Computer Vision
and
Machine Learning

(Neural Network-1)

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Regression

- Regression is a technique to establish relation between independent variables or primary observations (features) and dependent variables.
- Depending on type of relation between dependent variable (*decision or prediction*) and independent variable(s), we may classify the regression as
  - *Linear regression*
  - *Logistic regression*

Machine learning tasks T

- **Classification:** To decide which of the k classes the given input belongs to. Learning system tries to develop a mapping (function)

$$f: R^n \rightarrow \{1, 2, \cdots, k\}$$

- **Prediction:** To predict a numerical value for the given input. So the task is similar to classification except the representation of output. Thus the mapping (function) is

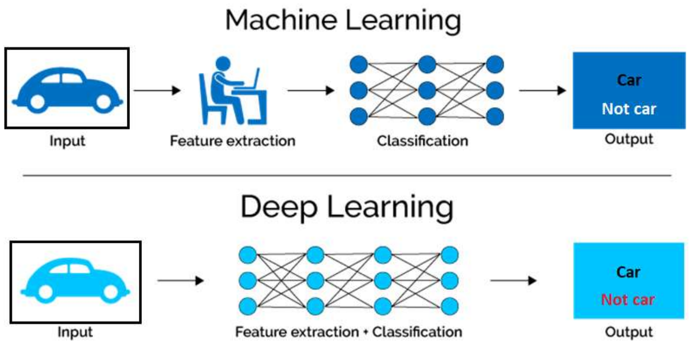
$$f: R^n \rightarrow R$$

Idea of machine learning

- A system (here, machine or computer) is said to have learned
  - to do some ***task*** T
  - from a set of ***examples*** E
  - in terms of a ***performance measure*** P,

if its performance improves
  - as measured by the same P
  - to carry out the same task T
  - by dealing with the example set E.

Machine learning vs. Deep learning



Linear regression

- Task is to build a system to predict a scalar value $y \in R$ as output from the given input $\boldsymbol{x} \in R^n$.
- Suppose $\hat{y}$ is the value predicted by the system, i.e.,

$$\hat{y} = \boldsymbol{w}^T \boldsymbol{x}$$

  where $\boldsymbol{w} \in R^n$ is parameter vector that controls behaviour of system.
- Assume $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$, similarly $\boldsymbol{w} = (w_1, w_2, \cdots, w_n)$

Linear regression (contd.)

- A more general relation between $\boldsymbol{x} \in R^n$ and $y \in R$ may be expressed as

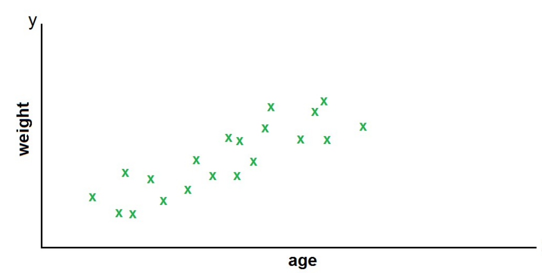
$$\hat{y} = \boldsymbol{w}^T \boldsymbol{x} + b$$

- If we append a '1' to $\boldsymbol{x}$ and including 'b' as a weight
  - Relation between y and $\boldsymbol{x}$ becomes affine, but
  - Relation between y and $\boldsymbol{w}$ remains linear.
- Consider $\boldsymbol{x} = (x_0, x_1, x_2, \cdots, x_n)$ and $\boldsymbol{w} = (w_0, w_1, w_2, \cdots, w_n)$

  where $x_0 = 1$ and $w_0 = b$.

Linear regression (contd.)

Example:



Linear regression (contd.)

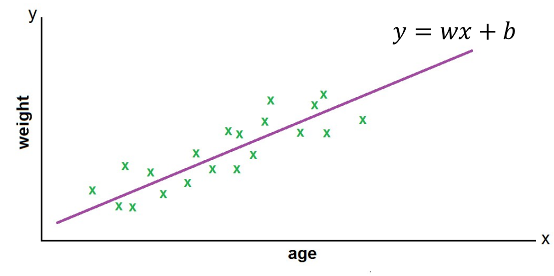
- x → independent variable (e.g., age of a deer, time in quarter, etc.)
- y → dependent variable (resp., weight of a deer, pairs of shoes sold)
- Let us consider relation between x and y may be modeled as a straight line:

$$y = wx + b$$

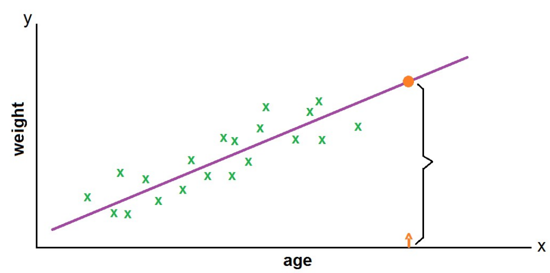
- Exploiting linear regression technique, we estimate

$$w = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2} \qquad b = \bar{y} - w\bar{x}$$

Linear regression (contd.)



