Linked Lists

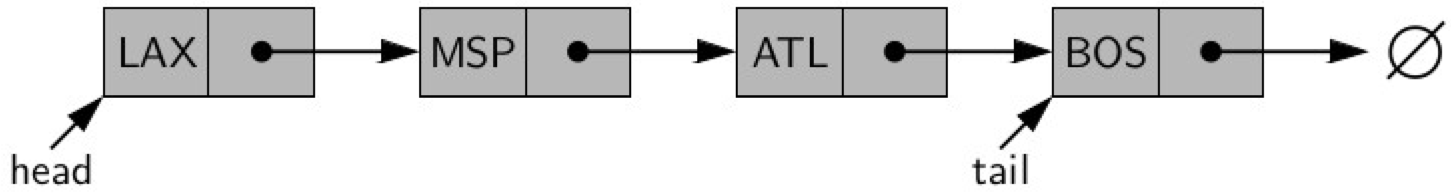
Limitations of array based implementations

1) The length is usually fixed apriori or if it is a dynamic array, it might be longer than the actual number of elements that it stores.

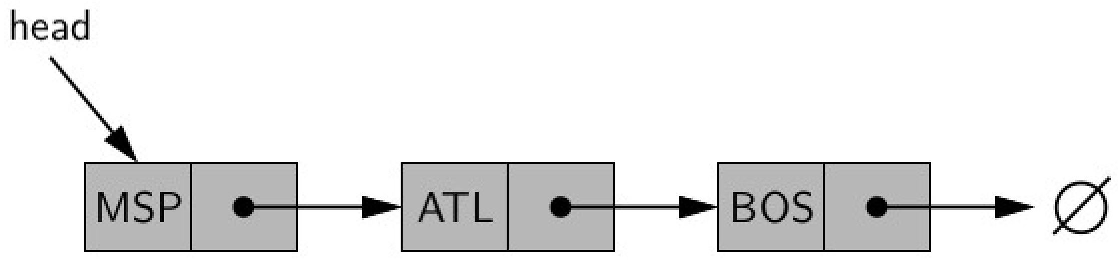
2) Amortized bounds for operations may be unacceptable in real-time systems

3) Insertions and deletions at interior positions of an array are expensive.

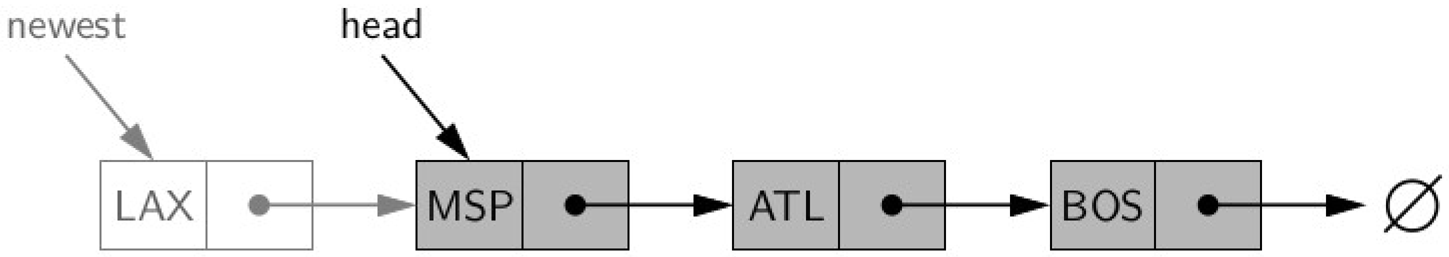
Singly Linked Lists



Inserting an Element at the Head of a Singly Linked List



(a)



