MAE 195 FINAL REPORT

Sam Hince 19295309

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Contents

Chapter 1

Overview of Propeller Theory

1.1 Nomenclature

a	=	Axial interference factor
a'	=	Rotational interference factor
B	=	Number of blades
C_d	=	Coefficient of drag
C_l	=	Coefficient of lift
C_P	=	Coefficient of power
C_T	=	Coefficient of thrust
D	=	Propeller diameter
F	=	Momentum loss factor
n	=	RPS
P_C	=	Power Coefficient
r	=	Radial station
R	=	Radius
T	=	Thrust force
T_C	=	Thrust Coefficient
J	=	Advance ratio
ρ	=	Air density
ξ	=	r/R
η	=	Efficiency
ω	=	Airflow angular velocity
Ω	=	Angular velocity
Γ	=	Circulation
σ	=	Local solidity
ϕ	=	Flow angle
ϕ_t	=	Flow angle

1.2 Simple Momentum

1.2.1 Actuator Disk

Let us first consider the propeller simply as an actuator disk. Thrust is uniformly distributed across the disk and there is a change in velocity in and pressure across the disk.

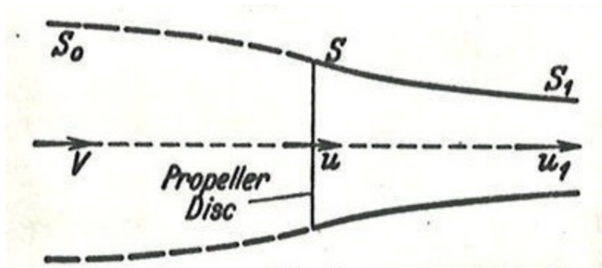

Figure 1.1: Propeller actuator disk [1]

Considering a stationary propeller with air moving past it. The thrust generated will be

$$T = \dot{m}\Delta V = \rho u_1 S_1 (u_1^2 - V^2) \tag{1.1}$$

We can say that the power absorbed by propeller or actuator disk is change in kinetic energy of per unit time, which is also equal to the work done on the air.

$$P = \frac{1}{2}\rho u_1 S_1 (u_1^2 - V^2) = uT \tag{1.2}$$

Next we must consider a propeller moving through a fluid where the power absorbed by propeller is

$$P = VT + E \tag{1.3}$$

Where VT is the work done by the actuator disk and E may be obtained by

$$E = \frac{1}{2}\dot{m}(\Delta V)^2 = \frac{1}{2}\rho u_1 S_1 (u_1 - V)^2 \tag{1.4}$$

Plugging in equations 1.1 and 1.4 into 1.3 and solving gives

$$P = \frac{1}{2}\rho u_1 S_1 (u_1^2 - V^2) \tag{1.5}$$

Note that this is the same result as for the stationary propeller.
From equation 1.2 we can obtain that

$$P = uT = \frac{1}{2}\rho u_1 S_1 (u_1 - v)^2 = 1/2(u_1 - v)T \tag{1.6}$$

Plugging in equation 1.1 gives the following, showing the velocity at the propeller is the average of the upstream and downstream velocities

$$u = \frac{1}{2}(u_1 + V) \tag{1.7}$$

1.2.2 Efficiency

We can consider the efficiency of the propeller or actuator disk to be the ratio of the work done on the air to the power absorbed by the propeller

$$\eta = \frac{VT}{P} \tag{1.8}$$

Note that the definition of efficiency presented in equation 1.8 does not take into account viscous effects by continuing to treat the propeller as a disk that imparts work directly onto the air.
Next, we plug equations 1.1 and 1.2 into 1.8:

$$\eta = \frac{V \rho u_1 S_1 (u_1^2 - V^2)}{\frac{1}{2}\rho u_1 S_1 (u_1^2 - V^2)} = \frac{2V}{u_1 + V} \tag{1.9}$$

For ease we can use equation 1.7 to redefine the axial velocities

$$\begin{aligned} u &= V(1+a) \\ u_1 &= V(1+b) = V(1+2a) \end{aligned} \tag{1.10}$$

Which allow efficiency to be written as

$$\eta = \frac{1}{1+a} \tag{1.11}$$

Using equation 1.10 thrust and power become

$$T = 2S\rho V^2 (1+a)a \tag{1.12}$$

$$P = 2S\rho V^3 (1+a)^2 a \tag{1.13}$$

From which we can define thrust and power coefficients

$$T_C = \frac{T}{S\rho V^2} = \frac{2(1-\eta)}{\eta^2} \tag{1.14}$$

$$P_C = \frac{P}{S\rho V^3} = \frac{2(1-\eta)}{\eta^3} \tag{1.15}$$

Figure 1.2 (a plot of η vs P_C) reveals that even without accounting for viscous effects a propeller does not operate at perfect efficiency. It also demonstrates that with a larger disk area or a faster velocity we can expect efficiency to increase.

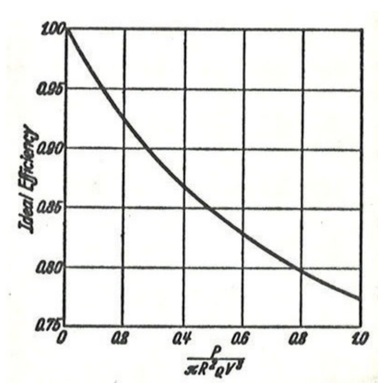

Figure 1.2: η vs P_C [1]

1.3 Vortex Model

In situations where the specific flow at the propeller disc is not of interest it may be advantageous to model the propeller and its slipstream in a simpler fashion. The vortex model of a propeller satisfies this need by providing a relatively accurate understanding of the wake of a propeller without needing to model the individual blades.

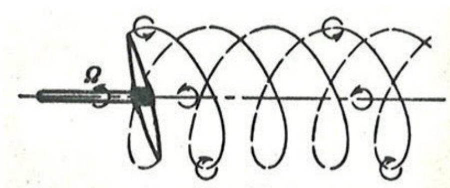

Figure 1.3: Blade shedding vortex [1]

At any blade station r a vortex is shed. These shed vertices follow a roughly helical downstream in the wake as shown in Figure 1.3.

If it is assumed that there are many blades we can model the flow field as a series of vortex rings and vortex lines.

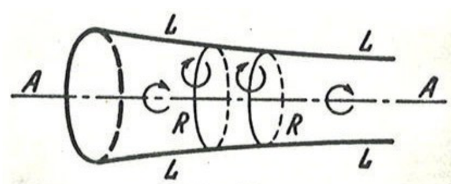

Figure 1.4: Vortex rings and threads [1]

Where the rings derive from the induced axial velocity and the lines from the induced rotational velocity.

Here the vortex rings are unbroken and the lines travel in a U shape up the side of the slipstream along a propeller blade and down the center of the slipstream in a similar fashion to how lift for a wing is modeled without breaking Helmholtz's vortex theorem.

1.4 Blade Elements

1.4.1 Velocity Relations

In order to understand the physics of the propeller in more depth we must examine the blade elements that comprise it. The blades are aerofoils and generate lift and drag just like a wing.

