Supervised Learning

Decision Trees

Road Map

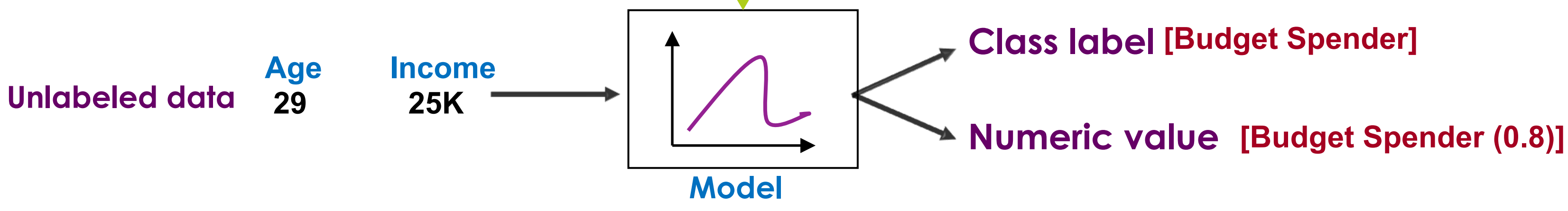
1. Basic Concepts of Classification
2. Decision Tree Induction
3. Attribute Selection Measures
4. Pruning Strategies

Definition

- Supervised Learning is also called **Classification (or Prediction)**
- **Principle**
  - Construct models (functions) based on training data
  - The training data are **labeled** data
  - New data (**unlabeled**) are classified using the training data

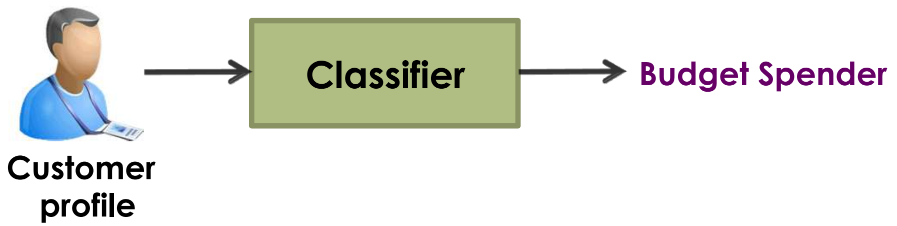
Training data

Age	Income	Class label
27	28K	Budget-Spenders
35	36K	Big-Spenders
65	45K	Budget-Spenders

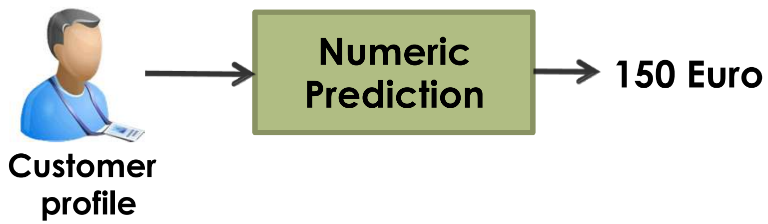


Classification vs Prediction

- **Classification** predicts categorical class labels (discrete or nominal)



- **Prediction** models continuous-valued functions, i.e., predicts unknown or missing values (ordered values)



  - **Regression** analysis is used for prediction

Entropy: Bits

- You are watching a set of independent random samples of X
- X has 4 possible values: A, B, C, and D
- The probabilities of generating each value are given by:

P(X=A)=1/4, P(X=B)=1/4, P(X=C)=1/4, P(X=D)=1/4

- You get a string of symbols ACBABBCDADDC…
- To transmit the data over binary link you can encode each symbol with bits (A=00, B=01, C=10, D=11)

http://www.cs.cmu.edu/~guestrin/Class/10701-S06/Handouts/recitations/recitation-decision_trees-adaboost-02-09-2006.ppt

Entropy: Bits

- Now someone tells you the probabilities are not equal

P(X=A)=1/2, P(X=B)=1/4, P(X=C)=1/8, P(X=D)=1/8

- In this case, it is possible to find coding that uses only 1.75 bits on the average
  - E.g., Huffman coding
- Compute the average number of bits needed per symbol

http://www.cs.cmu.edu/~guestrin/Class/10701-S06/Handouts/recitations/recitation-decision_trees-adaboost-02-09-2006.ppt

Entropy: General Case

- Suppose X takes n values, V_1, V_2,… V_n, and

$$P(X=V_1)=p_1,\ P(X=V_2)=p_2,\ \ldots\ P(X=V_n)=p_n$$

- The smallest number of bits, on average, per symbol, needed to transmit the symbols drawn from distribution of X is given by:

$$H(X) = -\sum_{i=1}^{m} p_i \log_2(p_i)$$

- H(X) = the **entropy** of X

http://www.cs.cmu.edu/~guestrin/Class/10701-S06/Handouts/recitations/recitation-decision_trees-adaboost-02-09-2006.ppt

Entropy Definition

- Entropy is a measure of the average information content one is missing when one does not know the value of the random variable
- **High Entropy**
  - X is from a uniform like distribution
  - Flat histogram
  - Values sampled from it are less predictable
- **Low Entropy**
  - X is from a varied (peaks and valleys) distribution
  - Histogram has many lows and highs
  - Values sampled from it are more predictable

http://www.cs.cmu.edu/~guestrin/Class/10701-S06/Handouts/recitations/recitation-decision_trees-adaboost-02-09-2006.ppt

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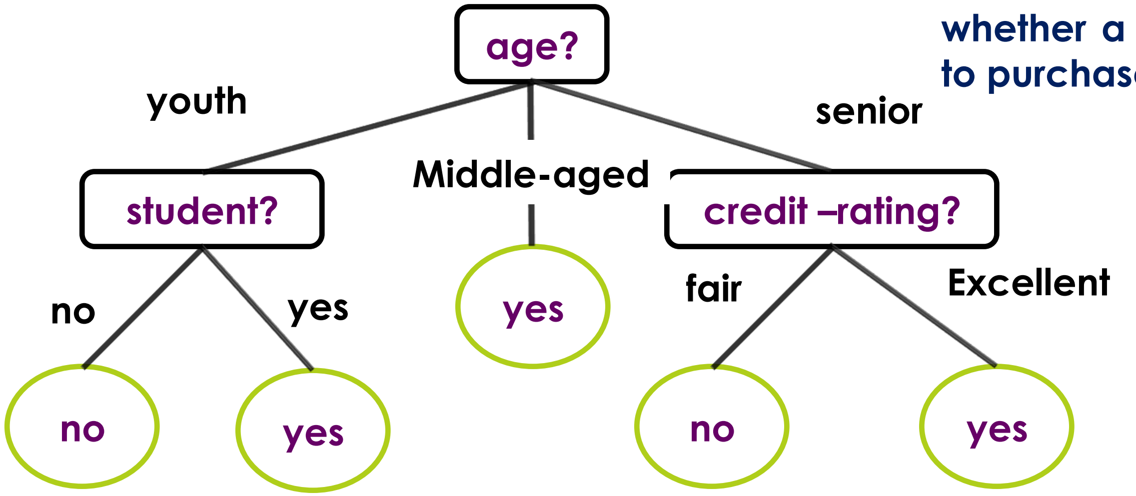
Decision Tree Induction

