Computer Vision and Image Understanding

(Image formation)

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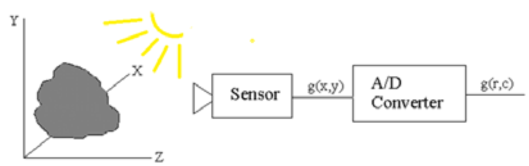
Outline

- Introduction
  - Image formation
    - Perspective projection
    - Photometric model

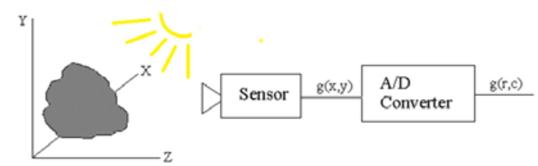
Image formation



Sensor grabs and forms two-dimensional intensity map $g(x,y)$ of the 3D scene, which is continuous in space and value.

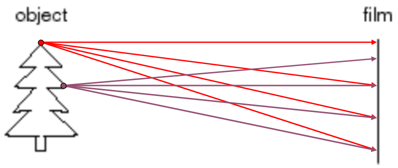
Example: Camera, Scanner, Ultrasound sensor, Infra-red sensor, MRI, CT, etc.

Image formation



- Two transformations that simultaneously take place in the camera
  - Geometric transformation (perspective projection)
  - Photometric transformation (light transportation)

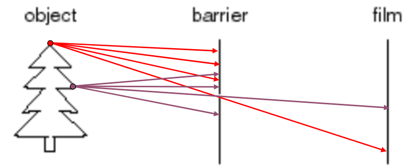
How does a camera see the world?



How a camera works:
- Put a piece of film in front of an object
- Light radiates from object in all possible directions.
- Do we get a reasonable image?

Slide inspired by by Steve Seitz

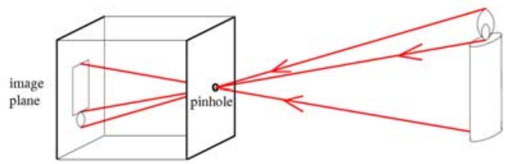
Pinhole camera



Add a barrier with a small hole to block off most of the rays
- This reduces blurring
- The opening known as the **aperture**

Slide inspired by by Steve Seitz

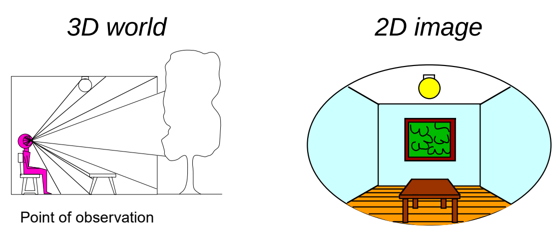
Pinhole camera model



Pinhole model:
- Captures **pencil of rays** – all rays through a single point
- The point is called **Center of Projection**
- The image is formed on the **Image Plane**

Slide by Steve Seitz

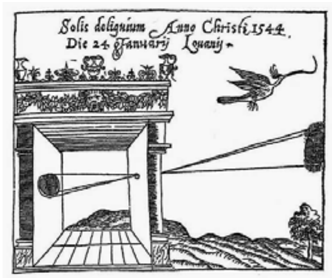
Dimensionality Reduction Machine (3D to 2D)



What have we lost?
- Distances (lengths)

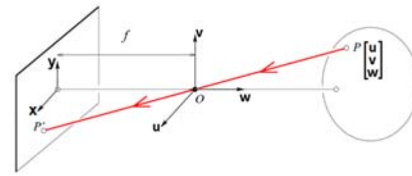
Slide by A. Efros
Figures © Stephen E. Palmer, 2002

Camera Obscura



- Basic principle known to Mozi (470-390 BCE), Aristotle (384-322 BCE)

Modeling the projection

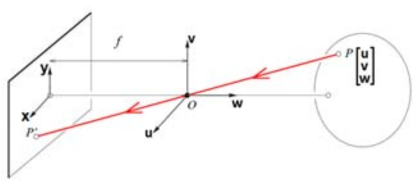


The coordinate system

- Optical center or center of projection ***O*** is at the origin
- Optical axis is in *w* direction
- The image plane (*xy*-plane) is parallel to *uv*-plane (perpendicular to *w* axis)

Source: J. Ponce, S. Seitz

Modeling projection



Projection equations

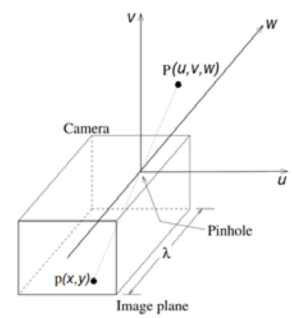
- Ray from ***P****(u, v, w)* through ***O*** intersects image plane at P'
- Derived using similar triangles

$$(u, v, w) \to (-f\frac{u}{w}, -f\frac{v}{w}) = (-x, -y)$$ **(Perspective projection)**

Source: J. Ponce, S. Seitz

Geometric transformation

- Consider pinhole camera model.
- Camera coordinate system coincides with world coordinate system.
- Origin *(0,0,0)* is at pinhole (optical centre)
- *z*-axis is same as optical axis of the camera and is perpendicular to image plane.
- λ is the focal length of the camera.
- *P(u,v,w)* and *p(x,y)* are world points an image point respectively.



