Task 8

→ Random variables   Video 1

Random Variables are really ways to map outcomes of random Processes to numbers.

Random Process like (Fliping a Coin or rolling dice or measuring rain that might Fall Tomorrow.

outcomes → numbers (quantifying)

random variable

$$X = \begin{cases} 1 & \text{if heads} \\ 0 & \text{if Tails} \end{cases}$$

ممكن اختار دول اي ارقام عادي براحتي

هنا عملت quantifing لـ Random process وحولتها لارقام عن طريق اني حددت لو

heads ← 1

Tails ← 0

Y = Sum of upward face after rolling 7 dice.

why we doing this (defining of random variable) ??

as soon as you quantify outcomes it become alittle bit more math on the outcomes -

((notes :- Random variables denoted by upper case))

— video 2

X = # of cars pass in an hour

Poisson Process : we need to Make some assumptions

[I] any other hour of aday is actually even within this hour

$E(X) = \lambda = n \cdot p$

# of cars pass in an hour

λ cars/hour = $\overbrace{60}^{n}$ min/hour $\underbrace{\frac{\lambda}{60} \text{ cars/minute}}_{p}$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{na} = \left(\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\right)^{a}$$

e

$$\to \lim_{k\to\infty}\left(1+\frac{a}{x}\right)^{k} = e^{a}$$

$$\to \frac{x!}{(x-k)!} = \frac{(x)}{1}\frac{(x-1)}{2}\frac{(x-2)}{3}\cdots\frac{(x-k+1)}{k\text{th}}$$

$$\frac{7!}{(7-2)!} = \cancel{7\times6\times5\times4\times3\times2}\ 7\times(7-2+) = 7\times6 = -$$

Ex:

video 3 lottery ticket two numbers and Letter

→ if two number matches and 1 letter drawn in order He wins 10,40 grand

→ if just his Letter match he win 100 $ (Small)

→ anything else he loss 5 $ Cost of ticket

he Chooses (04 R)

X = net Profit from Playing $\boxed{04R}$

$$E(X) = \underset{\frac{1}{2600}}{P(grand)}(10{,}405-5) + \underset{\frac{1}{26}-\frac{1}{2600}}{P(Small)}(100-5) + \underset{1-\frac{25}{26}}{P(neither)}(-5)$$

$$P(grand) = \frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{26} = \frac{1}{2600}$$

$$P(Small) = \frac{1}{26} - \frac{1}{2600} \to$$ علشان الكانيا من هاهو grand Prize

$$P(neither) = 1-P(Small)-P(grand) = 1-\left(\frac{1}{26}-\frac{1}{2600}\right)-\frac{1}{2600}$$

$$= 1-\frac{1}{26}+\frac{1}{2600}-\frac{1}{2600}$$

$$= \frac{25}{26}$$

$E(X) = 2.81$ $ net Profit

~~video 4.~~

Ex: expected value while fishing (10) trout and (10) Sunfish

Your friend Jermy.

Bet 1: if the next ~~three~~ fishes he catches are Sunfish you will pay him 100 \$ otherwise he will pay you 20 \$

Bet 2: if you cath at least 2 Sunfish of the next 3 catches ~~are~~ that you catch, he will pay you 50 \$ otherwise you will pay 25 \$

→ what is your expected value from bet 1?

X = what your profit from Bet 1

$$E(X) = P\left(\begin{smallmatrix}\text{Jermy}\\ \text{catches}\\ \text{3 Sun}\end{smallmatrix}\right)\cdot(-100) + \left(1 - P\left(\begin{smallmatrix}\text{Jermy}\\ \text{catches}\\ \text{3 Sun}\end{smallmatrix}\right)\right)(20)$$

$\frac{1}{8}$

$$P(\text{Jermy catches 3 Sun}) = \frac{10}{20} * \frac{10}{20} * \frac{10}{20} = \frac{1}{8}$$

$$E(X) = \frac{1}{8} * -100 + \left(1 - \frac{1}{8}\right)(20) = 5\ \$$$

→ what is your profit from bet 2?