Prediction



Prediction: multiple input

- So far we have discussed the cases where input is a single variable.

$$y = f(x)$$

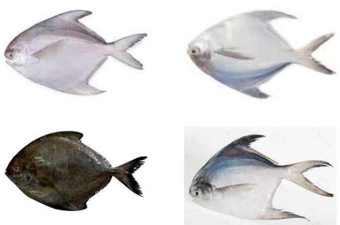
- No. of input variables (independent variables) may be more than 1.

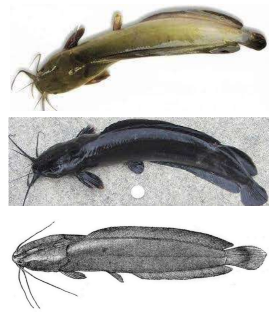
$$y = f(x_1, x_2, x_3, \ldots, x_n)$$

- A contrived example may have following input variables:

Variable	1	2	3	4	5	6	7	8	9	10
x_1	37	42	38	34	41	42	36	40	39	43
x_2	95	93	97	96	98	98	94	97	99	95
y	0	0	0	0	1	1	0	1	1	1

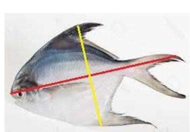
Binary classification: Pomfret and Magur

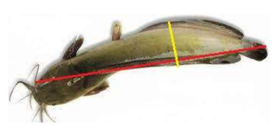




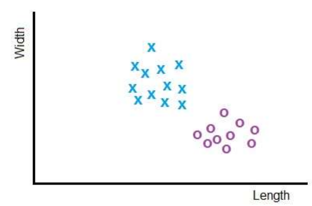
Two class problem: Pomfret and Magur

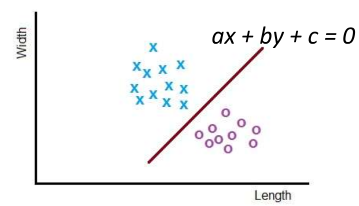
Features: Pomfret and Magur





Two-class problem: Feature space





Boundary function

- Find coefficients a, b and c of equation of a straight line

$$ax + by + c = 0$$

such that for all observation a feature pair (x, y):

$$ax + by + c > 0 \quad \text{if } (x,y) \text{ belongs to } C_1$$

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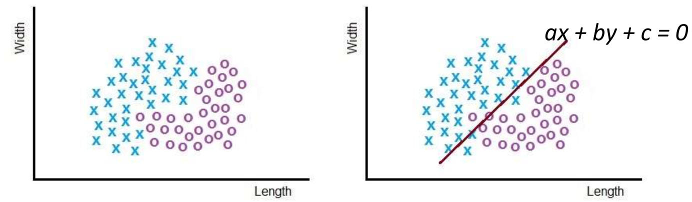
- If the desired condition is not satisfied for any feature-label pair we call a classification error has occurred.
- In general, decision boundary must be estimated to minimize this error.

Two-class problem



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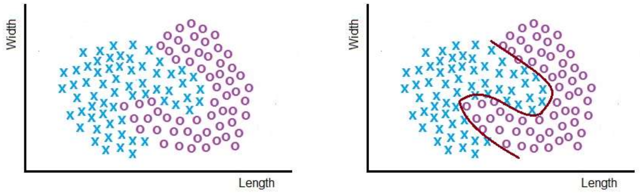


Generalization

- The ability to perform well on previously unseen data is called ***generalization***.
- The target of machine learning to keep ***generalization error*** or ***test error*** as low as possible.
  - Note that system is built by minimizing the train error.
  - Is there any relation between training error and test error?

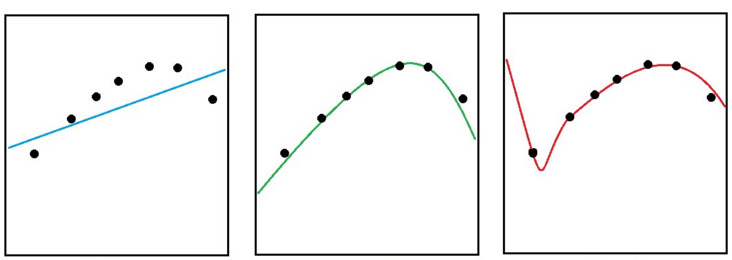
Generalization (contd.)

- Training and test data are accumulated by same data generating process.
  - Each example in training and test datasets are ***independent*** to each other.
  - The training and test datasets are ***identically distributed***.
- The *i.i.d.* assumption allows us to study the relationship between the training error and the test error.
  - Expected training error and the expected test error of a model are equal.

Overfitting and underfitting

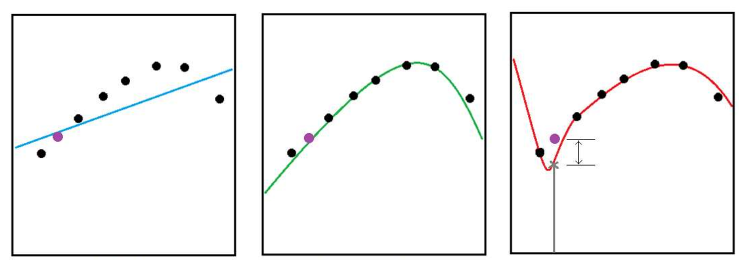
- ***Two criteria*** that determines how well a machine learning algorithm performs are its ability to
  1. make the training error small, and
  2. Make the gap between the training error and the test error small.
- These correspond to two problems: ***overfitting*** and ***underfitting***.
  - If the training error is not small → underfitting
  - If gap between training and test errors is not small → overfitting.

