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The Law of Averages

The roulette wheel has neither conscience nor memory.
—JOSEPH BERTRAND (FRENCH MATHEMATICIAN, 1822–1900)

1. WHAT DOES THE LAW OF AVERAGES SAY?

A coin lands heads with chance 50%. After many tosses, the number of heads should equal the number of tails: isn't that the law of averages? John Kerrich, a South African mathematician, found out the hard way. He was visiting Copenhagen when World War II broke out. Two days before he was scheduled to fly to England, the Germans invaded Denmark. Kerrich spent the rest of the war interned at a camp in Jutland. To pass the time he carried out a series of experiments in probability theory.[1] One experiment involved tossing a coin 10,000 times. With his permission, some of the results are summarized in table 1 and figure 1 (pp. 274–275 below). What do these results say about the law of averages? To find out, let's pretend that at the end of World War II, Kerrich was invited to demonstrate the law of averages to the King of Denmark. He is discussing the invitation with his assistant.

Assistant. So you're going to tell the king about the law of averages.

Kerrich. Right.

Assistant. What's to tell? I mean, everyone knows about the law of averages, don't they?

Kerrich. OK. Tell me what the law of averages says.

Assistant. Well, suppose you're tossing a coin. If you get a lot of heads, then tails start coming up. Or if you get too many tails, the chance for heads goes up. In the long run, the number of heads and the number of tails even out.

Kerrich. It's not true.

Assistant. What do you mean, it's not true?

Kerrich. I mean, what you said is all wrong. First of all, with a fair coin the chance for heads stays at 50%, no matter what happens. Whether there are two heads in a row or twenty, the chance of getting a head next time is still 50%.

Assistant. I don't believe it.

Kerrich. All right. Take a run of four heads, for example. I went through the record of my first 2,000 tosses. In 130 cases, the coin landed heads four times in a row; 69 of these runs were followed by a head, and only 61 by a tail. A run of heads just doesn't make tails more likely next time.

Assistant. You're always telling me these things I don't believe. What are you going to tell the king?

Kerrich. Well, I tossed the coin 10,000 times, and I got about 5,000 heads. The exact number was 5,067. The difference of 67 is less than 1% of the number of tosses. I have the record here in table 1.

Assistant. Yes, but 67 heads is a lot of heads. The king won't be impressed, if that's the best the law of averages can do.

Kerrich. What do you suggest?

Table 1. John Kerrich's coin-tossing experiment. The first column shows the number of tosses. The second shows the number of heads. The third shows the difference

number of heads − half the number of tosses.

Number of tosses	*Number of heads*	*Difference*	*Number of tosses*	*Number of heads*	*Difference*
10	4	−1	600	312	12
20	10	0	700	368	18
30	17	2	800	413	13
40	21	1	900	458	8
50	25	0	1,000	502	2
60	29	−1	2,000	1,013	13
70	32	−3	3,000	1,510	10
80	35	−5	4,000	2,029	29
90	40	−5	5,000	2,533	33
100	44	−6	6,000	3,009	9
200	98	−2	7,000	3,516	16
300	146	−4	8,000	4,034	34
400	199	−1	9,000	4,538	38
500	255	5	10,000	5,067	67

Assistant. Toss the coin another 10,000 times. With 20,000 tosses, the number of heads should be quite a bit closer to the expected number. After all, eventually the number of heads and the number of tails have to even out, right?

Kerrich. You said that before, and it's wrong. Look at table 1. In 1,000 tosses, the difference between the number of heads and the expected number was 2. With 2,000 tosses, the difference went up to 13.

Assistant. That was just a fluke. By toss 3,000, the difference was only 10.

Kerrich. That's just another fluke. At toss 4,000, the difference was 29. At 5,000, it was 33. Sure, it dropped back to 9 at toss 6,000, but look at figure 1. The chance error is climbing pretty steadily from 1,000 to 10,000 tosses, and it's going straight up at the end.

Assistant. So where's the law of averages?

Kerrich. With a large number of tosses, the size of the difference between the number of heads and the expected number is likely to be quite large in absolute terms. But compared to the number of tosses, the difference is likely to be quite small. That's the law of averages. Just like I said, 67 is only a small fraction of 10,000.