Y = what you profit from Bet 2

$$E(Y) = P\left(\begin{smallmatrix}\text{you catch}\\ \text{at least}\\ \text{2 Sun}\end{smallmatrix}\right)(50) + \left(1 - P\left(\begin{smallmatrix}\text{you catch}\\ \text{at least}\\ \text{2 Sun}\end{smallmatrix}\right)\right)(-25)$$

$\frac{1}{2}$ $\quad$ $\frac{1}{2}$

$$= \boxed{12.5\ \$}$$

→ SSS ✓
→ SST ✓
→ STS ✓
→ Stt
→ tSS ✓
→ tSt
→ ttS
→ ttt

$$P\left(\begin{smallmatrix}\text{You catch}\\ \text{at least 2 Sun}\end{smallmatrix}\right) = \frac{4}{8} = \frac{1}{2}$$

الملاحظات

③ your Friend says he is willing to take bothe bets combined total of 50 times if you wand to maximize your exspeded value. what shoult you do.

- O Take bet 1 all 50 times
- ✓ Take bet 2 all 50 times ← choose this
- O Take bet 1 twenty but bet 2 thirty
- O Take neighter bet

Video5 : Comparing insurance with the expeded value

Choose between two deductable.

[1] low deductible plan, he will have to pay the First 1000 $ of any medical costs. it costs 8000 $ a year.

[2] high deductible Plan. will have to pay First 2500 $ of any medical costs. it costs 7500 $ a year

medical cost	Probability
$0	30%
$1000	25%
$4000	20%
$7000	20%
15000	5%

X = Cost of low deductible cost

هنا انا بجيب التكلفة عليه هو مش على الشركة السنوى

$E(X) = \underline{8000} + .3*0 + .25 \times 1000$ اقصى 1000

$+ 0.2 * 1000 \quad 0.2 * 1000 + .05 * 1000$

$= 8700 \$$

Y: Cost of high deductible cost

$E(Y) = 7500 + 0.3 \times 0 + 0.25 \times 1000 + .2 * 2500 + 0.05 * 2500$

$= 8870 \$$

الأقصى

~~video 6:~~

Probability of making 2 shots in 6 attempts:

Prob(score) = 70% or 0.7
Prob(missing) = 30% or 0.3
P(Exactly two scors in 6 attempts) $= \binom{6}{2}(0.7)^2(0.3)^4$
$= .059535$ #

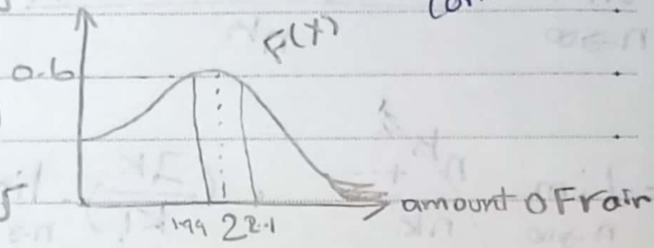
$_6C_2 = \binom{6}{2} = \frac{6!}{2!\,(6-2)!} = \frac{6 \cdot 5}{2} = 15$

15 different ways to make choose 2 things out of 6

video 7:

discrete Random variables ← Finite number
continuous Random variables ← infinite number

Y = exact amount of rain tomorrow



$P(Y=2) = 0$
Exactly 2
not 1.9999
not 2.0001

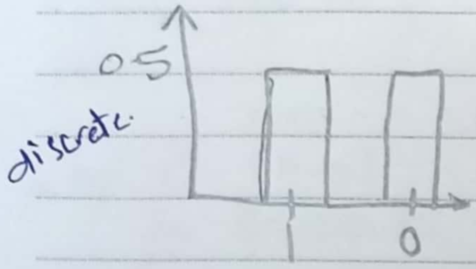
لأن كله هيكون مساحه تحت النقطه
لكن هسأل مساحه خط ملهوش width.