- **Decision tree induction** is the learning of decision trees from class-labeled training tuples
- A decision tree is a flowchart-like tree structure
  - **Internal nodes** (non leaf node) denotes a test on an attribute
  - **Branches** represent outcomes of tests
  - **Leaf nodes** (terminal nodes) hold class labels
  - **Root node** is the topmost node



A decision tree indicating whether a customer is likely to purchase a computer

Class-label Yes: The customer is likely to buy a computer
Class-label no: The customer is unlikely to buy a computer

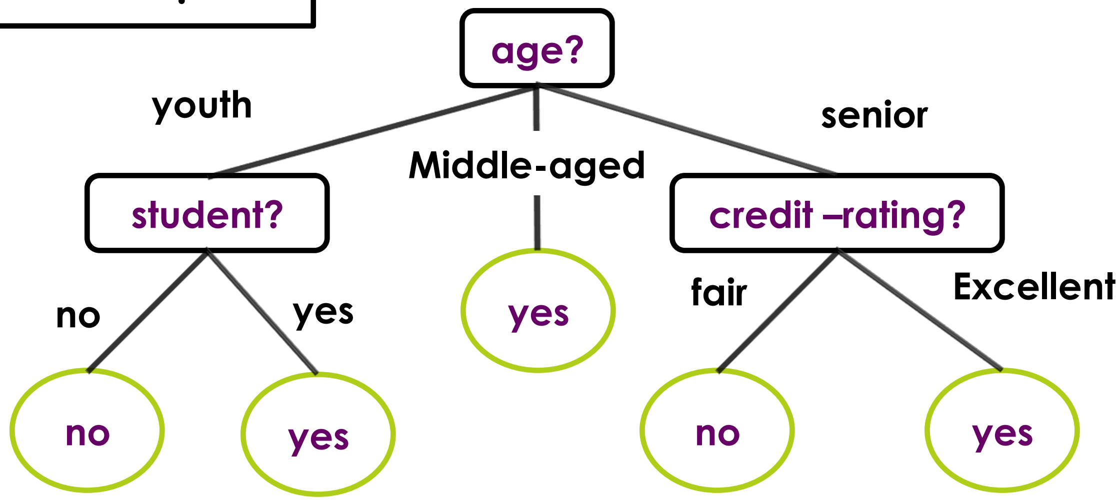
Decision Tree Induction

- **How are decision trees used for classification?**
  - The attributes of a tuple are tested against the decision tree
  - A path is traced from the root to a leaf node which holds the prediction for that tuple

- **Example**

RID	age	income	student	credit-rating	Class
1	youth	high	no	fair	?

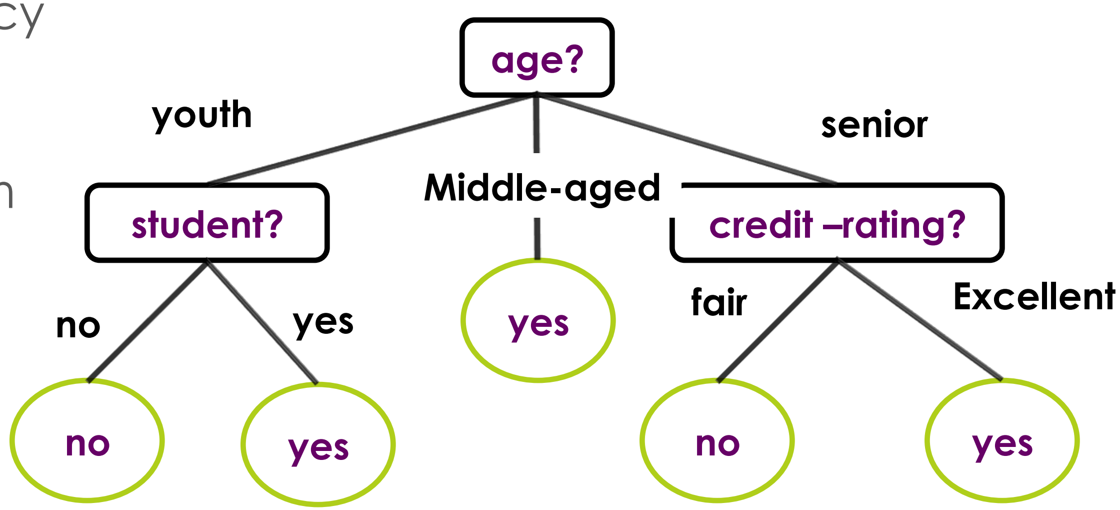
- Test on age: youth
- Test of student: no
- Reach leaf node
- **Class NO:** the customer Is unlikely to buy a computer



A decision tree indicating whether a customer is likely to purchase a computer

Decision Tree Induction

- **Why decision trees classifiers are so popular?**
  - The construction of a decision tree does not require any domain knowledge or parameter setting
  - They can handle high dimensional data
  - Intuitive representation that is easily understood by humans
  - Learning and classification are simple and fast
  - They have a good accuracy
- **Note**
  - Decision trees may perform
  Differently depending on
  the data set
- **Applications**
  - Medicine, astronomy
  - Financial analysis, manufacturing
  - Many other applications



A decision tree indicating whether a customer is likely to purchase a computer

The Algorithm

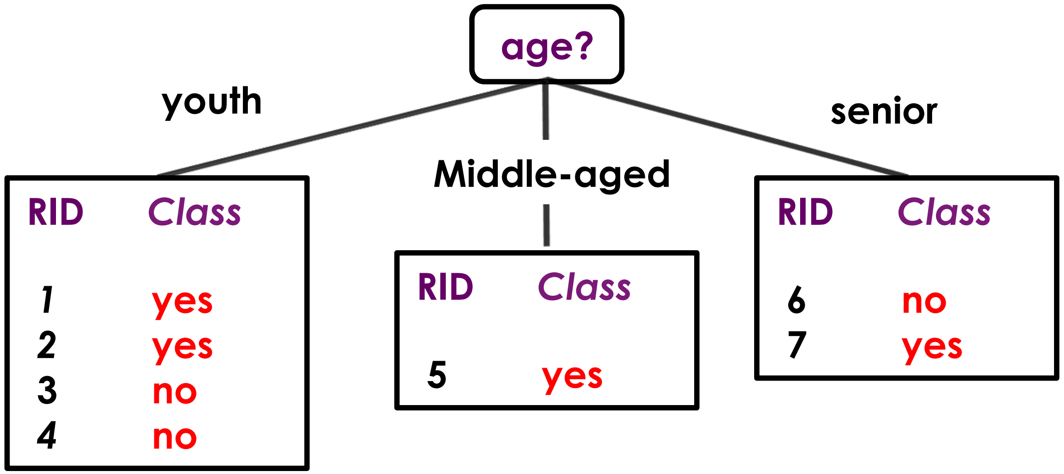
Principle

- Basic algorithm (adopted by ID3, C4.5 and CART): a **greedy algorithm**
- Tree is constructed in a top-down recursive divide-and-conquer manner