Assistant. I don't understand.

Kerrich. Look. In 10,000 tosses you expect to get 5,000 heads, right?

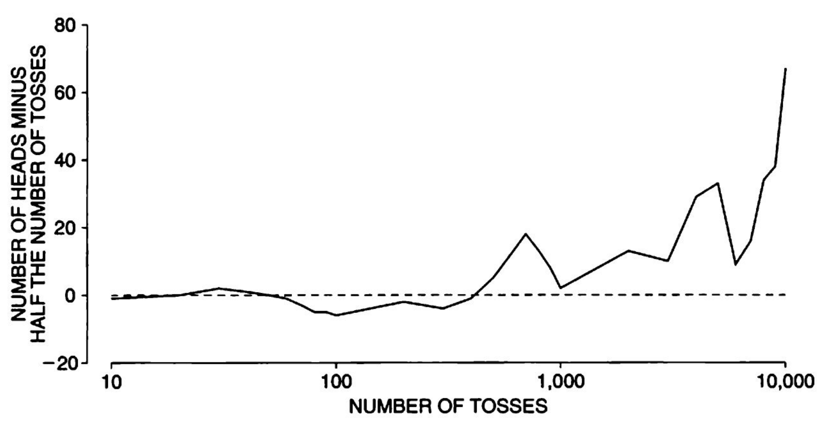
Assistant. Right.

Kerrich. But not exactly. You only expect to get around 5,000 heads. I mean, you could just as well get 5,001 or 4,998 or 5,007. The amount off 5,000 is what we call "chance error."

Figure 1. Kerrich's coin-tossing experiment. The "chance error" is

number of heads − half the number of tosses.

This difference is plotted against the number of tosses. As the number of tosses goes up, the size of the chance error tends to go up. The horizontal axis is not to scale and the curve is drawn by linear interpolation.



Assistant. Can you be more specific?

Kerrich. Let me write an equation:

number of heads = half the number of tosses + chance error.

This error is likely to be large in absolute terms, but small compared to the number of tosses. Look at figure 2. That's the law of averages, right there.

Assistant. Hmmm. But what would happen if you tossed the coin another 10,000 times. Then you'd have 20,000 tosses to work with.

Kerrich. The chance error would go up, but not by a factor of two. In absolute terms, the chance error gets bigger.[2] But as a percentage of the number of tosses, it gets smaller.

Assistant. Tell me again what the law of averages says.

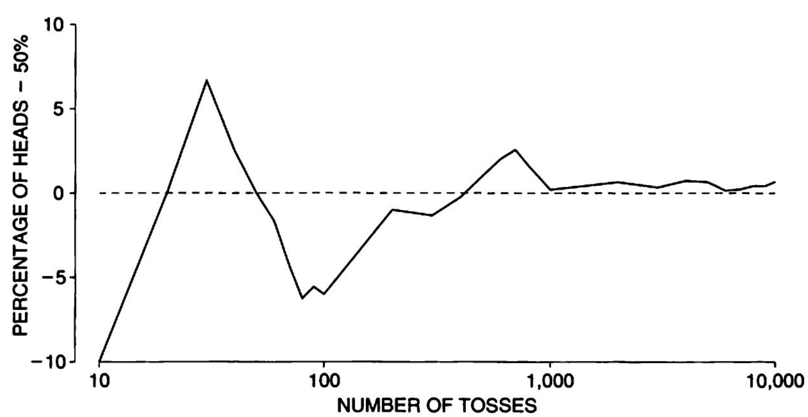
Kerrich. The number of heads will be around half the number of tosses, but it will be off by some amount—chance error. As the number of tosses goes up, the chance error gets bigger in absolute terms. Compared to the number of tosses, it gets smaller.

Assistant. Can you give me some idea of how big the chance error is likely to be?

Kerrich. Well, with 100 tosses, the chance error is likely to be around 5 in size. With 10,000 tosses, the chance error is likely to be around 50 in size. Multiplying the number of tosses by 100 only multiplies the likely size of the chance error by $\sqrt{100} = 10$.