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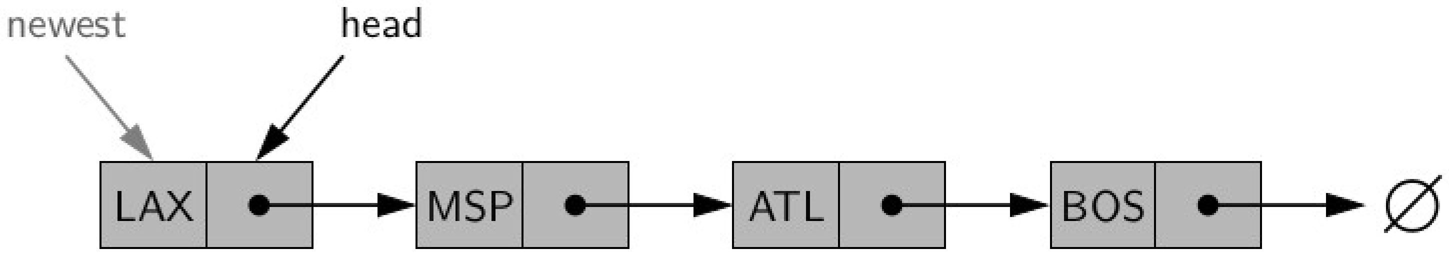
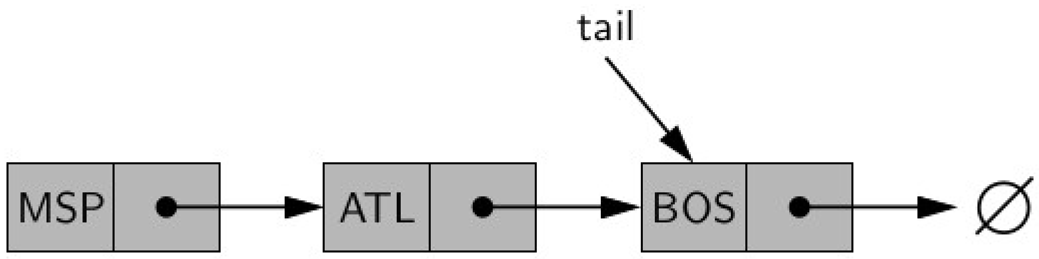
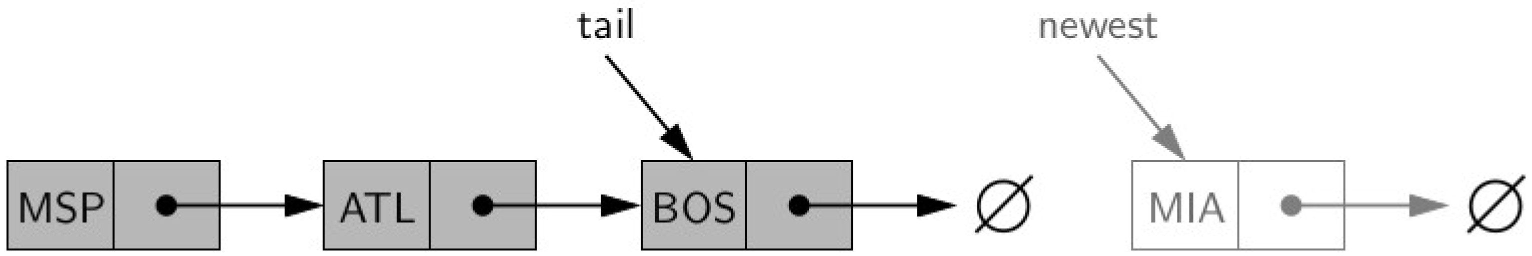


(c)

Inserting an Element at the Tail of a Singly Linked List



(a)



