Computer Vision and Machine Learning

(Segmentation: Region extraction)

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Outline

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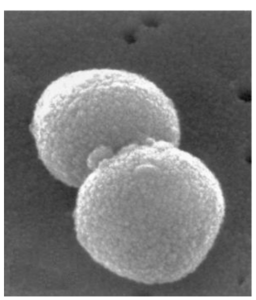
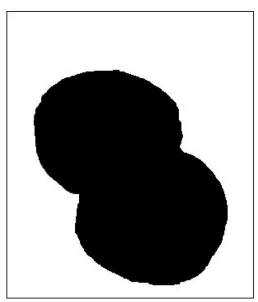
  - Definition
- Parallel approach
  - Graylevel thresholding
  - Threshold selection
  - Otsu's method
- Sequential approach
  - Region growing
  - Regin splitting, merging and region adjacency graph

Segmentation: Objective

Original graylevel image



Segmented image: Abstracted



Segmentation: Definition

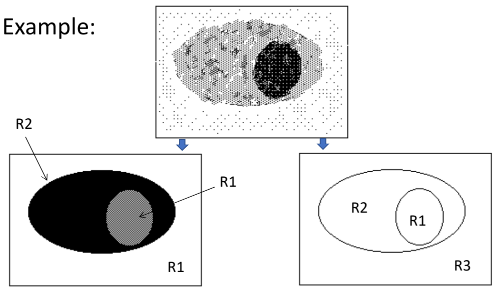
- Objective
  - Dividing image domain into meaningful regions.
- Definition
  - Segmentation technique divides image domain D into, say, n regions denoted by $R_1, R_2, \ldots, R_n$ such that

$$\bigcup_{i=1}^{n} R_i = D, \;\; R_i \cap R_j = \emptyset, \;\; Prop.(R_i) = True \;\; \text{for all } i, \text{ and}$$

$$Prop.\left(R_i \cup R_j\right) = False, \qquad \text{if } R_i \; and \; R_j \; are \; adjacent.$$

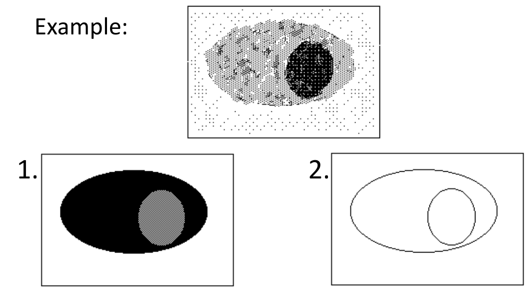
Segmentation

Example:



Segmentation

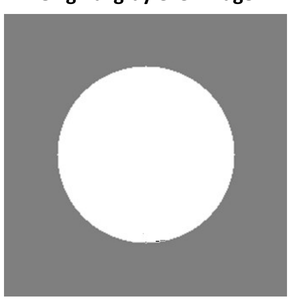
Example:



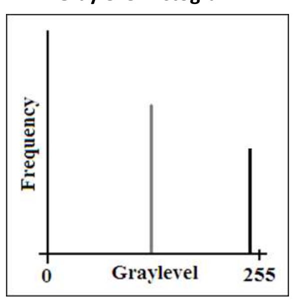
1. Region extraction (based on some measure of homogeneity).
2. Edge detection (based on abrupt change in some feature).