Let us first consider the velocities induced by the propeller. The axial velocity is as presented in equation 1.7, but the angular velocity ω' cannot be derived from the actuator disk theory.

Each blade on the propeller has circulation about it similar to a wing. In figure 1.3 we consider the propeller to be still and the air to be rotating with some angular velocity ω' as it approaches. The circulation Γ causes the angular velocity to doubled across the blade.

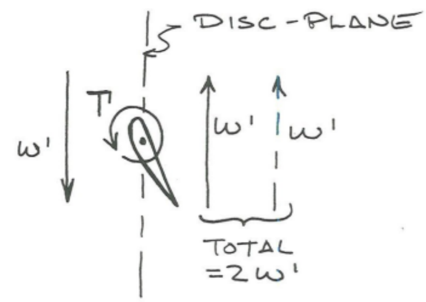

Figure 1.5: Angular velocity relation about a blade element [2]

If we call the total induced angular velocity ω then at the disk plane
the induced angular velocity is $\frac{\omega}{2}$. This allows us to sketch the flow seen by a segment of the propeller as shown

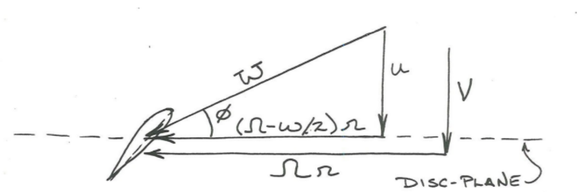

Figure 1.6: Velocity relation about a blade element [2]

Redefining as follows:

$$\begin{aligned} u &= V(1+a) \\ \omega &= 2\Omega a' \end{aligned} \tag{1.16}$$

Allows the figure to be simplified to

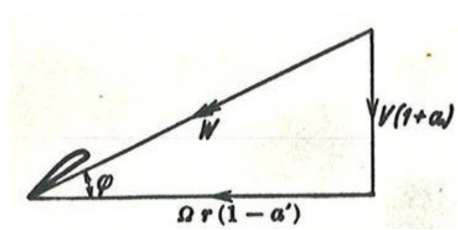

Figure 1.7: Simplified velocity relation about a blade element [1]

From here we can define the flow W seem by the blade element where the axial velocity is

$$Wsin(\phi) = u = V(1+a) \tag{1.17}$$

And the rotational velocity is

$$Wsin(\phi) = (\Omega - \frac{\omega}{2})r = \Omega r(1-a') \tag{1.18}$$

Combining gives

$$tan(\phi) = \frac{V}{\Omega r}\frac{1+a}{1-a'} \tag{1.19}$$

In the equations presented above a and a' are the axial and rotational interference factors respectively and are mathematical terms that correct for the between fluid movement in the slipstream. In other words these terms account for the induced velocities.

Next we can examine the downstream behaviors of the wake downstream.

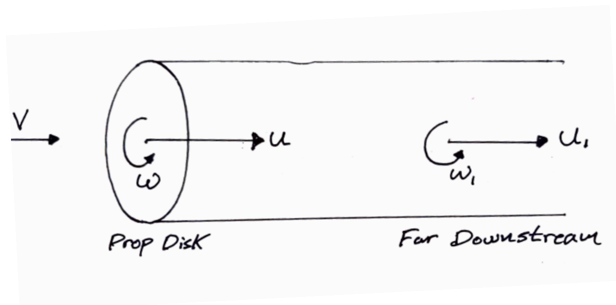

Figure 1.8: Wake progression

In addition to the conditions at the propeller disk found in equation 1.16, for the general case we can define

$$\begin{aligned} u_1 &= V(1+b) \\ \omega_1 &= 2\Omega b' \end{aligned} \tag{1.20}$$

Where $a \neq \frac{1}{2}b$. However, in the specific case of a lightly to moderately loaded propeller where slipstream contraction is minimal we can assume that $a = \frac{1}{2}b$. We will be using this assumption in the remainder of this document.

1.4.2 Momentum Relations

From the simple expression of $Force = \dot{m}\Delta V$ and equations 1.16 and 1.20 it can be derived that

$$\frac{dT}{dr} = 4\pi r\rho V^2 a(1+a)F \tag{1.21}$$

Similarly for $Torque = \dot{m}r\Delta\Omega r$ we get

$$\frac{1}{r}\frac{dQ}{dr} = 4\pi r^2\rho V\Omega a'(1+a)F \tag{1.22}$$

Note that in these equations F is a momentum loss factor to account for radial flow.

1.4.3 Forces

Knowing the velocity seen by a station on the blade, it is simple enough to determine the lift and drag experienced be the airfoil section.

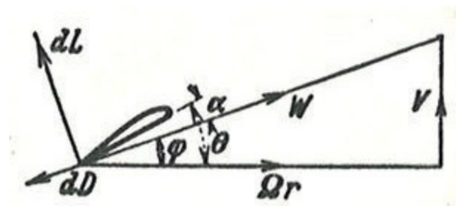

Figure 1.9: Lift and drag on a blade element [1]

Decomposing these forces into forces in the X and Y directions gives

$$C_Y = C_l cos(\phi) - C_d sin(\phi) \tag{1.23}$$

$$C_X = C_l sin(\phi) + C_d cos(\phi) \tag{1.24}$$

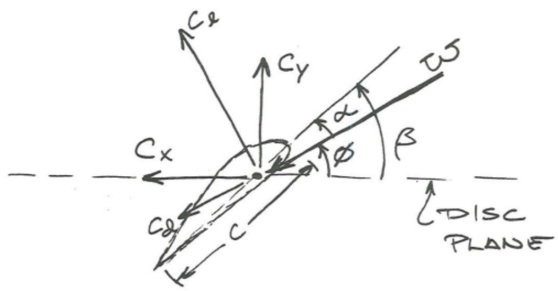

Figure 1.10: Forces on a blade element [2]

These force coefficients can be used to derive the following thrust and torque

$$\frac{dT}{dr} = \frac{1}{2}\rho W^2 B c C_Y \tag{1.25}$$

$$\frac{1}{r}\frac{dQ}{dr} = \frac{1}{2}\rho W^2 B c C_X \tag{1.26}$$

Where c in the chord at a given station and along the blade and B is the number of blades.

By equating the momentum equations 1.21 and 1.22 with the force equations 1.25 and 1.26 we can obtain the following:

$$\frac{a}{1+a} = \frac{1}{4}\frac{Bc}{2\pi r}\frac{1}{F}\frac{C_Y}{sin^2\phi} \tag{1.27}$$

$$\frac{a'}{1-a'} = \frac{1}{4}\frac{Bc}{2\pi r}\frac{1}{F}\frac{C_X}{sin\phi cos\phi} \tag{1.28}$$

These equations can be used to find a and a' given the geometry at a blade station r.

Chapter 2

Procedures

2.1 Analysis Procedure

In this section we will outline and walk through the procedure used to analyse a given propeller geometry. This procedure is the same as that which is used to create the analysis code presented in section 3.1.

