is the domain of the signals.
Plainly, f is time-invariant if and only if for any signal pair that y has a time sift of x, the system output $f(y)$ and $f(x)$ will remain the same time sift.
prop. **LTI (linear time-invariant)** of **system**
A system f is LTI $\iff f$ is linear and f is time-invariant
At this article, it will be restricted to LTI system.
The linear algebra tells us that if a function is linear, it might have eigen vectors. Let's find out the eigen vectors of LTI system.