Homework 4

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5-1

a) Hohmann Transfer

$$\Delta v_1 = v_1^+ - v_1^- = \sqrt{\frac{\mu}{r_1}}\left[\sqrt{\frac{2r_2}{r_1 + r_2}} - 1\right]$$

$$v_{c1} = \sqrt{\frac{\mu}{r_1}}$$

$$R = \frac{r_2}{r_1}$$

$$\Delta v_1 = v_{c1}\left[\sqrt{\frac{2R}{1+R}} - 1\right]$$

$$\boxed{\frac{\Delta v_1}{v_{c1}} = \left[\sqrt{\frac{2R}{1+R}} - 1\right]}$$

$$\Delta v_2 = v_2^+ - v_2^- = \sqrt{\frac{\mu}{r_2}}\left[1 - \sqrt{\frac{2r_1}{r_1 + r_2}}\right]$$

$$v_{c1} = \sqrt{\frac{\mu}{r_1}}$$

$$R = \frac{r_2}{r_1}$$

$$\Delta v_2 = v_{c1}\sqrt{\frac{1}{R}}\left[1 - \sqrt{\frac{2}{1+R}}\right]$$

$$\boxed{\frac{\Delta v_2}{v_{c1}} = \sqrt{\frac{1}{R}}\left[1 - \sqrt{\frac{2}{1+R}}\right]}$$

b) Bi-Elliptic Transfer

$$\Delta v_1 = v_1^+ - v_1^- = \sqrt{\frac{\mu}{r_1}}\left[\sqrt{\frac{2r_i}{r_1 + r_i}} - 1\right]$$

$$v_{c1} = \sqrt{\frac{\mu}{r_1}}$$

$$R = \frac{r_2}{r_1}$$

$$S = \frac{r_i}{r_2}$$

$$\Delta v_1 = v_{c1}\left[\sqrt{\frac{2RS}{1 + RS}} - 1\right]$$

$$\boxed{\frac{\Delta v_1}{v_{c1}} = \left[\sqrt{\frac{2RS}{1 + RS}} - 1\right]}$$

$$\Delta v_2 = v_2^+ - v_2^- = \sqrt{\frac{\mu}{r_i}}\left[\sqrt{\frac{2r_2}{r_2 + r_i}} - \frac{2r_1}{r_1 + r_i}\right]$$

$$v_{c1} = \sqrt{\frac{\mu}{r_1}}$$

$$R = \frac{r_2}{r_1}$$

$$S = \frac{r_i}{r_2}$$

$$\Delta v_2 = v_{c1}\sqrt{\frac{1}{RS}}\left[\sqrt{\frac{2}{1 + S}} - \sqrt{\frac{2}{1 + RS}}\right]$$

$$\boxed{\frac{\Delta v_2}{v_{c1}} = \sqrt{\frac{1}{RS}}\left[\sqrt{\frac{2}{1 + S}} - \sqrt{\frac{2}{1 + RS}}\right]}$$

$$\Delta v_3 = |v_3^+ - v_3^-| = \sqrt{\frac{\mu}{r_2}}\left[\sqrt{\frac{2r_i}{r_2 + r_i}} - 1\right]$$

$$v_{c1} = \sqrt{\frac{\mu}{r_1}}$$

$$R = \frac{r_2}{r_1}$$

$$S = \frac{r_i}{r_2}$$

$$\Delta v_3 = v_{c1}\left[\sqrt{\frac{2S}{R+RS}} - \sqrt{\frac{1}{R}}\right]$$

$$\boxed{\frac{\Delta v_3}{v_{c1}} = \left[\sqrt{\frac{2S}{R+RS}} - \sqrt{\frac{1}{R}}\right]}$$

c) Bi- Parabolic Transfer

$$\frac{\Delta v_1}{v_{c1}} = \left[\sqrt{\frac{2RS}{1+RS}} - 1\right] = \left[\sqrt{\frac{2R}{1/S+R}} - 1\right]$$

