ISyE 6333 - Fall 2023
Project Report

Network Monitoring

Submitted by:

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Problem Statement

In this project, a water utility company, and more specifically a network operator, is interested in allocating pressure sensors on nodes in order to detect pipe bursts in its water distribution Network.

Problem Parameters:

- The network is composed of 1123 pipes and 811 nodes.
- It provides 2.09 million gallons of water per day to its 5,010 customers.
- Each node can receive at most one sensor.

Notations: We denote V the set of 811 nodes/sensor locations, and E the set of 1123 network pipes. We are given a detection matrix, denoted $F \in \{0,1\}\|E\| \times \|V\|$, that represents the sensing capabilities of the pressure sensors.
F is a binary matrix of size $\|E\| \times \|V\| = 1123 \times 811$, and is such that $f_{e,v} = 1$ if a sensor placed at location $v \in V$ can detect a burst of pipe $e \in E$.

Solutions:

(a) Determine the minimum number of sensors, and where to locate them, so that if any pipe bursts, then at least one sensor will detect it.

Decision Variable: Let S_v be a binary variable to indicate 1 if a sensor is placed at node v and 0 if it is not.

Objective Function: Minimizing the total number of sensors on all nodes

$$\min \sum_{v \in V} S_v$$

Constraints:

- At least one sensor should detect any pipe bursts:

$$\sum_{v \in V} f_{e,v} S_v \geq 1 \; \forall \; e \in E$$

- S_v is a binary decision variable:

$$S_v \in \{0,1\} \; \forall \; v \in V$$

(b) As per the optimal solution, we need 19 sensors to detect any pipe burst. The sensors are placed on following nodes:

$$[16, 78, 104, 206, 233, 277, 392, 395, 424, 426, 430, 438, 454, 482, 651, 705, 712, 748, 786]$$

(c) For this part, we have b pressure sensors and we need to determine their location such that it maximizes the expected number of pipe bursts that are detected. The probability of each pipe bursting independently is 0.1.

Decision Variable: Let S_v be a binary variable to indicate 1 if a sensor is placed at node v and 0 if it is not.
Let y_e be a binary variable to indicate 1 if pipe burst is detected for pipe e and 0 if it is not detected.

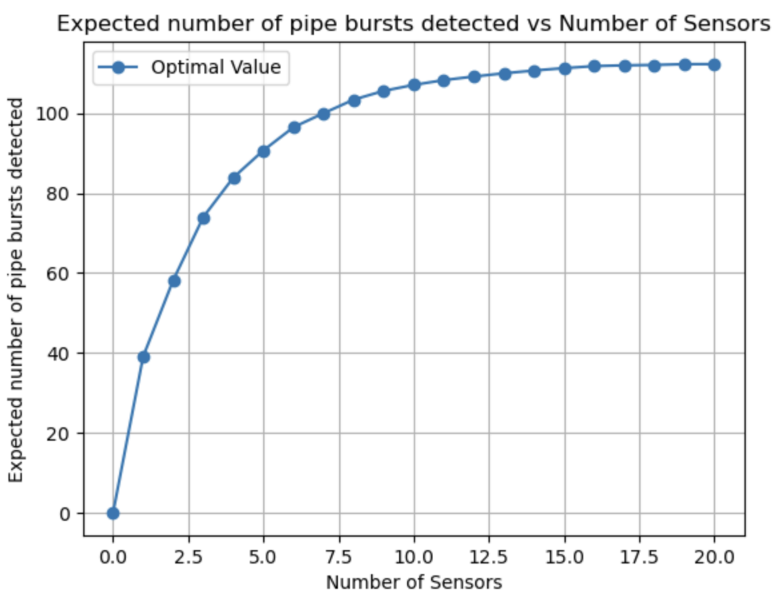
Objective Function: Maximizing the number of pipe bursts that are detected

$$\max \sum_{e \in E} p y_e$$

Constraints:

- Availability of sensors to place:
$$\sum_{v \in V} S_v \leq b$$
- Detection y_e should be 0 if none of the sensors placed can detect the pipe burst at pipe e:
$$\sum_{v \in V} f_{e,v} S_v \geq y_e \; \forall \; e \in E$$
- S_v and y_e are binary decision variables:
$$S_v, y_e \in \{0, 1\} \; \forall \; v \in V \text{and } e \in E$$

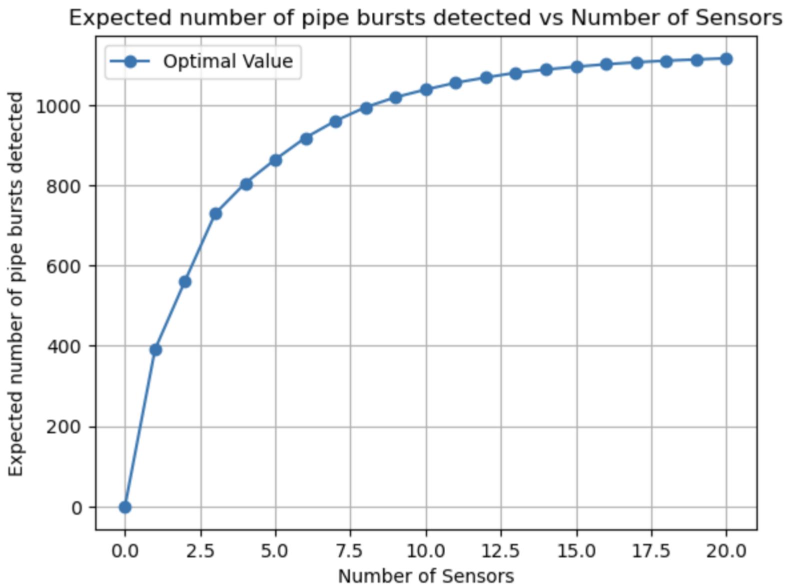
(d) Expected number of pipe bursts that are detected as a function of the number of sensors b