Assistant. What you're saying is that as the number of tosses goes up, the difference between the number of heads and half the number of tosses gets

Figure 2. The chance error expressed as a percentage of the number of tosses. When the number of tosses goes up, this percentage goes down: the chance error gets smaller relative to the number of tosses. The horizontal axis is not to scale and the curve is drawn by linear interpolation.



bigger; but the difference between the percentage of heads and 50% gets smaller.

Kerrich. That's it.

Exercise Set A

1. A machine has been designed to toss a coin automatically and keep track of the number of heads. After 1,000 tosses, it has 550 heads. Express the chance error both in absolute terms and as a percentage of the number of tosses.

2. After 1,000,000 tosses, the machine in exercise 1 has 501,000 heads. Express the chance error in the same two ways.

3. A coin is tossed 100 times, landing heads 53 times. However, the last seven tosses are all heads. True or false: the chance that the next toss will be heads is somewhat less than 50%. Explain.

4. (a) A coin is tossed, and you win a dollar if there are more than 60% heads. Which is better: 10 tosses or 100? Explain.
   (b) As in (a), but you win the dollar if there are more than 40% heads.
   (c) As in (a), but you win the dollar if there are between 40% and 60% heads.
   (d) As in (a), but you win the dollar if there are exactly 50% heads.

5. With a Nevada roulette wheel, there are 18 chances in 38 that the ball will land in a red pocket. A wheel is going to be spun many times. There are two choices:
   (i) 38 spins, and you win a dollar if the ball lands in a red pocket 20 or more times.
   (ii) 76 spins, and you win a dollar if the ball lands in a red pocket 40 or more times.

   Which is better? Or are they the same? Explain.

The next three exercises involve drawing at random from a box. This was described in section 1 of chapter 13 and is reviewed in section 3 below.

6. A box contains 20% red marbles and 80% blue marbles. A thousand marbles are drawn at random with replacement. One of the following statements is true. Which one, and why?
   (i) Exactly 200 marbles are going to be red.
   (ii) About 200 marbles are going to be red, give or take a dozen or so.
7. Repeat exercise 6, if the draws are made at random without replacement and the box contains 50,000 marbles.
8. One hundred tickets will be drawn at random with replacement from one of the two boxes shown below. On each draw, you will be paid the amount shown on the ticket, in dollars. (If a negative number is drawn, that amount will be taken away from you.) Which box is better? Or are they the same?

   (i) $\boxed{-1}\ \boxed{-1}\ \boxed{1}\ \boxed{1}$ (ii) $\boxed{-1}\ \boxed{1}$
9. (Hard.) Look at figure 1. If Kerrich kept on tossing, would the graph ever get negative?

The answers to these exercises are on pp. A71–72.

2. CHANCE PROCESSES

Kerrich's assistant was struggling with the problem of chance variability. He came to see that when a coin is tossed a large number of times, the actual number of heads is likely to differ from the expected number. But he didn't know how big a difference to anticipate. A method for calculating the likely size of the difference will be presented in the next chapter. This method works in many different situations. For example, it can be used to see how much money the house should expect to win at roulette (chapter 17) or how accurate a sample survey is likely to be (chapter 21).

What is the common element? All these problems are about chance processes.[3] Take the number of heads in Kerrich's experiment. Chance comes in with each toss of the coin. If you repeat the experiment, the tosses turn out differently, and so does the number of heads. Second example: the amount of money won or lost at roulette. Spinning the wheel is a chance process, and the amounts won or lost depend on the outcome. Spin again, and winners become losers. A final example: the percentage of Democrats in a random sample of voters. A chance process is used to draw the sample. So the number of Democrats in the sample is determined by the luck of the draw. Take another sample, and the percentages would change.

To what extent are the numbers influenced by chance? This sort of question must be faced over and over again in statistics. A general strategy will be presented in the next few chapters. The two main ideas:

- Find an analogy between the process being studied (sampling voters in the poll example) and drawing numbers at random from a box.

- Connect the variability you want to know about (for example, in the estimate for the Democratic vote) with the chance variability in the sum of the numbers drawn from the box.

The analogy between a chance process and drawing from a box is called a *box model*. The point is that the chance variability in the sum of numbers drawn from a box will be easy to analyze mathematically. More complicated processes can then be dealt with through the analogy.

