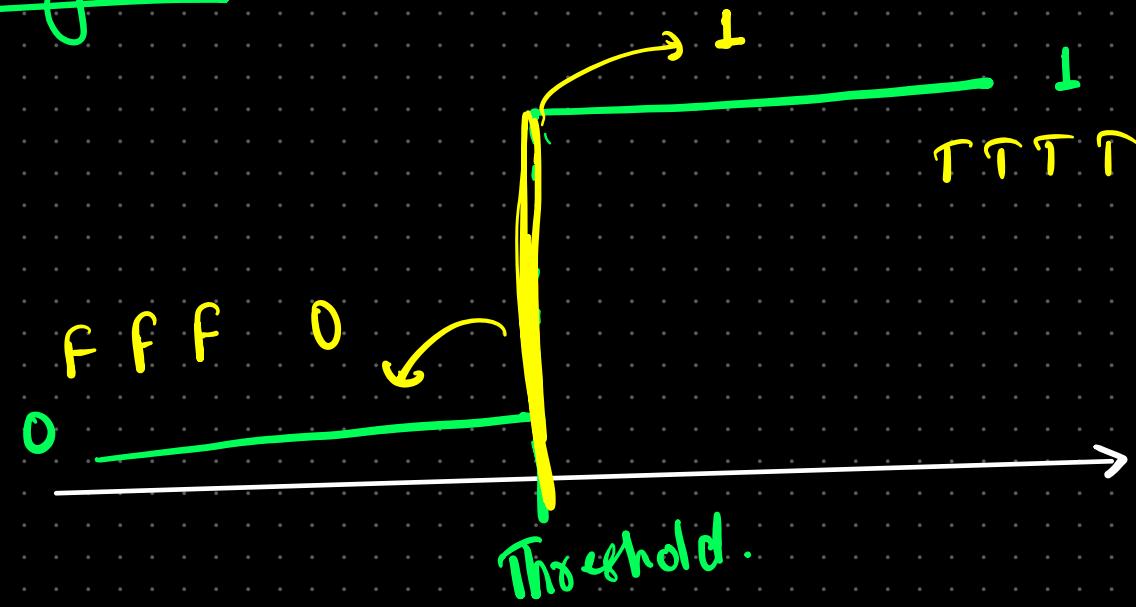
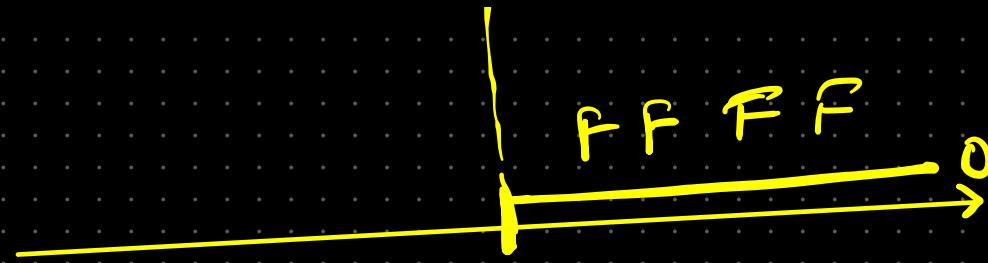


Day 2: Binary Search:

Predicate f_x :



L. T T T T



~~any f_x~~ :

$T T T T | F F F F$
 ↑ Threshold

OS
 $F F F F | T T T T$

0 1 2 2 3 4 5 6

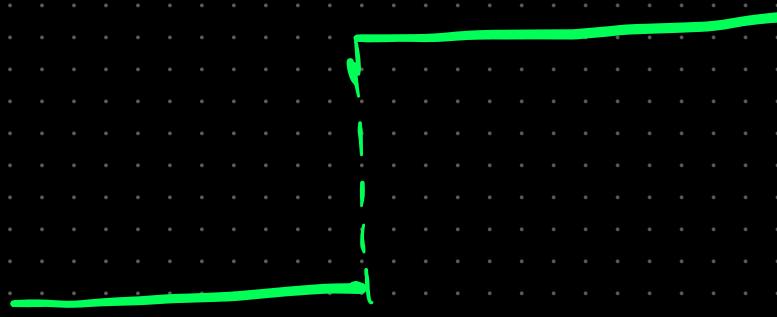
1 2 3 5 7 9 11

target = 3

start = 0

end = 6.

mid = $6/2 = 3$.



if (arr[mid] >= target) {

ans = mid.

end = mid - 1

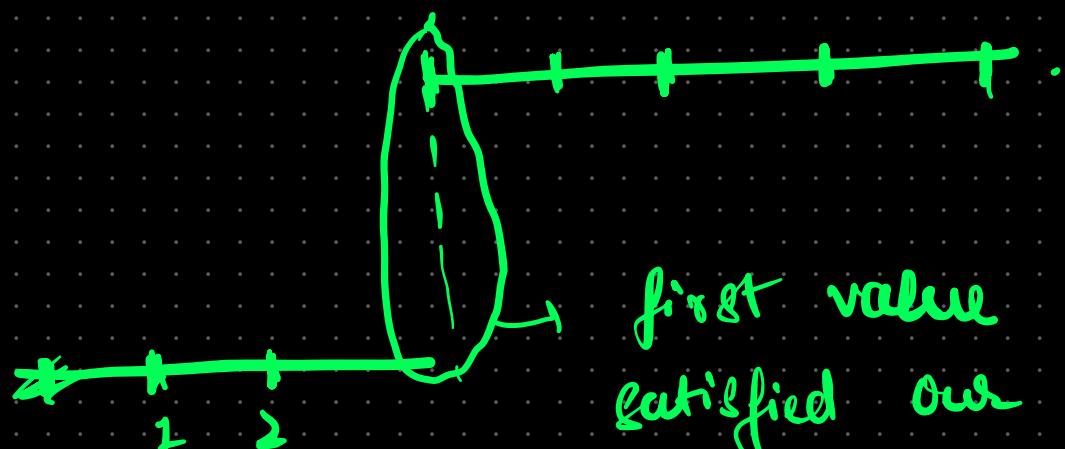
2

3

else $\text{start} = \text{mid} + 1$.

What is predicate here?

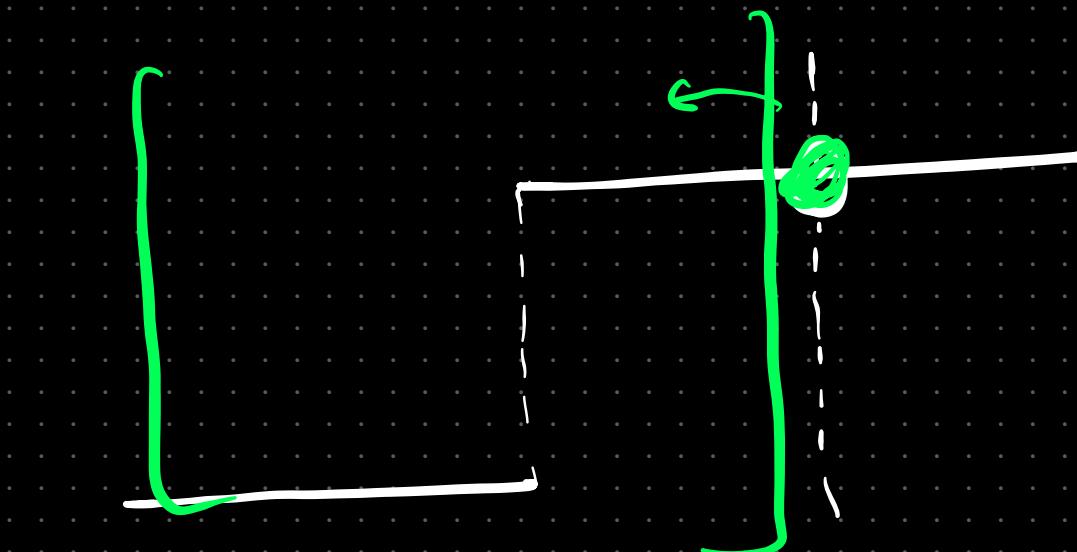
is the element in array
~~target~~ indeed at $\text{mid} \geq \text{target}$?