Segmentation: Example

Original graylevel image

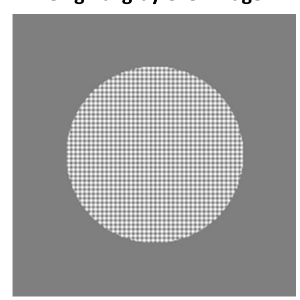


Graylevel histogram

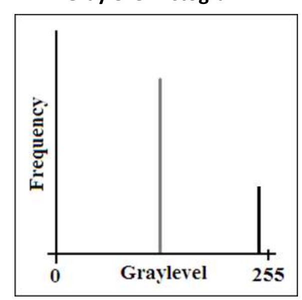


Segmentation: Example

Original graylevel image

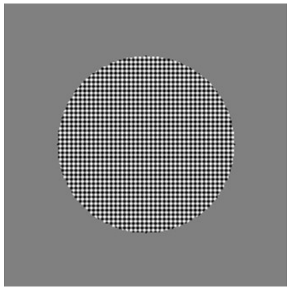


Graylevel histogram

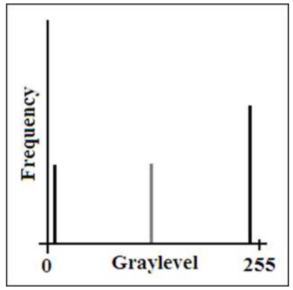


Segmentation: Example

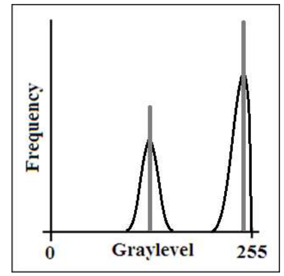
Original graylevel image

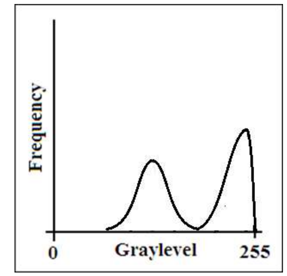


Graylevel histogram



Bimodal histogram: Exapmles



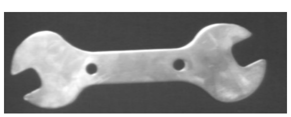


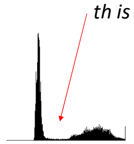
Region extraction

- Pixel classification approach based on the local (pixel) property.
- Graylevel (property) thresholding:

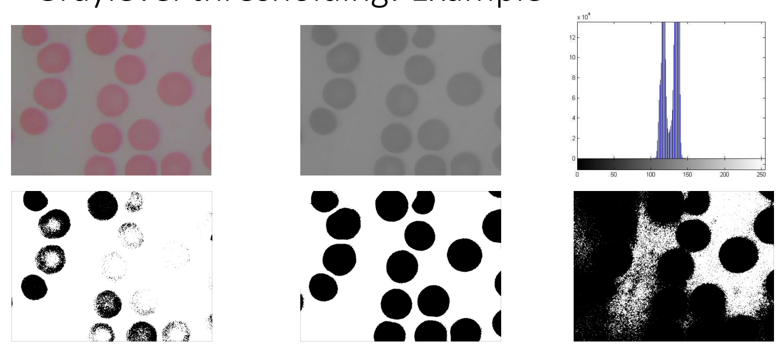
$$b(x,y) = \begin{cases} 1 & if\ g(x,y) > th \\ 0 & otherwise \end{cases}$$

th is the threshold value





Graylevel thresholding: Example



Th = 115   Th = 125   Th = 135

Optimum threshold selection

- Suppose $z = f(x, y)$ denotes the graylevel in the image.
- Let the graylevels in region (of type-1) R_1 follows Gaussian distribution with $mean = \mu_1$ and $variance = \sigma_1^2$, then

$$p_1(z) = \frac{1}{\sqrt{2\pi}\sigma_1} \exp(-\frac{(z-\mu_1)^2}{2\sigma_1^2})$$

- Similarly for region (of type-2) R_2,

$$p_2(z) = \frac{1}{\sqrt{2\pi}\sigma_2} \exp(-\frac{(z-\mu_2)^2}{2\sigma_2^2})$$

Optimum threshold selection (contd.)

- Let the probability of a pixel belonging to region (or type-1) R_1 is P_1, and that for region (or type-2) R_2 is P_2.
- Probability density function (p.d.f.) of graylevel in the image is

$$p(z) = P_1 p_1(z) + P_2 p_2(z)$$

- Assuming threshold t satisfies $\mu_1 \leq t \leq \mu_2$, then error in thresholding:

$$e(t) = \int_{-\infty}^{t} P_2 p_2(z) dz + \int_{t}^{\infty} P_1 p_1(z) dz$$

Optimum threshold selection (contd.)

- By minimizing the error $e(t)$ we get

$$t = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a = \sigma_1^2 - \sigma_2^2$, $b = 2(\mu_1\sigma_2^2 - \mu_2\sigma_1^2)$ and

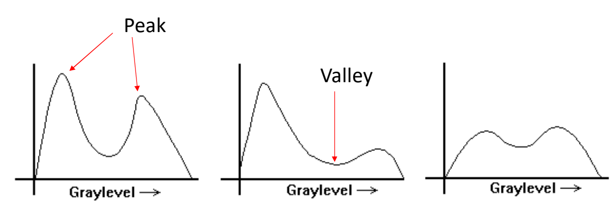
$$c = \mu_2\sigma_1^2 - \mu_1\sigma_2^2 + 2\sigma_1^2\sigma_2^2 \ln\frac{P_2\sigma_1}{P_1\sigma_2}$$

- If $\sigma_1 = \sigma_2 = \sigma$, we have

$$t = \frac{\mu_1 + \mu_2}{2} - \frac{\sigma^2}{\mu_2 - m_1} ln\frac{P_2}{P_1}$$

Threshold selection from histogram

- Single threshold (if no. of types of regions = 2)
- Multiple thresholds (if number of types of regions > 2)
- if number of types of regions = *n*, number of threshold = *n-1*.



