Probability

① distribution: The Possible Values a variable can Take and how Frequently they occur

Y: actual output of an event
y: one of the possible outcomes

$P(Y=y)$ $P(y)$ Probability function

Example: Y → The number of red Marbels we draw out of a bag.
y → 5
$P(Y=5)$ $P(5)$

We define distribution using only two characteristics:
mean: average value (μ)
variance: how spread out the data is (σ^2)

We need when we analysing distribution. it is important to understand what kind of data we deal with.

Population — Sample data.
(all data) (Just a part of it)

when we use Sample data.
Sample mean: $\bar{x}$ لو مثلا sec
Sample variance: S^2 sec^2
Sample standard deviation: S sec

$\sigma^2 = E((Y-\mu)^2) = E(Y^2) - \mu^2$

/ /

Types of Probability Distributions

[1] Discrete distribution

Like drawing cards from a deck or flipping a coin.

→ All outcomes are equally Likely → Equi probable.
uniform distribution.

A- Bernoulli

- events with only two Possible outcomes.
  « They Follow Bernoulli distribution »

we simple assign one of them to be true and other one to be False.

B- Binomial distribution:-   زي Bernoulli بس

↳ Two outcomes by iteration.
↳ Many iterations

C- Poisson distribution يختبر احتمالية حدث معين غير معتاد في وقت محدد

Test out how unusual an event frequency is for a given interval

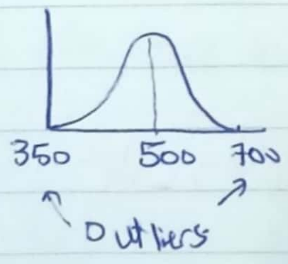
[2] Continuous distribution.

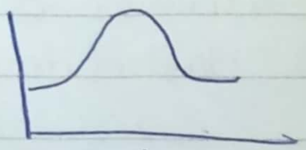
Probability distribution will be a curve

A- Normal distribution often observed in nature.

مثلا وزن الدب القطبي
متوسط 500

Normal.





Student's- T

الأقصى

ch. squared.

→ Asymmetric

→ only consists of non negative values

→ doesn't often mirror real Life events
used in hypothesis Testing
Goodness of fit

Exponential distribution: "online news articles"
events that are rapidly changed early on.

Logistic distribution:
useful in Forecast analysis
useful for determining a cutt off point for a successful outcome.

Video 3- discrete uniform distribution.

U(a,b)

a, b: range of the values in data set

X ~ U(3,7) variable x following discrete uniform distribution in range 3 to 7

→ All out comes has equal Probability. (الزهر)
P(1) = P(2) = P(3) = P(4) = P(5) = P(6)

→ Both Mean and variance are uninterpretable
→ No predictive power

~~Video 4~~

~~Bernoulli discrete distribution~~

Binomial distribution.

Notation $B(n, p)$

n: number of trials

P: probability in success in each one.

$X \sim B(10, .6)$

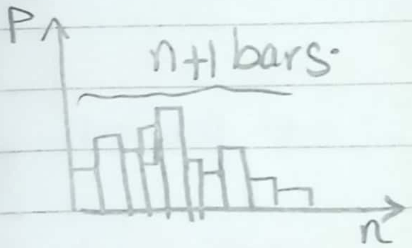
Variable x Followed a Bionomial distribution with 10 Trials and Likelihood of Success 0.6 of each individual trial

Bernoulli

$Bern(p) = B(1, p)$

مثلاً عندي امتحان مكون من عشر اسئلة كلهم T/F

كل سؤال بالنسبة ليا Bernoulli لكن الامتحان كله بوصفه Binomial

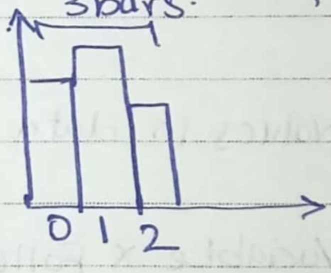


Coin → to check for tail.

Flipped two Times.

ممكن يظهر ٠، ١، ٢

3 bars.