3. THE SUM OF DRAWS

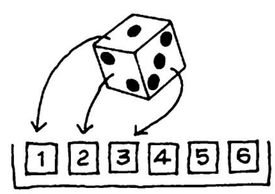
The object of this section is to illustrate the following process. There is a box of tickets. Each ticket has a number written on it. Then some tickets are drawn at random from the box, and the numbers on these tickets are added up. For example, take the box

$$\boxed{1}\ \boxed{2}\ \boxed{3}\ \boxed{4}\ \boxed{5}\ \boxed{6}$$

Imagine drawing twice at random with replacement from this box. You shake the box to mix up the tickets, pick one ticket at random, make a note of the number on it, put it back in the box. Then you shake the box again, and make a second draw at random. The phrase "with replacement" reminds you to put the ticket back in the box before drawing again. Putting the tickets back enables you to draw over and over again, under the same conditions. (Drawing with and without replacement was discussed in section 1 of chapter 13.)

Having drawn twice at random with replacement, you add up the two numbers. For example, the first draw might be $\boxed{3}$ and the second $\boxed{5}$. Then the sum of the draws is 8. Or the first draw might be $\boxed{3}$ and the second $\boxed{3}$ too, so the sum of the draws is 6. There are many other possibilities. The sum is subject to chance variability. If the draws turn out one way, the sum is one thing; if they turn out differently, the sum is different too.

At first, this example may seem artificial. But it is just like a turn at Monopoly—you roll a pair of dice, add up the two numbers, and move that many squares. Rolling a die is just like picking a number from the box.



Next, imagine taking 25 draws from the same box

$$\boxed{1}\ \boxed{2}\ \boxed{3}\ \boxed{4}\ \boxed{5}\ \boxed{6}$$

Of course, the draws must be made with replacement. About how big is their sum going to be? The most direct way to find out is by experiment. We programmed the computer to make the draws.[4] It got 3 on the first draw, 2 on the second, 4 on the third. Here they all are:

3 2 4 6 2   3 5 4 4 2   3 6 4 1 2   4 1 5 5 6   2 2 2 5 5

The sum of these 25 draws is 88.

Of course, if the draws had been different, their sum would have been different. So we had the computer repeat the whole process ten times. Each time, it made 25 draws at random with replacement from the box, and took their sum. The results:

88 84 80 90 83 78 95 94 80 89

Chance variability is easy to see. The first sum is 88, the second drops to 84, the third drops even more to 80. The values range from a low of 78 to a high of 95.

In principle, the sum could have been as small as $25 \times 1 = 25$, or as large as $25 \times 6 = 150$. But in fact, the ten observed values are all between 75 and 100. Would this keep up with more repetitions? Just what is the chance that the sum turns out to be between 75 and 100? That kind of problem will be solved in the next two chapters.

The *sum of the draws* from a box is shorthand for the process discussed in this section:

- Draw tickets at random from a box.
- Add up the numbers on the tickets.[5]

Exercise Set B

1. One hundred draws are made at random with replacement from the box $\boxed{\boxed{1}\,\boxed{2}}$. Forty-seven draws turn out to be $\boxed{1}$, and the remaining 53 are $\boxed{2}$. How much is the sum?

2. One hundred draws are made at random with replacement from the box $\boxed{\boxed{1}\,\boxed{2}}$.
   (a) How small can the sum be? How large?
   (b) How many times do you expect the ticket $\boxed{1}$ to turn up? The ticket $\boxed{2}$?
   (c) About how much do you expect the sum to be?

3. One hundred draws are made at random with replacement from the box $\boxed{\boxed{1}\,\boxed{2}\,\boxed{9}}$.
   (a) How small can the sum be? How large?
   (b) About how much do you expect the sum to be?

4. One hundred draws will be made at random with replacement from one of the following boxes. Your job is to guess what the sum will be, and you win $1 if you are right to within 10. In each case, what would you guess? Which box is best? Worst?