Overfitting and underfitting (contd.)



Underfit **Appropriate** **Overfit**

Overfitting and underfitting (contd.)



Underfit **Appropriate** **Overfit**

How to set the boundary function

- Based on the training data set.
  - All at a time.
    - Linear discriminant analysis
  - One at a time.
    - Perceptron network, neural network

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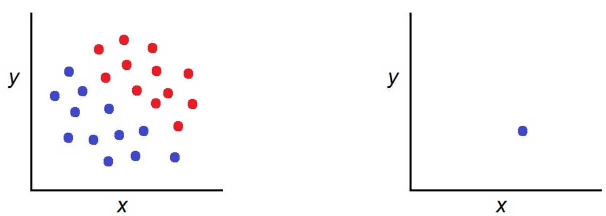
Forming the decision boundary



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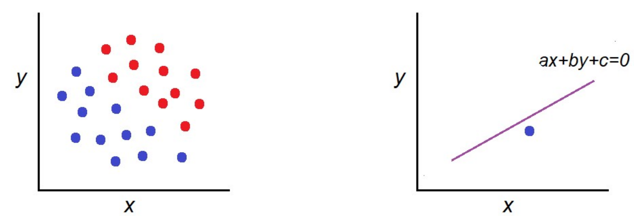

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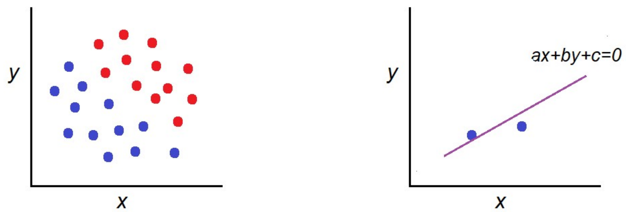

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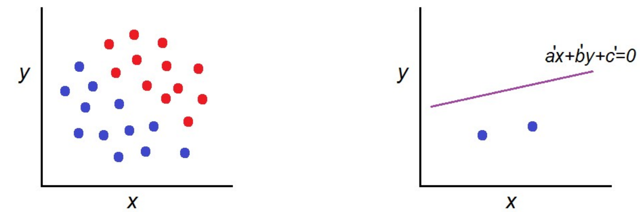

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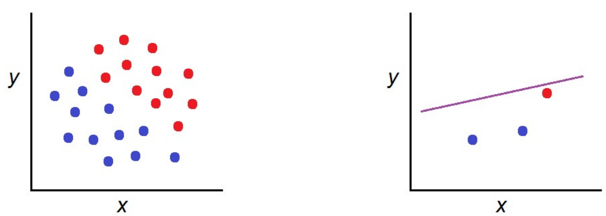

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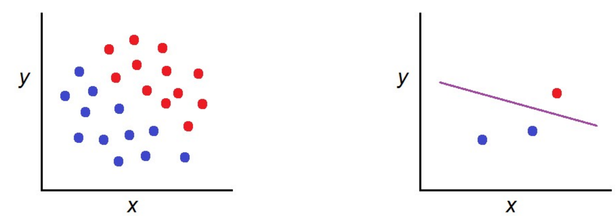

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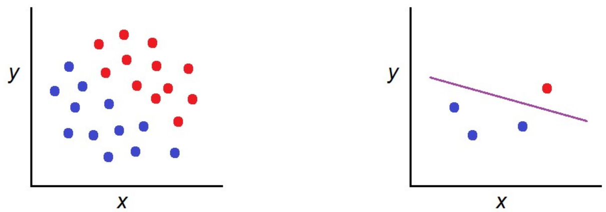

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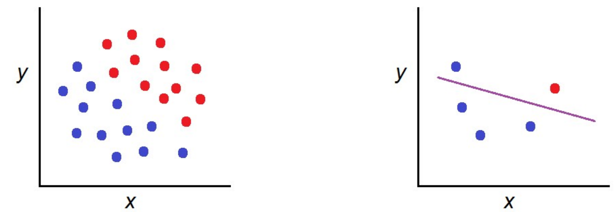

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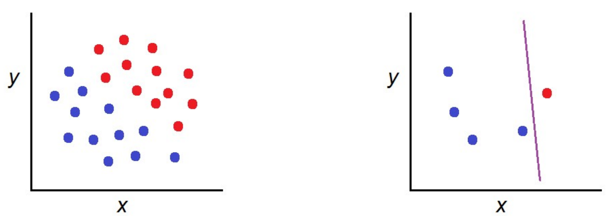

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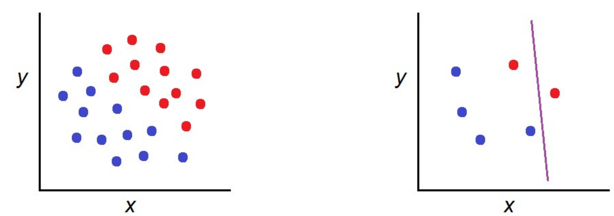