P(desired out come) = P

P(alternative outcome) = 1 − P

→ The number of ways in which 4 out of the 6 trials can be successful. = Picking 4 element out of a sample of space of 6

C^6_4 C^n_r

$C_2^3 = 3$ الاتي ٢ TAIL من ٣ قلبات للعملة

$P(y) = \binom{n}{y} \cdot p^y \cdot (1-p)^{n-y}$

Example

A Single stock of General Motors

$P(\uparrow) = 60\% = 0.6$

$P(\downarrow) = 40\% = 0.4$

· 3 increase in 5 days

$y \to 3$

$n \to 5$

$p \to 0.6$

$P(y) = \binom{n}{y} \cdot p^y \cdot (1-p)^{n-y}$

$= C_3^5 * (.6)^3 (1-.6)^{5-3} = 10 * .216 * .4^2$

$= 34.56\%$

Expected values

~~$E(x) = x_0 P(x_0) + x_1 P + x$~~

$E(x) = x_0 \cdot P(x_0) + x_1 (Px_1) + x_2 P(x_2) + \cdots + x_n (Px_n)$

$Y \sim B(n, p)$

$E(y) = n \cdot p$

$\sigma^2 = E(y^2) - E(y)^2$

$= n \cdot p \cdot (1-p) = 5 \cdot 0.6 \cdot (.4) = 1.2$

$\sigma = 1.1$

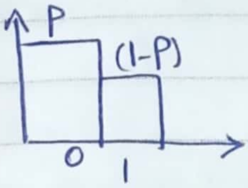
~~Video 5: Bernoulli distribution~~

Bern (P)

↳ 1 Trial

↳ 2 Possible outcomes.

P is known OR Past data indicating some experimental Probability.



We need To Assign which outcomes is 0, and which is 1

~~EXAMP~~

Conventionally. $P > 1-P$

Assign $P \leftarrow 1$

$1-P \leftarrow 0$

$\sigma^2 = P(1-P)$

→ unfair coin. 60% Tails → $P \leftarrow 1$

40% heads → $(1-P) \leftarrow 0$

$E(X) = 0.6$

$\sigma = 0.6(.4) = 0.24$

Video 6: Poisson distribution

$Po(\lambda)$

deals with the frequency with which an event occurs in a specific interval.

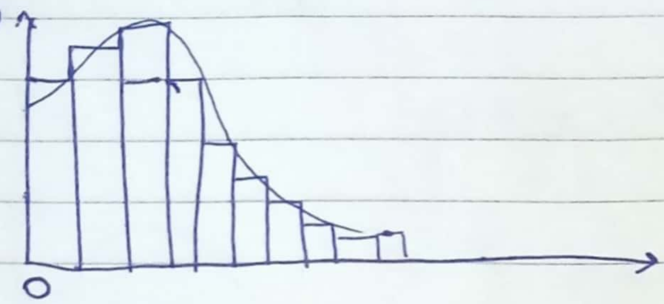
→ Known how often it occurs for a specific Period of time or distance.

FireFly exampe.

3 time in 10 sec   8 times in [illegible] sec.

$y \; Po(3)$

$P(\lambda)$



Q and A Exomple. $P(y=7) =$

Questions Per day

usually → 4 $\lambda = 4$

Yesterday → 7   interval → one day

$y = 7$

$$P(Y) = \frac{\lambda^{y} * e^{-\lambda}}{y!} = 0.06$$

$$E(y) = y_0 \cdot P(y_0) + y_1 P(y_1) + \dots = y_0 \frac{\lambda^{y_0} e^{-\lambda}}{y_0!} + y_1 \frac{\lambda^{y_1} e^{-\lambda}}{y_1!} + \dots$$

$\mu = \sigma^2 = \lambda$

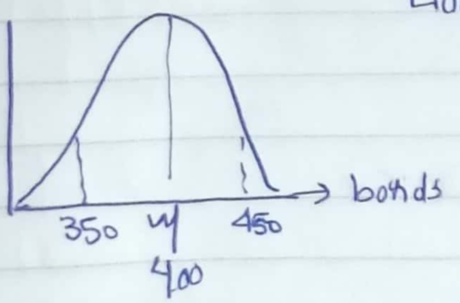
Video 7:

Normal distribution

$N(\mu, \sigma^2)$

distinct characteristics:

Lion (~~150~~ ~~350~~)

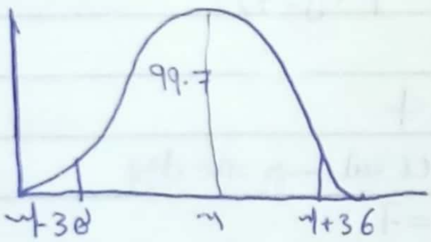


$E(X) = \mu$

$var(X) = \sigma^2$

$var(X) = E(X^2) - E(X)^2$

68, 95, 99.7 law



68% → with 1 Standerd $\mu \pm \sigma$

95% → with 2 Standerd $\mu \pm 2\sigma$

99.7 → with 3 Standerd $\mu \pm 3\sigma$

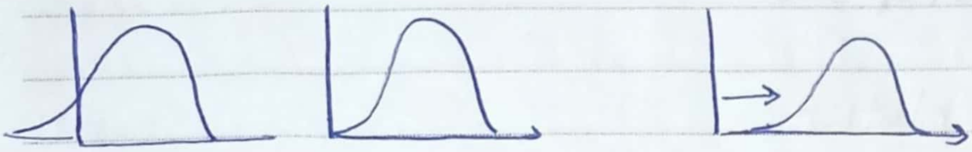
Video 8: Standard normal distribution.

Transformation: A way in which we can alter every element of a distribution to get a new distribution.

$X \sim N(\mu_1, \sigma_1^2)$

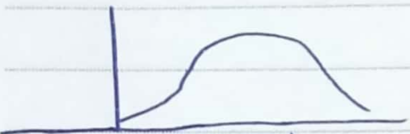
then $X+3 \sim N(\mu_2, \sigma_2^2)$

$y = f(x)$ $\quad$ $y = f(x+3)$

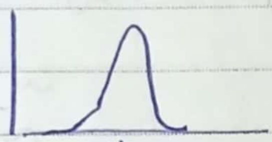


$F(X-3)$ $\quad$ $F(X)$ $\quad$ $F(X+3)$

$f(\frac{x}{c})$ $\quad\quad$ $y = f(x-c)$



expand.



Shrink

→ Standardizing.

A Special Kind of Transformation.

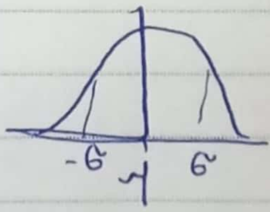
$E(X) = 0 \quad Var(X) = 1$

Standard normal distribution -

68, 95, 99.7 % Ruls → CDF table. (Z score table)

How to do Standardizing:

① نزحزح الـ graph لحد $\mu = 0$

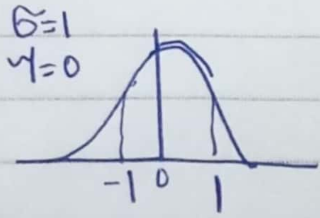


$y = f(x - \mu)$

$\sigma = 1$

$\sigma = 1$

$\mu = 0$



$y = f\left(\frac{x-\mu}{\sigma}\right)$

$Z \sim N(0, 1)$
$Y \sim N(\mu, \sigma^2)$

To get $Z = \frac{Y - \mu}{\sigma}$ → Standard normal distribution.

$y = \mu + 2.3\sigma$ $\quad Z = \frac{\mu + 2.3\sigma - \mu}{\sigma} = 2.3$

→ it requires alot of data.

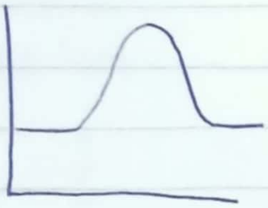
video 9.

→ Student T distribution.