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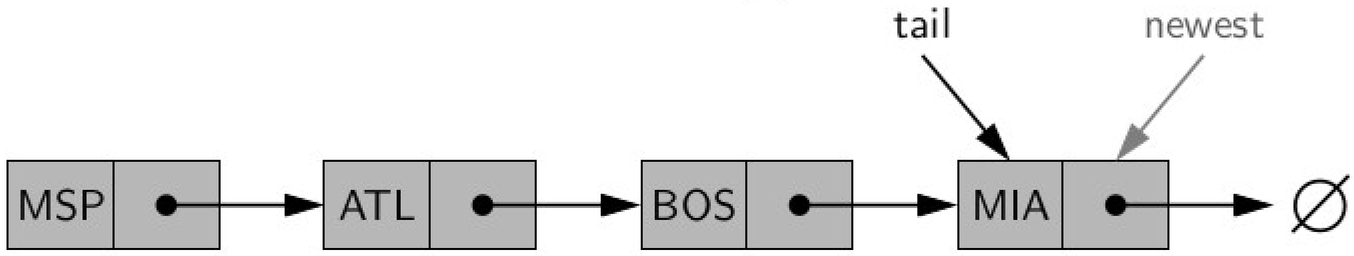
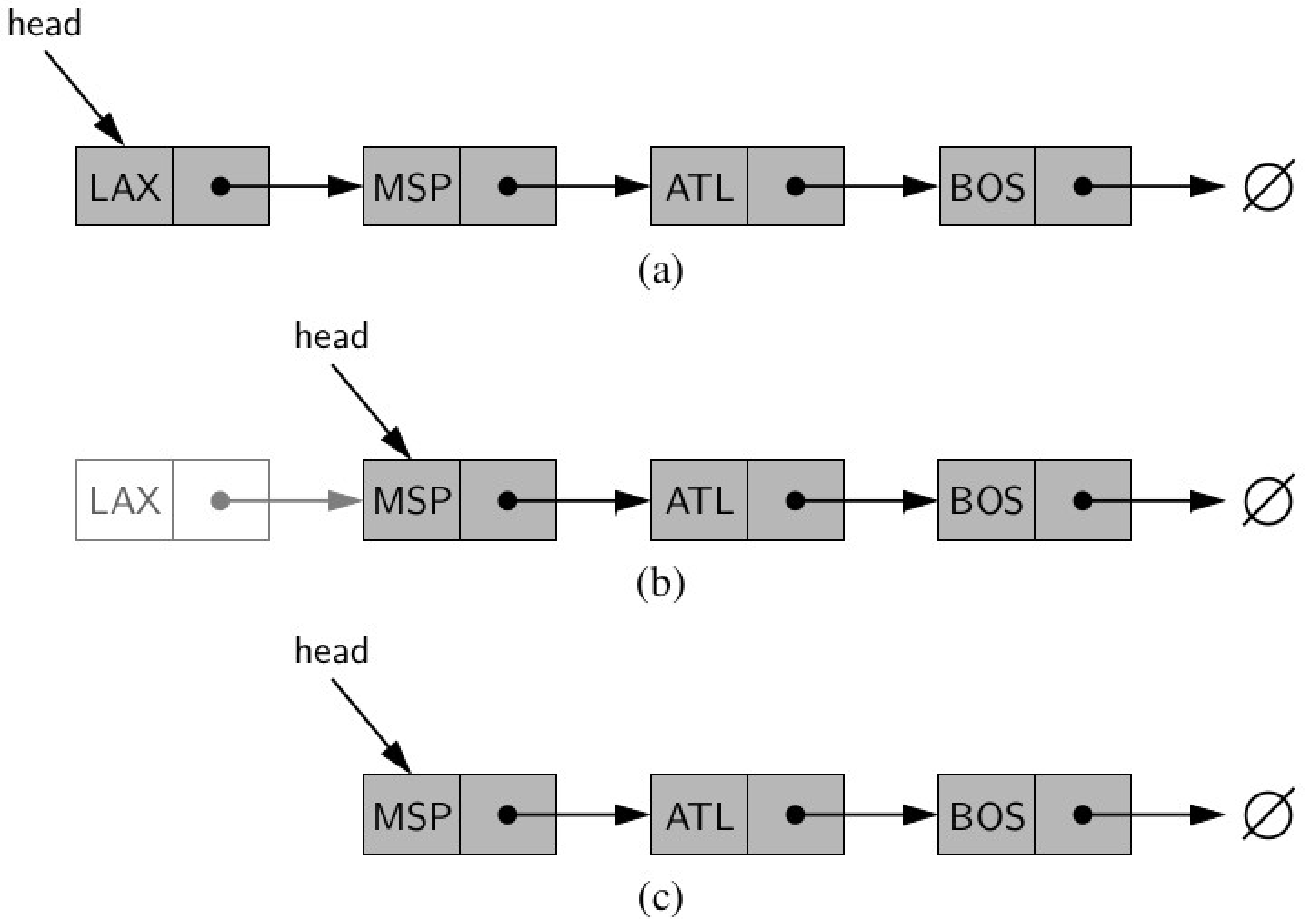


(c)