first value which
satisfied our predicate
in our

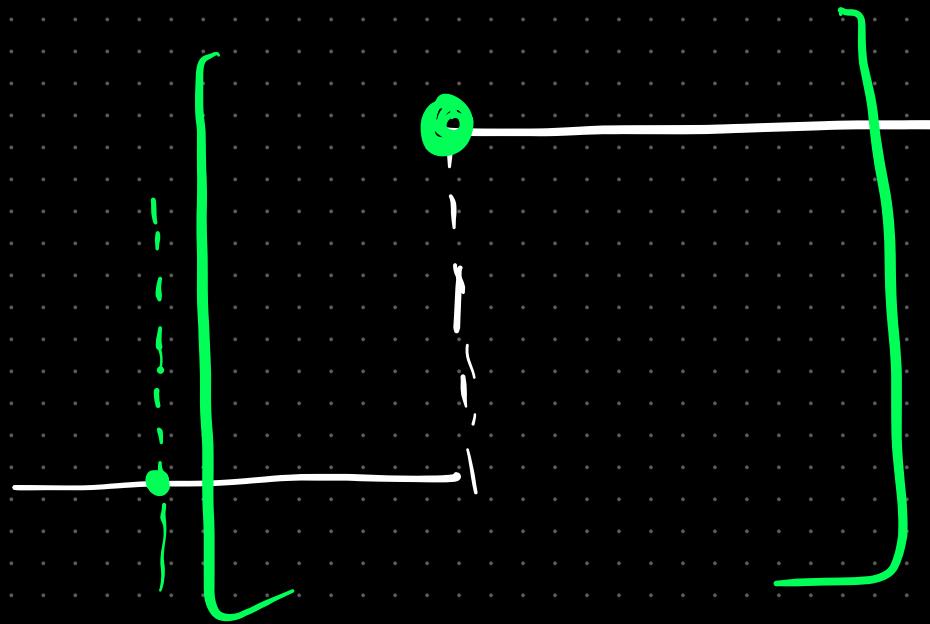
1 2 3 5 7 9 11 " answer.

boolean: True that our $f(x)$ is correct.
(assumption about mid is correct)



if correct -

$$\text{end} = \text{mid} - 1$$



start = mid + 1.

start = 0

λ

end = 1e6 = 10⁶

while (start <= end) {

mid = (start + end) / 2.

if (predicate(mid) == True) {

ans = mid.

start = mid + 1 / end = mid - 1

}.

else {

$\text{start} = \text{mid} + 1$ or $\text{end} = \text{mid} - 1$;

}

$m^2 \rightarrow n \log n$.

Question:

odd sized array (unsorted).

integers - K.

you can apply K operation.

arr [a₁ a₂ a₃ a₄ a₅].

one op: [a₁ a₂₊₁ a₃ a₄ a₅].

Aim: We have to maximize the median of this array.

$$n = 5$$

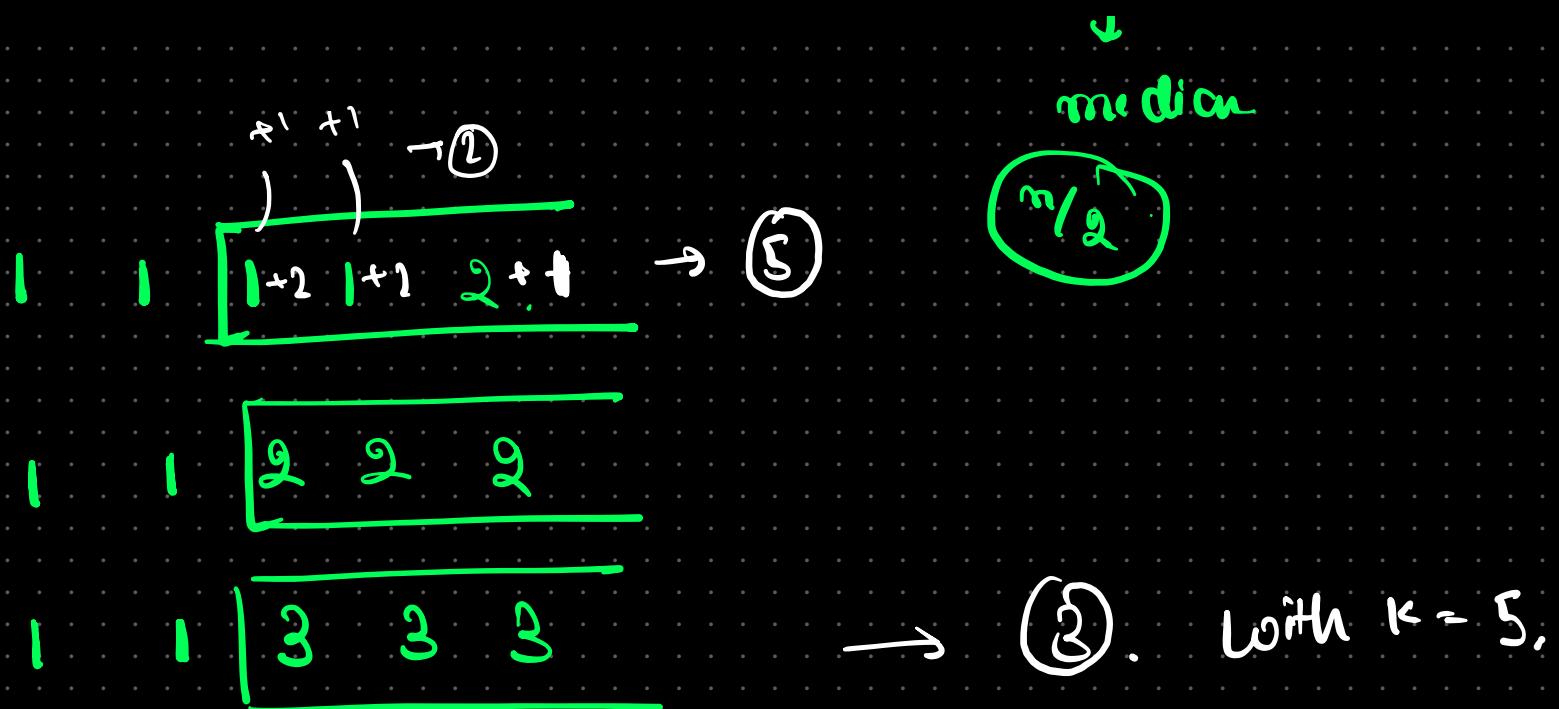
$$k = 5$$

1 2 1 4 1



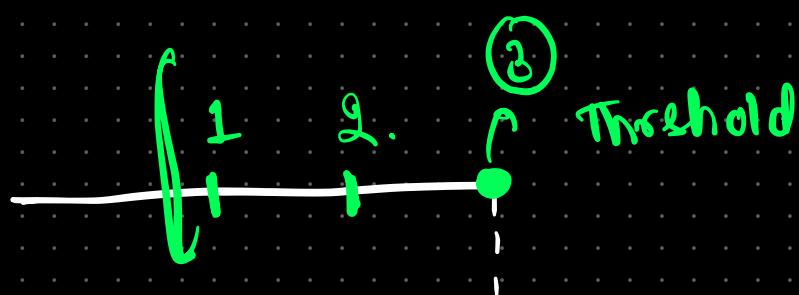
median :

