P1. $$P(h|v) = \frac{P(v,h)}{P(v)}$$

$$= \frac{1}{Z} \cdot \exp(-E(v,h)) \cdot \frac{1}{P(v)}$$

$$= \frac{1}{Z} \frac{1}{P(v)} \exp\left[-\frac{1}{2}\left([v-b] \oslash \sigma\right)^T\left([v-b] \oslash \sigma\right)\right] \exp\left(h^T W(v \oslash \sigma) + \alpha^T h\right)$$

$$f(v) = \frac{1}{Z} \frac{1}{P(v)} \exp\left[-\frac{1}{2}\left([v-b] \oslash \sigma\right)^T\left([v-b] \oslash \sigma\right)\right]$$

$$\Rightarrow P(h|v) = f(v) \exp\left[h^T\left(W(v \oslash \sigma) + \alpha\right)\right]$$

$$= f(v) \exp\left[\sum_i \sum_j h_i \left(W_{ij} \frac{v_i}{\sigma_i} + \alpha_j\right)\right]$$

$$\Rightarrow P(h_j = 1 | v) = \frac{\cancel{f(v)} \exp\left[\sum_i W_{ij} \frac{v_i}{\sigma_i} + \alpha_j\right]}{\cancel{f(v)}\left(\underbrace{\exp[0]}_{1} + \exp\left[\sum_i W_{ij} \frac{v_i}{\sigma_i} + \alpha_j\right]\right)}$$

$$= \boxed{\text{sigmoid}\left(\sum_i W_{ij} \frac{v_i}{\sigma_i} + \alpha_j\right)}$$

$$P(v|h) = \frac{P(v,h)}{P(h)}$$

$$\Rightarrow g(h) = \frac{1}{Z}\frac{1}{P(h)}\exp(\alpha^T h)$$

$$\Rightarrow P(v|h) = g(h)\exp\left[h^T W(v \oslash \sigma) - \frac{1}{2}\left([v-b] \oslash \sigma\right)^T\left([v-b] \oslash \sigma\right)\right]$$

$$= g(h)\exp\left[\sum_i \sum_j \left(w_{ij} h_j \frac{v_i}{\sigma_i} - \frac{(v_i - b_i)^2}{2\sigma_i^2}\right)\right]$$

$$\Rightarrow P(v_i = x|h) = \frac{\cancel{g(h)}\exp\left(\sum_j \left(w_{ij} h_j \frac{x}{\sigma_i} - \frac{(x-b_i)^2}{2\sigma_i^2}\right)\right)}{\cancel{g(h)}\int_{-\infty}^{\infty}\exp\left[\sum_j \left(w_{ij} h_j \frac{x}{\sigma_i} - \frac{(x-b_i)^2}{2\sigma_i^2}\right)\right]dx}$$

$$= \boxed{\frac{\exp\left[\sum_j \left(w_{ij} h_j \frac{x}{\sigma_i} - \frac{(x-b_i)^2}{2\sigma_i^2}\right)\right]}{\int_{-\infty}^{\infty}\exp\left[\sum_j \left(w_{ij} h_j \frac{x}{\sigma_i} - \frac{(x-b_i)^2}{2\sigma_i^2}\right)\right]dx}}$$