Step 1: We begin by assuming a value for ϕ that can be used as an initial starting condition.

$$\phi = tan^{-1}\left(\frac{V}{\Omega r}\right) \tag{2.1}$$

Step 2: Using the following equation α can be gotten from the assumed ϕ and the β given by the propeller geometry

$$\alpha = \beta - \phi \tag{2.2}$$

Calculate the Reynolds number and W

$$W = V\frac{1+a}{sin(\phi)} \tag{2.3}$$

$$Re = \frac{\rho W c}{\mu} \tag{2.4}$$

Step 3: Using α and Reynolds number, go to an airfoil performance chart to get C_l and C_d.

Step 4: Then, using ϕ and the results from Step 3 find the coefficients of force in x and Y.

$$C_X = C_l sin(\phi) + C_d cos(\phi) \tag{2.5}$$

$$C_Y = C_l cos(\phi) - C_d sin(\phi) \tag{2.6}$$

Step 5: Calculate the momentum loss factor using

$$\phi_t = tan^{-1}(\xi tan\phi) \tag{2.7}$$

$$f = \frac{B}{2}\frac{1-\xi}{sin^2\phi_t} \tag{2.8}$$

$$F = \frac{2}{\pi}tan^{-1}((e^{2f}-1)^{\frac{1}{2}}) \tag{2.9}$$

Where $\xi = r/R$.

Step 6: Use the following to obtain a and a':

$$a = \frac{\sigma}{4F}\frac{C_Y}{sin^2\phi} \Big/ \left(1 - \frac{\sigma}{4F}\frac{C_Y}{sin^2\phi}\right) \tag{2.10}$$

$$a' = \frac{\sigma}{4F}\frac{C_X}{sin\phi cos\phi} \Big/ \left(1 + \frac{\sigma}{4F}\frac{C_X}{sin\phi cos\phi}\right) \tag{2.11}$$

Step 7: Recalculate phi from a and a' using

$$\phi = tan^{-1}\left(\frac{V(1+a)}{\Omega r(1-a')}\right) \tag{2.12}$$

Step 8: Repeat from Step 2 after adjusting ϕ using the following equation:

$$\phi = \phi_o ld + 0.4(\phi_n ew - \phi_o ld) \tag{2.13}$$

Continue to iterate until ϕ converges.

Step 9: Obtain overall propeller performance by numerical integration across blade stations. The equations used in the integrands are defined in equations 1.25 and 1.26.

$$C_T = \frac{T}{\rho n^2 D^4}\int_{R_0}^{R} \frac{dT}{dr} dr \tag{2.14}$$

$$C_P = \frac{T}{\rho n^3 D^5}\int_{R_0}^{R} \Omega\frac{dQ}{dr} dr \tag{2.15}$$

$$J = \frac{V}{nD} \tag{2.16}$$

$$\eta = \frac{C_T J}{C_P} \tag{2.17}$$

This step is also where any specifications of interest can be calculated. Equations for several such performance statistics can be found in section 2.2 Step 11.

This process should work for any propeller. Some minor changes are needed for a windmill, which are presented in section 2.3.

2.2 Design Procedure

Here we will introduce the procedure for designing an optimal propeller.

There is a significant amount of additional theory behind the following procedure. If anything is unclear or you require additional explanation, it is recommended that you reference Design of Optimum Propellers by Charles Adkins and Robert Liebeck which contains considerable detail on the topic [4].

For the purposes of prefacing the following process let us simply state the goal as determining the propeller geometry which will develop maximum thrust with the minimum power required for a given set of parameters. This is accomplished by an iterative process of finding the flow speed at a series of stations along the blade from which we can find a chord and twist angle using the desired C_l and airfoil characteristic. The geometry in turn effects the flow velocities, and so we repeat the process until results converge.

Step 1: Begin by choosing an initial value ζ. In most cases $\zeta = 0$ is a good choice.

Step 2: Split the blades up into a series of blade stations. For each of the stations determine a value for both F and ϕ

$$F = \frac{2}{\pi} tan^{-1}((e^{2f} - 1)^{\frac{1}{2}}) \tag{2.18}$$

$$\phi = tan^{-1}\Big(\frac{tan\phi_t}{\xi}\Big) \tag{2.19}$$

Where

$$\xi = \frac{r}{R} \tag{2.20}$$

$$\phi_t = tan^{-1}\Big(\frac{V}{\Omega R}\Big)\Big(1 + \frac{\zeta}{2}\Big) \tag{2.21}$$

$$f = \frac{B}{2}\frac{1-\xi}{sin\phi_t} \tag{2.22}$$

Given that $rtan\phi = Rtan\phi_t = constant$ we can obtain an initial value for ϕ:

Step 3: Use the given equations to solve for W_c and Reynolds number.

$$W_c = \frac{4\pi V^2 G\zeta}{B\Omega C_l} \tag{2.23}$$

$$Rn = \frac{\rho W_c}{\mu} \tag{2.24}$$

Where

$$X = \frac{\Omega r}{V} \tag{2.25}$$

$$G = FXsin\phi cos\phi \tag{2.26}$$

Step 4: Using plots or functions for the characteristics of the chosen airfoil to determine $alpha$ and C_d using the desired or given C_l which is usually around 0.7. From C_l and C_d find ϵ

$$\epsilon = \frac{C_d}{C_l} \tag{2.27}$$

Step 5: Calculate the interference factors a and a'

$$a = \frac{\zeta}{2} cos^2\phi(1 - \epsilon tan\phi) \tag{2.28}$$

$$a' = \frac{\zeta}{2X} cos\phi sin\phi\Big(1 + \frac{\epsilon}{tan\phi}\Big) \tag{2.29}$$

Step 6: Calculate W, the flow velocity encountered by the blade

$$W = V\frac{1+a}{sin\phi} \tag{2.30}$$

Step 7: From W_c calculate the chord for each station.

$$c = \frac{W_c}{W} \tag{2.31}$$

Now calculate the blade twist at each station:

$$\beta = \alpha + \phi \tag{2.32}$$

Step 8: Use numerical integration to calculate I_1, I_2, J_1, J_2 from $\xi = \xi_0$ to $\zeta = 1$

$$I_1 = \int_{\xi_0}^{1} 4\xi G(1 - \epsilon tan\phi)d\xi \tag{2.33}$$

$$I_2 = \int_{\xi_0}^{1} \lambda 2G(1 - \epsilon tan\phi)\Big(1 + \frac{\epsilon}{tan\phi}\Big)sin\phi cos\phi d\xi \tag{2.34}$$

$$J_1 = \int_{\xi_0}^{1} 4\xi G\Big(1 + \Big(\frac{\epsilon}{tan\phi}\Big)\Big)d\xi \tag{2.35}$$

$$J_2 = \int_{\xi_0}^{1} 2\xi G\Big(1 + \Big(\frac{\epsilon}{tan\phi}\Big)\Big)(1 - \epsilon tan\phi)cos^2\phi d\xi \tag{2.36}$$

Where

$$\lambda = \frac{V}{\Omega R} \tag{2.37}$$

Step 9: Next we must find the new *zeta* and thrust or power coefficient. This step varies depending on if thrust or power has been specified as a target.