Negative to positive

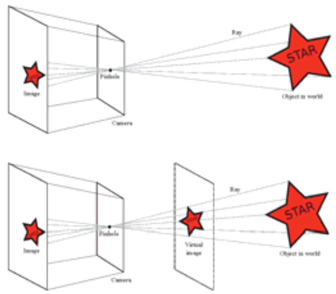
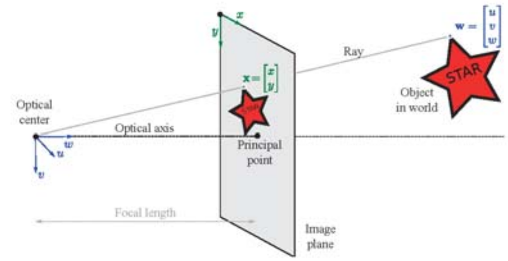


Image and scene coordinate



$$(u, v, w) \rightarrow (f\frac{u}{w}, f\frac{v}{w}) = (x, y)$$ **(Perspective projection)**

Homogeneous coordinates

Cartesian coordinate   Homogeneous coordinate

$$(x, y) \rightarrow (\kappa x, \kappa y, \kappa)$$
$$(x/w, y/w) \leftarrow (x, y, w)$$

$$(x, y, z) \rightarrow (\kappa x, \kappa y, \kappa z, \kappa)$$
$$(x/w, y/w, z/w) \leftarrow (x, y, z, w)$$

Special case: $(x, y, z) \rightarrow (x, y, z, 1)$

Geometric transformation (perspective projection)

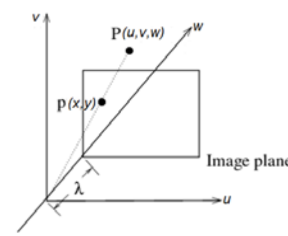
- Consider image plane in front of the pinhole (to avoid negative coordinate)

$$x = \frac{\lambda u}{w} \text{ and } y = \frac{\lambda v}{w}$$

- In matrix-vector form:

$$\begin{bmatrix} u \\ v \\ w/\lambda \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/\lambda \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

- *(u, v, w/ λ)* is homogeneous coordinate in image plane, to get Cartesian coordinate divide first two (u and v) by the third (w/λ).



Exercises

- Height of a lamppost at a distance of 20 m. from a camera is 4 m. Focal length of the came lens is 2.5 cm. What is height of the image of the lamppost on the image plane?
- A flat cardboard is placed at a distance of 15 m. in such a way that the optical axis of a camera passes through the centre of the cardboard and is perpendicular to it. Size of the cardboard is 3 m. x 5 m. Assuming that sides of the cardboard is parallel to the horizontal and vertical axes of the image plane of the camera. Given that focal length of the camera is 3 cm., calculate the area of the image of the cardboard.

Exercises

- Image of a tree of 5 m. height is taken with a camera having focal length 1.5 cm. Then the camera is moved 5 m. back and again the image of the same tree is taken. If due to this movement the height of the image of the tree is reduced by 0.05 cm., calculate the initial distance of the tree from the camera.

Properties of perspective projection

- Straight lines in 3D scene (world) map into straight lines in image.
- Distant objects (in scene) appear smaller in the image.
- A set of parallel lines in 3D, not perpendicular to z-axis maps to a set of concurrent lines in 2d image.
  - Point through which the concurrent lines pass is called *vanishing point*.

Photometric model

- Suppose light intensity $\mathscr{I}$ is transported ideally to image plane *(ξ, ζ)*.
- Scene reflectance $\mathfrak{R}$ is also transferred appropriately to image plane.
- Thus image intensity mapped is ideal one and is given by

$$f(\xi, \zeta) = \mathfrak{R}(\xi, \zeta)\, \mathscr{I}(\xi, \zeta)$$
$$(\xi, \zeta) \equiv (x, y)$$

- However, light energy transport to real image plane undergoes some transformation. This may be modeled by photometric transformation.

$$g(x, y) = \mathrm{T}[f(x, y)]$$

that satisfies the condition: $f(x, y) \geq 0$ and $g(x, y) \geq 0$

Thank you!

Any question?