

Signal and System

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def. Signal

Everything that carry information can be says as a signal, e.g., R , C , R^R , N^N .

def. System

x is a System $\iff x \in A^A$ where A is sets.

anno. $x \in A^B$ is another presentation of " x is a function mapping from B to A ". In this article, the A^B presentation will be used

def. continuous time signal

x is continuous time signal $\iff x \in A^R$ where A is a set.

At this article, it will be restricted to C^R or R^R

def. discrete time signal

x is discrete time signal $\iff x \in A^Z$ where A is a set.

At this article, it will be restricted to C^Z or R^Z

oper. addition of continuous time signal

The addition of the two continuous time signal is $f+g$ generated by the equation $(\forall x \in R)[(f+g)(x) = f(x) + g(x)]$.

oper. scaling of continuous time signal

The scaling of the continuous signal f with factor $a \in R$ is af generated by the equation $(\forall x \in R)[(af)(x) = a(f(x))]$.

oper. addition of discrete time signal

The addition of the two discrete time signal is $f+g$ generated by the equation $(\forall x \in R)[(f+g)(x) = f(x) + g(x)]$.

oper. scaling of discrete time signal

The scaling of the discrete signal f with factor $a \in R$ is af generated by the equation $(\forall x \in R)[(af)(x) = a(f(x))]$.

oper. addition of system

The addition of the two systems is $f+g$ generated by the equation $(\forall x \in A)[(f+g)(x) = f(x) + g(x)]$. A is the signal field.

oper. scaling of system

The scaling of the system f with factor a is af generated by the equation $(\forall x \in A)[(af)(x) = a(f(x))]$. A is the signal field.

oper. composition of system

The composition of the two system f, g is $f \circ g$ generated by the equation $(\forall x \in A)[(f \circ g)(x) = f(g(x))]$. A is the signal field.

prop. linearity of system

A system f is linear $\iff (\forall (x, y) \in A^2)[f(x+y) = f(x) + f(y)] \wedge (\forall (a, x) \in R \times A)[f(ax) = af(x)]$. A is the signal field.

prop. time-invariant of system

A system f is time-invariant

$\iff (\forall (x, y) \in A^2) [(\exists t_0 \in B)[(\forall t \in B)[x(t) = y(t+t_0)]]$

$\rightarrow (\exists t_0 \in B)[(\forall t \in B)[x(t) = y(t+t_0)] \wedge (\forall t \in B)[(f(x))(t) = (f(y))(t+t_0)]]$. A is the signal field, and B is the domain of the signals.

Plainly, f is time-invariant if and only if for any signal pair that y has a time sift of x , the system output $f(y)$ and $f(x)$ will remain the same time sift.

prop. LTI (linear time-invariant) of system

A system f is LTI $\iff f$ is linear and f is time-invariant

At this article, it will be restricted to LTI system.

The linear algebra tells us that if a function is linear, it might have eigen vectors. Let's find out the eigen vectors of LTI system.