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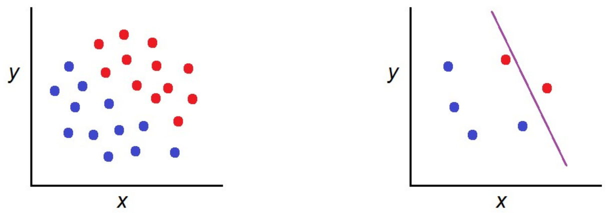

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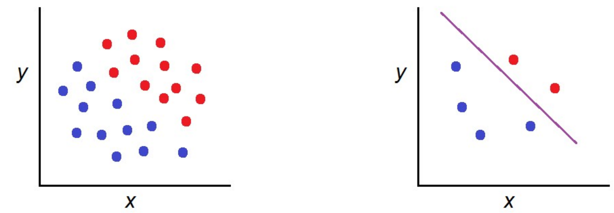

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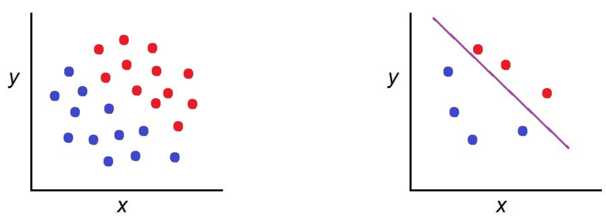

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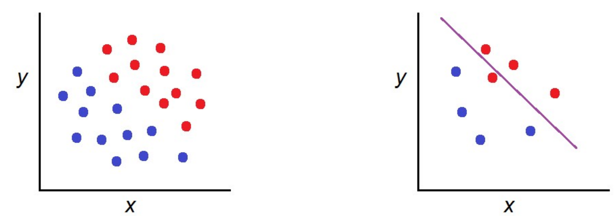

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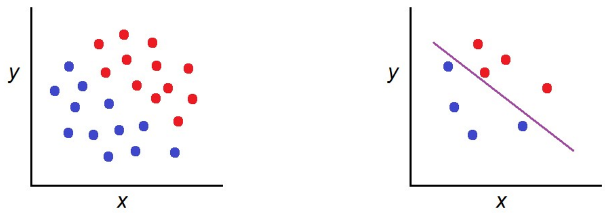

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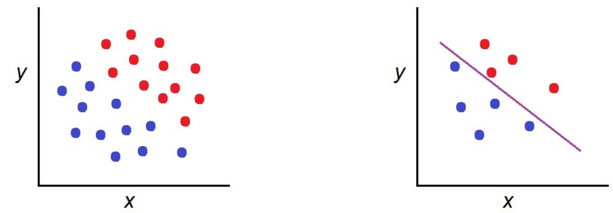

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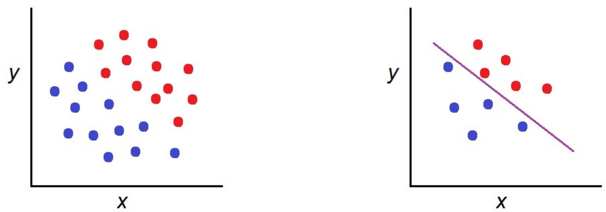

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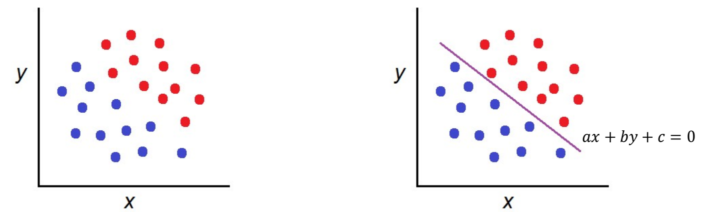

Boundary function

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such that for all observation a feature pair (x, y):

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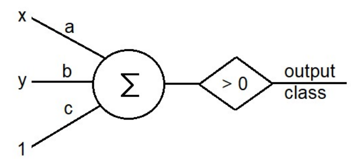
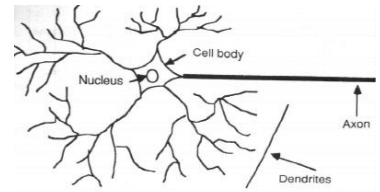
Linear classifier and neuron



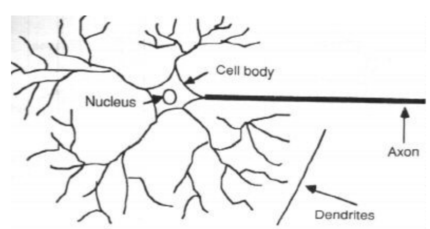


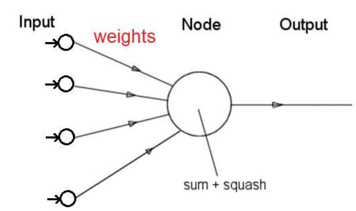
What are Artificial Neural Networks?

- Mimics the function of the brain and nervous system
- Highly parallel
  - Process information much more like the brain than a serial computer
- Learning

- Very simple principles
- Very complex behaviours

Neuron versus Node

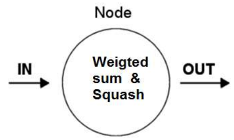


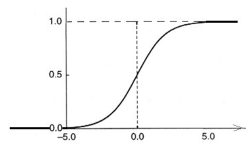


Function of a node

- At node
  Output $O = f(\sum w_i x_i)$
  where $f(.)$ is a squashing function.