If thrust has been given as a specification:

$$T_c = \frac{2T}{\rho V^2 \pi R^2} \tag{2.38}$$

$$\zeta = \frac{I_1}{2I_2} - \Big(\Big(\frac{I_1}{2I_2}\Big)^2 - \frac{T_c}{I_2}\Big)^{\frac{1}{2}} \tag{2.39}$$

$$P_c = \zeta J_1 + \zeta^2 J_2 \tag{2.40}$$

Where ζ is an updated value based on the conditions calculated in previous steps.

If power has been given as a specification:

$$P_c = \frac{2P}{\rho V^3 \pi R^2} \tag{2.41}$$

$$\zeta = \frac{-J_1}{2J_2} + \Big(\Big(\frac{J_1}{2J_2}\Big)^2 + \frac{P_c}{J_2}\Big)^{\frac{1}{2}} \tag{2.42}$$

$$T_c = \zeta I_1 - \zeta^2 I_2 \tag{2.43}$$

Step 10: If the ζ calculated in step 9 has not converged to within a small distance of the initial ζ (initially the value selected in step 1) then begin again from step two using the new ζ value. A reasonable criteria for convergence may be that the two ζ values are within 0.1%.

Step 11: Determine features of interest about the propeller performance such as efficiency and solidity. The following are a few equations for possible items of interest:

$$P = P_c \rho V^3 \pi R^2 \frac{1}{2} \tag{2.44}$$

$$T = T_c \rho V^2 \pi R^2 \frac{1}{2} \tag{2.45}$$

$$J = \frac{V}{nD} \tag{2.46}$$

$$\eta = \frac{T_c}{P_c} \tag{2.47}$$

$$\sigma = \frac{Bc}{2\pi r} \tag{2.48}$$

$$ActivityFactor = \frac{100000}{16}\int_{\xi_0}^{1} \frac{c}{D}\xi^3 d\xi \tag{2.49}$$

$$Mach = \frac{\Omega r}{\sqrt{\gamma R_{sp} Temp}} \tag{2.50}$$

Step 12: Create a table of the propeller geometry. This geometry is defined by r, $c(r)$, and $\beta(r)$. This geometry along with the airfoil section can then be analysed using the process laid out in section 2.1.

2.3 Windmills

Windmills function in almost the same way as propellers and the processes presented in 2.1 and 2.2 may easily be modified to work in designing and analysing a windmill. With a windmill the airflow is driving the blades instead of being driven.

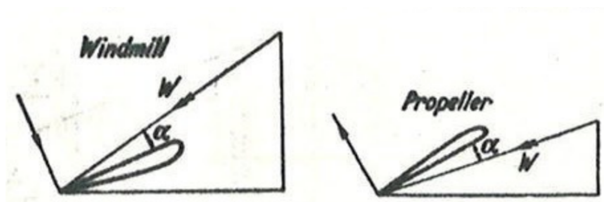

Figure 2.1: Windmill vs. Propeller [1]

This leads to two notable changes that must be made. Equations 2.2 and 2.32 must be modified to account for angle of attack being reversed. For a propeller we had that

$$\alpha = \beta - \phi \tag{2.51}$$

And for a windmill we now have

$$\alpha = \phi - \beta \tag{2.52}$$

Secondly, C_l will act in the opposite direction, so its sign must be changed in Step 3 of the analysis procedure and Step 4 of the design procedure. This change will reverse the signs of P_c, T_c, a, a', and a few other will be reversed by this change.

When designing a windmill the goal has become attaining the minimum thrust especially at a high V or wind speed. This is remarkably challenging to attain.

Chapter 3

Analysis Tools

Over the course of Winter Quarter 2021 I have prepared a number of tools for the design and analysis of propellers using the procedures presented in chapter two. The following will be a user manual for these tools along with examples of their possible uses.

As well as being presented in the appendices of this report, the code used below is located on GitHub at: https://github.com/samHince/MAE195

All of the code is written in R for speed and portability. Plotting is accomplished using ggplot2.

3.1 Analysis

3.1.1 Overview

The analysis code titled analysis.R performs the steps outlined in section 2.1. It requires inputs to define the velocity and other conditions under which the propeller is operating as well as the propeller geometry itself.

At each blade station the code will use the geometry and conditions to calculate the forces on the blade element using tools outlined in 1.4.3 and then numerically integrate the results from each blade station in order to generate overall performance metrics for the propeller.

This process is nested inside two additional loops which provide features not previously mentioned.

First is the ability to feather the propeller. In many cases the propeller being testing is not running at the conditions it was originally designed for. This feature will incrementally rotate the blades in order to achieve the maximum performance possible without changing the blade geometry. This should only be used for cases when the propeller being analysed is variable pitch in its final application.

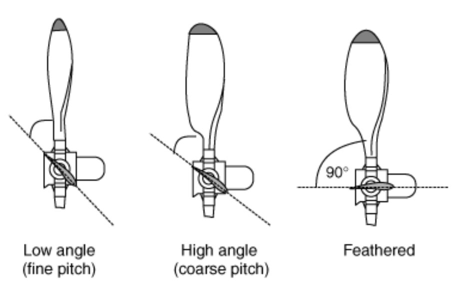

Figure 3.1: Propeller feathering [3]

Feathering has three settings: *none*, *thrust*, and *power*. When turned to *none* the code will treat the propeller being tested as fixed pitch. When set to *thrust*, the propeller will be feathered to reach a desired thrust output, and when set to *power* the propeller will be feathered to use all of the power available. This final feature is useful for finding maximum performance under various conditions with a given propeller / engine combination.

The second outer loop is titled "complex plotting loop" and is provisioned for creating plots of a propeller over varying conditions. This loop is commented out in the version uploaded to GitHub but may be used anytime such analysis is needed. It contains a section for configuring the incrementation beginning on line 102, and a section for recording and plotting the data of interest beginning on line 298. An example will be given at different velocities.