1 1 2 3 4



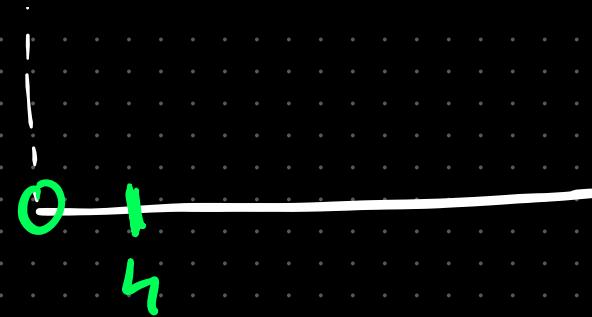
given K operation only:

$$K = 5 \rightarrow \textcircled{3}$$

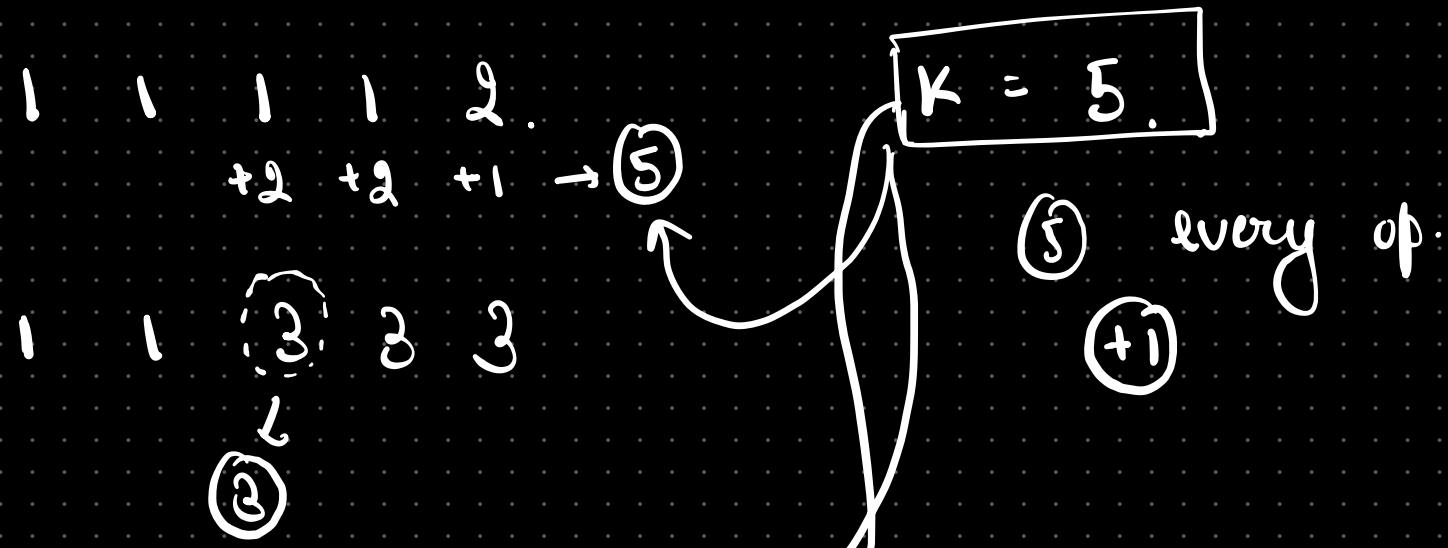


for $K=5$
median = 2

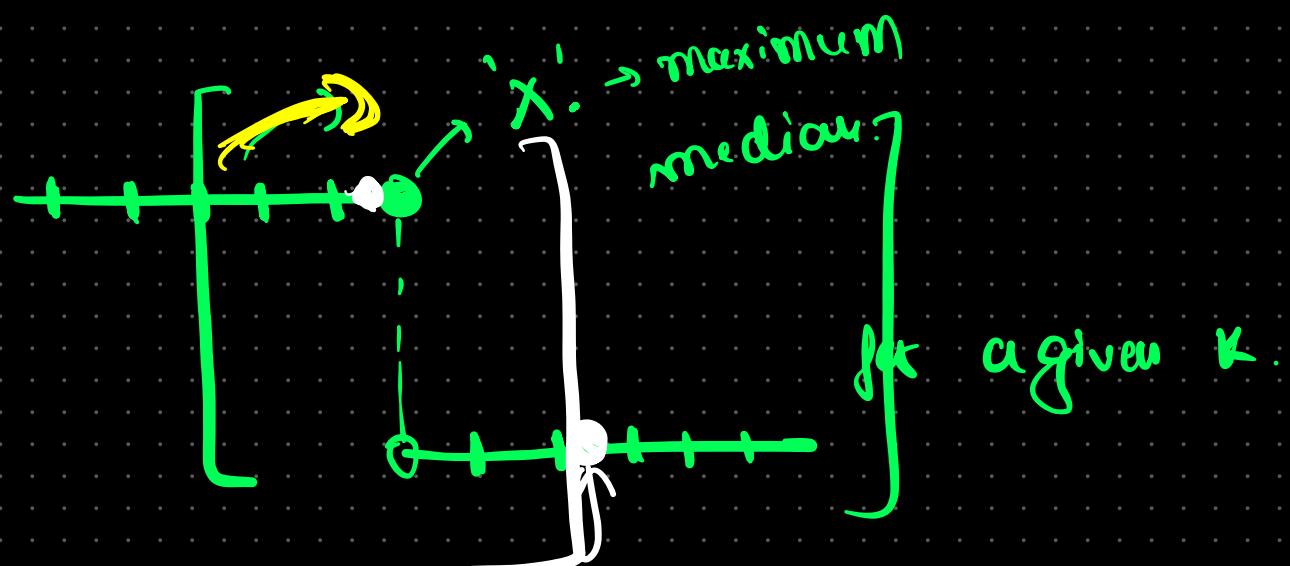
maximum - 7
not possible



Predicate: is median of 'x' possible
with given 'K'.
(x maximum)



$\begin{matrix} 1 & 1 & 1 & 1 & 2 \\ +3 & +3 & +2 \end{matrix} \rightarrow$ & only $K = 5$ allowed.
 $\alpha.$



for $K = 5$
 we can make
 max med = 5.