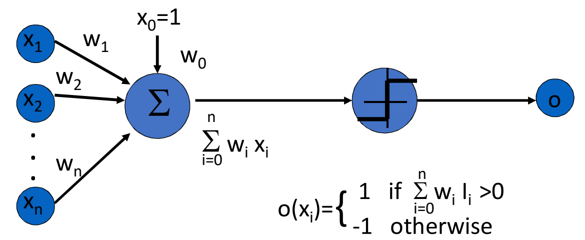
- Squashing function limits node output.





Perceptron

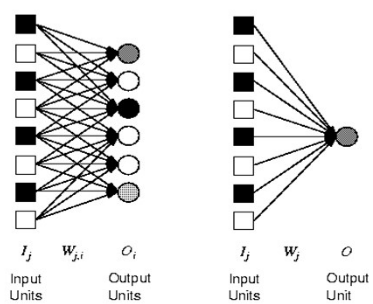
- Linear treshold unit (LTU)



Perceptron network

- Synonym for single layer, feed-forward network capable of learning.
- Output $O = f\left(\sum_j W_j I_j + b_j\right)$

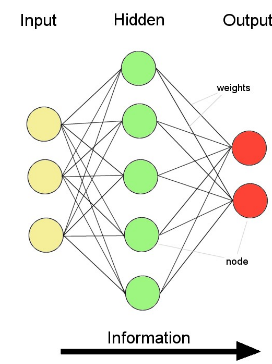
where 'b' is bias, which however, may be included as additional weight.



Perceptron Network **Single Perceptron**

Feed-forward nets

- Information flow is unidirectional
  - Data is presented to *Input layer*
  - Passed on to *Hidden Layer*
  - Passed on to *Output layer*
- Information is distributed
- Information processing is parallel
- True while testing new data



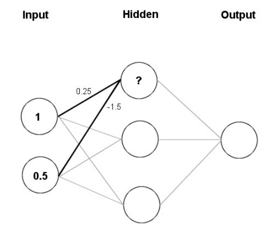
Standard activation functions

- The hard-limiting threshold function
  - Corresponds to the biological paradigm
    - either fires or not ***(Perceptron)***
- Sigmoid functions ('S'-shaped curves)
  - The hyperbolic tangent (symmetrical)
  - Both functions have a simple differential
  - Only the shape is important ***(Neuron)***

$$\phi(x) = \frac{1}{1 + e^{-ax}}$$

Example: node function

- Feeding data through the net:



$(1 \times 0.25) + (0.5 \times (-1.5)) = 0.25 + (-0.75) =$ **- 0.5**

Squashing: $\frac{1}{1+e^{0.5}} = 0.3775$

Loss function or Error or Cost function

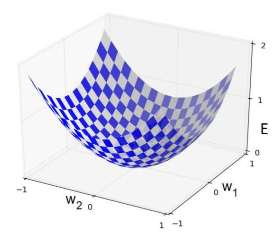
- Training sample is composed of
  - Input data (feature vector) and
  - Actual class label (also known as groundtruth)
- Given the input, feed forward network predicts class label
  - based on current parameters
  - Loss or error or cost is measured as total deviation from groundtruth

<u>Cost</u> or <u>Loss</u> or <u>Error</u>: $E(\boldsymbol{w}) = \sum(\text{Predicted label} - \text{Actual label})^2$

where $\boldsymbol{w}$ is parameter vector.

Training the network

- Means setting correct weights (including bias) or parameters of the network.
- Backpropagation
  - Requires training set (input / output pairs)
  - Starts with small random weights
  - Compute error between predicted label and actual label (groundtruth)
  - Error is used to adjust weights (supervised learning)
  - → Gradient descent on error landscape



*E(**w**)*

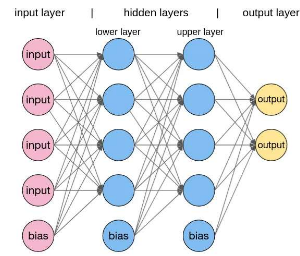
Machine learning network

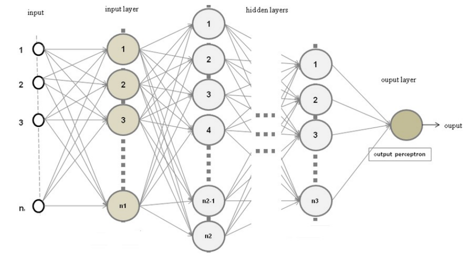
Machine learning models for classification have followings are common:

- ***Input layer:*** quantitative representation of object features
- ***Hidden layer(s):*** apply transformations with nonlinearity
- ***Output layer:*** Result for classification, regression etc.

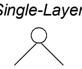
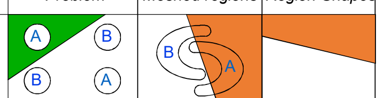
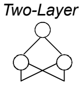
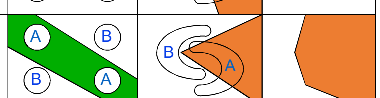
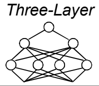
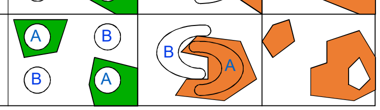
- The models are trained through ***supervised learning***.
  - Training data are explicitly labelled (known output).
  - Weights are updated to minimize error between prediction and the groundtruth.