limit S $\to \infty$

$$\boxed{\frac{\Delta v_1}{v_{c1}} = \sqrt{2} - 1}$$

$$\frac{\Delta v_2}{v_{c1}} = \sqrt{\frac{1}{RS}}\left[\sqrt{\frac{2}{1+S}} - \sqrt{\frac{2}{1+RS}}\right] = \sqrt{\frac{1}{RS}}\left[\sqrt{\frac{2/S}{1/S+1}} - \sqrt{\frac{2/S}{1/S+R}}\right]$$

limit S $\to \infty$

$$\boxed{\frac{\Delta v_2}{v_{c1}} = 0}$$

$$\frac{\Delta v_3}{v_{c1}} = \left[\sqrt{\frac{2S}{R+RS}} - \sqrt{\frac{1}{R}}\right] = \left[\sqrt{\frac{2}{R/S+R}} - \sqrt{\frac{1}{R}}\right]$$

limit S $\to \infty$

$$\boxed{\frac{\Delta v_3}{v_{c1}} = \sqrt{\frac{1}{R}}\left[\sqrt{2} - 1\right]}$$

d)

See figures 1 and 2.

5-2

a)

$$\boxed{R = 11.9384}$$

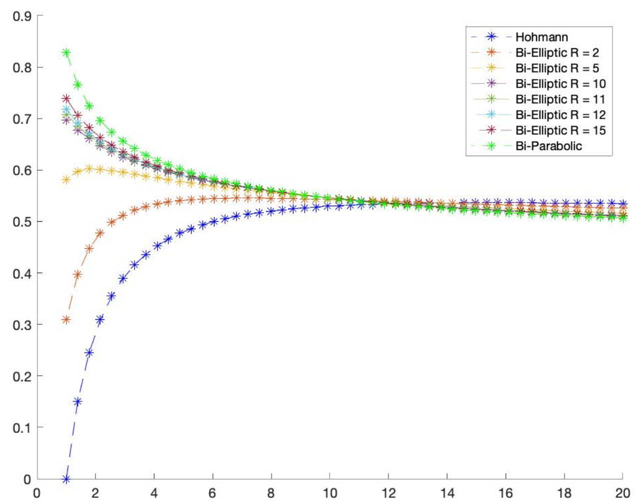

Figure 1: Question 1

S	**R**
2	13.8075544397706
5	12.7394717716265
10	12.3486720808664
11	12.3119346884137
12	12.2812283783872
15	12.2133732070721

Table 1: Question 1

b)

See table 1.

c)

See figure 3.

5-3

By rearranging the speed of spacecraft equation and taking into account that the radius of a circular orbit is equal to its semi-major axis, we get the following equations:

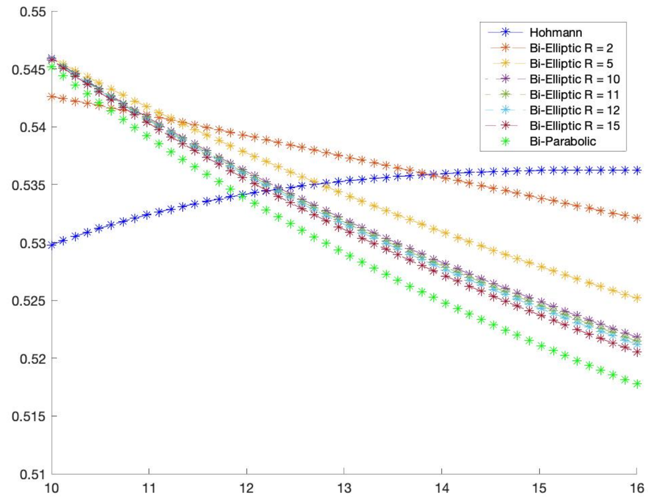

Figure 2: Question 1

$$r_1 = \frac{\mu}{v_1^2}$$

$$r_2 = \frac{\mu}{v_2^2}$$

The ratio between these two radii:

$$R = \frac{r_2}{r_1}$$

Plugging in the following values:

$$v_1 = 1$$

$$v_2 = .5$$

We get:

$$R = 4$$

From looking at the graph we made in the first problem, we can see that when $R = 4$ the orbit that accomplishes the orbit transfer with the lowest impulse is the Hohmann transfer.