- **Iterations**
  - At start, all the training tuples are at the root
  - Tuples are partitioned recursively based on selected attributes
  - Test attributes are selected on the basis of a heuristic or statistical measure (e.g., **information gain**)

- **Stopping conditions**
  - All samples for a given node belong to the same class
  - There are no remaining attributes for further partitioning – **majority voting** is employed for classifying the leaf
  - There are no samples left

Example



RID	age	student	credit-rating	Class: buys_computer
1	youth	yes	fair	yes
2	youth	yes	fair	yes
3	youth	yes	fair	no
4	youth	no	fair	no
5	middle-aged	no	excellent	yes
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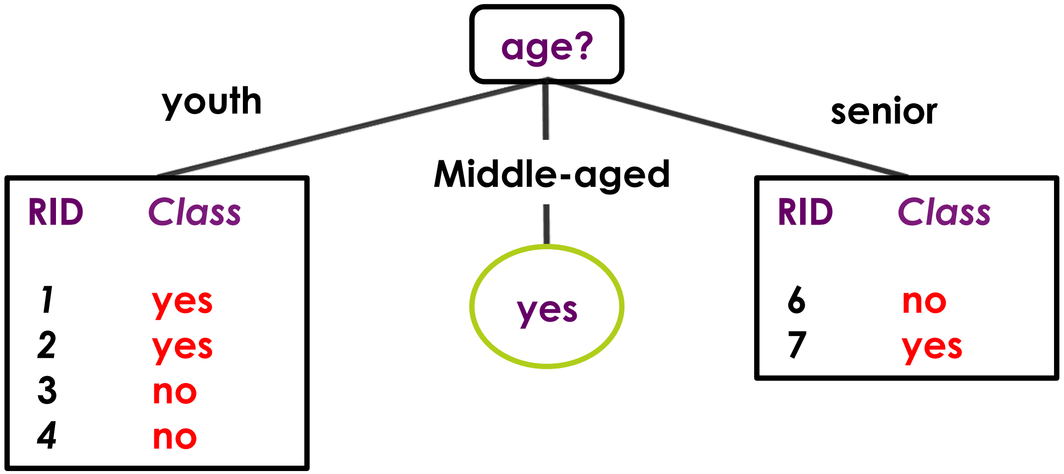
Example



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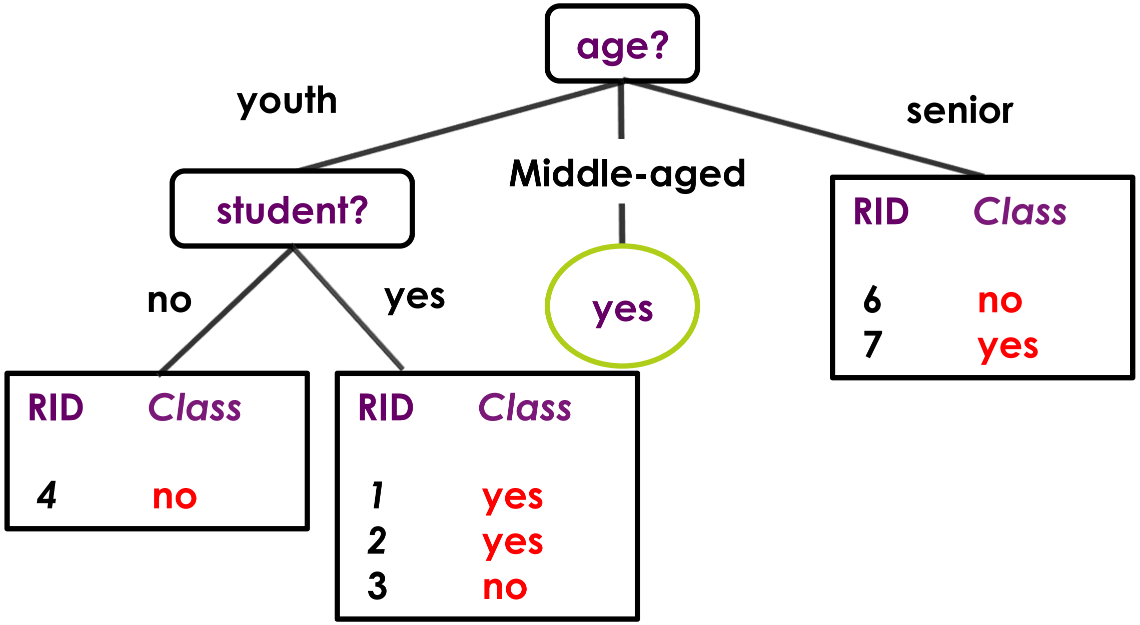

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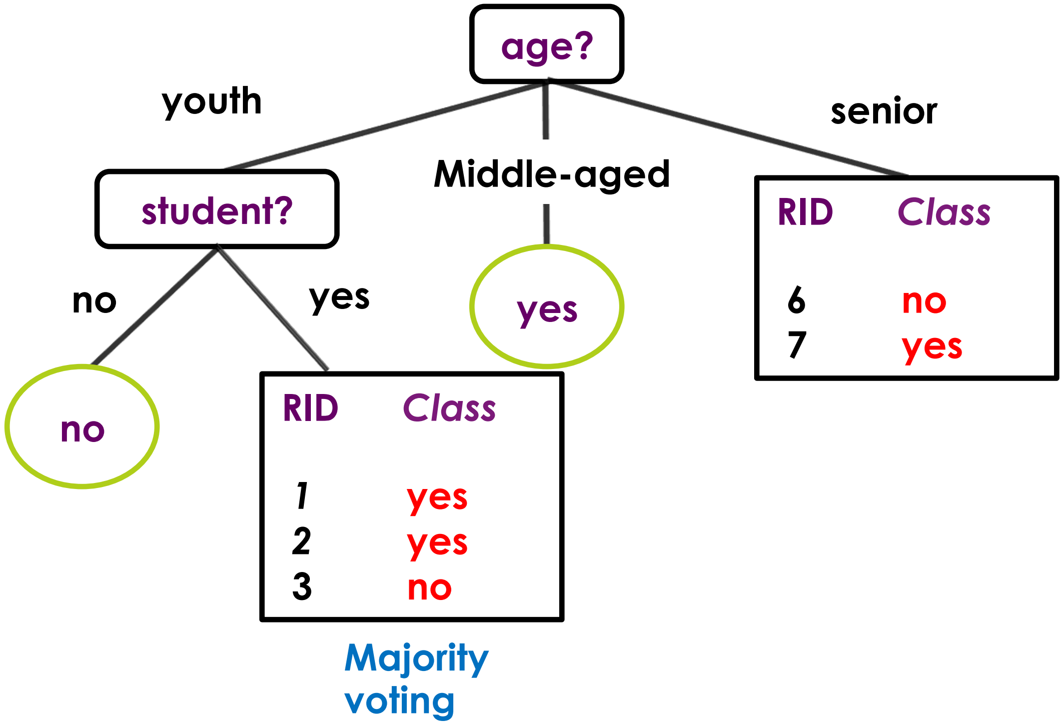