Removing an Element from a Singly Linked List



The Node class

```
class _Node:
  """Lightweight, nonpublic class for storing a singly linked node."""
  __slots__ = '_element', '_next'          # streamline memory usage

  def __init__(self, element, next):       # initialize node's fields
    self._element = element                # reference to user's element
    self._next = next                      # reference to next node
```

```
class LinkedStack:
  """LIFO Stack implementation using a singly linked list for storage."""

  #------------------------- nested _Node class -------------------------
  class _Node:
    """Lightweight, nonpublic class for storing a singly linked node."""
    __slots__ = '_element', '_next'         # streamline memory usage

    def __init__(self, element, next):      # initialize node's fields
      self._element = element               # reference to user's element
      self._next = next                     # reference to next node

  #------------------------------- stack methods -------------------------------
  def __init__(self):
    """Create an empty stack."""
    self._head = None                       # reference to the head node
    self._size = 0                          # number of stack elements

  def __len__(self):
    """Return the number of elements in the stack."""
    return self._size

  def is_empty(self):
    """Return True if the stack is empty."""
    return self._size == 0
```

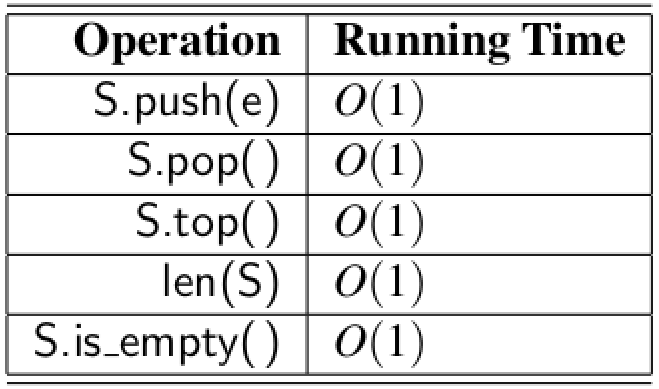
Stack Implementation using a Singly Linked List

```
def push(self, e):
  """Add element e to the top of the stack."""
  self._head = self._Node(e, self._head)      # create and link a new node
  self._size += 1

def top(self):
  """Return (but do not remove) the element at the top of the stack.

  Raise Empty exception if the stack is empty.
  """
  if self.is_empty():
    raise Empty('Stack is empty')
  return self._head._element                  # top of stack is at head of list
```

Stack Implementation using a Singly Linked List

```
  def pop(self):
    """Remove and return the element from the top of the stack (i.e., LIFO).

    Raise Empty exception if the stack is empty.
    """
    if self.is_empty():
      raise Empty('Stack is empty')
    answer = self._head._element
    self._head = self._head._next                # bypass the former top node
    self._size -= 1
    return answer
```

Performance of the LinkedStack implementation



Operation	Running Time
S.push(e)	$O(1)$
S.pop()	$O(1)$
S.top()	$O(1)$
len(S)	$O(1)$
S.is_empty()	$O(1)$

All bounds are worst-case and the space usage is O(n), where n is the current number of elements in the stack.

Queue Implementation using a Singly Linked List

```
class LinkedQueue:
  """FIFO queue implementation using a singly linked list for storage."""

  class _Node:
    """Lightweight, nonpublic class for storing a singly linked node."""
    (omitted here; identical to that of LinkedStack._Node)

  def __init__(self):
    """Create an empty queue."""
    self._head = None
    self._tail = None
    self._size = 0                          # number of queue elements

  def __len__(self):
    """Return the number of elements in the queue."""
    return self._size

  def is_empty(self):
    """Return True if the queue is empty."""
    return self._size == 0