See figure 4.

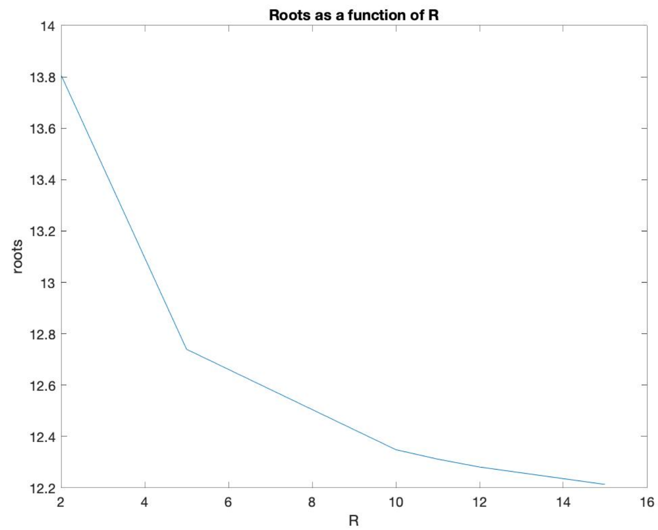

Figure 3: Question 2

5-4

a)

$$\frac{\Delta v_1}{v_{c1}} = \left[\sqrt{\frac{2R}{1+R}} - 1\right]$$

$$\frac{\Delta v_2}{v_{c1}} = \sqrt{\frac{1}{R}}\left[1 - \sqrt{\frac{2}{1+R}}\right]$$

$$R = \frac{r_2}{r_1}$$

$$r1 = 6371 + 300\ km$$

$$r1 = 6671\ km$$

We can use this equation to find r_2:

$$\tau = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$r_2 = a$$

$$r_2 = 42164\ km$$

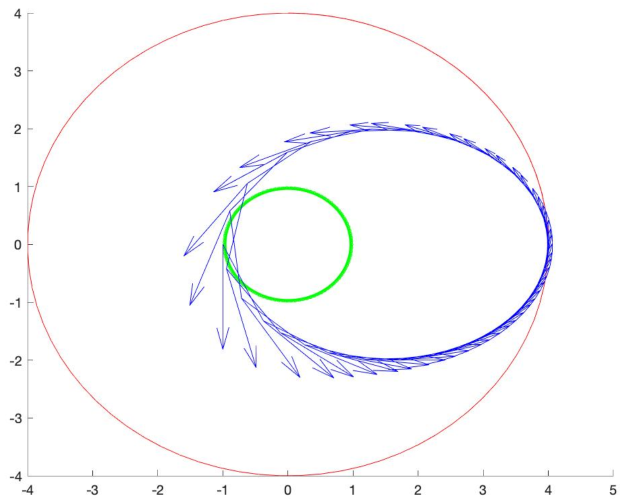

Figure 4: Question 3

$$R = 6.3204$$

$$\frac{\Delta v_1}{v_{c1}} = .3141$$

$$\frac{\Delta v_2}{v_{c1}} = .1899$$

b)

$$\frac{\Delta V}{v_{cl}} = \frac{\Delta v_1}{v_{c1}} + \frac{\Delta v_2}{v_{c1}} = .5039$$

$$v_{cl} = \sqrt{\frac{\mu}{r_1}} = 7.7299$$

$$\Delta V = 3.8953$$

c)

