Last Time:

- LQG
- Kalman

Today:

- Rocket Soft Landing

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* The Rocket Soft-Landing Problem:

  - Go from some initial state x_0 to some final position r_f with $z_f = 0$, $v_f \approx 0$

  - Minimize some combination of fuel consumption and/or landing position error $\|r_f - r_g\|$

  - Respect thrust limits + safety constraints

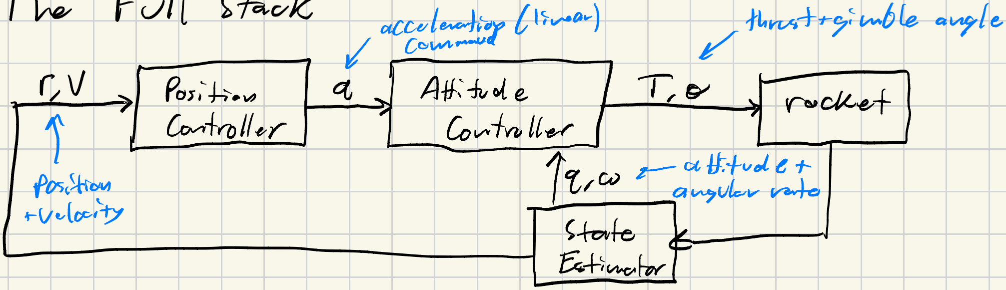
* Examples:

  - NASA Curiosity "Sky Crane" (2012)

  - SpaceX Falcon, Starship

  - NASA Perseverance w/TRN (2021)

* The "Full Stack"



- State Estimation

SpaceX: GPS + IMU with good filtering ~1 m position accuracy, < 1 cm/s velocity, ~1° attitude.

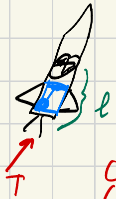
Mars: No GPS. IMU + Radar Altimeter + Vision ~30 meter Accuracy. Avoid Boulders.

- Decoupled Control Loop:

High-Level Position Controller: Uses a point-mass model. Reasons about safety, thrust limits, and fuel. Generating acceleration commands. Runs at ~ Hz

Low-Level Attitude Controller: Reasons about attitude, flexible modes, fluid slosh, generates thrust + gimble commands to ~~track~~ track desired acceleration. Runs at ~ 10 Hz

Rocket Dynamics



- Rigid Body

Position Control (slow)

$$\dot{v} = -g + \frac{T}{m} \quad \leftarrow \text{point mass}$$

$$\dot{m} = -\alpha T \quad \leftarrow \text{fuel burn}$$

attitude controller (fast)

$$\underset{\text{Inertia}}{J}\dot{\omega} + \omega \times J\omega = \underbrace{\ell \times T}_{\text{torque}} \quad \leftarrow \text{attitude}$$

- Fuel can be 80% + of initial vehicle mass. Have to account for this.

- Fluid slosh: Highly nonlinear, time-varying, hard to model. Standard model: pendulum.

- Flexible Modes: Rockets are built to be light $\Rightarrow$ not stiff $\Rightarrow$ low-frequency bending modes. First modes ~Hz. Dealt with by adding notch filters to the attitude controller at bending frequencies.

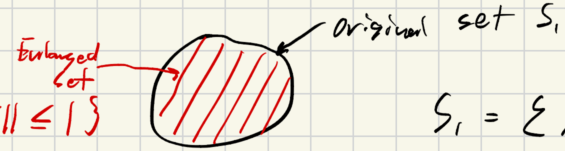
- Aerodynamic Forces: Mostly ignore.

- Lots of model error $\Rightarrow$ Linear robust control ideas are used in the attitude controller.

* Background: Convex Relaxation

- Sometimes we have a nonconvex constraint that can be expressed as the boundary of a convex set!

$S_2 = \{x \mid \|x\| \le 1\}$



$$S_1 = \{x \mid \|x\| = 1\}$$

$$S_1 = \partial S_2$$

- Replacing the original constraint with larger convex one is called "convex relaxation"

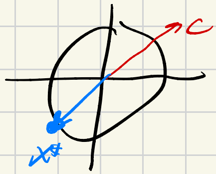
- Sometimes if the cost is "nice" we can still get the answer to the original problem by solving the relaxed version:

$$\min \; c^T x$$

$$\text{s.t. } \|x\| = 1$$

$$\downarrow$$

$$\|x\| \le 1$$

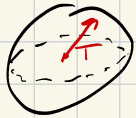


- When this happens we call it a "tight" relaxation.

* Convex Relaxation of Thrust Constraint

- Maximum thrust constraint

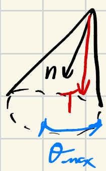
$$T \in \mathbb{R}^3, \quad \|T\| \le T_{max}$$



(convex)

- Thrust angle constraint

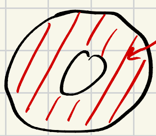
$$n^T \frac{T}{\|T\|} \le \cos(\theta_{max})$$



(convex)

- Rocket engines also have a minimum thrust constraint!

$$T_{min} \le \|T\| \le T_{max}$$



Not Convex!

- Let's add a new "slack variable" $\Gamma \in \mathbb{R}$ that equals the thrust magnitude:

$$1)\quad \|T\| = \Gamma \quad \} \text{ Boundary of a convex set (sphere)}$$

$$\left.\begin{array}{l} 2)\quad T_{min} \leq \Gamma \leq T_{max} \\ 3)\quad n^T T \leq \Gamma \cos(\theta_{max}) \end{array}\right\} \text{ Convex}$$

- Now we can convexify the constraint by relaxing 1)!

$$1^*)\quad \|T\| \leq \Gamma$$

$$2)\quad T_{min} \leq \Gamma \leq T_{max}$$

$$3)\quad n^T T \leq \Gamma \cos(\theta_{max})$$

- The paper proves that this relaxation is tight using Pontryagin's minimum principle.