```

Queue Implementation using a Singly Linked List

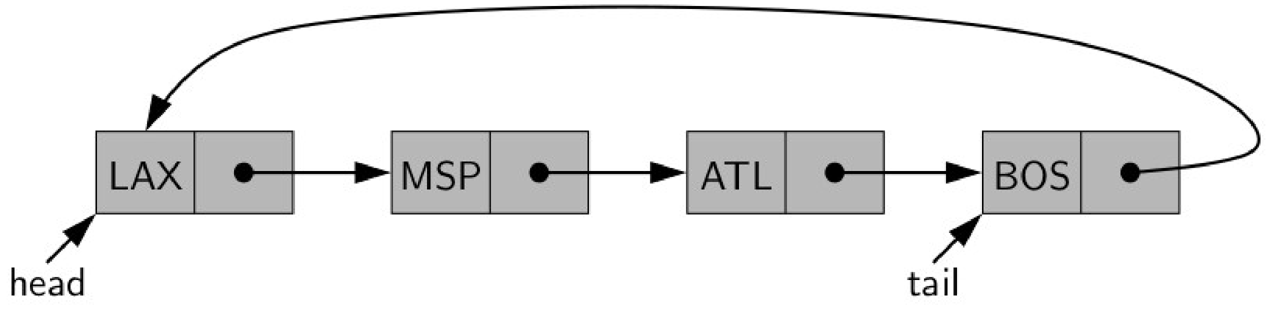
```

    def first(self):
        """Return (but do not remove) the element at the front of the queue."""
        if self.is_empty():
            raise Empty('Queue is empty')
        return self._head._element                  # front aligned with head of list

    def dequeue(self):
        """Remove and return the first element of the queue (i.e., FIFO).

        Raise Empty exception if the queue is empty.
        """
        if self.is_empty():
            raise Empty('Queue is empty')
        answer = self._head._element
        self._head = self._head._next
        self._size -= 1
        if self.is_empty():                         # special case as queue is empty
            self._tail = None                       # removed head had been the tail
        return answer
```

Queue Implementation using a Singly Linked List

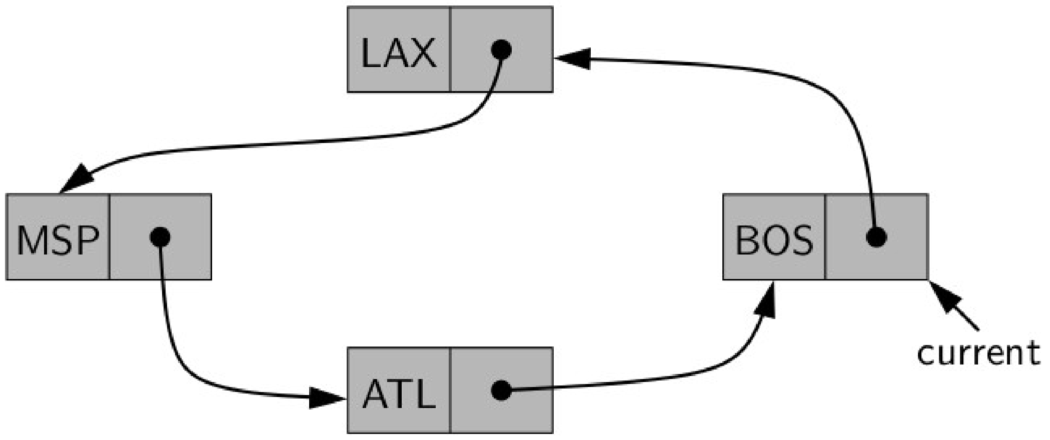
```
def enqueue(self, e):
  """Add an element to the back of queue."""
  newest = self._Node(e, None)          # node will be new tail node
  if self.is_empty():
    self._head = newest                 # special case: previously empty
  else:
    self._tail._next = newest
  self._tail = newest                   # update reference to tail node
  self._size += 1
```

Circularly Linked Lists

Example of a singly linked list with circular structure



Example of a circular linked list, with current denoting a reference to a select node.



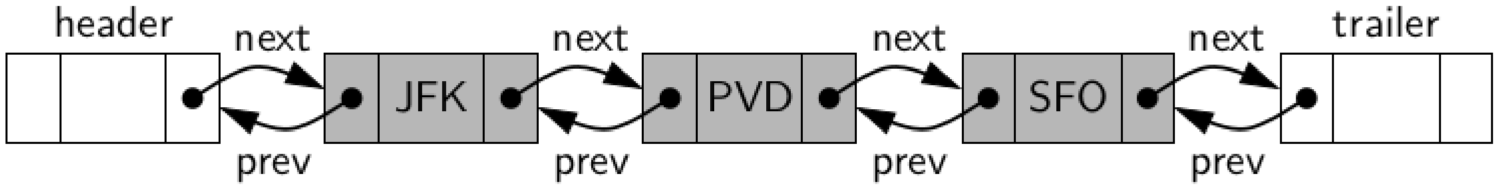
Circularly Linked Lists – key methods code walk-through

```
def first(self):
  """Return (but do not remove) the element at the front of the queue.

  Raise Empty exception if the queue is empty.
  """
  if self.is_empty():
    raise Empty('Queue is empty')
  head = self._tail._next
  return head._element

def dequeue(self):
  """Remove and return the first element of the queue (i.e., FIFO).

  Raise Empty exception if the queue is empty.
  """
  if self.is_empty():
    raise Empty('Queue is empty')
  oldhead = self._tail._next
  if self._size == 1:                      # removing only element
    self._tail = None                      # queue becomes empty
  else:
    self._tail._next = oldhead._next       # bypass the old head
  self._size -= 1
  return oldhead._element
```