Otsu's method for threshold selection

- Segmentation problem may be viewed as pixel classification problem.
  - Classification is done based on local properties (e.g., graylevel).
- Otsu's method maximizes between class variance.
- Let the image graylevel is denoted by $i = f(x,y)$ $(i = 0,1,\dots,L-1)$ and histogram is denoted by n_i.
- Total number of pixels $N = \sum_{i=0}^{L-1} n_i$
- Normalized histogram $p_i = \frac{n_i}{N}$ and $\sum_{i=0}^{L-1} p_i = 1$

Otsu's method for threshold selection

- Suppose, based on graylevel i, the threshold t classifies image pixels into two classes C_1 and C_2 such that
  - $i = f(x,y) < t$, then $(x,y) \in C_1$.
  - $i = f(x,y) \geq t$, then $(x,y) \in C_2$.
- Probability of pixels assigned to C_1: $P_1(t) = \sum_{i=0}^{t-1} p_i$
- Probability of pixels assigned to C_2: $P_2(t) = \sum_{i=t}^{L-1} p_i$
- Thus $P_1(t) + P_2(t) = 1$

Otsu's method for threshold selection

- Mean of pixels values assigned to C_1: $\mu_1(t) = \frac{1}{P_1(t)} \sum_{i=0}^{t-1} i\, p_i$
- Mean of pixels values assigned to C_2: $\mu_2(t) = \frac{1}{P_2(t)} \sum_{i=t}^{L-1} i\, p_i$
- Mean of all pixels values: $\mu = \sum_{i=0}^{L-1} i\, p_i$
  - So $P_1(t)\mu_1(t) + P_2(t)\mu_2(t) = \mu$

Otsu's method for threshold selection

- Variance of pixels assigned to C_1: $\sigma_1^2(t) = \frac{1}{P_1(t)} \sum_{i=0}^{t-1} (i - \mu_1(t))^2 p_i$
- Variance of pixels assigned to C_2: $\sigma_2^2(t) = \frac{1}{P_2(t)} \sum_{i=t}^{L-1} (i - \mu_2(t))^2 p_i$
- *Within class or intra class* variance may be given by

$$\sigma_w^2(t) = P_1(t)\sigma_1^2(t) + P_2(t)\sigma_2^2(t)$$

- Minimizing $\sigma_w^2(t)$ is equivalent to maximizing *between class or inter-class* variance $\sigma_b^2(t)$ given by

$$\sigma_b^2(t) = P_1(t)(\mu_1(t) - \mu)^2 + P_2(t)(\mu_2(t) - \mu)^2$$

Otsu's method for threshold selection

- We calculate $\sigma_b^2(t)$ for $t = 1, 2, \ldots, L-1$.
- The value of t, say, t^* for which the between class variance is maximum is taken to be desired threshold.
  - If there are multiple t^*, we take average of these t^* as desirable threshold.
- The *between class or inter-class* variance may be given efficiently by

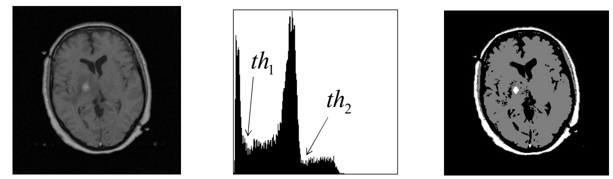
$$\sigma_b^2(t) = P_1(t)P_2(t)(\mu_1(t) - \mu_2(t))^2 = \frac{(\mu P_1(t) - \mu_1(t))^2}{P_1(t)[1 - P_1(t)]}$$

- Goodness of threshold selection is measured by $\eta(t) = \sigma_b^2(t)/\sigma^2$

Multiple threshold

- Number of types of regions present > 2.

$$b(x,y) = \begin{cases} 2 & \text{if } g(x,y) > th_2 \\ 1 & \text{if } th_2 \geq g(r,c) > th_1 \\ 0 & otherwise \end{cases}$$



Thank you !

Any question?