$t(K)$

K: degrees of freedom.

it is bell shaped and symmetric



μ
σ
k

if $K > 2$
$E(Y) = \mu$
$var(Y) = \frac{S^2 \cdot k}{K - 2}$

Video 10: Chi distribution

$\chi^2(k) =$

k: degrees of freedom.

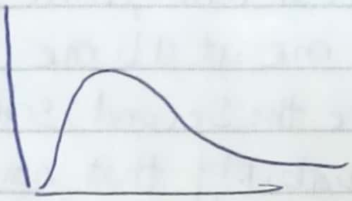
↳ Statistical analysis

↳ Hypothesis testing.

↳ Computing confidence intervals

Goodness of fit

~~no~~ is Asymmetric



$Y \sim t(k)$

$Y^2 \sim \chi^2(k)$

$E(X) = K$

$Var(X) = 2k$

② / /

~~Conditional Propability~~

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$S = \{1, 2, 3, 4, 5, 6\}$ $\quad A = \{1, 3, 5\}$ $\quad B = \{1, 2\}$

$A \cap B = \{1\}$

$$P(B|A) = \frac{1}{3} = \frac{P(A \cap B)}{P(A)} = \frac{1/6}{3/6} = \frac{1}{3}$$

Product rule:

$$P(E \cap F) = P(F) \cdot P(E|F)$$

علاقة مستنتجة من Conditional propability

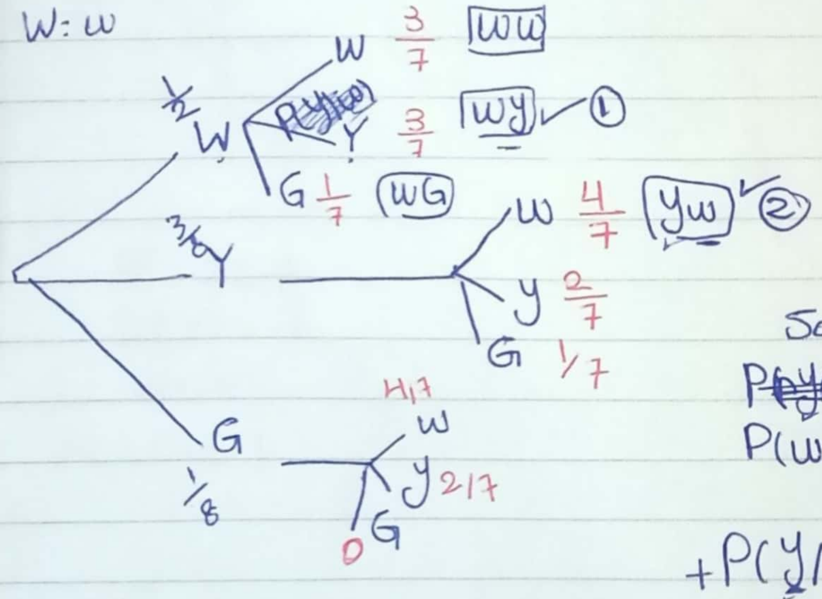
Example: box contain four white, three yellow, one green ball two balls are drawn one at a time with out replacing the first ball before the second is drawn. Used tree diagram to find probability that one white one yellow.

balls = 8 balls. one white, one yellow

مهم سيتم تبديلها

$wy - yw$

W: w



~~P(W+Y)~~

Sol

~~P(Y∩W)~~ = ~~P(W+Y) P(Y)~~

$P(W \cap Y) = P(W_{2nd} | Y_{1st}) \cdot P(Y_{1st}) + P(Y|W) \cdot P(W)$

$$= \frac{3}{7} \cdot \frac{1}{2} + \frac{3}{8} \times \frac{4}{7}$$

$$\frac{3}{7} \times \frac{1}{2} + \frac{3}{8} \times \frac{4}{7} = \frac{3}{7}$$