1 1 1 1 1 2

K = 5.

{ start = 1] → range b/w which our
end = 9 [max median lies.

main

[1 1]

'x' ≤ x ≤ x.
[1 1 2]

sum = 0

if (num[i] < x)

sum += (5 - 1)

= 4

[Sum = 4]

mid = $\frac{(a+1)}{2}$
= '5'.
↓

(assumed
median
'x').

$$\text{Sum} + = (S-1) = 4+4 \rightarrow 8$$

$$\begin{aligned}\text{Sum} + &= (S-2) = 4+4+3 \\ &= 11\end{aligned}$$

$$\boxed{\text{Sum} = 11} \times$$

$$\boxed{K = 5}$$

$$\boxed{\text{end} = \text{mid} - 1}$$

$$\boxed{\text{Sum} \leq K}$$

$$\boxed{\text{ans} = \text{mid}}$$

We know:



$\Theta(n^2)$

$\log n + n$

$m \log n$

$\xrightarrow{\text{ans}}$ \rightarrow max median

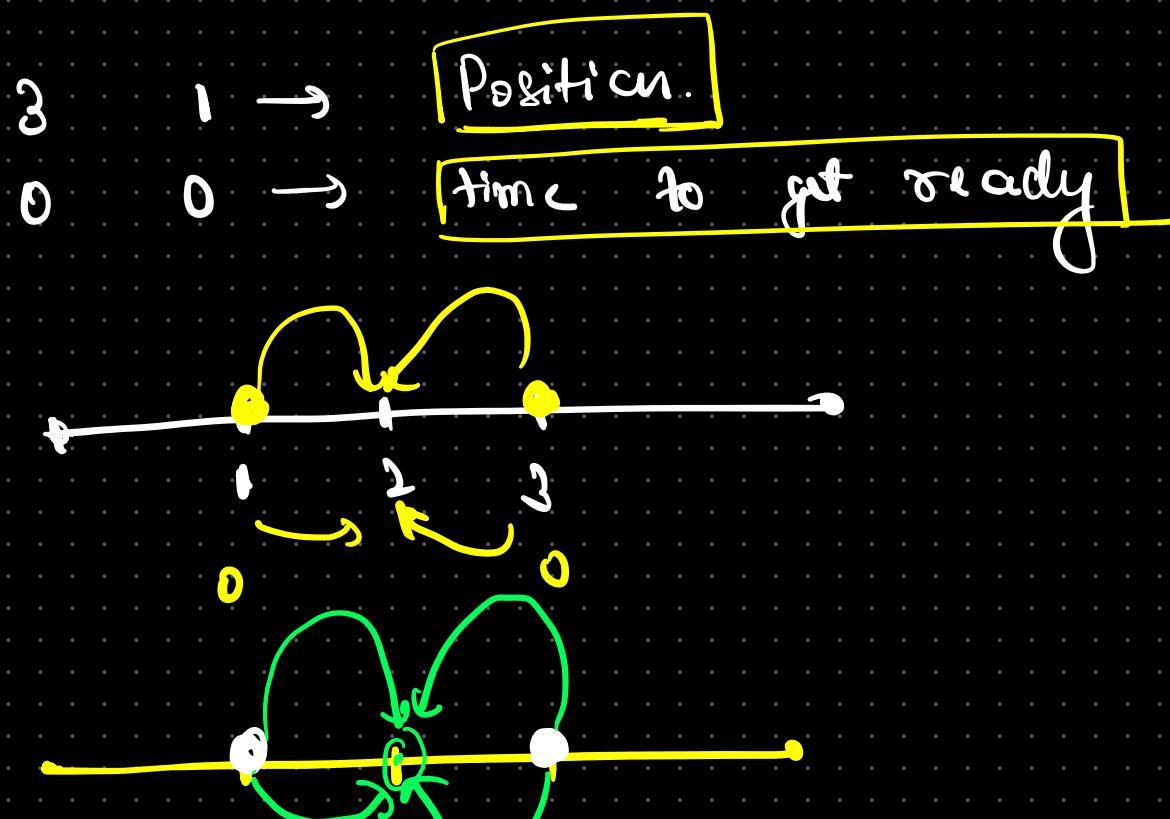
$\boxed{\text{mid}}$

Question :

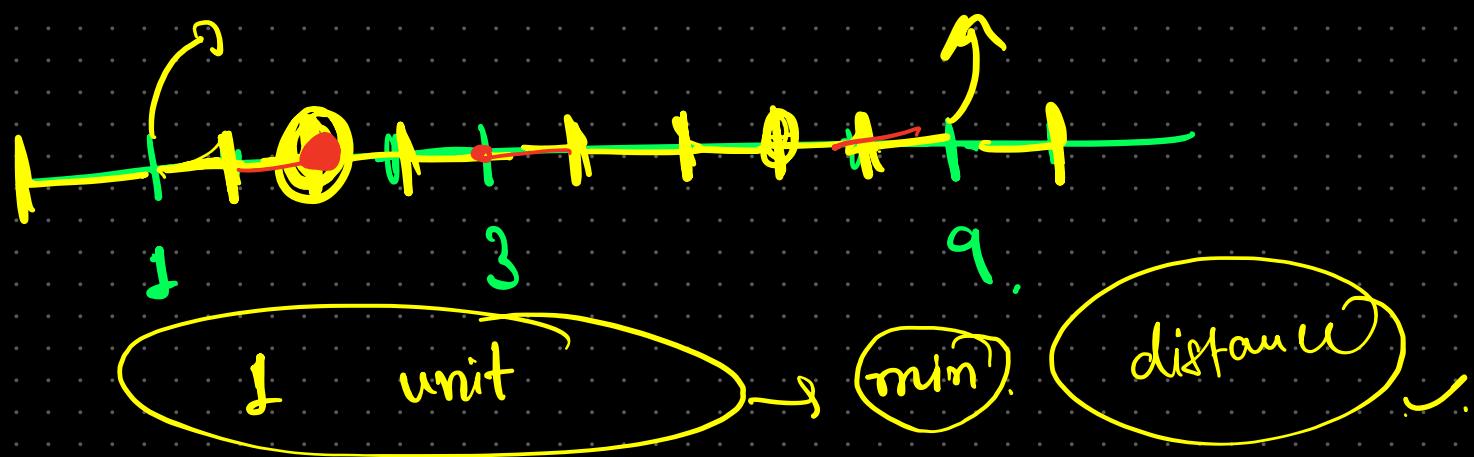


all must come to a single point
for meeting.