$$t_H = \pi\sqrt{\frac{a^3}{\mu}}$$

$$t_H = 18986\ s$$

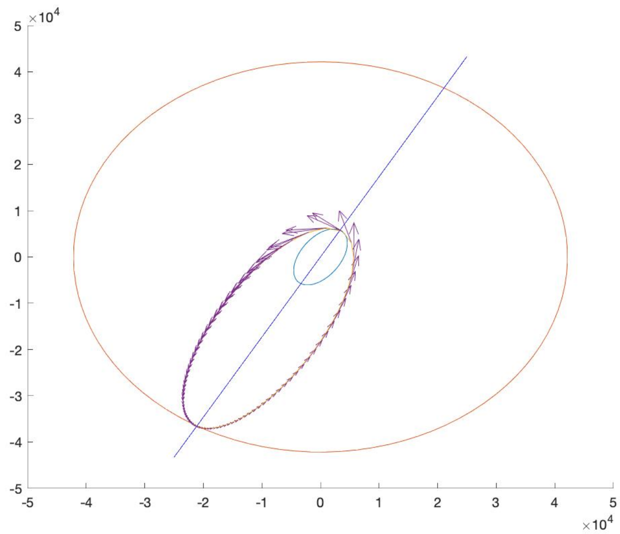

Figure 5: Question 4

$$t_H = 5.2738\ hours$$

d)

$$\Delta v = g_0 I_{sp} \ln\left(\frac{m(t_0)}{m(t)}\right)$$

$$g_0 = 9.80665\ m/s^2$$

$$I_{sp} = 320\ s$$

$$ratio\ 1 = 1.0001$$

$$ratio\ 2 = 1.0001$$

e)

See figure 5.

f)

Yes.

5-5

a)

$$[0.8660 \quad 0.5000 \quad 0]$$

b)

$$\mathbf{r}_1 = [5820.6 \quad 3360.5 \quad 0]$$
$$\mathbf{r}_2 = [-23000 \quad -13279 \quad 0]$$

c)

$$\Delta v = 2v sin(\theta/2)$$
$$\theta = cos^{-1}\left[\frac{{}^I\mathbf{h}_1 \cdot {}^I\mathbf{h}_2}{||{}^I\mathbf{h}_1||||{}^I\mathbf{h}_2||}\right] = .4712$$
$$\Delta v = 1.8088$$

d)

$$time\ of\ flight = 2.9668\ hours$$

e)

$$e = .5961$$

f)

$$\theta = .4712$$

g)

See figure 6.

5-6

a)

$$\Delta v = 2v sin(\theta/2)$$
$$\theta = cos^{-1}\left[\frac{{}^I\mathbf{h}_1 \cdot {}^I\mathbf{h}_2}{||{}^I\mathbf{h}_1||||{}^I\mathbf{h}_2||}\right] = .4974$$
$$v = 1.6044$$
$$\Delta v = .7898$$

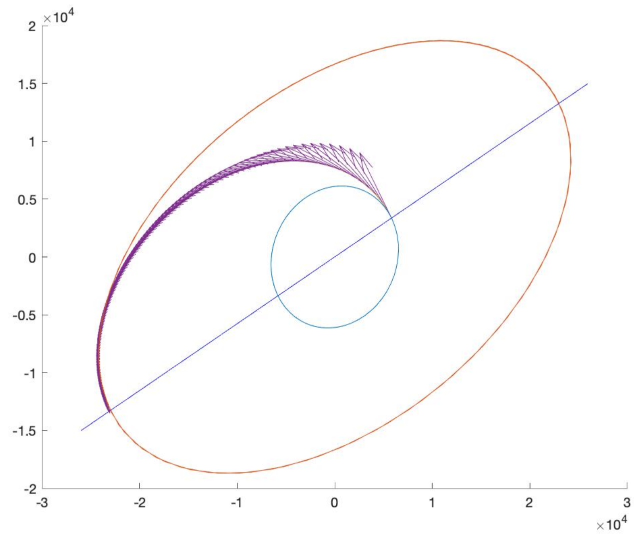

Figure 6: Question 5

b)

$$\Delta v = 3.8055$$

c)

See figure 7.

d)

When f = 0.

