

PROBLEM 1

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Description:

Beta Function $B(x, y)$ can be defined in terms of the Gamma Function or several integrals, also known as Euler integral of the first kind. The beta function is similar to gamma function, can have complex factors and is unclassified at 0 and negative integers. The beta function takes place in the evaluation of certain integrals and defines binomial co-efficient after adjusting indices.

Characteristics:

In mathematics, the Beta function (also known as the Euler integral of the first kind), is a special function defined by:

$$B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt, \quad (1)$$

The Beta function is symmetric, meaning that $B(x, y) = B(y, x)$. And the Beta function is related to the Gamma function by the following formula:

$$B(x, y) = \frac{\Gamma(x) \times \Gamma(y)}{\Gamma(x+y)} \quad (2)$$

Domain:

$\text{Re}(x) > 0, \text{Re}(y) > 0$

References

- [1]<https://ncalculators.com/statistics/beta-function-calculator.htm>
- [2]<https://www.miniwebtool.com/beta-function-calculator>
- [3]https://en.wikipedia.org/wiki/Beta_functionRelationship between gamma function and beta function

Assumptions

1. First Assumption

- Description = All inputs of x and y are real positive numbers.

2. Second Assumption

- Description = The user can change the input.

Requirements

(a) First Requirement

- ID = FR1
- Type = Functional Requirements
- Version = 1.0
- Difficulty = Easy
- Description = The System shall ask the user to type two numbers in specified boundaries.

(b) Second Requirement

- ID = FR2
- Type = Functional Requirements
- Version = 1.0
- Difficulty = Easy
- Description = The system shall show an error for out of boundry inputs.

(c) Third Requirement

- ID = FR3
- Type = Functional Requirements
- Version = 1.0
- Difficulty = Easy
- Description = User shall give an input from all positive real numbers .

PROBLEM 3 - BETA FUNCTION SAMANEH SHIRDEL FARI-MANI

Selected Algorithm:

There are different rules for calculating 'Integral'. I chose Simpson's rule. Following is a definition of Simpson's rule and some advantages and disadvantages. In numerical analysis, Simpson's rule is a method for numerical integration, the numerical approximation of definite integrals.[1] Simpson's rule is a method of numerical integration which is a good deal more accurate than the Trapezoidal rule[2] Simpson's rule approximates an integral by performing quadratic interpolation between the endpoints and midpoint of the interval of integration. It also divides the area under the function to be integrated, $f(x)$, into vertical strips, but instead of joining the points $f(x_i)$ with straight lines, every set of three such successive points is fitted with a parabola.[2]

Advantages:

1. Instead of joining the points $f(x_i)$ with straight lines, every set of three such successive points is fitted with a parabola.[2]
2. The other rules are more complex; consequently, greater computational effort is involved and rounding errors may become a more significant problem.[2]
3. In some other rules like trapezoidal if the curve is concave down, the trapezoids lie below the curve and over-estimate of the integral but there is not a such defect in Simpson's rule.[3]

Disadvantages:

1. The truncation errors indicate that some improvement in accuracy may be obtained by using some other rules rather than Simpson's rule.[2]

ALGORITHM 1: Algorithm to calculate Beta Function

```
1. function simpsonRule(a,b,f)  
  in: double number a, b function f  
  out: double number sum*h  
2.  $N \leftarrow 10000$   
3.  $h \leftarrow (b - a)/(N - 1)$   
4.  $h \leftarrow 1.0/3.0 * (f(a) + f(b))$   
5. for  $i = 1 \leq N - 1$  do  
6.    $x \leftarrow a + h * i$   
7.    $sum \leftarrow sum + 4.0/3.0 * f$   
8.    $i \leftarrow i + 2$   
9. end for  
10. for  $i = 2 \leq N - 1$  do  
11.   $x \leftarrow a + h * i$   
12.   $sum \leftarrow sum + 2.0/3.0 * f$   
13.   $i \leftarrow i + 2$   
14. end for  
15. return sum*h
```

References

- [1]https://en.wikipedia.org/wiki/Simpson27s_rule
- [2]<https://www.sciencedirect.com/topics/engineering/simpsons-rule>
- [3]<http://www.math.pitt.edu/sparling/052/23052/23052notes/23052notestojan14th/node3>