3

→ independent of F — event e is independent of f

$\boxed{P(E/F) = P(E)}$

second / first

Last Example

$P(\text{2nd } w / \text{1st } y) = P(w) = \frac{1}{2}$

→ Criterion For independent Events ←

$P(E \cap F) = P(E) \cdot P(F)$

→ independent Events

$(P(E_1 \cap E_2 \cap E_3 \cap \dots \cap E_n) = P(E_1) \cdot P(E_2) \cdot P(E_3) \cdot \dots$

Ex. ① die 1→6 ② Coin H T ③ B, y, B, P

$P(1 \cap H \cap y) = \frac{1}{6} \cdot \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{48} = P(1) \cdot P(H) \cdot P(y)$

4

Ex. Fair Coin is tossed repeatedly until the first tail appear what is the probability of getting the first tail at 5th trial.

H: head $1/2$ P(A) — احتمال انه يظهرلي الصوره ١/٢ مثلا

T: tail $1/2$ P(T)

$P(\text{First tail appears in 5th trial}) = P(H_1 \cap H_2 \cap H_3 \cap H_4 \cap H_5)$

$= P(H) \cdot P(H) \cdot P(H) \cdot P(H) \cdot P(T)$

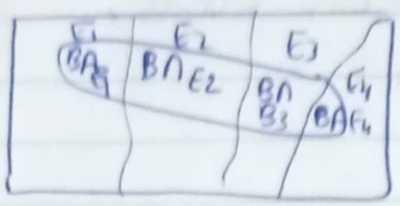
$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{32}$ #

الأقصى

law of total Probability concept and formula:

$$P(B) = P(B\cap E_1) + P(B\cap E_2) + \cdots P(B\cap E_k)$$

$$P(E_1)\cdot P(B|E_1) + P(E_2)P(B|E_2) + \cdots$$



$$B = (B\cap E_1)\cup(B\cap E_2)\cup(B\cap E_3)\cup(B\cap E_4)$$

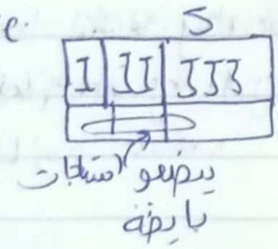
(6)

Ex:- Three machines I, II, III manufacture (0.4, 0.5, .1) of total Production Plant. The Percentage of defective items produced by I, II, III is 2%, 4%, 1%. For an item chosen at random, what is the probability that if it is defective.

لو هختار منتج عشوائي عايز اعرف هيكون فيه نسبة مشاكل قد اي.

D: The item is defective.

الخانة المشتركة هيتجمع



$P(D, I)$ 0.02

0.4 Machine 1 < defective 0.02 / no defective .98

0.5 Machine 2 < defective .04 $P(D/II)$ / no defective .96

.1 Machine 3 < defective .01 $P(D/III)$ / non defective .99

$P(D) =$

$$P(D) = P(D\cap I) + P(D\cap II) + P(D\cap III) \; \boxed{.029}$$

الآخر

$= 0.02\times.4 + .04\times.5$

$+ .01\times.1$

Bayes' theorem.

To get reversed conditional probability.

$$P(A_j|B) = \frac{P(A_j)\,P(E/A_j)}{P(E)} = \frac{P(A_j)\,P(E|A_j)}{P(A_1)\cdot P(E/A_1) + P(A_2)\,P(E/A_2) + P(A_3)\,P(E/A_n)}$$

Example.

Company gets its cars from three agencies with probabilities of 60%, 30%, 10%. if car is delivered - the probability that this car needs repairing 90% 1st, 2nd - 6% 3rd.

① Find the probability that delivered cars need ~~to~~ repairing.