Input is given to the analysis code in the form of a *.json* file with the format shown below. Several such files are included in the GitHub repository as examples.

```
> str(geom)
List of 19
 $ propName     : chr "mustan_from_json"
 $ diameter     : num 11.2
 $ hubDiameter  : num 1.5
 $ blades       : num 4
 $ airfoil      : chr "./propSpecs/NACA4415_RN500K_NCRIT9.csv"
 $ Cl           : num 0.7
 $ velocity     : num 587
 $ J            : num 3.15
 $ RPM          : num 1000
 $ power        : num 796
 $ thrust       : num 695
 $ Cp           : chr ""
 $ Ct           : chr ""
 $ solidity     : num 0.0531
 $ AF           : num 43
 $ alt          :List of 3
  ..$ density           : num 0.0015
  ..$ kinematicViscosity: num 0.000229
  ..$ speedofsound      : num 1057
 $ radialStation: num [1:21] 0.75 0.992 1.234 1.475 1.717 ...
 $ chord        : num [1:21] 0.0341 0.0581 0.0869 0.1197 0.1552 ...
 $ beta         : num [1:21] 84.5 82.2 79.9 77.6 75.4 ...
```

Figure 3.2: JSON Format

Output from the design code includes two items in addition to any advanced plot configured for the user. First is a table showing key statistics about the propeller, a table showing the blade geometry, and at the bottom a line indicating how far and which direction the blades were feathered if feathering is active.

```
[1] "Cp: 0.363384"
[1] "Ct: 0.107449"
[1] "Efficiency: 0.932022"
[1] "Advance Ratio: 3.15201"
[1] "Power (Hp): 795.802"
[1] "Thrust (lbs): 695.191"
[1] "RPM: 1000"
[1] "Solidity: 0.0530982"
   station      chord     beta
1        1 0.03409353 84.47649
2        2 0.05805707 82.15822
3        3 0.08694760 79.87133
4        4 0.11969946 77.62232
5        5 0.15515411 75.41699
6        6 0.19210760 73.26041
7        7 0.22935290 71.15687
8        8 0.26571430 69.10991
9        9 0.30007159 67.12227
10      10 0.33137306 65.19598
11      11 0.35863648 63.33240
12      12 0.38093746 61.53227
13      13 0.39738361 59.79577
14      14 0.40707035 58.12261
15      15 0.40900897 56.51208
16      16 0.40200489 54.96314
17      17 0.38443117 53.47446
18      18 0.35374303 52.04448
19      19 0.30519186 50.67148
20      20 0.22690106 49.35360
21      21 0.00000000 48.08891
[1] "Total change in beta: 0 deg"
```

Figure 3.3: Analysis output

Additionally, a plot showing the blade's chord distribution is output.

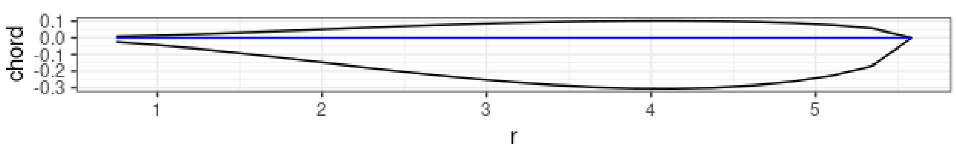

Figure 3.4: Plot of blade

This figure gives a good sanity check that everything is working properly.

3.1.2 Examples

1: Analysis of stock Cessna 150 propeller:

Using easily available data for the Cessna 150 propeller (JSON file available on GitHub) the following results appear.

```
[1] "Cp: 0.039485"
[1] "Ct: 0.0492479"
[1] "Efficiency: 0.874869"
[1] "Advance Ratio: 0.701435"
[1] "Power (Hp): 68.6464"
[1] "Thrust (lbs): 204.743"
[1] "RPM: 2400"
[1] "Solidity: 0.0577591"
```

Figure 3.5: Cessna 150 Prop Performance

With a full table of results as shown in Figure 3.6 being available by viewing the *df* variable.

	station	r	c	beta	alpha	phi	Re	a	aprime	f	F	W	Cx	Cy	L/D
1	1	0.500	0.3353	56.42	1.710545	54.70946	0.4351998	0.03295592	0.061383757	3.462013e+00	9.800282e-01	204.1662	0.5663488	0.3903595	80.27045
2	2	0.620	0.3966	50.50	1.751734	48.74827	0.5645297	0.04342060	0.052557944	3.284859e+00	9.761557e-01	223.9045	0.5273460	0.4510575	80.67098
3	3	0.740	0.4325	45.48	1.774731	43.70527	0.6757593	0.05260078	0.044702906	3.108570e+00	9.715560e-01	245.7733	0.4882553	0.4983606	80.93313
4	4	0.860	0.4484	41.24	1.792632	39.44737	0.7675405	0.06040978	0.038041418	2.932622e+00	9.660794e-01	269.2554	0.4517691	0.5354775	81.13643
5	5	0.970	0.4501	37.64	1.621153	36.01885	0.8352240	0.06394885	0.031589906	2.781313e+00	9.605316e-01	291.8923	0.4035918	0.5406631	79.39742
6	6	1.090	0.4423	34.57	1.652095	32.91791	0.8926762	0.06939337	0.027189160	2.604712e+00	9.528953e-01	317.4722	0.3762495	0.5655137	79.70127
7	7	1.210	0.4285	31.94	1.677890	30.26211	0.9365406	0.07394017	0.023549089	2.428443e+00	9.437937e-01	343.7989	0.3515732	0.5856093	79.95323
8	8	1.330	0.4110	29.66	1.692317	27.96768	0.9686167	0.07762920	0.020497925	2.252765e+00	9.329622e-01	370.7139	0.3288529	0.6010852	80.09362
9	9	1.450	0.3913	27.68	1.705290	25.97471	0.9902958	0.08072781	0.017965608	2.077214e+00	9.200350e-01	398.0923	0.3085957	0.6138703	80.21953
10	10	1.570	0.3704	25.95	1.718086	24.23191	1.0027669	0.08338188	0.015857279	1.901740e+00	9.045904e-01	425.8511	0.2905658	0.6246849	80.34343
11	11	1.690	0.3487	24.42	1.724974	22.69503	1.0062381	0.08555653	0.014067650	1.726562e+00	8.861425e-01	453.9181	0.2741032	0.6331867	80.41001
12	12	1.810	0.3265	23.06	1.728053	21.33195	1.0009609	0.08736067	0.012545353	1.551549e+00	8.640571e-01	482.2394	0.2591306	0.6399491	80.43973
13	13	1.920	0.3040	21.85	1.667913	20.18209	0.9825364	0.08720586	0.011133016	1.393362e+00	8.402960e-01	508.3979	0.2431369	0.6367279	79.85592
14	14	2.040	0.2810	20.77	1.681402	19.08860	0.9594611	0.08872912	0.010054302	1.217925e+00	8.087987e-01	537.0933	0.2316009	0.6431248	79.98744
15	15	2.160	0.2572	19.79	1.688078	18.10192	0.9253591	0.08994560	0.009108814	1.042729e+00	7.706692e-01	565.9369	0.2208025	0.6479500	80.05240
16	16	2.280	0.2324	18.90	1.691007	17.20899	0.8789475	0.09099457	0.008286610	8.676178e-01	7.240947e-01	594.9158	0.2108079	0.6517235	80.08089
17	17	2.400	0.2059	18.10	1.700125	16.39987	0.8168017	0.09205388	0.007581200	6.924672e-01	6.664166e-01	624.0063	0.2019696	0.6559209	80.16944
18	18	2.520	0.1768	17.36	1.699726	15.66027	0.7341691	0.09290382	0.006953728	5.174573e-01	5.934846e-01	653.1946	0.1934699	0.6584167	80.16556
19	19	2.640	0.1433	16.69	1.703120	14.98688	0.6217254	0.09392979	0.006420211	3.424129e-01	4.973401e-01	682.4661	0.1858502	0.6611269	80.19849
20	20	2.760	0.1006	16.07	1.696306	14.37369	0.4552322	0.09528665	0.005974865	1.674389e-01	3.582134e-01	711.8093	0.1785099	0.6621078	80.13236
21	21	2.875	0.0000	15.50	2.913799	12.58620	0.0000000	0.00000000	0.000000000	4.585421e-15	6.074243e-08	740.3576	0.1964943	0.8375404	92.84222