   (i) $\boxed{\boxed{1}\,\boxed{9}}$ (ii) $\boxed{\boxed{4}\,\boxed{6}}$ (iii) $\boxed{\boxed{5}\,\boxed{5}}$

5. One ticket will be drawn at random from the box

   $\boxed{\boxed{1}\,\boxed{2}\,\boxed{3}\,\boxed{4}\,\boxed{5}\,\boxed{6}\,\boxed{7}\,\boxed{8}\,\boxed{9}\,\boxed{10}}$

What is the chance that it will be 1? That it will be 3 or less? 4 or more?

6. Fifty draws will be made at random with replacement from one of the two boxes shown below. On each draw, you will be paid in dollars the amount shown on the ticket; if a negative number is drawn, that amount will be taken away from you. Which box is better? Or are they the same? Explain.

   (i) $\boxed{-1}\ \boxed{2}$ (ii) $\boxed{-1}\ \boxed{-1}\ \boxed{2}$

7. You gamble four times at a casino. You win \$4 on the first play, lose \$2 on the second, win \$5 on the third, lose \$3 on the fourth. Which of the following calculations tells how much you come out ahead? (More than one may be correct.)
   (i) $\$4 + \$5 - (\$2 + \$3)$
   (ii) $\$4 + (-\$2) + \$5 + (-\$3)$
   (iii) $\$4 + \$2 + \$5 - \$3$
   (iv) $-\$4 + \$2 + \$5 + \$3$

The answers to these exercises are on p. A72.

4. MAKING A BOX MODEL

The object of this section is to make some box models, as practice for later. The sum of the draws from the box turns out to be the key ingredient for many statistical procedures, so keep your eye on the sum. There are three questions to answer when making a box model:

- What numbers go into the box?
- How many of each kind?
- How many draws?

The purpose of a box model is to analyze chance variability, which can be seen in its starkest form at any gambling casino. So this section will focus on box models for roulette. A Nevada roulette wheel has 38 pockets. One is numbered 0, another is numbered 00, and the rest are numbered from 1 through 36. The croupier spins the wheel, and throws a ball onto the wheel. The ball is equally likely to land in any one of the 38 pockets. Before it lands, bets can be placed on the table (figure 3 on the next page).

One bet is *red or black*. Except for 0 and 00, which are colored green, the numbers on the roulette wheel alternate red and black. If you bet a dollar on red, say, and a red number comes up, you get the dollar back together with another dollar in winnings. If a black or green number comes up, the croupier smiles and rakes in your dollar.

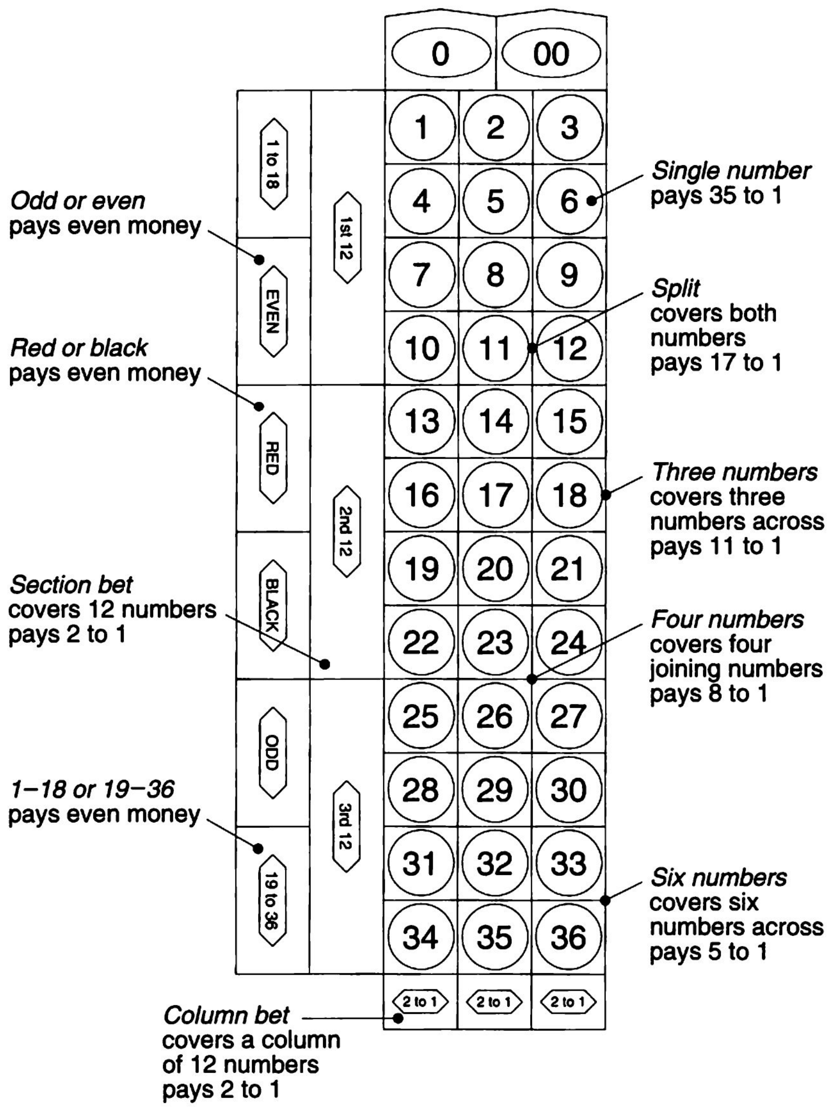
Suppose you are at the Golden Nugget in Las Vegas. You have just put a dollar on red, and the croupier spins the wheel. It may seem hard to figure your chances, but a box model will help. What numbers go into the box? You will either win a dollar or lose a dollar. So the tickets must show either $+\$1$ or $-\$1$.

