Signal and System

CHEN DING-SHAN

def. **Signal**
x is a Signal $\iff x \in A^B$ where A, B is sets, or it can be represented by x is a function, $f : B \to A$.
anno. the elements of A^B is functions $f : B \to A$.
def. **System**
x is a System $\iff x \in \left(A^B\right)^{\left(A^B\right)}$ where A, B is sets.
def. **continuous time signal**
x is continuous time signal $\iff x \in A^R$ where A is a set.
def. **discrete time signal**
x is discrete time signal $\iff x \in A^Z$ where A is a set.
At this scope, the codomain is typically R, e.g., a continuous time signal is a element in R^R.
oper. **addition** of **signal**
prer. the domain A and codomain B of the two signals f, g is the same, and B has addition operation.
The addition of the two signals is $f + g$ generated by the equation $(\forall x \in A)[(f + g)(x) = f(x) + g(x)]$.
oper. **scaling** of **signal**
prer. the codomain B of the signal f has scaling operation with scaling factor in the sets C.
The scaling of the signal f with factor a is af generated by the equation $(\forall x \in A)[(af)(x) = a(f(x))]$ where A is the domain of f.
oper. **addition** of **system**
prer. the domain and codomain A of the two systems f, g is the same, and A has addition operation.
The addtition of the two systems is $f + g$ generated by the equation $(\forall x \in A)[(f + g)(x) = f(x) + g(x)]$.
oper. **scaling** of **system**
prer. the domain and codomain A of the system f has scaling operation with scaling factor in the sets B.
The scaling of the system f with factor a is af generated by the equation $(\forall x \in A)[(af)(x) = a(f(x))]$.
oper. **composition** of **system**
prer. the domain and codomain A of the two systems f, g is the same.
The composition of the two system f, g is $f \circ g$ generated by the equation $(\forall x \in A)[(f \circ g)(x) = f(g(x))]$.
prop. **stability** of **system**
prer. A is the domain and codomain of the system f. The codomain B of A is measurable to $R^+ \cup \{\infty\}$.
f is stable $\iff (\forall x \in A)[(\forall y \in B)[|x(y)| < \infty] \to (\forall y \in B)[|f(x(y))| < \infty]]$.
anno. $|x|$ is the measurement of x
thm. $((f \text{ is stable}) \land (g \text{ is stable})) \to ((f \circ g) \text{ is stable})$
prer. $\exists(f \circ g)$, the domain of f and g is measurable to $R^+ \cup \{\infty\}$.