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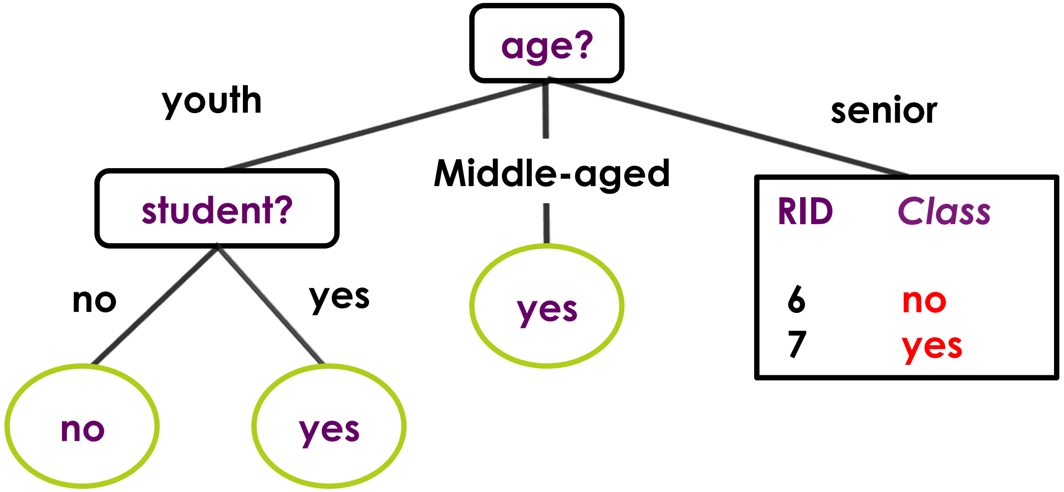

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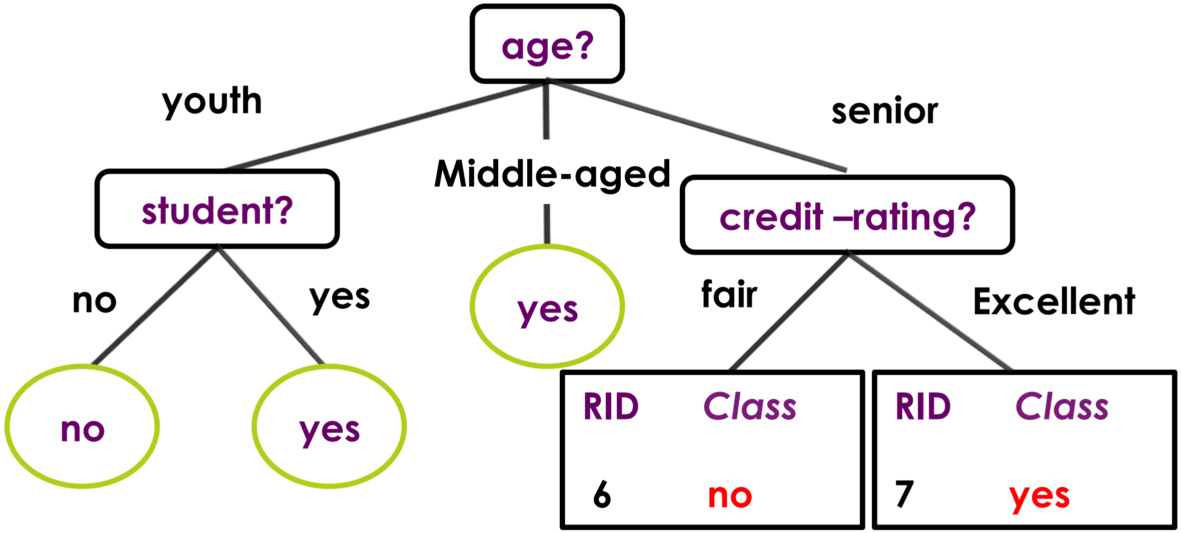

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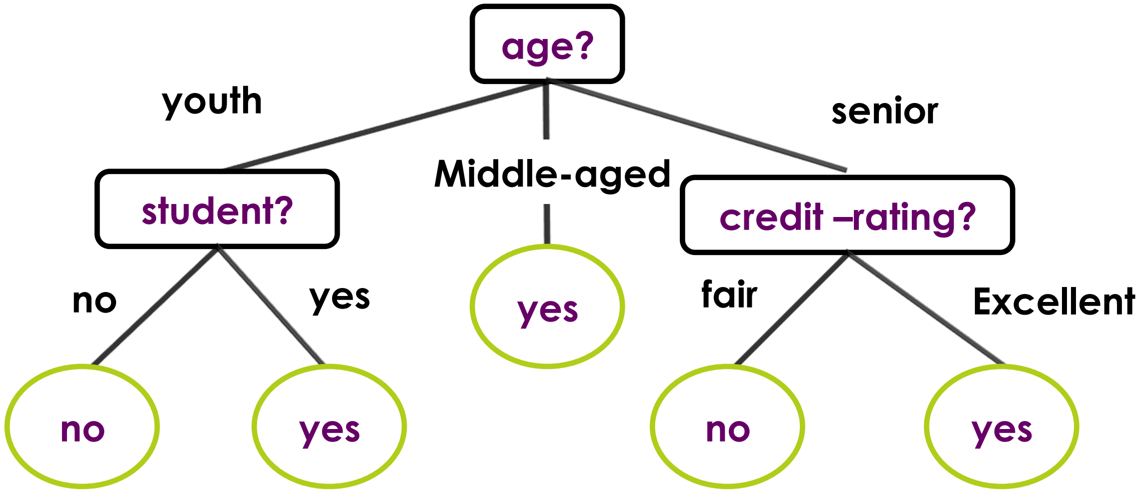