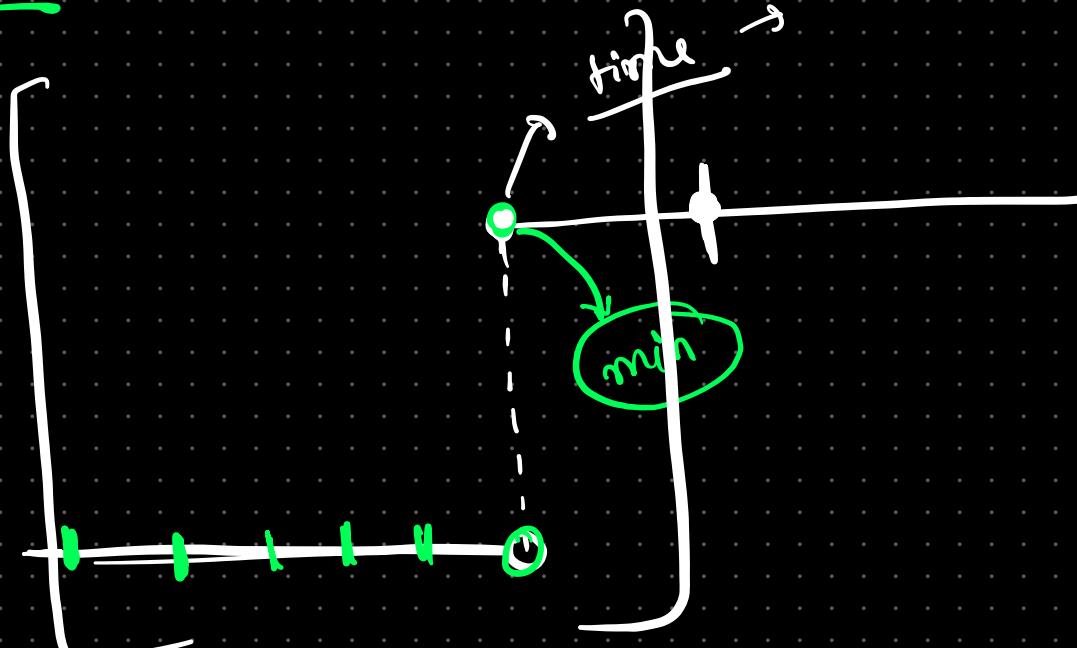
ti



1 — 2 3
max time \rightarrow 1 unit.



Predicate:



$$[1, 5] \cap [4, a] \cap [11, 15].$$

intervallchen

$$\ell = -1e9 = -10^9$$

$$\sigma = +1e9 = 10^9$$

val
[ℓ, σ]

$$A \cap B = \emptyset$$

~~Intersection~~

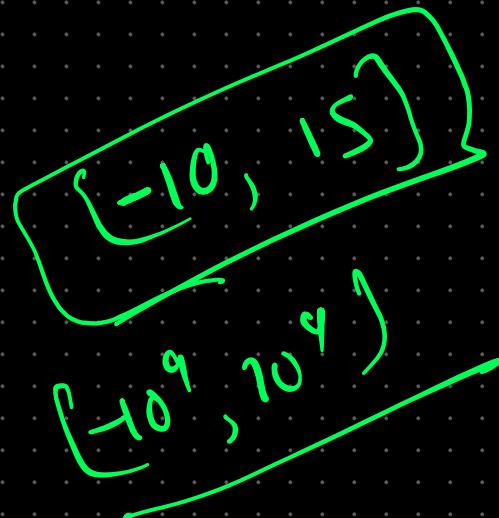
$$\{1, 5\}$$

$$[\ell, \delta] \rightarrow [-10^9, 10^9]$$

' \emptyset '

$$[-10^9, 10^9]$$

range
{U} → intersection



$$(1, 5)$$

$$(4, 5)$$

$$(11, 15]$$

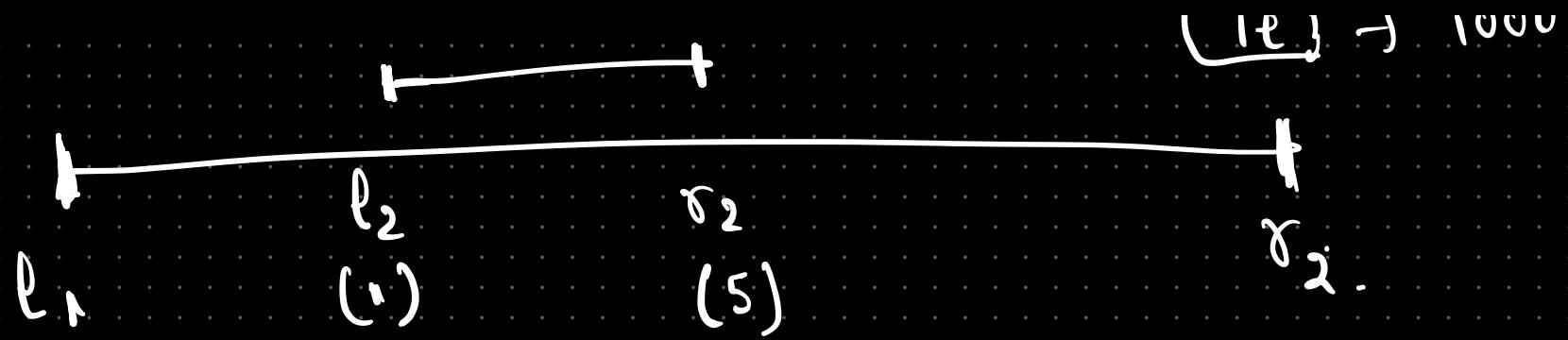
$$\ell = -10^9$$

$$\delta = 10^9$$

$$10^9 = 10^9$$

$$10^1 \rightarrow 10$$

$$10^2 \rightarrow 100$$



$$\text{new-left} = \max(\text{left}, l_2)$$

$$= \max(-1e9, 1) = 1.$$

$$\text{new-left} = 1.$$

$$\text{new-right} = \min(\text{right}, r_2)$$

$$= \min(1e9, 5) = 5.$$

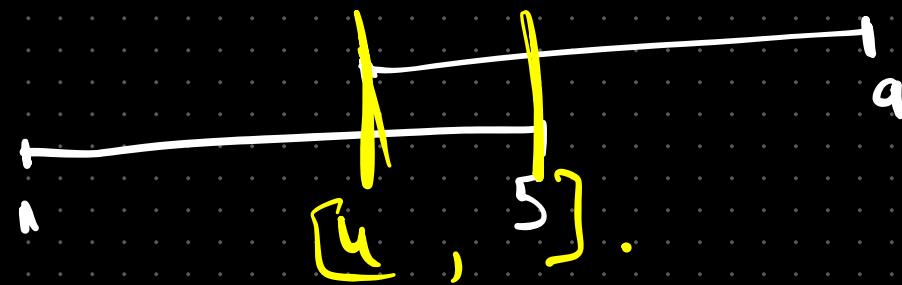
~~1000~~

$$\text{new } (l, r) = \underline{[1, 5]} \rightarrow [\underline{[4, 9]}]$$

$$\text{new-left} = \max(1, 4) = 4.$$

$$\text{new-right} = \min(5, 9) = 5$$

$$\text{new inter} = [4, 5].$$



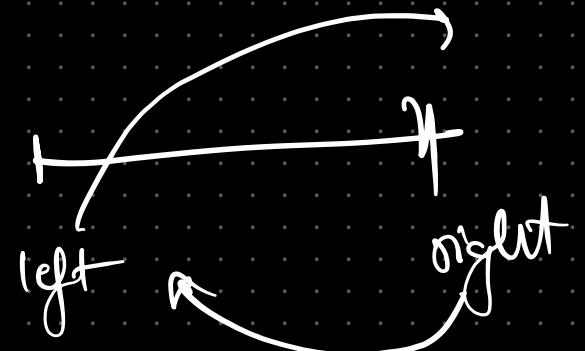
$$\text{range} \rightarrow [4, 5] \cap [11, 15].$$

$$\text{new_left} = \max(4, 11) = 11$$

$$\text{new_right} = \min(5, 15) = 5$$

new left > new right.

left > right



When ever :

left > right

→ null ' \emptyset '
 $\{\emptyset\}$.