Figure 3.6: Complete C150 output

Performing the same analysis with viscous effects turned off by setting C_d to 0 the following output is obtained:

```
[1] "Cp: 0.0382443"
[1] "Ct: 0.0495457"
[1] "Efficiency: 0.908713"
[1] "Advance Ratio: 0.701435"
[1] "Power (Hp): 66.4894"
[1] "Thrust (lbs): 205.981"
[1] "RPM: 2400"
[1] "Solidity: 0.0577591"
```

Figure 3.7: Cessna 150 Prop no Viscous Effects

Predictably, we see that power required decreases and efficiency increases. However, efficiency does not go to 100% due to the theory outlined in section 1.2.2. Essentially, some of the energy put into the propeller goes into angular velocity instead of axial and therefore does not contribute to thrust.

2: Changing number of blades for a Cessna 150 propeller

Using the advanced plotting loop various trends can be analysed. In this example, the number of blades on a stock Cessna 150 propeller was varied and plotted against the efficiency. Feathering was set to maintain constant thrust even as the number of blades increased. The results shows that the required power slowly increases thus decreasing the efficiency of the propeller.

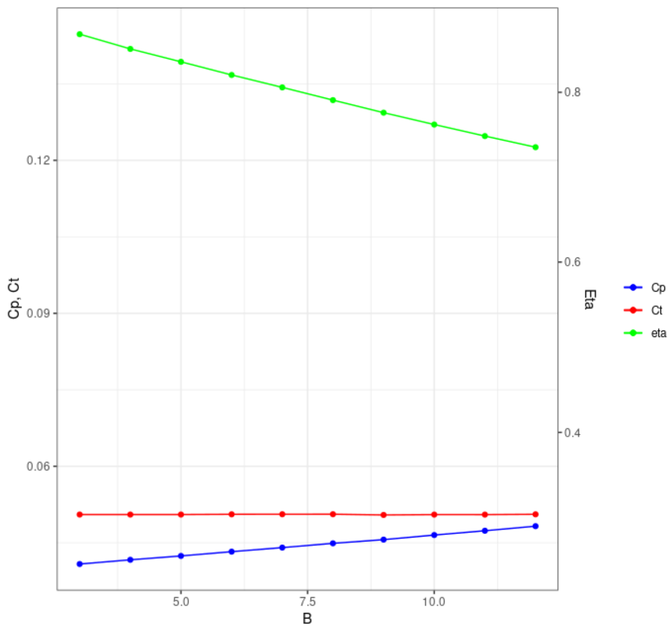

Figure 3.8: Blades vs. efficiency with feathering

Here we can see the viscous effects at work. More blades increase drag along with the possible thrust. By feathering back the blades we maintain the same thrust get grad increases due to viscous effects on the additional blades. Therefore we observe an increase in C_p and a decrease in η.

If we run the same analysis but without feathering the prop we get the following results

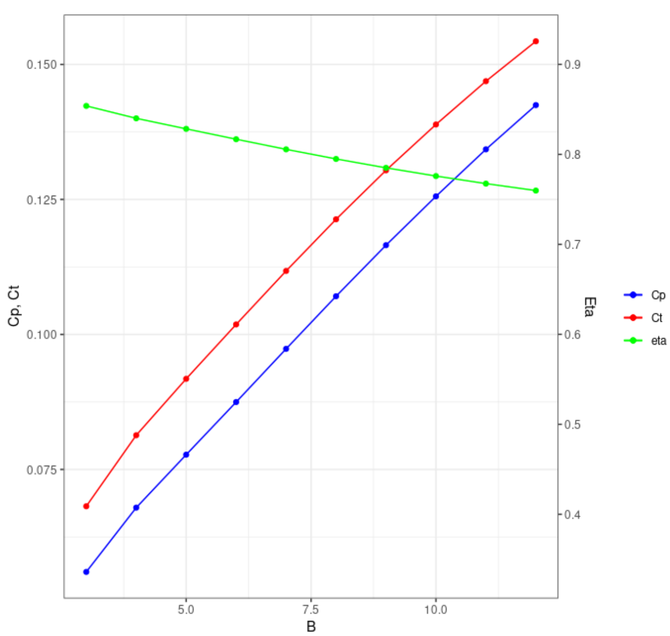

Figure 3.9: Blades vs. efficiency

Here both C_t and C_p increase, which would be necessary for a more powerful engine or larger aircraft. However, due to the viscous effects and interference between the blades η still decreases.

3: Changing airspeed for a Cessna 150 propeller

Another interesting result is obtained by varying the onset velocity for a propeller at fixed pitch and plotting the resulting efficiency and advance ratio.

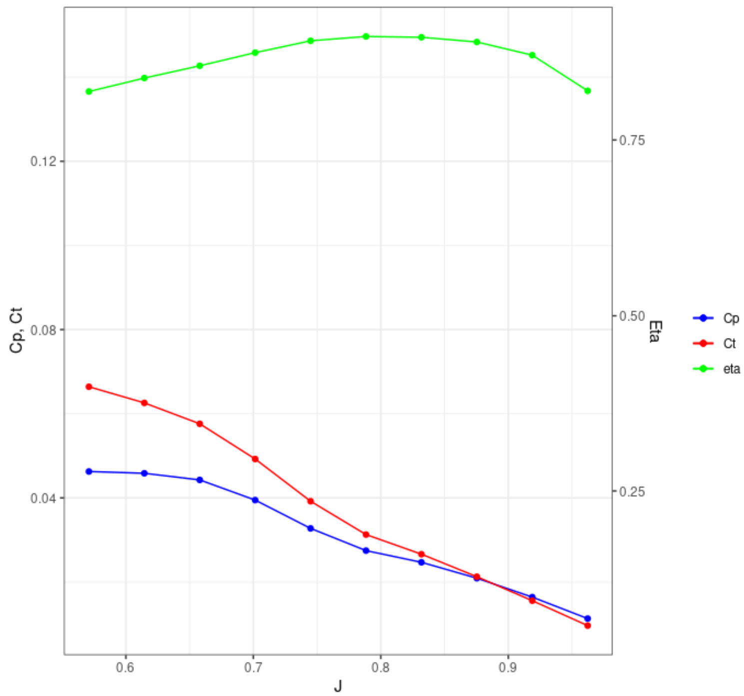

Figure 3.10: Varying onset velocity

Here we can see that the propeller performs at its peak efficiency for a certain speed when not allowed to feather, thus showing why variable pitch is important for aircraft that operate over a wide speed envelope.