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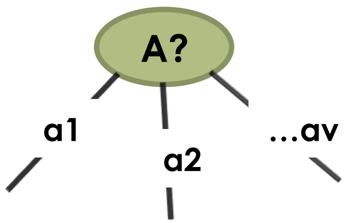
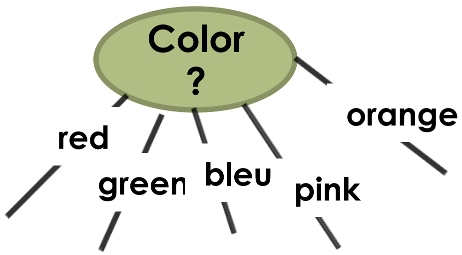
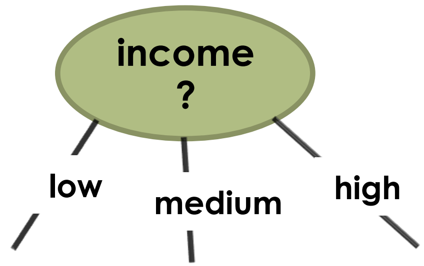
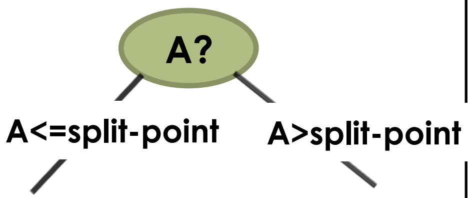
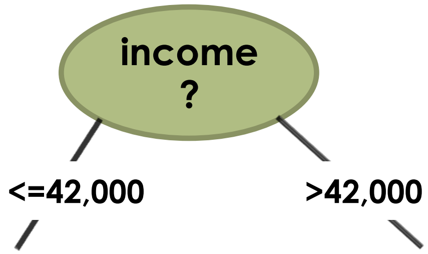
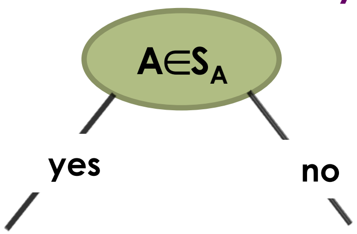
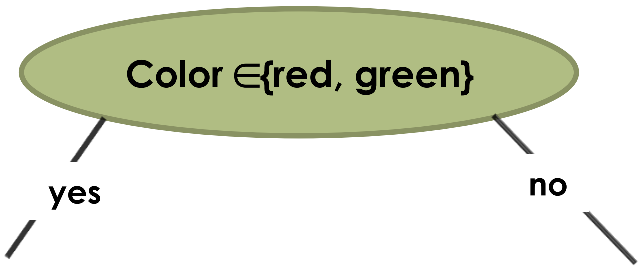
Three Possible Partition Scenarios

Partitioning scenarios	Examples	
Discrete-valued 		
Continuous-valued 		
Discrete-valued+ binary tree 		

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Attribute Selection Measures

- An **attribute selection measure** is a heuristic for selecting the splitting criterion that "best" separates a given data partition **D**

Ideally

- Each resulting partition would be pure
- A **pure** partition is a partition containing tuples that all belong to the same class

- Attribute selection measures (splitting rules)
  - Determine how the tuples at a given node are to be split
  - Provide ranking for each attribute describing the tuples
  - The attribute with highest score is chosen
  - Determine a **split point** or a **splitting subset**

- **Methods**
  - Information gain
  - Gain ratio
  - Gini Index

Quiz

- In both pictures **A** and **B** the child is eating a soup

Low Entropy **High Entropy**

The values (locations of the soup) sampled entirely from within the soup ball

A

The values (locations of the soup) almost unpredictable...almost uniformly sampled throughout the living room

- Which situation (A or B) has a high/low entropy in terms of the locations of the soup?

From: http://www.autonlab.org/tutorials/infogain11.pdf

Information Gain Approach

- **D**: the current partition
- **N**: represent the tuples of partition D
- Select the attribute with the highest information gain (based on the work by Shannon on information theory)
- This attribute
  - minimizes the information needed to classify the tuples in the resulting partitions
  - reflects the least randomness or "impurity" in these partitions
- **Information gain** approach minimizes the expected number of tests needed to classify a given tuple and guarantee a simple tree

First Step

- Compute **Expected information** (entropy) needed to classify a tuple in partition **D**

$$Info(D) = -\sum_{i=1}^{m} p_i \log_2(p_i)$$

  - **m**: the number of classes
  - **p_i**: the probability that an arbitrary tuple in **D** belongs to class **C_i** estimated by: **$|C_i,D| / |D|$** (proportion of tuples of each class)
  - A **log** function to the base 2 is used because the information is encoded in bits

- **Info(D)**
  - The average amount of information needed to identify the class label of a tuple in D
  - It is the **entropy**

Example

RID	age	income	student	credit-rating	class:buy_computer
1	youth	high	no	fair	no
2	youth	high	no	excellent	no
3	middle-aged	high	no	fair	yes
4	senior	medium	no	fair	yes
5	senior	low	yes	fair	yes
6	senior	low	yes	excellent	no
7	middle-aged	low	yes	excellent	yes
8	youth	medium	no	fair	no
9	youth	low	yes	fair	yes
10	senior	medium	yes	fair	yes
11	youth	medium	yes	excellent	yes
12	middle-aged	medium	no	excellent	yes
13	middle-aged	high	yes	fair	yes
14	senior	medium	no	excellent	no

In partition D

m=2 (the number of classes)
N= 14 (number of tuples)

9 tuples in class yes
5 tuples in class no

$$Info(D) = -\frac{9}{14}\log_2\left(\frac{9}{14}\right) - \frac{5}{14}\log_2\left(\frac{5}{14}\right) = 0.940 \text{ bits}$$

Second Step

- For each attribute, compute the amount of information needed to arrive at an exact classification after portioning using that attribute
- Suppose that we were to partition the tuples in D on some attribute **A** **{$a_1,\ldots,a_v$}**
  - Split **D** into v partitions **{D1,D2,…Dv}**
  - Ideally **Di** partitions are pure but it is unlikely
- The amount of information needed to arrive at an exact classification is measured by:

$$Info_A(D) = \sum_{j=1}^{v} \frac{|D_j|}{|D|} \times Info(D_j)$$

  - **|Dj|/|D|**: the weight of the jth partition
  - **Info(Dj)**: the entropy of partition **Dj**
  - The smaller the expected information still required, the greater the purity of the partitions

Example

RID	age	income	student	credit-rating	class:buy_computer
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11	youth	medium	yes	excellent	yes
12	middle-aged	medium	no	excellent	yes
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Using attribute age

Part1(youth) **D1** has **2 yes** and **3 no**
Part2(middle-aged) **D2** has **4 yes** and **0 no**
Part3(senior) **D3** has **3 yes** and **2 no**

$$Info_{age}(D) = \frac{5}{14} Info(D_1) + \frac{4}{14} Info(D_2) + \frac{5}{14} Info(D_3) = 0.694$$

Third Step

- Compute Information Gain
- Information gain by branching on A is:

$$Gain(A) = Info(D) - Info_A(D)$$

- Information gain is the expected reduction in the information requirements caused by knowing the value of A
- The attribute **A** with the **highest information gain** (Gain(A)), is **chosen** as the splitting attribute at node N

Example

RID	age	income	student	credit-rating	class:buy_computer
1	youth	high	no	fair	no
2	youth	high	no	excellent	no
3	middle-aged	high	no	fair	yes
4	senior	medium	no	fair	yes
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$$Gain(age) = Info(D) - Info_{age}(D) = 0.246$$