The second question is, how many of each kind? You win if one of the 18 red numbers comes up, and lose if one of the 18 black numbers comes up. But you also lose if 0 or 00 come up. And that is where the house gets its edge. Your chance of winning is only 18 in 38, and the chance of losing is 20 in 38. So there are 18 $\boxed{+\$1}$'s and 20 $\boxed{-\$1}$'s. The box is

$$\boxed{\text{18 tickets } \boxed{+\$1} \quad \text{20 tickets } \boxed{-\$1}}$$

As far as the chances are concerned, betting a dollar on red is just like drawing a ticket at random from the box. The great advantage of the box model is that all

Figure 3. A Nevada roulette table.



Roulette is a pleasant, relaxed, and highly comfortable way to lose your money.
—JIMMY THE GREEK

the irrelevant details—the wheel, the table, and the croupier's smile—have been stripped away. And you can see the cruel reality: you have 18 tickets, they have 20.

That does one play. But suppose you play roulette ten times, betting a dollar on red each time. What is likely to happen then? You will end up ahead or behind by some amount. This amount is called your *net gain*. The net gain is positive if you come out ahead, negative if you come out behind.

To figure the chances, the net gain has to be connected to the box. On each play, you win or lose some amount. These ten win-lose numbers are like ten draws from the box, made at random with replacement. (Replacing the tickets keeps the chances on each draw the same as the chances for the wheel.) The net gain—the total amount won or lost—is just the sum of these ten win-lose numbers. Your net gain in ten plays is like the sum of ten draws made at random with replacement from the box

| 18 tickets $\boxed{+\$1}$ 20 tickets $\boxed{-\$1}$ |

This is our first model, so it is a good idea to look at it more closely. Suppose, for instance, that the ten plays came out this way:

R R R B G R R B B R

(R means red, B means black, and G means green—the house numbers 0 and 00). Table 2 below shows the ten corresponding win-lose numbers, and the net gain.

Table 2. The net gain. This is the cumulative sum of the win-lose numbers.

Plays	R	R	R	B	G	R	R	B	B	R
Win-lose numbers	+1	+1	+1	−1	−1	+1	+1	−1	−1	+1
Net gain	1	2	3	2	1	2	3	2	1	2

Follow the net gain along. When you get a red, the win-lose number is +1, and the net gain goes up by 1. When you get a black or a green, the win-lose number is −1, and the net gain goes down by 1. The net gain is just the sum of the win-lose numbers, and these are like the draws from the box. That is why the net gain is like the sum of draws from the box. This game had a happy ending: you came out ahead $2. To see what would happen if you kept on playing, read the next chapter.