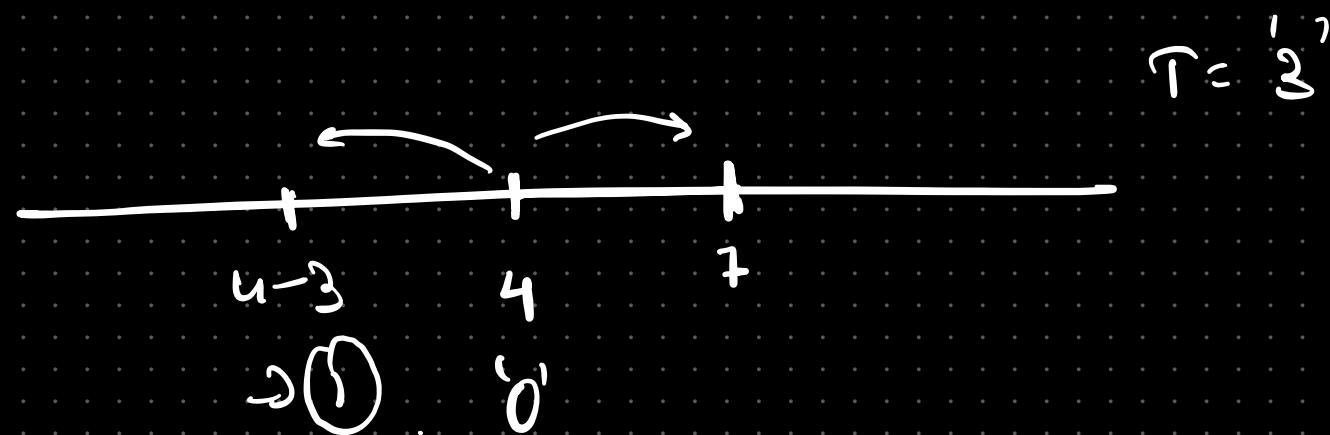
predicate: that is the time ' T ' possible
for everyone to meet at
a point.

$$n_1 \quad n_2 \quad n_3. \quad [^T].$$

$$\begin{aligned} & (n_1 - T, \quad n_1 + T) \\ & (n_2 - T, \quad n_2 + T) \\ & (n_3 - T, \quad n_3 + T) \end{aligned}$$

intersection
of all $\{ \}$
to done.

if intersection $\{\emptyset\}$



$[1, 7] \rightarrow$ anywhere b/w
 $[1, 7]$.

True $(l, r) \rightarrow \boxed{l \leq r} \rightarrow$

if $\boxed{l > r} \rightarrow \emptyset \rightarrow \boxed{\text{False}}$.

start = 0

end = $1e9$

while (Start <= $\frac{\text{end}}{2}$)

mid = (start + end) / 2.

if (pred (mid) == true) {

end = mid - 1 ;

}

else start = mid + 1;

Predicate (mid) { $t_i + (n_i - n_0)$

$l = -1e9$

$r = 1e9$

for ($i = 0 \rightarrow n$) {

$l = \max(l, \text{Pos}[i] - (\text{mid} - t[i]))$

$r = \min(r, \text{Pos}[i] + (\text{mid} - t[i]))$

if ($l > r$) return false

}

return True;

min time:

$$l = -1e9$$

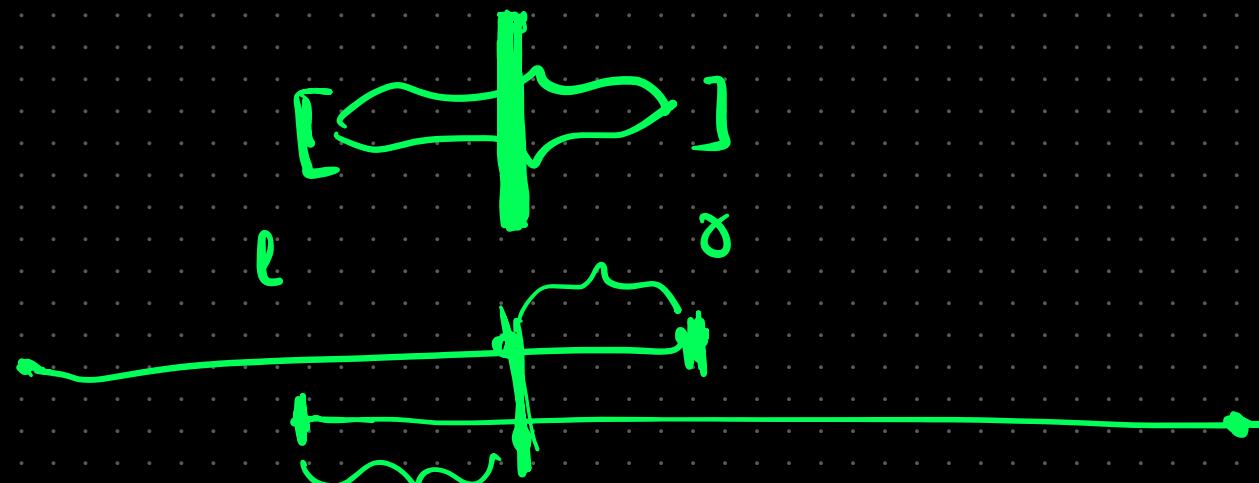
$$r = 1e9$$

positions

$$(x[i] + \tau, n[i] + \tau)$$

$$(l, r)$$

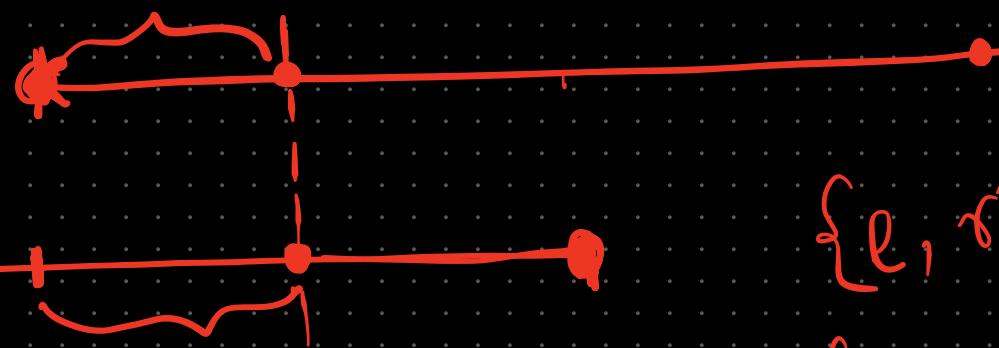
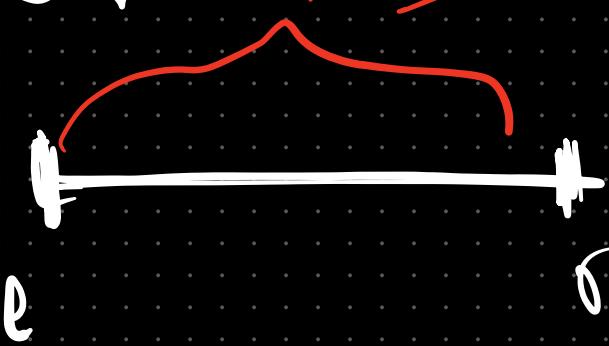
$$\frac{(l+r)}{2}$$