Gain(income)=0.029,
Gain(student)=0.151
Gain(credit_rating)=0.048

"Age" has the highest gain ⇒ It is chosen as the splitting attribute

Note on Continuous Valued Attributes

- Let attribute **A** be a continuous-valued attribute
- Must determine the ***best split point*** for A
  - Sort the values of A in increasing order
  - Typically, the midpoint between each pair of adjacent values is considered as a possible split point
    - $(a_i+a_{i+1})/2$ is the midpoint between the values of a_i and a_{i+1}
  - The point with the minimum expected information requirement for A is selected as the split point
- **Split**
  - **D1** is the set of tuples in D satisfying **A ≤ split-point**
  - **D2** is the set of tuples in D satisfying **A > split-point**

Gain Ratio Approach

- Problem of Information Gain
  - Biased towards tests with many outcomes (attributes having a large number of values)
  - E.g: attribute acting as a unique identifier
    - Produce a **large number of partitions** (1 tuple per partition)
    - Each resulting partition D is **pure** Info(D)=0
    - The **information gain** is **maximized**
- Extension to Information Gain
  - Use **gain ratio**
  - Overcomes the bias of Information gain
  - Applies a kind of normalization to information gain using a **split information** value

Split Information

- The **split information value** represents the potential information generated by splitting the training data set **D** into **v** partitions, corresponding to v outcomes on attribute **A**

$$SplitInfo_A(D) = -\sum_{j=1}^{v} \frac{|D_j|}{|D|} \times \log_2\left(\frac{|D_j|}{|D|}\right)$$

  - **High split Info**: partitions have more or less the same size (uniform)
  - **Low split Info**: few partitions hold most of the tuples (peaks)

- The gain ratio is defined as:

$$GainRatio(A) = \frac{Gain(A)}{SplitInfo(A)}$$

- The attribute with the maximum gain ratio is selected as the splitting attribute

Example

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Using attribute income

Part1 (low) **: 4 tuples ,** Part2 (medium): **6 tuples,** Part3 (high): **4 tuples**

$$SplitInfo_{income}(D) = -\frac{4}{14}\log_2(\frac{4}{14}) - \frac{6}{14}\log_2(\frac{6}{14}) - \frac{4}{14}\log_2(\frac{4}{14}) = 0.926$$

$$Gain(income) = 0.029$$

$$GainRatio(income) = \frac{0.029}{0.926} = 0.031$$

Gini Index Approach

- Measures the impurity of a data partition **D**

$$Gini(D) = 1 - \sum_{i=1}^{m} p_i^{\,2}$$

  - **m**: the number of classes
  - **p_i**: the probability that a tuple in D belongs to class C_i

- The Gini Index considers a **binary split** for each attribute A, say **D_1** and **D_2**. The Gini index of **D** given that partitioning is:

$$Gini_A(D) = \frac{|D_1|}{|D|} Gini(D_1) + \frac{|D_2|}{|D|} Gini(D_2)$$

- The reduction in impurity is given by:

$$\Delta Gini(A) = Gini(D) - Gini_A(D)$$

- The attribute that maximizes the reduction in impurity is chosen as the splitting attribute

Binary Split

- **Continuous Values Attributes**
  - Examine each possible split point. The midpoint between each pair of (sorted) adjacent values is taken as a possible split-point
  - For each split-point, compute the weighted sum of the impurity of each of the two resulting partitions (D_1: A<=split-point, D_2: A> split-point)
  - The point that gives the minimum Gini index for attribute A is selected as its split-point

- **Discrete Attributes**
  - Examine the partitions resulting from all possible subsets of $\{a_1,\ldots,a_v\}$
  - Each subset S_A is a binary test of attribute **A** of the form "$A \in S_A$?"
  - 2^v possible subsets. We exclude the power set and the empty set, then we have 2^v-2 subsets
  - The subset that gives the minimum Gini index for attribute A is selected as its splitting subset

Example

RID	age	income	student	credit-rating	class:buy_computer
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Compute the Gini index of the training set D: 9 tuples in class yes and 5 in class no

$$Gini(D) = 1 - \left[\left(\frac{9}{14}\right)^2 + \left(\frac{5}{14}\right)^2\right] = 0.459$$

Using attribute income: there are three values: **low, medium** and **high**

Choosing the subset **{low, medium}** results in two partitions:

D1 (income $\in$ {low, medium}): 10 tuples

D2 (income $\in$ {high}): 4 tuples

Example

$$Gini_{income \in \{low, medium\}}(D) = \frac{10}{14} Gini(D_1) + \frac{4}{14} Gini(D_2)$$

$$= \frac{10}{14}\left(1 - \left(\frac{6}{10}\right)^2 - \left(\frac{4}{10}\right)^2\right) + \frac{4}{14}\left(1 - \left(\frac{1}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right)$$

$$= 0.450$$

$$= Gini_{income \in \{high\}}(D)$$

The Gini Index measures of the remaining partitions are:

$$Gini_{\{low, high\}\, and\, \{medium\}}(D) = 0.315$$

$$Gini_{\{medium, high\}\, and\, \{low\}}(D) = 0.300$$

The best binary split for attribute income is on **{medium, high}** and **{low}**

Comparing Attribute Selection Measures

- **Information Gain**
  - Biased towards multivalued attributes

- **Gain Ratio**
  - Tends to prefer unbalanced splits in which one partition is much smaller than the other

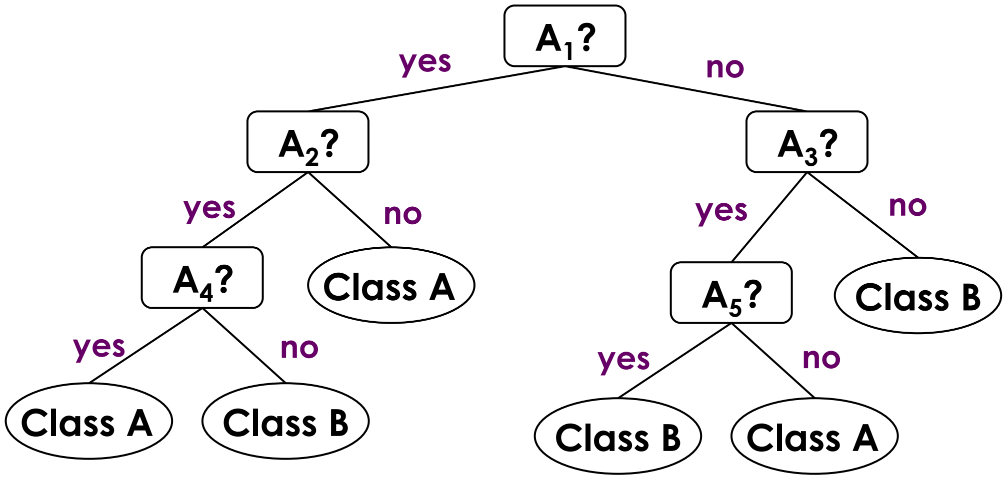
- **Gini Index**
  - Biased towards multivalued attributes
  - Has difficulties when the number of classes is large
  - Tends to favor tests that result in equal-sized partitions and purity in both partitions