Circularly Linked Lists – key methods code walk-through

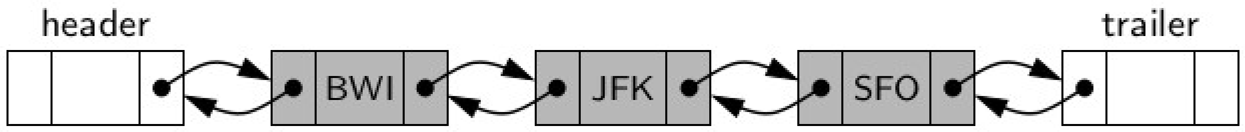
```
def enqueue(self, e):
  """Add an element to the back of queue."""
  newest = self._Node(e, None)              # node will be new tail node
  if self.is_empty():
    newest._next = newest                   # initialize circularly
  else:
    newest._next = self._tail._next         # new node points to head
    self._tail._next = newest               # old tail points to new node
  self._tail = newest                       # new node becomes the tail
  self._size += 1

def rotate(self):
  """Rotate front element to the back of the queue."""
  if self._size > 0:
    self._tail = self._tail._next           # old head becomes new tail
```

Doubly Linked Lists

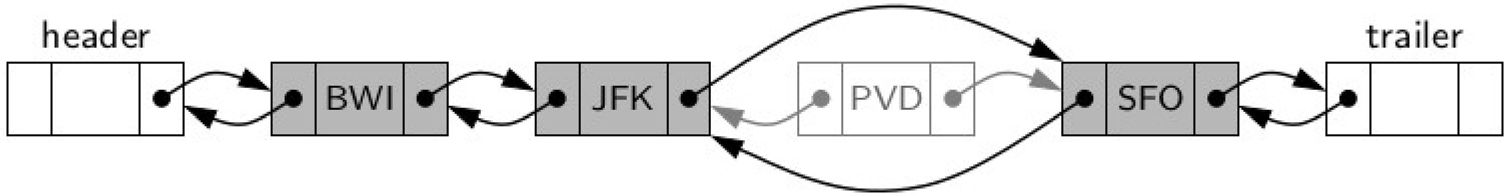


A doubly linked list representing the sequence { JFK, PVD, SFO }, using sentinels header and trailer to demarcate the ends of the list.

Adding an element to a doubly linked list (DLL) with header and trailer sentinels



(a)