Scripts

Script for 5-1

```
clc
clear all
close all

S = [2 5 10 11 12 15]; %for bi-elliptic transfer
Rcell = {[linspace(1, 20, 50)] [linspace(10, 16, 50)]}; %ratio of radii

%%
%Hohmann
```

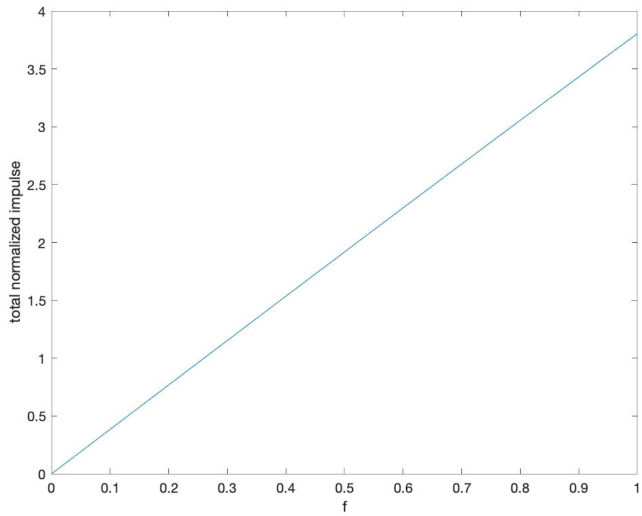

Figure 7: Question 6

```
for j = 1:length(Rcell)
    hohmann = [];
    for i = 1:length(Rcell{j})
        R = [Rcell{j}];
        imp1norm = sqrt(2*R(i)/(1+R(i))) - 1;
        imp2norm = sqrt(1/R(i))*(1 - sqrt(2/(1+R(i))));
        hohmann(i) = imp1norm + imp2norm;
    end
    hohmanncell{j} = [hohmann];
end

%%
%bi-elliptic
SE = [2 5 10 11 12 15]; %for bi-elliptic transfer
RcellE = {[linspace(1, 20, 50)] [linspace(10, 16, 50)]};
for j = 1:length(Rcell)
    biElliptic = [];
    for i = 1:length(RcellE{j})
        for m = 1:length(SE)
        R = [RcellE{j}];
        RS = S(m)*R(i);
        imp1normE = sqrt(2*RS/(1+RS)) - 1;
        imp2normE = sqrt(1/RS)*(sqrt(2/(1+SE(m)))- sqrt(2/(1+RS)));
        imp3normE = sqrt(2*SE(m)/(R(i)+RS)) - sqrt(1/R(i));
        biElliptic(i, m) = imp1normE + imp2normE + imp3normE;
        end
```

```
    end
    biEllipticcell{j} = [biElliptic];
end
%%
%Bi Parabolic Transfer
S = [2 5 10 11 12 15]; %for bi-elliptic transfer
Rcell = {[linspace(1, 20, 50)] [linspace(10, 16, 50)]};
for j = 1:length(Rcell)
    biParabolic = [];
    for i = 1:length(Rcell{j})
        R = [Rcell{j}];
        imp1norm = sqrt(2) - 1;
        imp3norm = sqrt(1/R(i))*(sqrt(2) - 1);
        biParabolic(i) = imp1norm + imp3norm;
    end
    biParaboliccell{j} = [biParabolic];
end


%%
%graphs

biElliptic1 = biEllipticcell{1};
biElliptic2 = biEllipticcell{2};


figure
hold on
plot(Rcell{1}, hohmanncell{1},'b*--')
plot(Rcell{1}, biElliptic1(:,1), '*--')
plot(Rcell{1}, biElliptic1(:,2), '*--')
plot(Rcell{1}, biElliptic1(:,3), '*--')
plot(Rcell{1}, biElliptic1(:,4), '*--')
plot(Rcell{1}, biElliptic1(:,5), '*--')
plot(Rcell{1}, biElliptic1(:,6), '*--')
plot(Rcell{1}, biParaboliccell{1}, 'g*--')
legend('Hohmann', 'Bi-Elliptic R = 2', 'Bi-Elliptic R = 5'...
    , 'Bi-Elliptic R = 10', 'Bi-Elliptic R = 11', 'Bi-Elliptic R = 12'...
    , 'Bi-Elliptic R = 15', 'Bi-Parabolic');


figure
hold on
plot(Rcell{2}, hohmanncell{2}, 'b*--')
plot(Rcell{2}, biElliptic2(:,1), '*--')
plot(Rcell{2}, biElliptic2(:,2), '*--')
plot(Rcell{2}, biElliptic2(:,3), '*--')
plot(Rcell{2}, biElliptic2(:,4), '*--')
plot(Rcell{2}, biElliptic2(:,5), '*--')
plot(Rcell{2}, biElliptic2(:,6), '*--')
```

```
plot(Rcell{2}, biParaboliccell{2}, 'g*')
ylim([.51 .55])
legend('Hohmann', 'Bi-Elliptic R = 2', 'Bi-Elliptic R = 5'...
    , 'Bi-Elliptic R = 10', 'Bi-Elliptic R = 11', 'Bi-Elliptic R = 12'...
    , 'Bi-Elliptic R = 15', 'Bi-Parabolic');
```