Road Map

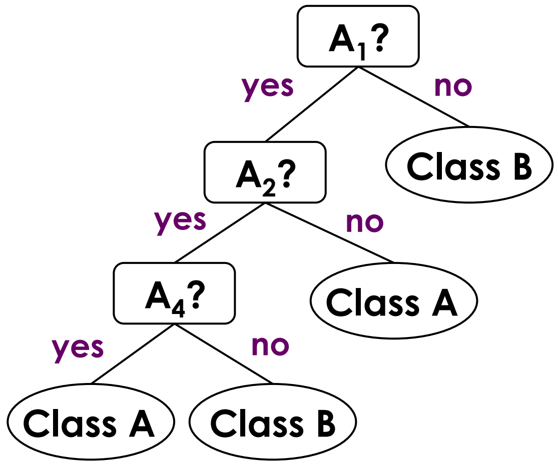
1. Basic Concepts of Classification
2. Decision Tree Induction
3. Attribute Selection Measures
4. Pruning Strategies

Overfitting

- Many branches of the decision tree will reflect anomalies in the training data due to noise or outliers
- Poor accuracy for unseen samples
- Solution: **Pruning**
  - Remove the least reliable branches



Before Pruning



After Pruning

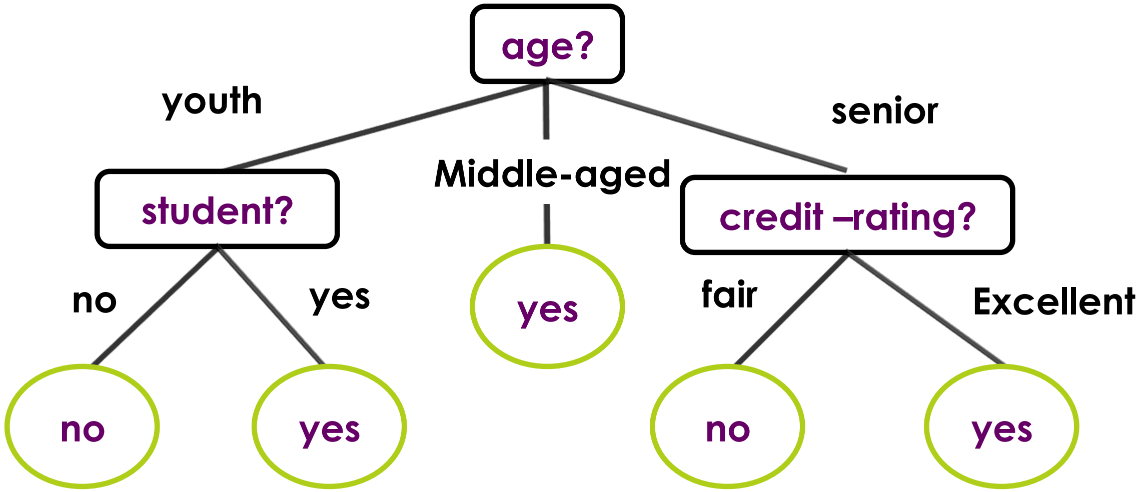
Tree Pruning Strategies

- **Prepruning**
  - Halt tree construction early—do not split a node if this would result in the goodness measure falling below a threshold
  - Statistical significance, information gain, Gini index are used to assess the goodness of a split
  - Upon halting, the node becomes a leaf
  - The leaf may hold the most frequent class among the subset tuples
- **Postpruning**
  - Remove branches from a "fully grown" tree:
  - A subtree at a given node is pruned by replacing it by a leaf
  - The leaf is labeled with the most frequent class

Cost Complexity Pruning Algorithm

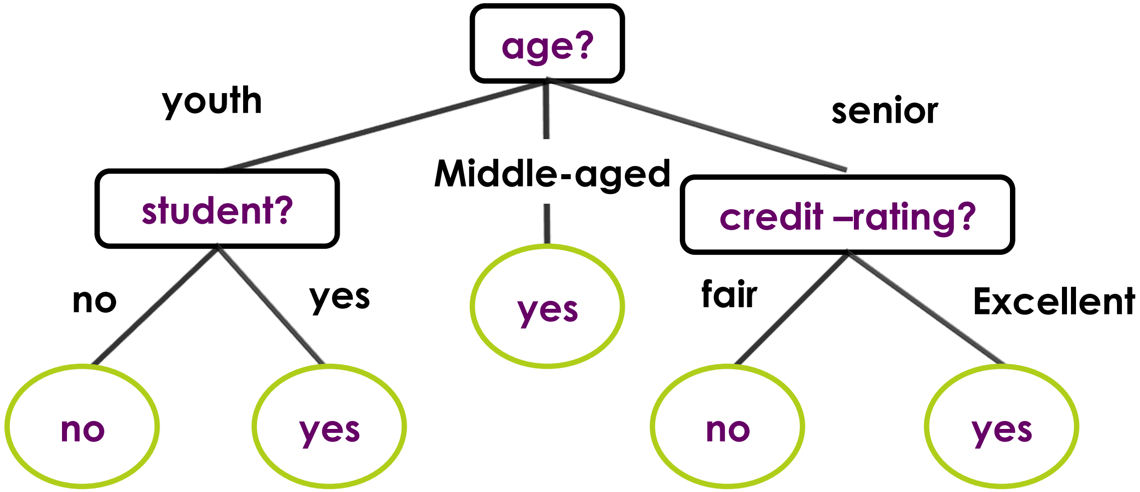
- Cost complexity of a tree is a function of the the number of leaves and the error rate (percentage of tuples misclassified by the tree)
- At each node N compute
  - The cost complexity of the subtree at N
  - The cost complexity of the subtree at N if it were to be pruned
- If pruning results in smaller cost, then prune the subtree at N
- Use a set of data different from the training data to decide which is the "best pruned tree"