R = car needs repair

$P(A_1) = 0.6$ $\quad$ $P(R/A_1) = 0.9$

$P(A_2) = 0.3$ $\quad$ $P(R/A_2) = 0.2$

$P(A_3) = 0.1$ $\quad$ $P(R/A_3) = 0.06$

$P(R) = P(A_1 \cap R) + P(A_2 \cap R) + P(A_3 \cap R)$

$P(R) = P(A_1)\cdot P(R/A_1) + P(A_2)\cdot P(R/A_2) + P(A_3)\cdot P(R/A_3)$

$0.6 \times .9 + 0.3 \times .2 + 0.1 \times .06 = 0.606$

② Find the delivered car needs repairing what is the propapility it ~~needs repai~~ came agency 3?

$$P(A_3/R) = \frac{P(A_3)\cdot P(R/A_3)}{P(R)} = \frac{0.1 * .06}{.612} = \checkmark$$

③ if the delived car doesn't need repairing what is probability from Agency 3

$$P(A_3/R') = \frac{P(A_3)\cdot P(R'/A_3)}{P(R')} = \frac{0.1 * (1 - .06)}{(1 - .606)} = 0.2423$$

($P(R'/A_3) = 1 - P(R/A_3)$)

Probability: قياس احتمالية وقوع الحدث

$0 \to 1$

$0\% \to 100\%$

↳ Sample Spaces and Event + outcome

Coin

$S = \{H, T\}$ Sample Space.

عدد النواتج outcome = عدد مرات التكرار (عدد النواتج)

① Toss coin two times or toss two coin in on time

Find S and outcome.

① اختار الصيغة اللى تبين عدد مرات التكرار

$S = \{HT, HH, TH, TT\}$

H T ①

H T H T ②

outcome = $4 = ($ عدد النواتج من العملية الواحدة $)^{\text{عدد مرات التكرار}}$

$= (2)^2 = 4$

② Toss dice two time or Toss dice onse

1 2 3

~~S = {11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 27, 28, 29, 2...}~~

1
2 $S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$
3 $(2,1), (2,2), (2,3), \ldots (6,6)$
4
5 Out come $= (6)^2 = 36$
6

Events

① Union.

$A \cup B$ Keyword → OR

② intersection.

$A \cap B$ Keyword → And.

③ Complement

④ probability of events

$P(\ldots) = \dfrac{\text{عدد ..}}{\text{Sample space.}}$

$A = \{1, 2, 3\}$ $B = \{2, 3, 4\}$ $S = \{0, 1, 2, 3, 4\}$

$A \cap B = \{2, 3\}$ $P(A \cap B) = \frac{2}{5}$ $P(S) = 1$

$A \cup B = \{1, 2, 3, 4\}$

$A' = \{0, 4\}$ موجودة في S ومش $P(A) = \frac{3}{5}$

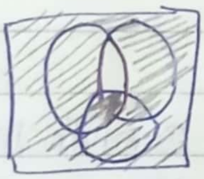
$B' = \{0, 1\}$ موجودة في A

$P(B') = \frac{2}{5}$

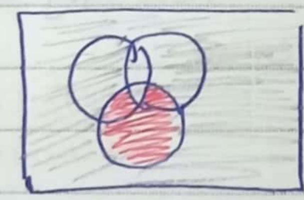
②

Venn diagram.

$(A \cap B)' \cup C$



② $(A \cap B)' \cap C$



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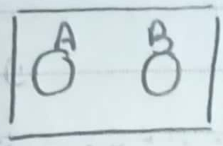
Mutually exclusive

شيئين لا يعتمد على بعض



$A \cap B = \emptyset$

$P(A \cap B) = 0$

→ Counting techniques

- Multiplication
- Permutation
  - of subsets
  - of similar objects
  - of elements

  P
- Combination

  C

الحل

① اقرأ السؤال واحدد keyword

② اكتب القانون

اذا طلب Prob

اقسم ع Total

☐ ① Multiplication (شئ واحد بس)

باخد احتمالات كل اختيار واضربهم في بعض

Ex: Coffe shop has 4 types of Sandwhichies 5 Types of Coffe 2 Cake Types how May ways choose 1 Sandwich 1 Coffe and one Cake.