min time

$[l, \delta]$

meeting



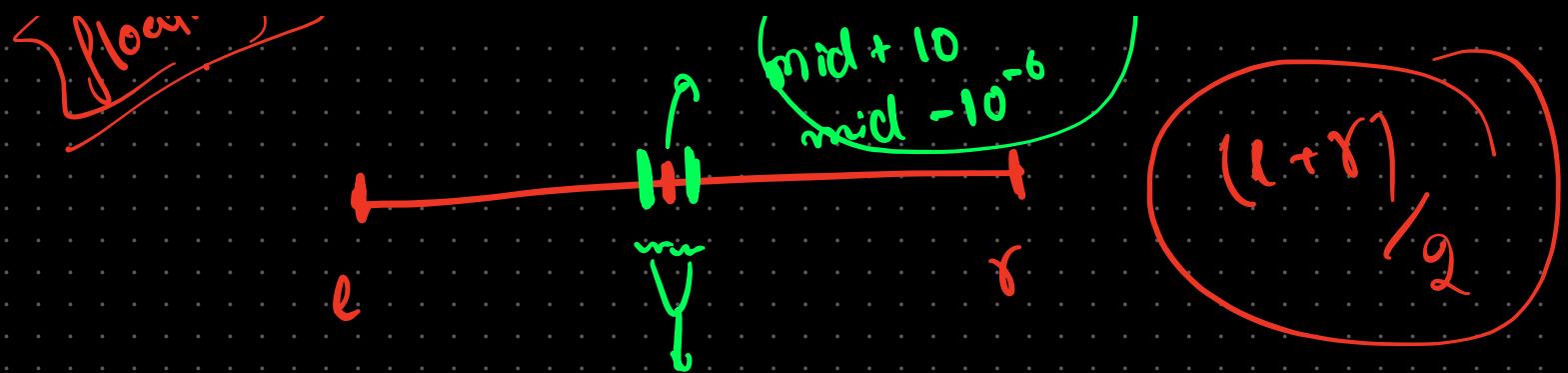
mid
Home

$$(1+3)_2 \rightarrow 0. \quad \{1, 3\}$$

$\{l, \delta\}$

$\{\delta, \infty\}$

-6



$k \rightarrow$ no. of splits.

array of size n .

maximum
seen that is
possible in
one of
the subarray

7	2 5 10 8
---	----------

$$k = 2$$

7 25

7 2 | 5 10 8

9

7 2 5 | 10 8 .

minimum sum
possible in
subarray ✓

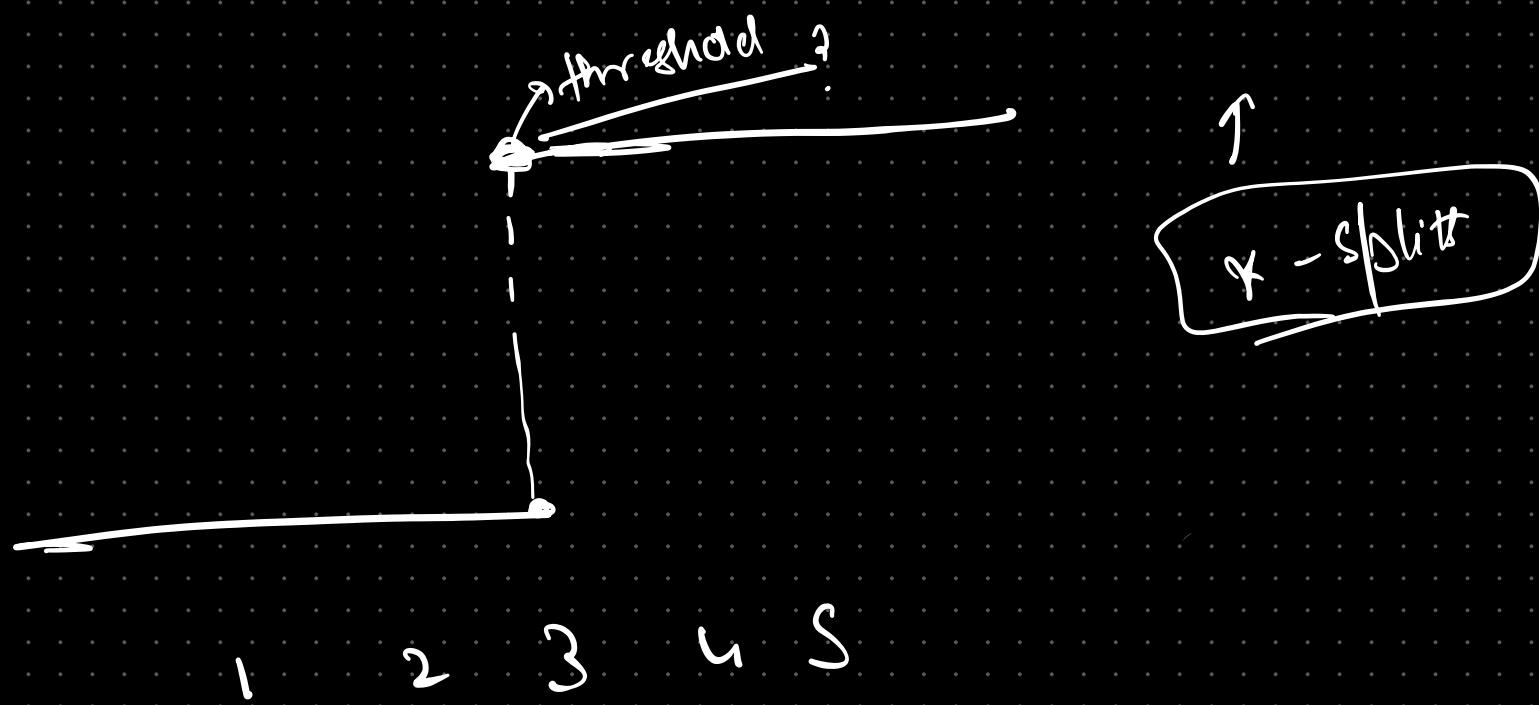
min

w

18

minimum subarray sum obtained after
K splits.

mid \rightarrow sum (potential min)



Bit manipulation:

Remember: any no. can be represented as a sum of power of 2.

0 0 1
0 1 0
0 1 1
1 0 0
:
→ in 0's & 1's.

: J

integer: Signed: -2^{30} to $2^{31} - 1$

total 32 bits are used for int.

unsigned int!

$0 \text{ to } 2^{32} - 1$

and: ' & ' → if all one's then
' 1 '

otherwise ' 0 '

' | ' → if any bit is one

'~' then '1'
'^' if odd no. of '1's
then 1

if even no. of 1's then 0.

<< left shift .