Script for 5-2

```
clc
clear all
close all

%Hohmann and BiParabolic
fun = @(R) -(sqrt(2)-1)*(1+sqrt(1/R)) + sqrt(2*R/(1+R)) - 1 + sqrt(1/R)*(1 - sqrt(2/(1+R)));
R0 = 12;
R = fsolve(fun,R0);


%Hohmann and BiElliptic
S2 = [2 5 10 11 12 15]; %for bi-elliptic transfer
for i = 1:length(S2)
    S = S2(i);
    fun2 = @(R) sqrt((2*R*S)/(1+R*S)) + sqrt(1/(R*S))*sqrt(2/(1+S)) -
    sqrt(1/(R*S))*sqrt(2/(1+R*S)) + sqrt((2*S)/(R+R*S)) - sqrt(2*R/(1+R)) + sqrt(1/R)*sqrt(2/(1+R)) -
    2*sqrt(1/R);
    R02 = 13;
    R2(i) = fsolve(fun2, R02);
end

plot(S2, R2)
xlabel('R')
ylabel('roots')
title('Roots as a function of R')
```

Script for 5-3

```
clc
clear all
close all

%given
mu = 1;
v1 = 1; %speed in canonical units
v2 = .5; %speed in canonical units

%time/N
t0 = 0;
tf = 20*pi;
```

```
N = 100;
trange = linspace(t0, tf, 50);

%first orbit
r1 = mu/(v1^2); %radius of first orbit
r01 = [r1,0,0];
v01 = [0,v1,0];
[tspan, rv1, vv1, w1, w2, w3] = propogateOnCircle(r01, v01, t0, tf, mu, N);

%second orbit
r2 = mu/(v2^2); %radius of second orbit
r02 = [r2,0,0];
v02 = [0,v2,0];
[tspan, rv2, vv2, w1, w2, w3] = propogateOnCircle(r02, v02, t0, tf, mu, N);

%transfer orbit
a = (r2 + r1)/2;
e = (r2 - r1)/(r2 + r1);
oet = [a, e, 0,0,0,0];
[rt,vt]  = oe2rv_Harris_Samantha(oet,mu);
[rt,vt,E0,nu0,E,nu] = propagateKepler(t0,tf, trange, rt,vt,mu);

figure
hold on
plot3(rv1(:,1),rv1(:,2),rv1(:,3), 'g')
plot3(rv2(:,1),rv2(:,2),rv2(:,3), 'r')
plot3(rt(:,1),rt(:,2),rt(:,3), 'b')
quiverScale = 2;
quiver3(rt(:,1),rt(:,2),rt(:,3),vt(:,1),vt(:,2),vt(:,3), quiverScale, 'b')
```