$P(|Y-2| < 0.1)$
$P(1.9 < Y < 2.1)$ ← area under curve.

$= \int_{1.9}^{2.1} F(x)\,dx$

كل ما أكبر الـ interval كل ما الـ Probability تزيد

note $\int_{0}^{\infty} F(x)\,dx = 1$ احتمالية curve كامل من الارض
↳ sum of all probabilitys.



X → 1 Heads
X → 0 Tails

مجموعهم بواحد بردو

X : number of cars pass in an hour

Success in hour

$E(X) = \lambda = n \cdot P$

مره معينة

# of Trials

مره اكل من دي

Probability of Success of Trial

نفس سرعة دي

$P(X=K) = \lim_{n \to \infty} \binom{n}{K} \cdot \left(\frac{\lambda}{n}\right)^K \left(1 - \frac{\lambda}{n}\right)^{n-K}$

Success

Failure

~~3 cars in Particular hour~~

$= \lim_{n \to \infty} \frac{n!}{(n-K)!\,K!} \cdot \left(\frac{\lambda^K}{n^K}\right) \left(1 - \frac{\lambda}{n}\right)^n \left(1 - \frac{\lambda}{n}\right)^{-K}$

1 2 3 K^{th}

$= \lim_{n \to \infty} \frac{n \cdot (n-1) \cdot (n-2) \cdots (n-K+1)}{n^K} \cdot \frac{\lambda^K}{K!} \left(1 - \frac{\lambda}{n}\right)^n \left(1 - \frac{\lambda}{n}\right)^{-K}$

$= \lim_{n \to \infty} \underbrace{\frac{\cancel{n^K} + \cdots}{n^K}}_{1} \cdot \left(\frac{\lambda^K}{K!}\right) \cdot \underbrace{\lim_{n \to \infty} \left(1 - \frac{\lambda}{n}\right)^n}_{e^{\lambda}} \lim_{n \to \infty} \left(1 - \frac{\lambda}{n}\right)^{-K}$

$1 \cdot \frac{\lambda^K}{K!} \cdot \cancel{e^{n}}^{K} e^{\lambda} \cdot 1$

$= \frac{\lambda^K}{K!} \cdot e^{\lambda}$

X = 9 cars pass

$P(X=2) = \frac{9^2}{2!} \cdot e^{-9} = -$

Video 9: visuallizing a bionomial distribution

↳ Possability of Possible out comes of 5 fibls = $2^5 = 32$

Term Life insurance and death Probability

$ 1 million Policy

500/ year Perminmur

term - 20 Years

P (Sal's death in 20 years) $\leq \frac{1}{100}$

$50 \times 20 = \frac{10,000}{1,000,000} = \frac{1}{100}$

Break even if only one Sal dies.

video 11

expected value of binomial distribution

X = # of successes with Probability p after n Trials

$E(X) = n \cdot P$

X = # of baskets I make after 10 shots 40% Percent

$E(X) = \underset{n}{10} \cdot (\underset{p}{.4}) = 4$

$P(X=k) = \binom{n}{k} \cdot p^k (1-P)^{n-k}$

$E(X) = \sum_{k=0}^{n} k \cdot \binom{n}{k} p^k (1-P)^{n-k}$

$= 1\binom{n}{1} P^1 (1-P)^{n-1} + 2\binom{n}{2} P^2 (1-P)^{n-2} + \cdots + n\binom{n}{n} P^n (1-P)^{n-n}$

$E(X) = \sum_{k=1}^{n} k\binom{n}{k} P^k (1-P)^{n-k} = \sum_{k=1}^{n} k \cdot \frac{n!}{k!\,(n-k)!} P^k (1-P)^{n-k}$

$= np \sum_{k=1}^{n} \frac{(n-1)!}{(k-1)!(n-k)!} \cdot P^{k-1} (1-P)^{n-k}$

الأضحى

$a = k-1 \qquad b = n-1$

$$= np \boxed{\sum_{a=0}^{b} \frac{b!}{a!\,(b-a)!} \cdot p^{a} (1-p)^{(b-a)}}$$

$$= np \boxed{\sum_{a=0}^{b} \binom{b}{a} p^{a} (1-p)^{b-a}}$$

Summing of all Probabilites that a random value Can have it actully equal 1

$\therefore E(X) = np$ only True For bionomial distribution.