001 << 1

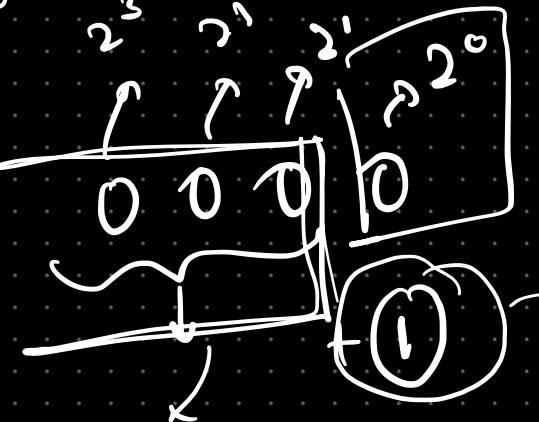
0010

0001 ~~→~~ when 001 >> 1

To find if an integer is odd

or even:

binu



even sum +1 if last bit is
0N.

mathematic

lax.b → time consuming.

if a is odd or even

$a \& 1$

$a \rightarrow 5$ ↗ odd

$$\begin{array}{r}
 0\ 0\ 0\ 1\ 0\ 1 \\
 \hline
 0\ 0\ 0\ 0\ 1
 \end{array}$$

0 0 0 0 0 1

$$\begin{array}{r}
 0\ 0\ 0\ 1\ 0\ 0 \\
 \hline
 0\ 0\ 0\ 0\ 1
 \end{array}$$

right shift it divides then by 2^k .

left shift multiply by 2^k .

100 << 1

1000 → '8'

100 >> 1

10 → '2'

$$\text{mid} = \lceil \frac{\text{start} + \text{end}}{2} \rceil$$

mul & div.

⊕ ⊖

$$\lceil (\text{start} + \text{end}) >> 1 \rceil \rightarrow$$

how many bits are left:

1 31

Count = 0.

```
for (i = 0 → 31)  
if(a & (1 << i)) {  
    count ++;  
}  
}.
```

$\leftarrow \ll \qquad \gg \rightarrow$
101100 loop 0 → 31

t k

10

A hand-drawn diagram on a grid background. It features a sequence of numbers: three '0's, a dashed line, three '0's, a large '1', another '1', and two '0's. Arrows point from the first '0' to the first '1', from the second '0' to the second '1', and from the third '0' to the final '0'. There is also a small arrow pointing upwards from the middle of the first '1'.

loop 0 to 31

(1240)

Ent + 1 + 1 + 1

→ (3)

101100

3

1 0 0 0 0 → 16 → 2⁴

$$\begin{array}{r} 01111 \\ + 1 \\ \hline 10000 \end{array}$$

$$\begin{array}{r} 10000 \\ - 1 \\ \hline 01111 \end{array}$$

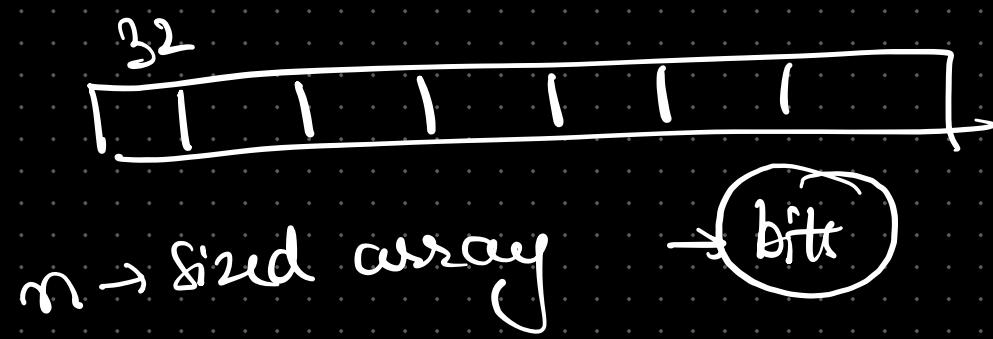
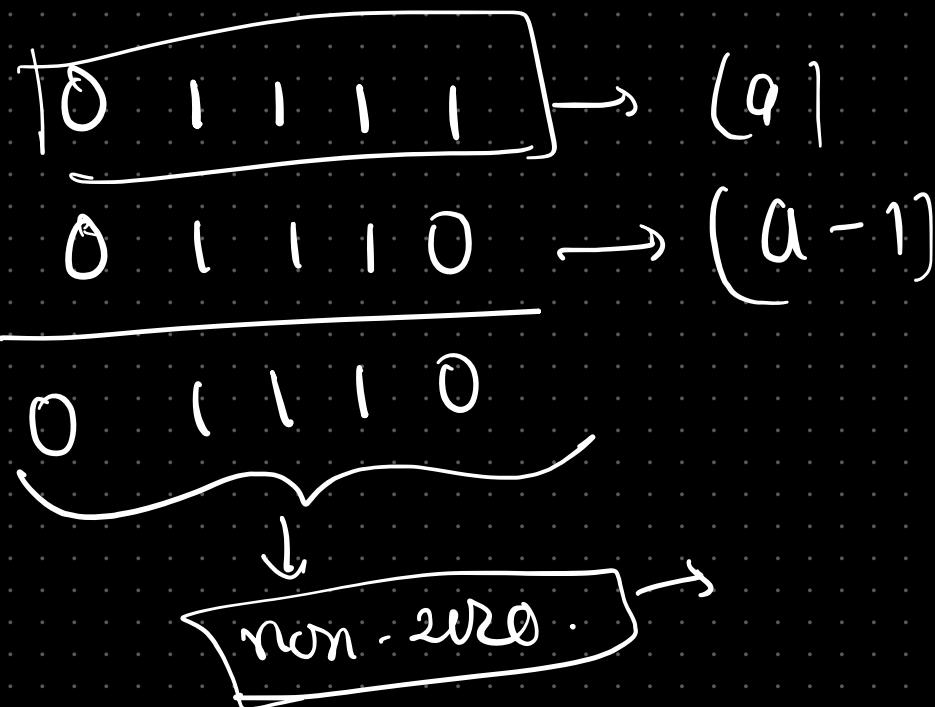
$$a^8 (a-1) = 0$$

$$\begin{array}{r} 10000 \\ 01111 \\ \hline 00000 \end{array}$$

$$a^8 (a-1) = 0$$

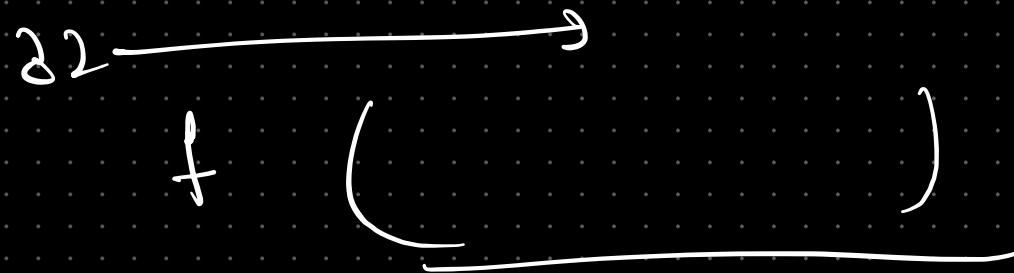
then it is
powers of

1 2. 0



32

$n \times 32$



~~100~~ → '8' >> →

100 → '4'

'5' → $\delta_{1/2}$ ②.

~~10~~

10X >> 1

mid