Example 1. If you bet a dollar on a single number at Nevada roulette, and that number comes up, you get the $1 back together with winnings of $35. If any other number comes up, you lose the dollar. Gamblers say that a single number *pays 35 to 1*. Suppose you play roulette 100 times, betting a dollar on the number 17 each time. Your net gain is like the sum of ______ draws made at random with replacement from the box ______. Fill in the blanks.

Solution. What numbers go into the box? To answer this question, think about one play of the game. You put a dollar chip on 17. If the ball drops into the pocket 17, you'll be up $35. If it drops into any other pocket, you'll be down $1. So the box has to contain the tickets $\boxed{\$35}$ and $\boxed{-\$1}$.

The tickets in the box show the various amounts that can be won or lost on a single play.

How many tickets of each kind? Keep thinking about one play. You have only 1 chance in 38 of winning, so the chance of drawing $\boxed{\$35}$ has to be 1 in 38. You have 37 chances in 38 of losing, so the chance of drawing $\boxed{-\$1}$ has to be 37 in 38. The box is

$$\underline{\text{| 1 ticket } \boxed{+\$35} \quad \text{37 tickets } \boxed{-\$1}\text{ |}}$$

The chance of drawing any particular number from the box must equal the chance of winning that amount on a single play. ("Winning" a negative amount is the mathematical equivalent of what most people call losing.)

How many draws? You are playing 100 times. The number of draws has to be 100. Tickets must be replaced after each draw, so as not to change the odds.

The number of draws equals the number of plays.

So, the net gain in 100 plays is like the sum of 100 draws made at random with replacement from the box

$$\underline{\text{| 1 ticket } \boxed{+\$35} \quad \text{37 tickets } \boxed{-\$1}\text{ |}}$$

This completes the solution.

Exercise Set C

1. Consider the following three situations.
   (i) A box contains one ticket marked "0" and nine marked "1." A ticket is drawn at random. If it shows "1" you win a panda bear.
   (ii) A box contains ten tickets marked "0" and ninety marked "1." One ticket is drawn at random. If it shows "1" you win the panda.
   (iii) A box contains one ticket marked "0" and nine marked "1." Ten draws are made at random with replacement. If the sum of the draws equals 10, you win the panda.

   Assume you want the panda. Which is better—(i) or (ii)? Or are they the same? What about (i) and (iii)?

2. A gambler is going to play roulette 25 times, putting a dollar on a *split* each time. (A split is two adjacent numbers, like 11 and 12 in figure 3 on p. 282.) If either

number comes up, the gambler gets the dollar back, together with winnings of $17. If neither number comes up, he loses the dollar. So a split pays 17 to 1, and there are 2 chances in 38 to win. The gambler's net gain in the 25 plays is like the sum of 25 draws made from one of the following boxes. Which one, and why?

(i) | $\boxed{0}$ $\boxed{00}$ 36 tickets numbered $\boxed{1}$ through $\boxed{36}$ |

(ii) | $\boxed{\$17}$ $\boxed{\$17}$ 34 tickets $\boxed{-\$1}$ |

(iii) | $\boxed{\$17}$ $\boxed{\$17}$ 36 tickets $\boxed{-\$1}$ |

3. In one version of chuck-a-luck, 3 dice are rolled out of a cage. You can bet that all 3 show six. The house pays 36 to 1, and the bettor has 1 chance in 216 to win. Suppose you make this bet 10 times, staking $1 each time. Your net gain is like the sum of ______ draws made at random with replacement from the box ______. Fill in the blanks.

The answers to these exercises are on p. A72.

5. REVIEW EXERCISES

1. A box contains 10,000 tickets: 4,000 $\boxed{0}$'s and 6,000 $\boxed{1}$'s. And 10,000 draws will be made at random with replacement from this box. Which of the following best describes the situation, and why?
   (i) The number of 1's will be 6,000 exactly.
   (ii) The number of 1's is very likely to equal 6,000, but there is also some small chance that it will not be equal to 6,000.
   (iii) The number of 1's is likely to be different from 6,000, but the difference is likely to be small compared to 10,000.

2. Repeat exercise 1 for 10,000 draws made at random without replacement from the box.

3. A gambler loses ten times running at roulette. He decides to continue playing because he is due for a win, by the law of averages. A bystander advises him to quit, on the grounds that his luck is cold. Who is right? Or are both of them wrong?

