

# Signal and System

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*def.* **Signal**

$x$  is a Signal  $\iff x \in A^B$  where  $A, B$  is sets, or it can be represented by  $x$  is a function,  $f : B \rightarrow A$ .

*anno.* the elements of  $A^B$  is functions  $f : B \rightarrow A$ .

*def.* **System**

$x$  is a System  $\iff x \in (A^B)^{(A^B)}$  where  $A, B$  is sets.

*def.* **continuous time signal**

$x$  is continuous time signal  $\iff x \in A^R$  where  $A$  is a set.

*def.* **discrete time signal**

$x$  is discrete time signal  $\iff x \in A^Z$  where  $A$  is a set.

At this scope, the codomain is typically  $R$ , e.g., a continuous time signal is a element in  $R^R$ .

*oper.* **addition of signal**

*prer.* the domain  $A$  and codomain  $B$  of the two signals  $f, g$  is the same, and  $B$  has addition operation.

The addition of the two signals is  $f + g$  generated by the equation  $(\forall x \in A)[(f + g)(x) = f(x) + g(x)]$ .

*oper.* **scaling of signal**

*prer.* the codomain  $B$  of the signal  $f$  has scaling operation with scaling factor in the sets  $C$ .

The scaling of the signal  $f$  with factor  $a$  is  $af$  generated by the equation  $(\forall x \in A)[(af)(x) = a(f(x))]$  where  $A$  is the domain of  $f$ .

*oper.* **addition of system**

*prer.* the domain and codomain  $A$  of the two systems  $f, g$  is the same, and  $A$  has addition operation.

The addition of the two systems is  $f + g$  generated by the equation  $(\forall x \in A)[(f + g)(x) = f(x) + g(x)]$ .

*oper.* **scaling of system**

*prer.* the domain and codomain  $A$  of the system  $f$  has scaling operation with scaling factor in the sets  $B$ .

The scaling of the system  $f$  with factor  $a$  is  $af$  generated by the equation  $(\forall x \in A)[(af)(x) = a(f(x))]$ .

*oper.* **composition of system**

*prer.* the domain and codomain  $A$  of the two systems  $f, g$  is the same.

The composition of the two system  $f, g$  is  $f \circ g$  generated by the equation  $(\forall x \in A)[(f \circ g)(x) = f(g(x))]$ .

*prop.* **stability of system**

*prer.*  $A$  is the domain and codomain of the system  $f$ . The codomain  $B$  of  $A$  is measurable to  $R^+ \cup \{\infty\}$ .

$f$  is stable  $\iff (\forall x \in A)[(\forall y \in B)[|x(y)| < \infty] \rightarrow (\forall y \in B)[|f(x(y))| < \infty]]$ .

*anno.*  $|x|$  is the measurement of  $x$

*thm.*  $((f \text{ isstable}) \wedge (g \text{ isstable})) \rightarrow ((f \circ g) \text{ isstable})$