Gamma Distribution

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Definition

The **Gamma Distribution** is continuous and is the sum of exponentials, where you determine the probability it will take a certain amount of time to see a fixed number of events occur. This distribution takes two parameters, the number of events and the rate, υ, at which the events occur. We would say

$$W_r \sim Gamma(r, \upsilon)$$

Alternatively, you could also exchange the rate with the mean, θ, such that $\theta = \frac{1}{\upsilon}$. In this case,

$$W_r \sim Gamma(r, \theta)$$

Probability Density Function

The probability of waiting w time to see r events occur is:

$$f(w) = \frac{\upsilon^r w^{r-1}}{\Gamma(r)} e^{-\upsilon w}$$

This pdf can be adapted so that the parameter is the mean:

$$f(w) = \frac{w^{r-1}}{\theta^r \Gamma(r)} e^{-w/\theta}$$

For $r \in \mathbb{Z}^+, \Gamma(r) = (r-1)!$

Descriptive Statistics

The Gamma distribution is simply the sum of independent exponentials ($W_r = Y_1 + Y_2 + ... + Y_r$ where Y_i's are independent). Therefore, the mean of the gamma distribution is:

$$E[W_r] = E[\sum_{i=1}^r Y_i] = \sum_{i=1}^r \frac{1}{\upsilon} = \frac{r}{\upsilon} = r\theta$$

And the variance of the gamma distribution is:

$$Var[W_r] = Var[\sum_{i=1}^r Y_i] = \sum_{i=1}^r \frac{1}{\upsilon^2} = \frac{r}{\upsilon^2} = r\theta^2$$