Signal and System

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def. **Signal**
Everything that carry information can be says as a signal, e.g., R, C, R^R, N^N.
def. **System**
x is a System $\iff x \in A^A$ where A is sets.
anno. $x \in A^B$ *is another presentation of "*x *is a function mapping from* B *to* A*". In this article, the* A^B *presentation will be used*
def. **continuous time signal**
x is continuous time signal $\iff x \in A^R$ where A is a set.
At this article, it will be restricted to C^R or R^R
def. **discrete time signal**
x is discrete time signal $\iff x \in A^Z$ where A is a set.
At this article, it will be restricted to C^Z or R^Z
oper. **addition** of **continuous time signal**
The addtition of the two continuous time signal is $f+g$ generated by the equation $(\forall x \in R)[(f+g)(x) = f(x) + g(x)]$.
oper. **scaling** of **continuous time signal**
The scaling of the continuous signal f with factor $a \in R$ is af generated by the equation $(\forall x \in R)[(af)(x) = a(f(x))]$.
oper. **addition** of **discrete time signal**
The addtition of the two discrete time signal is $f+g$ generated by the equation $(\forall x \in R)[(f+g)(x) = f(x) + g(x)]$.
oper. **scaling** of **discrete time signal**
The scaling of the discrete signal f with factor $a \in R$ is af generated by the equation $(\forall x \in R)[(af)(x) = a(f(x))]$.
oper. **addition** of **system**
The addtition of the two systems is $f+g$ generated by the equation $(\forall x \in A)[(f+g)(x) = f(x) + g(x)]$. A is the signal field.
oper. **scaling** of **system**
The scaling of the system f with factor a is af generated by the equation $(\forall x \in A)[(af)(x) = a(f(x))]$. A is the signal field.
oper. **composition** of **system**
The composition of the two system f, g is $f \circ g$ generated by the equation $(\forall x \in A)[(f \circ g)(x) = f(g(x))]$. A is the signal field.
prop. **linearity** of **system**
A system f is linear $\iff \left(\forall (x,y) \in A^2\right)[f(x+y) = f(x) + f(y)] \wedge (\forall (a,x) \in C \times A)[f(ax) = af(x)]$. A is the signal field.
prop. **time-invariant** of **system**
A system f is time-invariant
$\iff \left(\forall (x,y) \in A^2\right)[(\exists t_0 \in B)[(\forall t \in B)[x(t) = y(t+t_0)]]$
$\rightarrow (\exists t_0 \in B)[(\forall t \in B)[x(t) = y(t+t_0)] \wedge (\forall t \in B)[(f(x))(t) = (f(y))(t+t_0)]]]$. A is the signal field, and B is the domain of the signals.
Plainly, f is time-invariant if and only if for any signal pair that y has a time sift of x, the system output $f(y)$ and $f(x)$ will remain the same time sift.
prop. **LTI (linear time-invariant)** of **system**
A system f is LTI $\iff f$ is linear and f is time-invariant
At this article, it will be restricted to LTI system.
Linear algebra tells us that if a function is linear, it might have eigen vectors. Let's find out the eigen vectors of LTI system.
thm. $(\forall x \in C)[(f(t) = e^{xt})$is a eigen vector of LTI systems$]$
pf. Given arbitrary real numbers t_0, complex number a, and LTI system F, consider two functions $f(t) = e^{at}$ and $g(t) = e^{a(t+t_0)}$.

$f(t+t_0) = g(t)$ and F is time-invariant $\implies (F(f))(t+t_0) = (F(g))(t)$.
$e^{at_0}f(t) = g(t)$ and F is linear $\implies e^{at_0}(F(f))(t) = (F(g))(t)$.
$\implies (F(f))(t+t_0) = e^{at_0}(F(f))(t)$
$\because t_0$ is an arbitrary real number $\therefore (\forall x \in R)[(F(f))(t+x) = e^{ax}(F(f))(t)]$
By plugging in $t=0$, $(\forall x \in R)[(F(f))(x) = e^{ax}(F(f))(0)]$
$\implies (F(f))(t) = e^{at}(F(f))(0) = (F(f))(0)f(t)$
$\implies f(t) = e^{at}$ is a eigen vector of the LTI system F.
$\because a$ is an arbitrary complex number $\therefore (\forall x \in C)[(f(t) = e^{xt})$is a eigen vector of LTI system$]$