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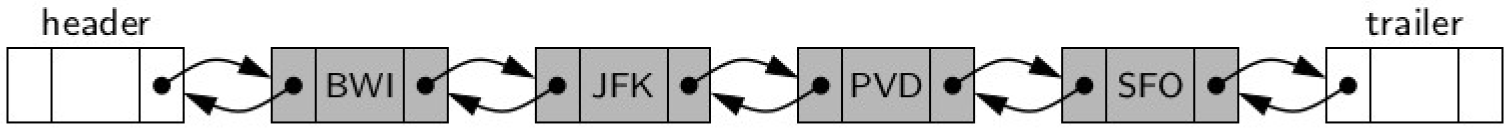
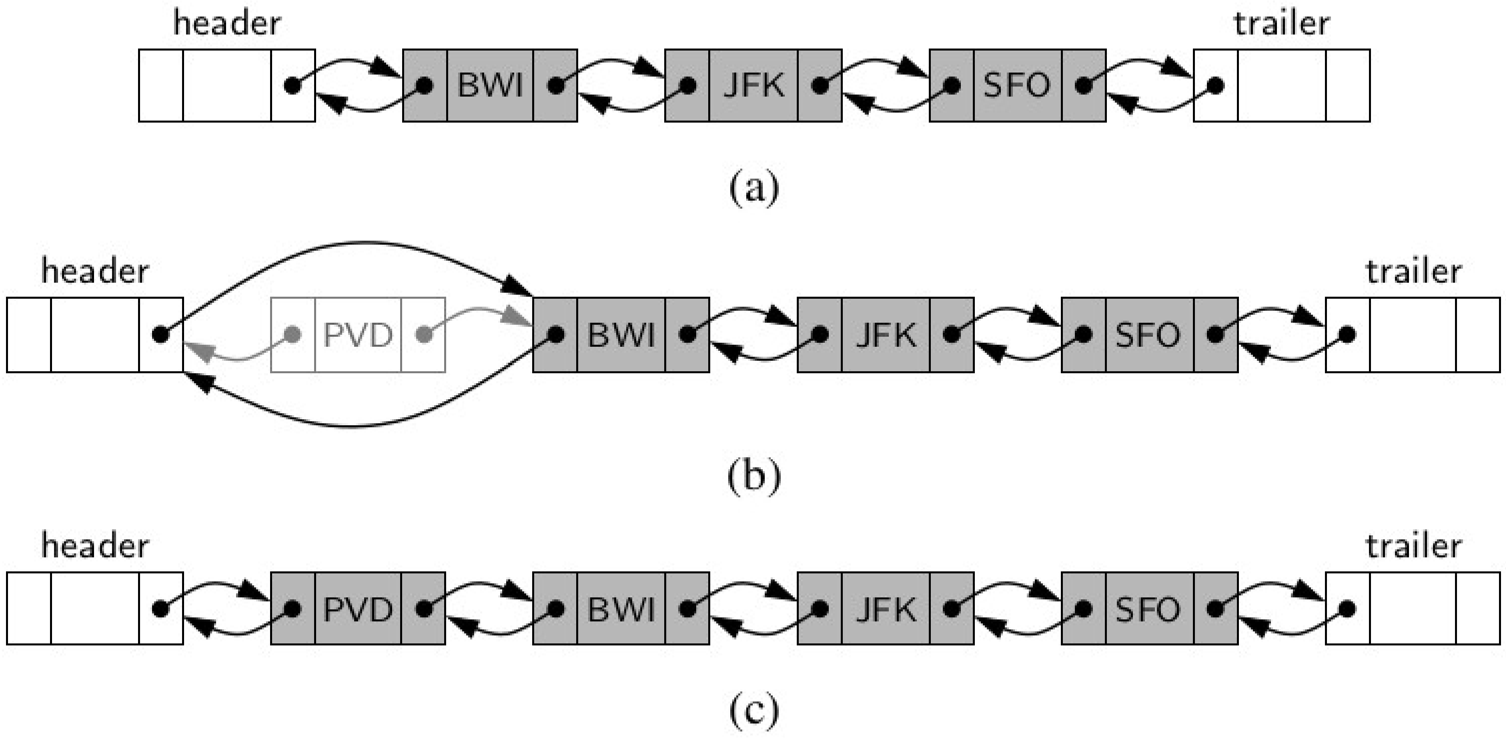
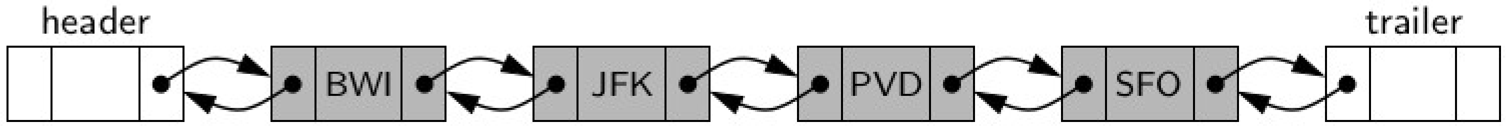


(c)

