1 Equivalence Principles

Equation 1.1 (Newton's Law of Gravitation)**.** The differential form of Newtonian gravity is

$$\Delta\Phi = 4\pi G\rho. \tag{1.1}$$

The integral solution to this is

$$\varphi(t, \mathbf{x}) = -G \int_{\mathbb{R}^3} \mathrm{d}^3 y \, \frac{\rho(t, \mathbf{y})}{|\mathbf{x} - \mathbf{y}|.} \tag{1.2}$$

2 Manifolds and Tensors

3 The Metric Tensor

Equation 3.1 (Geodesic Lagrangian)**.** To find geodesics on a Lorentzian manifold, we use a functional formula for the proper time, treating this as an action, the 'Lagrangian' is

$$G(x(\lambda), \dot{x}(\lambda)) \coloneqq \sqrt{-g_{\mu\nu}(x)\dot{x}^\mu \dot{x}^\nu}. \tag{3.1}$$

The proper time for a curve $x : [0, 1] \hookrightarrow M$ is then

$$\tau[x] = \int_0^1 G(x(\lambda), \dot{x}(\lambda))\mathrm{d}\lambda. \tag{3.2}$$

Equation 3.2 (Geodesic Equation)**.** The Euler-Lagrange equations for 0.2 reduce to the **geodesic equation**

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho} \frac{dx^\nu}{d\tau} \frac{dx^\rho}{d\tau} = 0, \tag{3.3}$$

where the **Christoffel symbols** are defined by

$$\Gamma^\mu_{\nu\rho} \coloneqq \tfrac{1}{2} g^{\mu\sigma}(g_{\sigma\nu,\rho} + g_{\sigma\rho,\nu} - g_{\nu\rho,\sigma}). \tag{3.4}$$

4 Covariant Derivative

Equation 4.1 (Tensor Coordinate Transformation)**.** The generalisation of **??** for an arbitrary (r, s) tensor field is simply

$$T'^{\mu_1 \cdots \mu_r}{}_{\nu_1 \cdots \nu_s} = \left(\frac{\partial x'^{\mu_1}}{\partial x^{\rho_1}}\right) \cdots \left(\frac{\partial x'^{\mu_r}}{\partial x^{\rho_r}}\right) \left(\frac{\partial x^{\sigma_1}}{\partial x'^{\nu_1}}\right) \cdots \left(\frac{\partial x^{\sigma_s}}{\partial x'^{\nu_s}}\right) T^{\rho_1 \cdots \rho_r}{}_{\sigma_1 \cdots \sigma_s} \tag{4.1}$$

5 The Levi-Civita Connexion