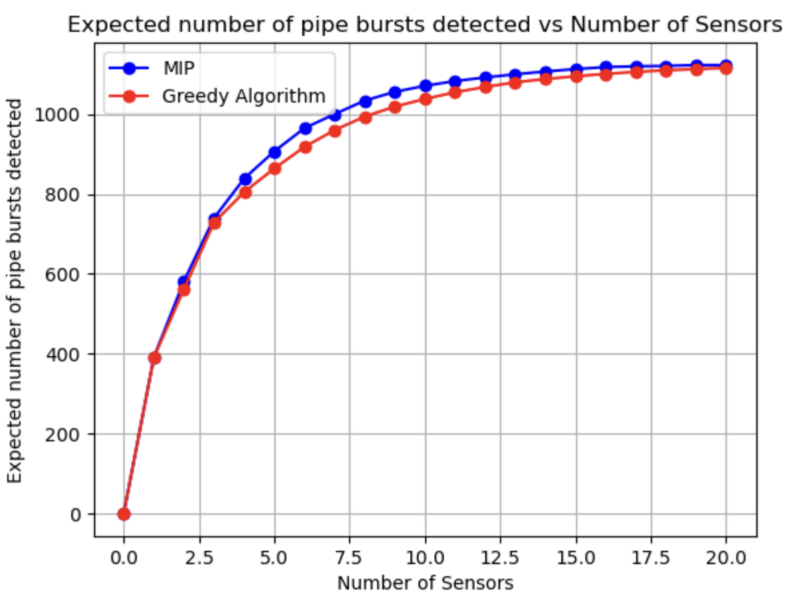
Here as well, all pipes are detected with 19 sensors.

(e) For this part, we have an iterative algorithm (greedy algorithm), the operator takes their first sensor, and places it at the node that detects the maximum expected number of pipe bursts. Then, the operator takes the second sensor, and places it at the node that detects the maximum expected number of pipe bursts that were not previously detected, and so on until all sensors are positioned.

Expected number of pipe bursts that are detected by that solution, as a function of the number of sensors available



Plot Comparison:



(f) For this part we have criticality level w_e for each pipe e and we need to position b sensors as to minimize the highest criticality of a pipe that is not detected by any sensor.

Decision Variable: Let S_v be a binary variable to indicate 1 if a sensor is placed at node v and 0 if it is not.
Let y_e be a binary variable to indicate 1 if pipe burst is detected for pipe e and 0 if it is not detected.

Our objective is to minimize the highest criticality of a pipe that is not detected. So, we take the maximum criticality from all the pipes that are not detected for pipe burst and minimize that:

$$\min \max_{e \in E} (1 - y_e) w_e$$

Replacing

$$\max_{e \in E} (1 - y_e) w_e$$

with z representing the highest criticality level from all the pipes that are not detected for pipe burst.

Objective Function:

$$\min z$$

Constraints:

- Additional constraint because of max replacement:

$$z \geq (1 - y_e) w_e \ \forall \ e \in E$$

- Availability of sensors to place:

$$\sum_{v \in V} S_v \leq b$$

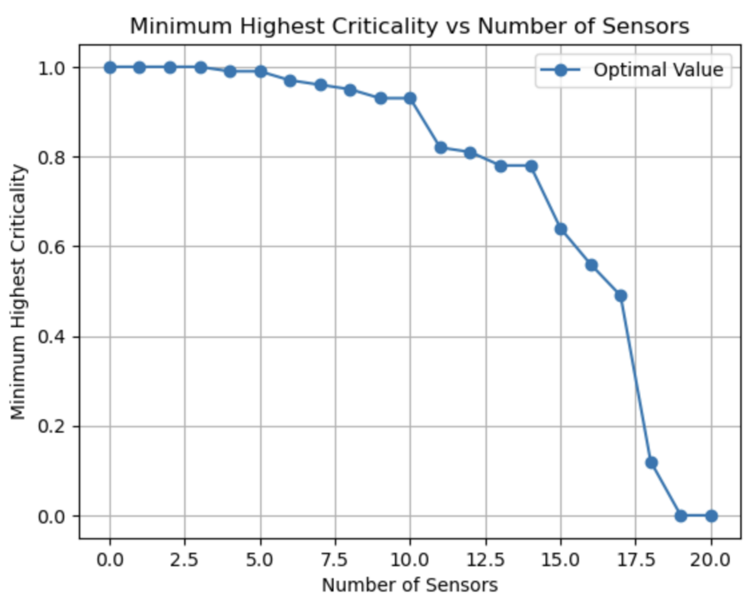
- Detection y_e should be 0 if none of the sensors placed can detect the pipe burst at pipe e:

$$\sum_{v \in V} f_{e,v} S_v \geq y_e \ \forall \ e \in E$$

- S_v and y_e are binary decision variables:

$$S_v, y_e \in \{0, 1\} \ \forall \ v \in V and e \in E$$

(g) Minimum highest criticality level as a function of the number of sensors b.



Here as well, highest criticality goes to 0 after 19 sensors are placed.