Script for 5-4

```
clc
clear all
close all

%given
mu     =  398600;       %gravitational parameter
r1     = 6371 + 300;    %radius of first orbit
inc1   = deg2rad(57);   %inclination of first orbit
Omega1 = deg2rad(60);   %long. of ascend. node of first orbit
tau2   = 23.934*60*60;  %orbital period of second orbit
nu     = 0;             %starting true anomaly

%time
t0 = 0;
```

```
tf = 10000;
trange = linspace(t0, tf, 100);
trange2 = linspace(t0, tau2, 100);
tranget = linspace(t0, 50000, 100);

%magnitude of each impulse that contributes to the total impulse
r2 = nthroot(mu*(tau2/(2*pi))^2,3);
R = r2/r1;
imp1norm = sqrt(2*R/(1+R)) - 1;
imp2norm = sqrt(1/R)*(1 - sqrt(2/(1+R)));

%total impulse required to complete the transfer
totalimp =  imp1norm + imp2norm;

%time in hours required to complete the transfer
aTransfer = (r1 + r2)/2;
eTransfer = (r2 - r1)/(r2 + r1);
tauTransfer = pi*sqrt((aTransfer^3)/mu);
tauTransferHours = tauTransfer/60/60;
completeTransfer = tauTransferHours/2;

%ratio of initial and terminal masses
specificImpulse = 320; %in seconds
accGravity = 9.80665; %in km/s^2
ratio1 = exp(imp1norm/(specificImpulse*accGravity));
ratio2 = exp(imp2norm/(specificImpulse*accGravity));

%orbit 1
[r01,v01]  = oe2rv_Harris_Samantha([r1,0,Omega1,inc1,0,nu],mu);
[rv1,vv1,E0,nu0,E,nu] = propagateKepler(t0,tf, trange, r01,v01,mu);

%orbit 2 - geostationary
[r02,v02]  = oe2rv_Harris_Samantha([r2,0,0,0,0,nu],mu);
[rv2,vv2,E0,nu0,E,nu] = propagateKepler(t0,tf, trange2, r02,v02,mu);

%transfer orbit
[r0t,v0t]  = oe2rv_Harris_Samantha([aTransfer,eTransfer,Omega1,inc1,0,nu],mu);
[rt,vt,E0,nu0,E,nu] = propagateKepler(t0,tf, tranget, r0t,v0t,mu);

%l
h1 = cross(r01, v01);
h2 = cross(r02, v02);
cross = cross(h1, h2);
l = cross/norm(cross);
lx = linspace(-50000*l(1), 50000*l(1), 1000);
ly = linspace(-50000*l(2), 50000*l(2), 1000);
```

```
lz = linspace(-50000*l(3), 50000*l(3), 1000);

%plots
figure
hold on
plot3(rv1(:,1),rv1(:,2),rv1(:,3))
plot3(rv2(:,1),rv2(:,2),rv2(:,3))
plot3(rt(:,1),rt(:,2),rt(:,3))
quiver3(rt(:,1),rt(:,2),rt(:,3), vt(:,1),vt(:,2),vt(:,3), 2)
plot3(lx, ly, lz, 'b')
```