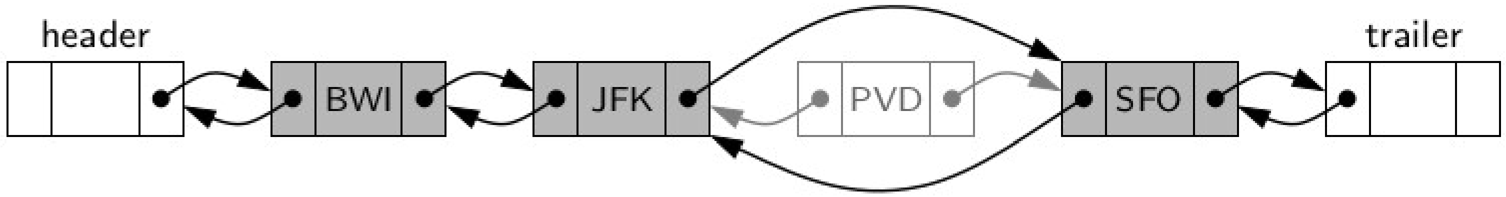
Adding an element to the front of a DLL



Deleting an element from a DLL



(a)



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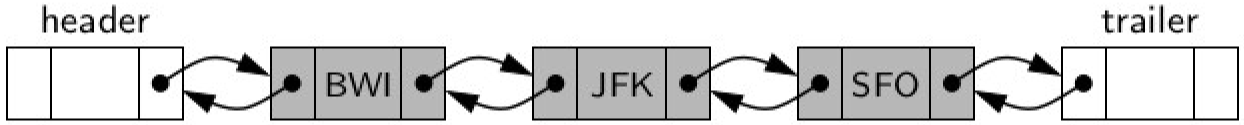

(c)

The Positional List Abstract Data Type – Basic Operations

L.first( ):	Return the position of the first element of L, or None if L is empty.
L.last( ):	Return the position of the last element of L, or None if L is empty.
L.before(p):	Return the position of L immediately before position p, or None if p is the first position.
L.after(p):	Return the position of L immediately after position p, or None if p is the last position.
L.is_empty( ):	Return True if list L does not contain any elements.
len(L):	Return the number of elements in the list.
iter(L):	Return a forward iterator for the *elements* of the list. See Section 1.8 for discussion of iterators in Python.

The Positional List Abstract Data Type – Basic Operations

The positional list ADT also includes the following ***update*** methods:

L.add_first(e):	Insert a new element e at the front of L, returning the position of the new element.
L.add_last(e):	Insert a new element e at the back of L, returning the position of the new element.
L.add_before(p, e):	Insert a new element e just before position p in L, returning the position of the new element.
L.add_after(p, e):	Insert a new element e just after position p in L, returning the position of the new element.
L.replace(p, e):	Replace the element at position p with element e, returning the element formerly at position p.
L.delete(p):	Remove and return the element at position p in L, invalidating the position.

The Positional List Abstract Data Type – Basic Operations

A position instance is a simple object, supporting only the following method:

p.element( ): Return the element stored at position p.

Exercise problems

- Give an algorithm for finding the second-to-last node in a singly linked list in which the last node's next member points to a NULL.
- Describe a good algorithm for concatenating two singly linked lists L and M, given only references to the first node of each list, into a single list L that contains all the nodes of L followed by all the nodes of M.

Exercise problems

Suppose that x and y are references to nodes of circularly linked lists, although not necessarily the same list. Describe a fast algorithm for telling if x and y belong to the same list.

Our CircularQueue class of Section 7.2.2 provides a rotate( ) method that has semantics equivalent to Q.enqueue(Q.dequeue( )), for a nonempty queue. Implement such a method for the LinkedQueue class of Section 7.1.2 without the creation of any new nodes.

Exercise problems

Describe a nonrecursive method for finding, by link hopping, the middle node of a doubly linked list with header and trailer sentinels. In the case of an even number of nodes, report the node slightly left of center as the "middle." (Note: This method must only use link hopping; it cannot use a counter.) What is the running time of this method?

Exercise problems

Given the set of element {a, b, c, d, e, f } stored in a list, show the final state of the list, assuming we use the move-to-front heuristic and access the elements according to the following sequence: (a, b, c, d, e, f , a, c, f , b, d, e).

References

Data Structures and Algorithms in Python
Michael T. Goodrich, Roberto Tamassia, Michael H. Goldwasser

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Algorithms, 4th Edition
Robert Sedgewick and Kevin Wayne

Few Images from the internet