Video 12

Law of Large numbers.

X, $E(X)$

$\bar{X}_n$ mean of n observations

$$\bar{X}_n = \frac{X_1 + X_2 + X_3 + \dots + X_n}{n}$$

$\bar{X}_n \to E(X)$

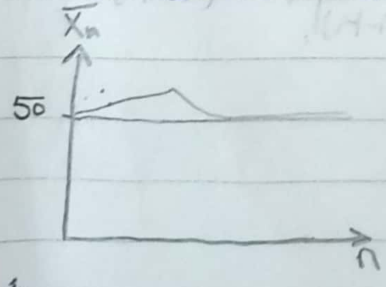
$\bar{X}_n \to \mu$ for $n \to \infty$

→ X = # of heads after 100 tosses of a fair Coin.

$E(X) = 100 * 0.5 = 50$

$$\bar{X}_n = \frac{55 + 65 + 45 + \dots + n}{n}$$

$\bar{X}_n \to 50$ as $n \to \infty$



Mean E(X) في النهاية هيساوي بعد infinite number of Trials

video 13

~~Binomial distribution:~~

X = # of heads From Flipping Coin 5 times.

Possible out Comes From 5 flips $= 2\cdot2\cdot2\cdot2\cdot2 = 2^5 = 32$

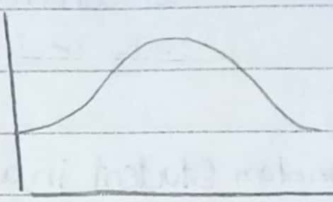
$P(X=0) = \frac{1}{32} = \frac{{}_5C_0}{32} =$

no heads

$P(X=1) = \frac{{}_5C_1}{32} = \frac{5}{32}$ $\quad$ $P(X=4) = \frac{{}_5C_4}{32} = \frac{5}{32}$

$P(X=2) = \frac{{}_5C_2}{32} = \frac{10}{32}$ $\quad$ $P(X=5) = \frac{{}_5C_5}{32} = \frac{1}{32}$

$P(X=3) = \frac{{}_5C_3}{32} = \frac{10}{32}$

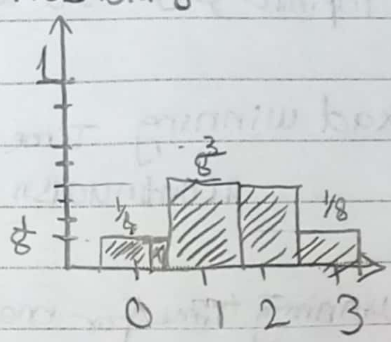


بعد تمثيلها ←
Graphically
خلتها و
عند ي هل كده

video 14

X = # of heads after 3 flips of a fair coin.

HHH
HHT $\quad P(X=0) = \frac{1}{8}$
HTH $\quad P(X=1) = \frac{{}_3C_1}{8} = \frac{3}{8}$
HTT
THH $\quad P(X=2) = \frac{{}_3C_2}{8} = \frac{3}{8}$
THT
TTH $\quad P(X=3) = \frac{{}_3C_3}{8} = \frac{1}{8}$
TTT



discrete Probability distribution.

video15.