Multilayer neural networks





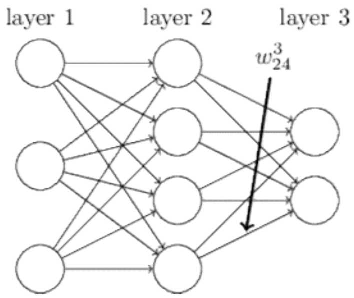
Different Non-Linearly Separable Problems

Structure	*Types of Decision Regions*	*Exclusive-OR Problem*	*Classes with Meshed regions*	*Most General Region Shapes*
Single-Layer 	*Half Plane Bounded By Hyperplane*			
Two-Layer 	*Convex Open Or Closed Regions*			
Three-Layer 	Arbitrary (Complexity Limited by No. of Nodes)			

Backpropagation

- Algorithm proposed in 1970.
- Became convincingly popular in 1986 due to a paper by [David Rumelhart](), [Geoffrey Hinton](), and [Ronald Williams]().
- At the core of backpropagation is an expression for the partial derivative of Error function with respect to weights, i.e., $\frac{\partial E}{\partial w}$

Weights of neural network



w_{jk}^{l} is the weight from the k^{th} neuron in the $(l-1)^{\text{th}}$ layer to the j^{th} neuron in the l^{th} layer

$w_{\textit{to neuron}, \textit{from neuron}}^{\textit{to the layer}}$

Output of j-th node at the l-th layer

- Input to l-th layer is coming from $(l-1)$-th layer, i.e., $\boldsymbol{y}^{(l-1)}$.
- Suppose there are K nodes in the $(l-1)$-th layer.
  - $\boldsymbol{y}^{(l-1)} = \left(1,\ y_1^{(l-1)}, y_2^{(l-1)}, y_3^{(l-1)}, \dots, y_{K-1}^{(l-1)}\right)^T$
- Weight of the connection from k-th node of the $(l-1)$-th layer to the j-th node of the l-th layer is $w_{jk}^{(l)}$.
  - $\boldsymbol{w}_j^{(l)} = \left(w_{j0}^{(l)}, w_{j1}^{(l)}, w_{j2}^{(l)}, \dots, w_{jK-1}^{(l)}\right)^T$, where $w_{j0}^{(l)}$ is the weight to the bias.

Output of j-th node at the l-th layer

- Output of the j-th node at the l-th layer is

$$y_j^{(l)} = \sigma\left(\left(\boldsymbol{w}_j^{(l)}\right)^T \boldsymbol{y}^{(l-1)}\right)$$

where $l = 1, 2, 3, \dots, L$ and that means the NN has $L-1$ hidden layers.

- Note that at the input layer, i.e., $\boldsymbol{y}^{(0)} = \boldsymbol{x}$ and output is $\boldsymbol{y}^{(L)} = \hat{\boldsymbol{y}}$.
- Let us decompose $y_j^{(l)} = \sigma\left(\left(\boldsymbol{w}_j^{(l)}\right)^T \boldsymbol{y}^{(l-1)}\right)$ into

$$y_j^{(l)} = \sigma\left(z_j^{(l)}\right) \text{ where } z_j^{(l)} = \sum_k w_{jk}^{(l)} y_k^{(l-1)}$$

Chain rule to compute

- Considering single output node, rewrite $y_j^{(l)} = \sigma\left(\left(\boldsymbol{w}_j^{(l)}\right)^T \boldsymbol{y}^{(l-1)}\right)$ as

$$y_j^{(l)} = \sigma\left(z_j^{(l)}\right) \text{ where } z_j^{(l)} = \sum_k w_{jk}^{(l)} y_k^{(l-1)}$$

- Following the chain rule:

$$\begin{aligned}
\hat{y}(\boldsymbol{x}, \boldsymbol{w}) = y^{(L)} &= \sigma\left(\sum_k w_{jk}^{(L)} y_k^{(L-1)}\right) \\
&= \sigma\left(\sum_k w_{jk}^{(L)} \sigma\left(z_k^{(L-1)}\right)\right) \\
&= \sigma\left(\sum_k w_{jk}^{(L)} \sigma\left(\sum_m w_{km}^{(L-1)} y_m^{(L-2)}\right)\right) \ \cdots
\end{aligned}$$

Derivative of function of functions

- Suppose we have $f(x) = log_e\left(sin(x^2)\right)$
- Consider $f(x) = f_1(y)$ where $y = sin(x^2)$
- then $y = f_2(z)$ where $\mathrm{z} = x^2$
- then $\mathrm{z} = f_3(x)$
- Thus $f(x) = f_1(y) \Rightarrow f(x) = f_1(f_2(z)) \Rightarrow f(x) = f_1(f_2(f_3(x)))$
- $\frac{df}{dx} = \frac{df_1}{df_2}\frac{df_2}{df_3}\frac{df_3}{dx}$ OR $\frac{df}{dx} = \frac{df}{dy}\frac{dy}{dz}\frac{dz}{dx}$ OR $\frac{df}{dx} = \frac{1}{sin(x^2)}\cos(x^2)2x$