4: Changing thrust for a Cessna 206 propeller

To explore the effects of velocity on feathering, a Cessna 206 propeller was analysed at increasing velocities over the design velocity. In response the propeller feathers, thus increasing the bite of air it takes with each revolution.

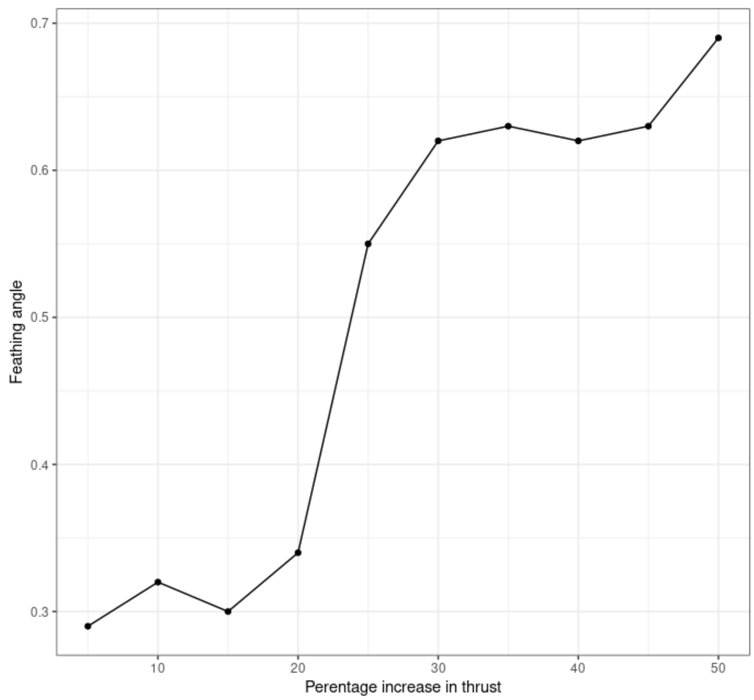

Figure 3.11: Feathering at different velocities

If this were to not occur then the thrust would drop off as the onset air exceeded the design condition. This is because the effective angle of attack of each blade element decreases. If feathering were not allowed and velocity was increased to high enough speeds, then the propeller would begin to act as a windmill with air forcing it around creating drag rather than thrust.

5: Feathering for constant power for a Cessna 206 propeller

In some cases such as air racing it may be desired to use all of the available power by feather the propeller to match the engine output rather than feathering for a desired thrust. To explore this idea we ran a test where feathering was set to maintain maximum power over a range of flight velocities.

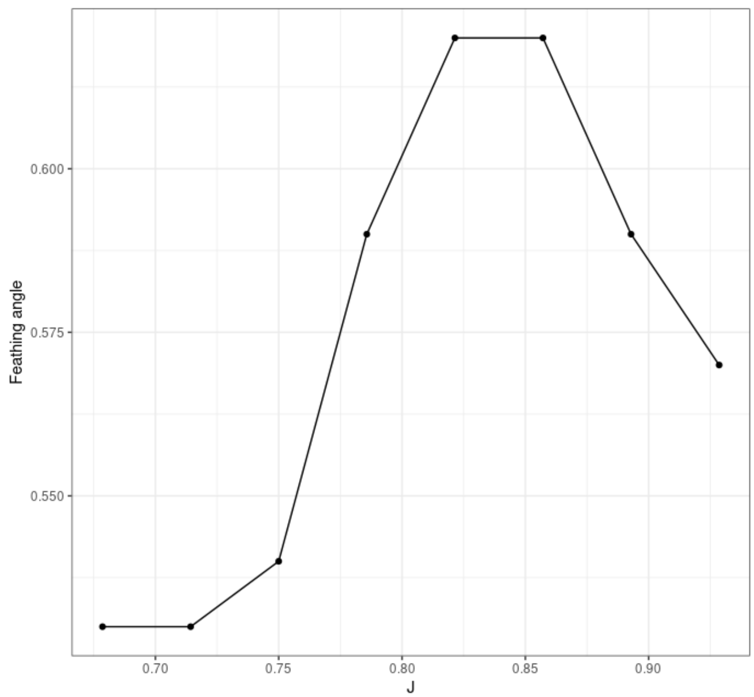

Figure 3.12: Constant power feathering angle

Here we can see that the feathering angle follows a similar arc to what is observed from η in figure 3.10. This is because where the propeller is most efficient it can be feathered further before exceeding the engine output horsepower.

3.2 Design

3.2.1 Overview

The design code performs the steps outlined in section 2.2. It is intended to optimize a blade chord distribution and twist angle to achieve the best possible performance for a given set of operating conditions. This is done through an iterative process where the flow speed and rotational velocity of the propeller are used to find W or the flow velocity encountered by a blade element. Using the airfoil performance and velocity information a twist angle may be determined to achieve a desired C_l (around 0.7 in these examples). Finally, a chord distribution is determined which will create the desired level of thrust or use a desired amount of power.

Inputs are given by setting a series of variables at the top of the script. In addition to setting physical characteristics of the propeller, power plant, and environment, the user is able to set a desired number of blade stations for analysis. A desired C_l and a threshold for convergence may also be set.

```
#givens
name <- "mustan_from_json"
D <- 11.17 #ft
D_hud <- 1.50 #ft
B <- 4 #number of blades
V <- 400 #mph
power <- 795.73 #BHp
# thrust <- 0
RPM <- 1000 #3200 # 1250
rho <- 0.00149620 #0.002378 20k ft
kinetic_viscosity <- 0.00022927
gamma <- 1.4
gas_const <- 1718
temp <- 464.514 # atmospheric temperature (currently: standard atm at 20k ft)
airfoil <- "./propSpecs/NACA4415_RN500K_NCRIT9.csv" # airfoil data

# settings
steps <- 21 # number of blade stations
Cl <- rep(0.7, steps) # desired Cl at each blade station
sucessThreshold <- 0.001 # criteria for convergence
```

Figure 3.13: Design inputs

Note that *power* or *thrust* may be defined, but not both. If a user defines both of these parameter then thrust will be used as the criteria for design.

The outputs from the design code include two items. First is a .json file which contains the information needed to run the analysis code presented in the previous step. The format is outlined in Figure 3.2. This file will be saved in "/propSpecs/DesignOutput.json" within the working directory. This behavior can be changed by modifying line 183 of the design code.

The second output is a plot similar to that shown in Figure 3.4 which shows the chord distribution for the designed propeller.

Once a propeller has been designed, the output JSON file may be fed directly into the analysis code presented in 3.1 to gather performance data.