Pruning Example



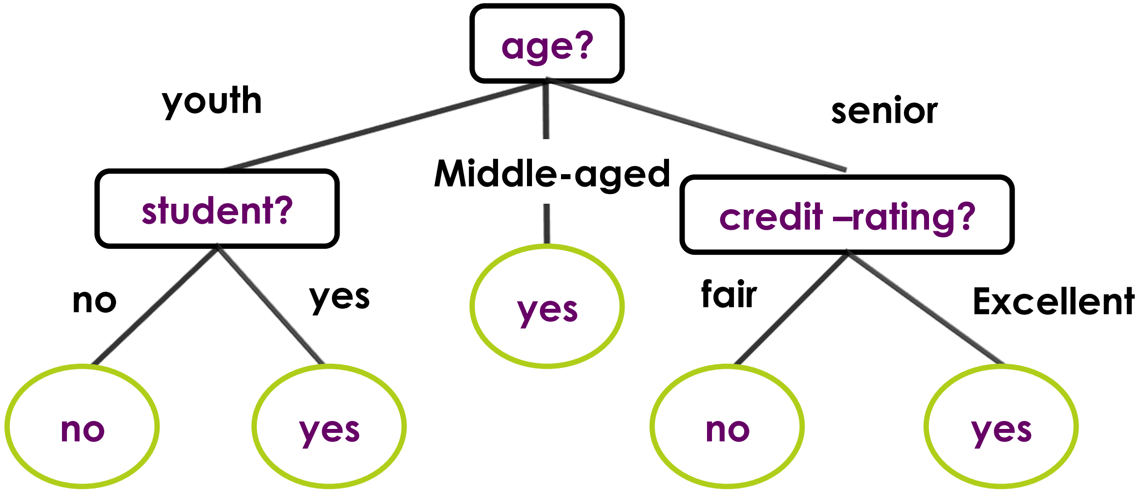
RID	age	student	credit-rating	Class: buys_computer
1	youth	yes	fair	yes
2	youth	yes	fair	yes
3	youth	yes	fair	no
4	youth	no	fair	no
5	middle-aged	no	excellent	yes
6	senior	yes	fair	no
7	senior	yes	excellent	yes

Optimistic Evaluation (Error =1/7)



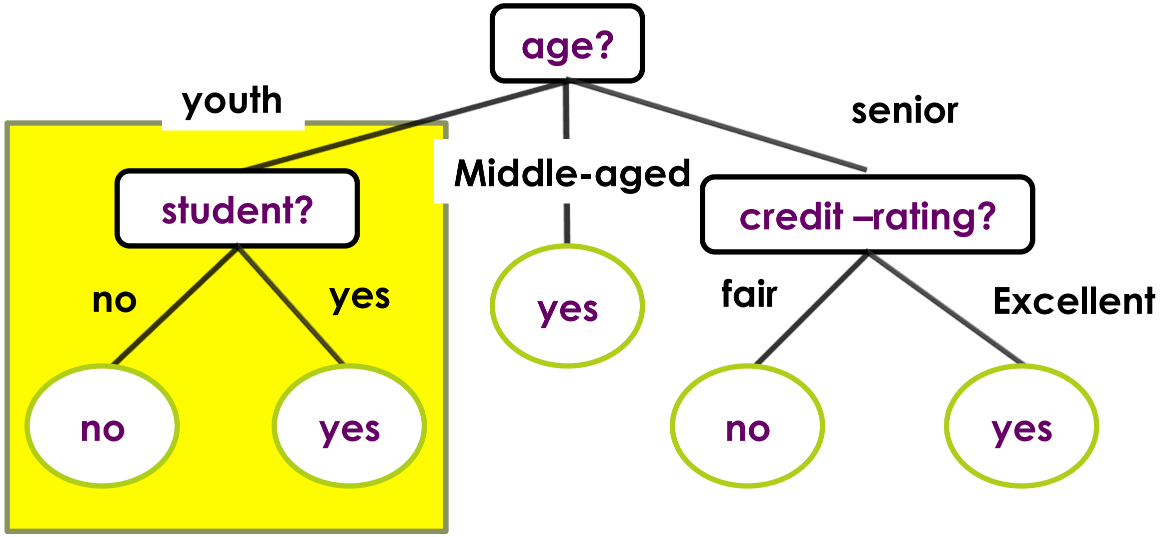
RID	age	student	credit-rating	Class: buys_computer
1	youth	yes	fair	yes
2	youth	yes	fair	yes
3	youth	yes	fair	no
4	youth	no	fair	no
5	middle-aged	no	excellent	yes
6	senior	yes	fair	no
7	senior	yes	excellent	yes

Evaluation using Validation Set (Error=3/7)



RID	age	student	credit-rating	Class: buys_computer
1	youth	no	excellent	yes
2	youth	yes	fair	yes
3	youth	yes	fair	no
4	youth	no	fair	yes
5	youth	no	fair	yes
6	middle-aged	no	excellent	yes
7	senior	yes	excellent	no

Pruning



RID	age	student	credit-rating	Class: buys_computer
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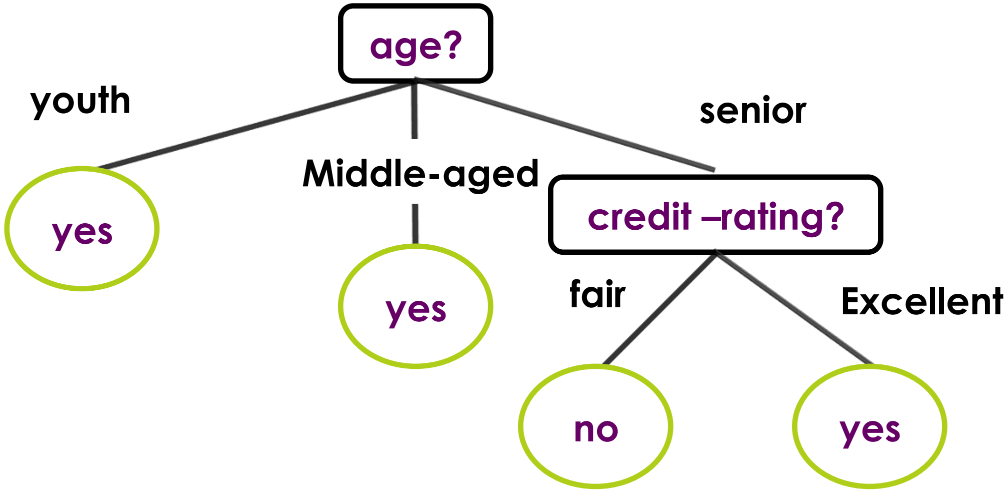
2	youth	yes	fair	yes
3	youth	yes	fair	no
4	youth	no	fair	yes
5	youth	no	fair	yes
6	middle-aged	no	excellent	yes
7	senior	yes	excellent	no

After Pruning (Error=1/7)



RID	age	student	credit-rating	Class: buys_computer
1	youth	no	excellent	yes
2	youth	yes	fair	yes
3	youth	yes	fair	no
4	youth	no	fair	yes
5	youth	no	fair	yes
6	middle-aged	no	excellent	yes
7	senior	yes	excellent	no

Summary

- Decision Trees have relatively **faster learning** speed than other methods
- Conversable to simple and **easy to understand** classification rules
- Information Gain, Ratio Gain and Gini Index are the most common methods of **attribute selection**
- **Tree pruning** is necessary to remove unreliable branches

Question

Given dataset D and the number of attributes n, show that the computational cost of growing a binary decision tree is at most

$$n \times |D| \times \log(|D|)$$