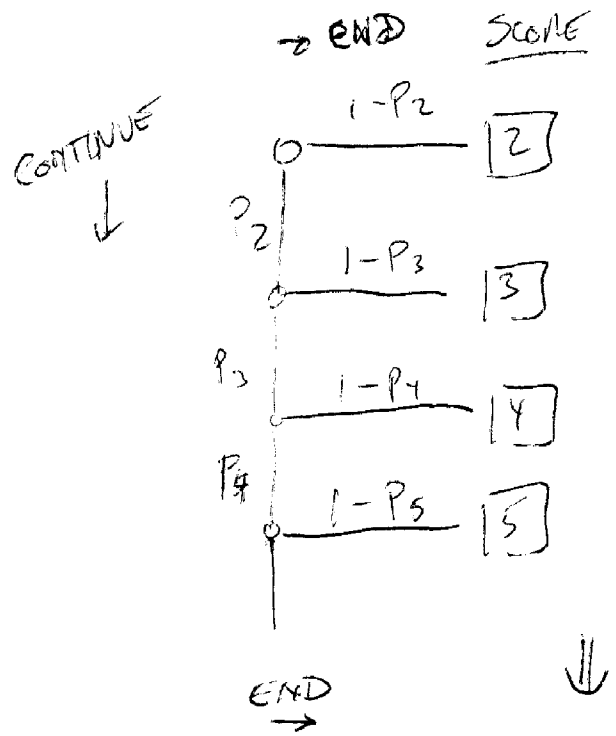
P_i = Probability that shot i is better than shot $i-1$

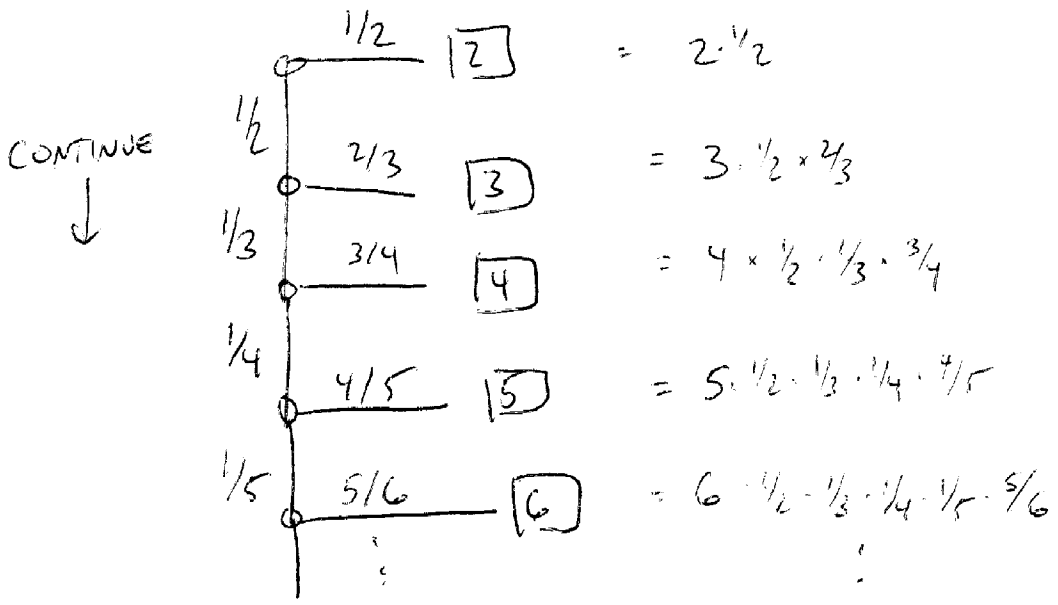


$P_2 = 1/2$

$P_3 = E(\min(\text{shot}_1, \text{shot}_2)) = 1/3$

$P_4 = E(\min(\text{shot}_1, \text{shot}_2, \text{shot}_3)) = 1/4$

Proof too long to fit in margin!



Expected Score =

$$2\times\frac{1}{2} + 3\times\frac{1}{2}\cdot\frac{2}{3} + 4\times\left(\frac{1}{2}\times\frac{1}{3}\times\frac{3}{4}\right) + 5\left(\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{4}\right)\frac{4}{5} + \dots$$

$$1 + 1 + \frac{1}{2} + \frac{1}{6} + \dots$$

$$= \sum_{N=1}^{\infty} N/N!$$

$$= e$$