$= 4 \times 5 \times 2 = 40$

عرفت منين ان عرالمطلع تجاب واحده

$r=4$
$n=10$

How Many License plates can be made consisting of two letters Followed by 3 numbers (assume that there are 26 letters)

$26 \times 26 \times 10 \times 10 \times 10 = 676\,000$ 5   ① عدد الخانات

26 حرف + 10 أرقام   ② عدد الاختيارات

② Permutation   كم طريقة اقدر ارتب

① ترتيب عناصر مجموعة (ordering element)

② عدد طرق ترتيب مجموعة كبيرة بالترتيب ← Permutation of Subsets

③ Permutation of similar object → $\dfrac{n!}{n_1!\, n_2!\, n_3! \ldots}$

by Factorial $P^n_r = \dfrac{n!}{(n-r)!}$

1 ordering elements

→ Six students are lining up out side the heads office
The number of different that they could queue up,

$6! = 720$

2 Permutation of Subsets

→ Eight Chefs enter a competition. The judges award a $1^{st}, 2^{nd}, 3^{rd}, 4^{th}$ Price The number of different way in which Prices can be awarded?

$P^8_4 = \dfrac{8!}{(8-4)!} = 1680$

Permutation of Similar objects $\frac{n!}{n!\,n!}$

How Many different orders [MINIMUM] $M \to 3$, $I \to 2$

$$\frac{7!}{3!\,2!} = 420$$

3 Combination. احتيار عناصر من مجموعة متشابهة

لو استخدمها لما يقول اختيار من Box

NCR

Ex: A bin of 50 part Contains 3 defective parts and 47 non defective parts. A Sample of 6 parts is selected from 50. How Many different Samples of Size 6 that Contain exactly 2 defective parts

50: 3D, 47nD → 6: 2D, 4ND

~~N=50 Both~~

اختيار ٢ بايظين و 4 شغالين

~~3C2 4~~

$3C2 * 47C4 = 535095$

→ what is the probability of this Case:

Total. نقسم ع

$$\frac{3C2 \times 47C4}{50C6} = .03367$$

at least one major defect

1 - P(no Major defect)

Rules: $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$P(A\cap B') = P(A) - P(A\cap B)$

$P(A'\cap B) = P(B) - P(A\cap B)$

$P(A'\cap B') = P(A\cup B)' = 1 - P(A\cup B)$

$(A\cup B)' = A'\cap B'$

$(A\cap B)' = (A'\cup B')$

$P(A) = \frac{1}{3}$ $P(B) = 0.5$ $P(A\cup B) = 0.75$

a- $(A\cap B) =$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$0.75 = \frac{1}{3} + 0.5 - P(A\cap B) = .08333$

b) $(A'\cup B') = P(A\cap B)' = 1 - P(A\cap B) = 1 - .08333 = .91667$

c) $P(A'\cap B) = P(B) - P(A\cap B) = 0.5 - .0833 = 0.41667$

Ex2: $P(A) = .3$

$P(B) = 0.2$

$P(A\cap B) = 0.1$

① $P(A') = 1 - 0.3 = 0.7$

② $P(A\cup B) = P(A) + P(B) - P(A\cap B) = 0.3 + 0.2 - 0.1 = 0.4$

③ $P(A'\cap B)' = 1 - P(A'\cap B) = 1 - [P(B) - P(A\cap B)]$

$= 1 - [0.2 - 0.1] = 0.9$

④ $P(A'\cap B) = P(B) - P(A\cap B) = 0.2 - 0.1 = 0.1$

⑤ $P(A\cup B)' = 1 - P(A\cup B) = 1 - .4 = 0.6$

⑥ $P(A'\cup B) = P(A') + P(B) - P(A'\cap B)$

$= 0.7 + 0.2 - 0.1 = 0.8$

→ Conditional Probability.

احتمل حدوث A P(A)

B P(B)

احتمال حدوث A و B P(A∩B)

P(A|B)

احتمال حدوث A بشرط

وقوع حدث B

جدول مباشر

القانون العام

فقره ومتها شوى (شجره حل)

النسب المطلوبه هي قيمه ال P( ) Prob.

اهم شئ لما ابدى حل اسمى كل حدث ، الحدث يكون جمله مثلا

Given that , Knowing that : Key word

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

تم تلخيصهم في

Playlist سابقه