$$\iint pdf\,(u_1, u_2)\,\partial A = \iint pdf\,(x(u_1,u_2), y(u_1,u_2)) \det|J|\,\partial A$$

$$x = R\cos\theta = \sqrt{-2\ln u_2}\,\cos(2\pi u_1)$$

$$y = R\sin\theta = \sqrt{-2\ln u_2}\,\sin(2\pi u_1)$$

$$\Rightarrow x^2 = -2\pi \ln u_2 \cos^2(2\pi u_1)$$

$$y^2 = -2\pi \ln u_2 \sin^2(2\pi u_1)$$

$$\Rightarrow \frac{y}{x} = \tan(2\pi u_1)$$

$$\Rightarrow u_1 = \frac{1}{2\pi}\tan^{-1}\left(\frac{y}{x}\right)$$

$$\Rightarrow x^2 + y^2 = -2\pi \ln u_2$$

$$\Rightarrow \frac{-(x^2+y^2)}{2} = \ln u_2$$

$$\Rightarrow u_2 = e^{-\frac{x^2+y^2}{2}}$$

$$pdf(u_1, u_2) = pdf(x(u_1,u_2), y(u_1,u_2)) \det|J|$$

$$\det|J| = \begin{vmatrix} \frac{\partial u_1}{\partial x} & \frac{\partial u_1}{\partial y} \\ \frac{\partial u_2}{\partial x} & \frac{\partial u_2}{\partial y} \end{vmatrix} = \begin{vmatrix} -\frac{1}{2\pi}\frac{y}{x^2+y^2} & \frac{1}{2\pi}\frac{x}{x^2+y^2} \\ -xe^{-\frac{(x^2+y^2)}{2}} & -ye^{-\frac{(x^2+y^2)}{2}} \end{vmatrix}$$

$$= \frac{1}{2\pi}\,\cancel{\frac{(x^2+y^2)}{x^2+y^2}}\,e^{-\frac{(x^2+y^2)}{2}} = \boxed{\frac{1}{2\pi}e^{-\frac{(x^2+y^2)}{2}}}$$

bivariate standard normal