Backpropagation

- We do not have groundtruth at the output of every layer, except the final layer, change in weight at any layer is related to the change in error ΔE as

$$\Delta E = \frac{\partial E}{\partial w_{jk}^{(l)}}\Delta w_{jk}^{(l)}$$

Recall that

$$\hat{y}(\boldsymbol{x}, \boldsymbol{w}) = y^{(L)} = \sigma\left(\sum_k w_{jk}^{(L)}\sigma\left(\sum_m w_{km}^{(L-1)}y_m^{(L-2)}\right)\right) \cdots$$

Backpropagation (contd.)

- However, to compute total change in error ΔE due to change in weights of k-th node of $(l-1)$-th layer connected to the j-th node of l-th layer, it is plausible that we should sum over all possible paths from k-th node of the $(l-1)$-th layer to the final layer, i.e.,

$$\Delta E \approx \sum\nolimits_{mnp\dots qr}\frac{\partial E}{\partial y_m^{(L)}}\frac{\partial y_m^{(L)}}{\partial y_n^{(L-1)}}\frac{\partial y_n^{(L-1)}}{\partial y_p^{(L-2)}}\cdots\frac{\partial y_q^{(l+1)}}{\partial y_r^{(l)}}\frac{\partial y_r^{(l)}}{\partial w_{jk}^{(l)}}\Delta w_{jk}^{(l)}$$

Backpropagation (contd.)

- Now combining following two equations:

$$\Delta E = \frac{\partial E}{\partial w_{jk}^{(l)}}\Delta w_{jk}^{(l)} \quad \text{and}$$

$$\Delta E \approx \sum\nolimits_{mnp\dots qr}\frac{\partial E}{\partial y_m^{(L)}}\frac{\partial y_m^{(L)}}{\partial y_n^{(L-1)}}\frac{\partial y_n^{(L-1)}}{\partial y_p^{(L-2)}}\cdots\frac{\partial y_q^{(l+1)}}{\partial y_r^{(l)}}\frac{\partial y_r^{(l)}}{\partial w_{jk}^{(l)}}\Delta w_{jk}^{(l)}$$

- We obtain

$$\frac{\partial E}{\partial w_{jk}^{(l)}} = \sum_{mn\ \dots qr}\frac{\partial E}{\partial y_m^{(L)}}\frac{\partial y_m^{(L)}}{\partial y_n^{(L-1)}}\frac{\partial y_n^{(L-1)}}{\partial y_p^{(L-2)}}\cdots\frac{\partial y_q^{(l+1)}}{\partial y_r^{(l)}}\frac{\partial y_r^{(l)}}{\partial w_{jk}^{(l)}}$$

Updating weight

- Error function: $E(\boldsymbol{w}) = \frac{1}{2n}\sum_{x} ||\hat{y}(\boldsymbol{x},\boldsymbol{w}) - y(\boldsymbol{x})||^2$

where $\hat{y}(\boldsymbol{x},\boldsymbol{w}) = \sigma\left(\sum_k w_{jk}^{(l)}\sigma\left(\sum_m w_{km}^{(l-1)} y_m^{(l-2)}\right)\right)\ \cdots$

- Earlier we had (for single layer): $w_k^{(t+1)} = w_k^{(t)} - \eta\frac{\partial E}{\partial w_k}$
- Now updating weight from k-th node of $(l-1)$-th layer to j-th node of l-th layer as

$$w_{jk}^{(l)(t+1)} = w_{jk}^{(l)(t)} - \eta\frac{\partial E}{\partial w_{jk}^{(l)(t)}}$$

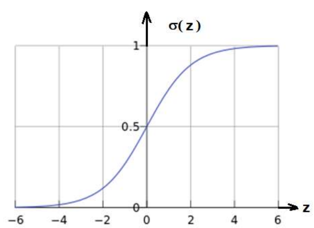
Standard activation functions

- Sigmoid functions ('S'-shaped curves)
  - The hyperbolic tangent (symmetrical)
  - Both functions have a simple differential
  - Only the shape is important ***(Neuron)***

$$\sigma(z) = \frac{1}{1 + e^{-az}}$$

Sigmoid function

- Sigmoid function
  - may be expressed as
  $$\sigma(z) = \frac{1}{1 + e^{-\alpha z}}$$
  - is one of the most popular activation function.
  - squashes the input value between 0 and 1.
  - is smooth and differentiable.
  - maximum slope is at $z = 0$



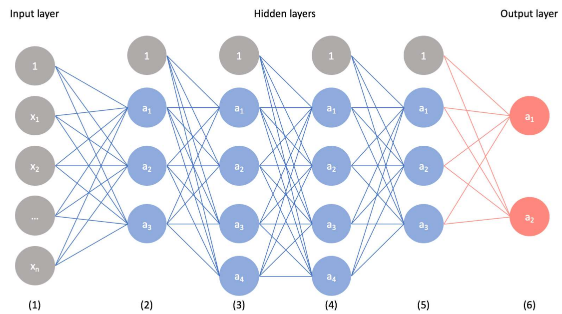
Derivative of sigmoid function

- We have $y = \sigma(z) = \frac{1}{1+e^{-\alpha}}$
- Derivative of $\sigma(z)$ at $z = 0$ may be written as

$$\frac{d\sigma}{dz}|_{z=0} = \frac{e^{-\alpha z}}{(1+e^{-\alpha z})^2}|_{z=0} = \frac{1}{(1+1)^2} = 0.25$$

- This is the maximum value of gradient for any z.