3.2.2 Examples

1: Design of stock Cessna 150 propeller

As a test for the design process the following is a design for a stock Cessna 150 propeller. The inputs are:

```
#givens
name <- "Cessna_150_design"
D <- 5.75 #ft
D_hud <- 1 #ft
B <- 2 #number of blades
V <- 110 #mph
power <- 70 #BHp
# thrust <- 0
RPM <- 2400 #3200 # 1250
rho <- 0.002377 # sea level ft
kinetic_viscosity <- 0.0001573
gamma <- 1.4
gas_const <- 1718
temp <- 464.514 # atmospheric temperature (currently: standard atm at 20k ft)
airfoil <- "./propSpecs/NACA4415_RN500K_NCRIT9.csv" # airfoil data

# settings
steps <- 21 # number of blade stations
Cl <- rep(0.7, steps) # desired Cl at each blade station
sucessThreshold <- 0.001 # criteria for convergence
```

Figure 3.14: Inputs for Cessna 150 design

When analysed the propeller designed from these inputs performs identically to the given geometry analysed in Section 3.1.2 Example 1.

2: Designing for different altitudes Cessna 150 propeller

In order to determine the effects of altitude on propeller design the same procedure was done for a Cessna 150 propeller at increasing altitude conditions.

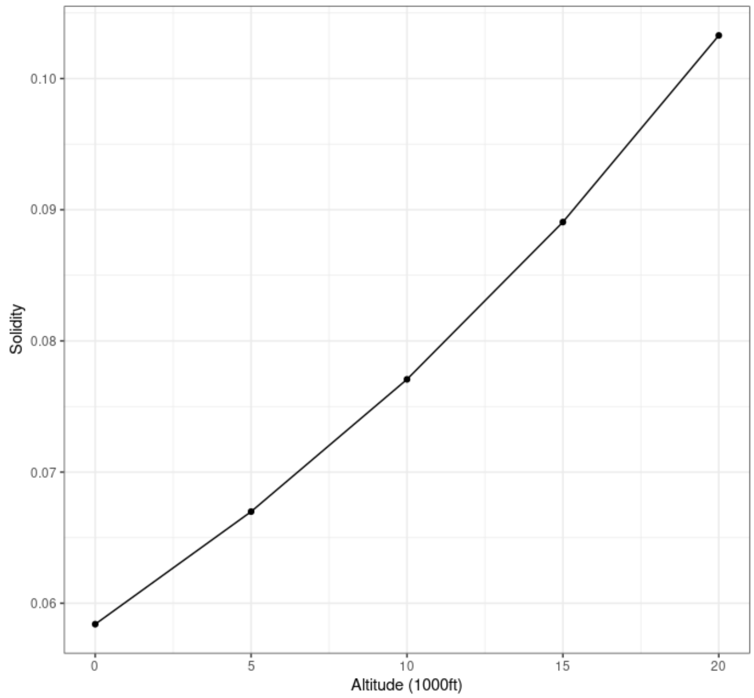

Figure 3.15: Altitude of design vs. solidity

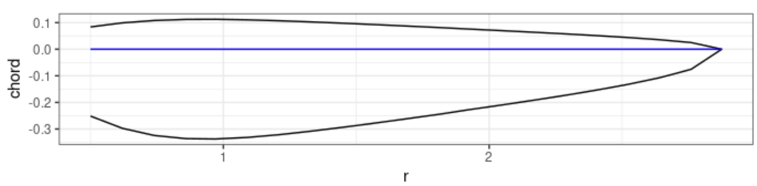

Figure 3.16: Sea level Cessna 150 design

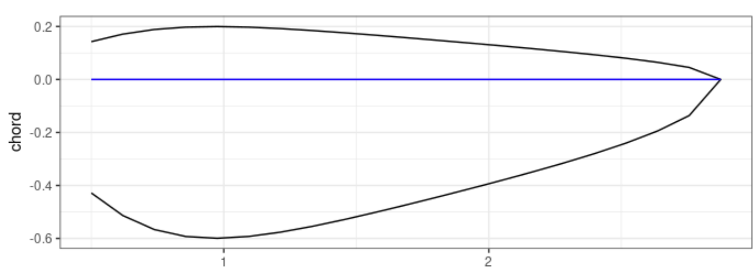

Figure 3.17: Cessna 150 design at altitude

As we can see, with increasing altitude the solidity of the propeller increases due to the thinner atmosphere. This takes the form of increasing blade chord.

3.3 Windmill

3.3.1 Overview

In addition to the propeller design and analysis code presented previously the GitHub contains versions for windmills. These scripts function the same as the design and analysis scripts discussed in 3.1 and 3.2. They require the same input parameters and yield the same outputs. The only difference is that they contain the modifications described in section 2.3 which adjust the α calculations and the sign of C_l.

3.3.2 Examples

1: Design of sample windmill

Using the windmill design code and the following inputs, a reasonable windmill blade design can be achieved.

```
#givens
name <- "windmill"
D <- 30 #ft
D_hud <- 2 #ft
B <- 3 #number of blades
V <- 25 #mph
power <- 10 #BHp
# thrust <- 0
RPM <- 158 #3200 # 1250
rho <- 0.002378 #
kinetic_viscosity <- 0.00022927
gamma <- 1.4
gas_const <- 1718
temp <- 464.514 # from standard atm at 20k ft
airfoil <- "./propSpecs/NACA4415_RN500K_NCRIT9.csv"

# settings
Cl <- rep(0.7, 21)
sucessThreshold <- 0.001
steps <- 21

```

Figure 3.18: Inputs for windmill design

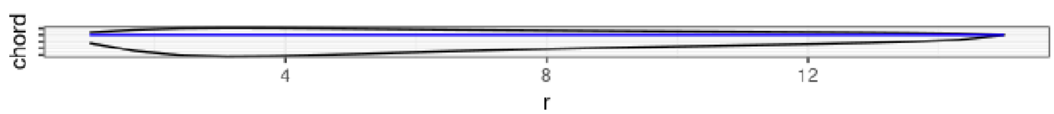

Figure 3.19: Design of windmill blade

Analysis shows this design operated at around $\eta = 0.9$ without any further optimization. Similar to the advanced analysis done for propellers there is significant analysis which could be performed under varying wind conditions, feathering angles etc. Such analysis is left to the reader.

Chapter 4

Acknowledgements and References

I would like to thank Professor Robert Liebeck for offering this course and providing exceptional lectures and clear notes. It has greatly enhanced my final year at UCI. In developing the software presented in this document Nathan Yeung's help has been invaluable. He took time beyond the hours of class and discussion to clarify directions and help find bugs. Finally, I would like to acknowledge the help of Shubham Sharma who worked in parallel and helped me catch many mistakes and improve the overall solution.

[1] Glauert, H., 1935. Airplane Propellers.

[2] Liebeck, R., 2021. MAE 195 Notes

[3] "Feathered Propeller," Academic Dictionaries and Encyclopedias, 2014.

[4] Liebeck, R. and Adkins, C., 1983. Design of Optimum Propellers.

Chapter 5

Appendices

List of scripts:

1: analysis.R for propeller analysis

2: design.R for propeller design

3: windmillAnalysis.R for windmill analysis

4: windmill.R for windmill design

Please see accompanying PDF or check https://github.com/samHince/MAE195