Random variables → Discrete ~~(distinct or separate values)~~
→ Continuous (any value in an interval)

$X = \begin{cases} \to 1 & \text{Heads} \\ \to 0 & \text{Tails} \end{cases}$ «discrete»

Y = exact mass of a random animal selected at New Orleans Zoo
«Continuous»

0 ———————————— 500 kg

Y = Year a random student in a class was born. «discrete»

Z = # of ants born tomorrow in the universe. «~~continuous~~»
«discrete»
ممكن يكونوا مش لو infinite

X = exact winning Time for men's 100 m to 2016 olympics
«Continuous»

X = winning Time for men's 100 m to 2016 olympics rounded to The nearest hundreds. «discrete RV»

→ PMF : Probability mass Function: discrete random values
→ Pdf : Probability density function : continuous random values
→ Cdf : Cumulative distribution Function
Mathematical Concept that describes the likelihood of a random variable taking on values less than or equal to a specific value.

video 16

Prob(score) = 70%

Prob(missing) = 30%

P(Exactly 2 Scores in 6 attemps) $= \binom{6}{2}(0.7)^2(0.3)^4$

P(Exactly k Scores in n attempss $= \binom{n}{k}(f)^k(1-f)^{n-k}$

X = # of made Freethraws when Taking 6 ft (assuming 70% ft)

6 Choose 0 = $_6C_0$

$P(X=0) = \binom{6}{0}(0.7)^0(.3)^6 = .000729 \simeq .001 = 0.1\%$

$P(X=1) = \binom{6}{1}(0.7)^1(0.3)^5 = .010206 = .01 \simeq 0.1\%$

$P(X=2) = \binom{6}{2}(0.7)^2(0.3)^4 = .059535 \simeq .06 \simeq 6\%$

$P(X=3) = \binom{6}{3}(0.7)^3(0.3)^3 = 0.18522 \simeq .19 \simeq 19\%$

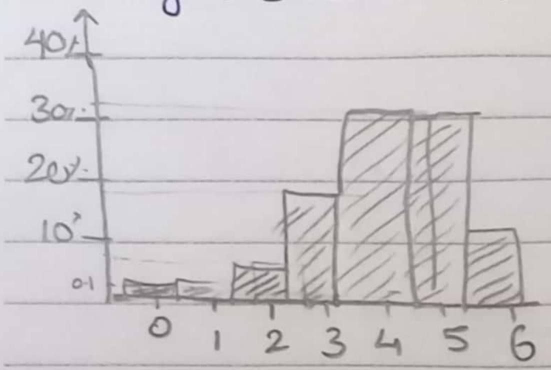
$P(X=4) = \binom{6}{4}(0.7)^4(0.3)^2 = 0.32 \simeq 32\%$

$P(X=5) = \binom{6}{5}(0.7)^5(0.3) = 0.303 \simeq 30\%$

$P(X=6) = \binom{6}{6}(0.7)^6(0.3)^0 = 0.117649 \simeq .118 = 11.8\%$

video ~~18~~ ~~P(Exactly~~ video 19:

Graphing basketball bionomal distribution.



Video 18

getting data from expected value.

→ james dad gives him a die for her birthday. She wan To make sure it is fair so she take her ~~fair~~ die to school and rolled it (500) time and kept track. Afterwards She calculated the expected value of sum of 20 rolls to be 67.4 . on her way home 2 values were deleted

die value	Absolute freq
1	75 $\boxed{A}$?
2	110
3	95
4	70
5	75
6	75 $\boxed{B}$?
total	500

Expected value of a roll = $\frac{67.4}{20}$ = 3

$\frac{A}{500}(1) + \frac{110}{500} * (2) + \frac{95}{500}(3) + \frac{70}{500}(4)$

$+ \frac{75}{500}(5) + (\frac{B}{500}(6) = 3.37$

$A + 110 * 2 + 95 \times 3 + 70 \times 4 + 75 \times 5 + B * 6 =$

$A + 220 + 285 + 280 + 375 + 6B = 1685$

$A + 6B = 525 \rightarrow$ 1

$A + 110 + 95 + 70 + 75 + B = 500$

$A + B = 150 \rightarrow$ 2

$A = 75 \qquad B = 75$