4. (a) A die will be rolled some number of times, and you win $1 if it shows an ace ($\boxed{\cdot}$) more than 20% of the time. Which is better: 60 rolls, or 600 rolls? Explain.
   (b) As in (a), but you win the dollar if the percentage of aces is more than 15%.
   (c) As in (a), but you win the dollar if the percentage of aces is between 15% and 20%.
   (d) As in (a), but you win the dollar if the percentage of aces is exactly $16\frac{2}{3}\%$.

5. True or false: if a coin is tossed 100 times, it is not likely that the number of heads will be exactly 50, but it is likely that the percentage of heads will be exactly 50%. Explain.

6. According to genetic theory, there is very close to an even chance that both children in a two-child family will be of the same sex. Here are two possibilities.

   (i) 15 couples have two children each. In 10 or more of these families, it will turn out that both children are of the same sex.
   (ii) 30 couples have two children each. In 20 or more of these families, it will turn out that both children are of the same sex.

   Which possibility is more likely, and why?

7. A quiz has 25 multiple choice questions. Each question has 5 possible answers, one of which is correct. A correct answer is worth 4 points, but a point is taken off for each incorrect answer. A student answers all the questions by guessing at random. The score will be like the sum of ______ draws from the box ______. Fill in the first blank with a number and the second with a box of tickets. Explain your answers.

8. A gambler will play roulette 50 times, betting a dollar on four joining numbers each time (like 23, 24, 26, 27 in figure 3, p. 282). If one of these four numbers comes up, she gets the dollar back, together with winnings of \$8. If any other number comes up, she loses the dollar. So this bet pays 8 to 1, and there are 4 chances in 38 of winning. Her net gain in 50 plays is like the sum of ______ draws from the box ______. Fill in the blanks; explain.

9. A box contains red and blue marbles; there are more red marbles than blue ones. Marbles are drawn one at a time from the box, at random with replacement. You win a dollar if a red marble is drawn more often than a blue one.[6] There are two choices:

   (A) 100 draws are made from the box.
   (B) 200 draws are made from the box.

   Choose one of the four options below; explain your answer.

   (i) A gives a better chance of winning.
   (ii) B gives a better chance of winning.
   (iii) A and B give the same chance of winning.
   (iv) Can't tell without more information.

10. Two hundred draws will be made at random with replacement from the box $\boxed{-3}\ \boxed{-2}\ \boxed{-1}\ \boxed{0}\ \boxed{1}\ \boxed{2}\ \boxed{3}$.

    (a) If the sum of the 200 numbers drawn is 30, what is their average?
    (b) If the sum of the 200 numbers drawn is -20, what is their average?
    (c) In general, how can you figure the average of the 200 draws, if you are told their sum?
    (d) There are two alternatives:
        (i) winning \$1 if the sum of the 200 numbers drawn is between -5 and $+5$.
        (ii) winning \$1 if the average of the 200 numbers drawn is between -0.025 and $+0.025$.

    Which is better, or are they the same? Explain.

6. SUMMARY

1. There is *chance error* in the number of heads:

number of heads = half the number of tosses + chance error.

The error is likely to be large in absolute terms, but small relative to the number of tosses. That is the *law of averages*.

2. The law of averages can be stated in percentage terms. With a large number of tosses, the percentage of heads is likely to be close to 50%, although it is not likely to be exactly equal to 50%.

3. The law of averages does not work by changing the chances. For example, after a run of heads in coin tossing, a head is still just as likely as a tail.

4. A complicated chance process for generating a number can often be modeled by drawing from a box. The sum of the draws is a key ingredient.

5. The basic questions to ask when making a box model:

- Which numbers go into the box?
- How many of each kind?
- How many draws?

6. For gambling problems in which the same bet is made several times, a box model can be set up as follows:

- The tickets in the box show the amounts that can be won (+) or lost (−) on each play.
- The chance of drawing any particular value from the box equals the chance of winning that amount on a single play.
- The number of draws equals the number of plays.

Then, the *net gain* is like the sum of the draws from the box.

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