Multilayer neural network: Example



Vanishing gradient problem

- Number of layers are usually approximates the degree of polynomial function it can realize.
- However, more layers means more neurons and consequently more time to train the network.
- Second, since the derivative of the activation function (resulting in output at each layer) ≤ 0.25,

$$\frac{\partial E}{\partial w_{jk}^{(l)}} = \sum_{mnp\ldots qr} \frac{\partial E}{\partial y_m^{(L)}} \frac{\partial y_m^{(L)}}{\partial y_n^{(L-1)}} \frac{\partial y_n^{(L-1)}}{\partial y_p^{(L-2)}} \cdots \frac{\partial y_q^{(l+1)}}{\partial y_r^{(l)}} \frac{\partial y_r^{(l)}}{\partial w_{jk}^{(l)}}$$

- May tend to zero. This is known as *vanishing gradient problem*.
- This problem is more evident as we deeper layers from output input.

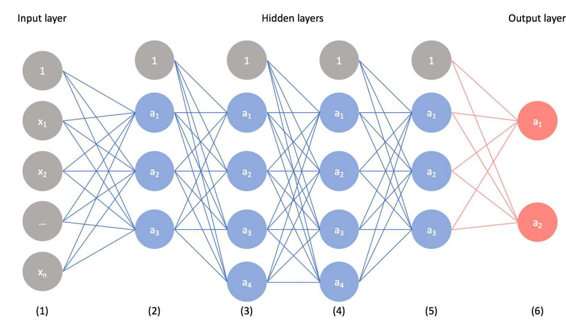
Vanishing gradient problem

- Recall the weight updating rule

$$w_{jk}^{(l)(t+1)} = w_{jk}^{(l)(t)} - \eta \frac{\partial E}{\partial w_{jk}^{(l)(t)}}$$

- If $\frac{\partial E}{\partial w_{jk}^{(l)(t)}} \to 0$, we have $w_{jk}^{(l)(t+1)} \approx w_{jk}^{(l)(t)}$
- The first layers are supposed to carry most of the information, but we see it gets trained the least.
- Hence, the problem of vanishing gradient eventually leads to the death of the network.

Vanishing gradient problem

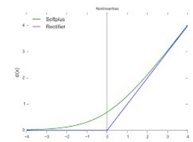


Exploding gradient problem

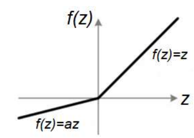
- Suppose vanishing gradient problem does not occur.
- Then $\frac{\partial E}{\partial w_{jk}^{(l)}} = \sum_{mn\ \ldots qr} \frac{\partial E}{\partial y_m^{(L)}} \frac{\partial y_m^{(L)}}{\partial y_n^{(L-1)}} \frac{\partial y_n^{(L-1)}}{\partial y_p^{(L-2)}} \cdots \frac{\partial y_q^{(l+1)}}{\partial y_r^{(l)}} \frac{\partial y_r^{(l)}}{\partial w_{jk}^{(l)}}$ implies that
- $\frac{\partial E}{\partial w_{jk}^{(l)}}$ is a sum of gradient magnitude along $m \times n \times p \times \cdots \times q \times r$ number of paths, where each gradient is greater than 0.
- Thus this sum could be significantly high resulting in *exploding gradient problem*.

Activation function (non-linear)

- Rectified Linear Unit (ReLU): $y = \max(0, x)$
  - Softplus function: $y = \log(1 + e^x)$



- Leaky ReLU:



Some hyperparameters

- ***Epoch:*** Suppose there are n samples in the training set. Passing (or using) all n samples to train the network is known as one epoch.
  - To train the network we need pass the training samples over and over again.
  - As the number of epoch increases network upgrades from underfitting to optimum to overfitting.
- ***Batch:*** If n is large, training set is divided into small batches or groups or sets of training data. *Batch size* is the number of training samples, say m, in each batch.
- ***Iteration:*** The number of batches that are passed through the network to complete one epoch, *i.e.*, n/m.

Batch normalization

- As the training progresses the network encounters (or being feed into) newer data
  - The statistical distribution of the input to layer(s) keeps changing.
  - The distribution of the output of each layer in different batches are different.
  - This reduces training efficiency.
- The input samples (in every batch) are normalized before feeding it into the next layer of the network.
  - The mean and variance of all such batches, instead of the entire data, are computed.
  - This is known as *batch normalization*.

Dropout

- This is used to overcome the overfitting problem.
- Often certain nodes in the network are randomly switched off, from some or all the layers of a neural network.
  - Hence, in every iteration, we get a new network.
  - The resulting network (obtained at the end of training) is a combination of all of them.
  - This is an way of implementing the *regularization*.

Thank you!

Any question?