Script for 5-5

```
clc
clear all
close all

%given
    mu = 398600;
    r1 = 6371 + 350; %in km
    r2 = 26558; %km
    inc1 = deg2rad(28);
    inc2 = deg2rad(27);

%arbitrary
    Omega = deg2rad(30);
    tol = 1e-8;

%time
    t0 = 0;
    tf = 10000;
    trange = linspace(t0, tf, 100);
    trange2 = linspace(t0, 10*tf, 100);
    tranget = linspace(t0, tf, 100);
%%
%orbit 1
    %oe = [a,    e,  Longitude,  inc, argument,    nu]
    oe1 = [r1,   0,      Omega, inc1,        0,     0];
    [r01,v01]  = oe2rv_Harris_Samantha(oe1,mu);
    [rv1,vv1,E0,nu0,E,nu] = propagateKepler(t0,tf, trange, r01,v01,mu);
%%
%orbit 2
    oe2 = [r2, 0, deg2rad(30), deg2rad(55), 0, 0];
    halfTau = pi*sqrt((r2^3)/mu);
    [r02,v02]  = oe2rv_Harris_Samantha(oe2,mu);
    [rv2,vv2,E0,nu0,E,nu] = propagateKepler(t0,tf, trange2, r02,v02,mu);
```

```
    trangeHalfTau = linspace(t0, halfTau, 100);
    [rv2Imp,vv2Imp,E0,nu0,E,nu] = propagateKepler(t0,halfTau,trangeHalfTau , r02,v02,mu);
    positionAtImpulse2 = rv2Imp(end, 1:3);
%%
%transfer
    a = (r1 + r2)/2;%km
    e = (r2 - r1)/(r2 + r1);
    tauTransferSec = pi*sqrt((a^3)/mu);
    tauTransferHour = tauTransferSec/60/60;
    oe2 = [a, e, Omega, inc2, 0, 0];
    [r0t,v0t]  = oe2rv_Harris_Samantha(oe2,mu);
    [rt,vt,E0,nu0,E,nu] = propagateKepler(t0,tf, tranget, r0t,v0t,mu);
%%
%l
h1 = cross(r01, v01);
h2 = cross(r02, v02);
cross = cross(h1, h2);
l = cross/norm(cross);
lx = linspace(-30000*l(1), 30000*l(1), 1000);
ly = linspace(-30000*l(2), 30000*l(2), 1000);
lz = linspace(-30000*l(3), 30000*l(3), 1000);
%%
%finding total impulse
h1norm = norm(h1);
h2norm = norm(h2);
theta = acos(dot(h1, h2)/(h1norm*h2norm));
speed = norm(vv2Imp(end, 1:3));
totalImpulse = 2*speed*sin(theta/2);


%%
figure
hold on
plot3(rv1(:,1),rv1(:,2),rv1(:,3))
plot3(rv2(:,1),rv2(:,2),rv2(:,3))
plot3(rt(:,1),rt(:,2),rt(:,3))
quiver3(rt(:,1),rt(:,2),rt(:,3), vt(:,1),vt(:,2),vt(:,3), 2)
plot3(lx, ly, lz, 'b')
```

Script for 5-6

```
clc
clear all
close all

%%
```

```
%given
mu = 398600;
r1 = 6371 + 300; %in km
inc1 = deg2rad(28.5);
tau2 = 24*60*60;

%%
%arbitrary
N = 1000;

%%
%time
t0 = 0;
tf = 10000;
trange = linspace(t0, tf, 100);
trange2 = linspace(t0, tau2, 100);
tranget = linspace(t0, 10*tf, 100);

%%
%orbit 1
    %oe = [a,    e,  Longitude,         inc, argument,    nu]
    oe1 = [r1,   0,deg2rad(30), inc1,        0,     0];
    [r01,v01]  = oe2rv_Harris_Samantha(oe1,mu);
    [rv1,vv1,E0,nu0,E,nu] = propagateKepler(t0,tf, trange, r01,v01,mu);

%%
%orbit 2
    r2 = nthroot(mu*(tau2/(2*pi))^2,3);
    oe2 = [r2, 0, 0, 0, 0, 0];
    [r02,v02]  = oe2rv_Harris_Samantha(oe2,mu);
    [tspan, rv2, vv2, w1, w2, w3] = propogateOnCircle(r02, v02, t0, tau2, mu, N);

%%
%transfer
    a = (r1 + r2)/2;%km
    e = (r2 - r1)/(r2 + r1);
    halfTau = pi*sqrt((a^3)/mu);
    trangeHalfTau = linspace(t0, halfTau, 100);
    oe2 = [a, e, deg2rad(30), deg2rad(28.5), 0, 0];
    [r0t,v0t]  = oe2rv_Harris_Samantha(oe2,mu);
    [rt,vt,E0,nu0,E,nu] = propagateKepler(t0,tf, tranget, r0t,v0t,mu);
    [rtImp,vtImp,E0,nu0,E,nu] = propagateKepler(t0,halfTau,trangeHalfTau , r0t,v0t,mu);
    positionAtImpulse2 = rtImp(end, 1:3);
    velocityAtImpulse2 = vtImp(end, 1:3);
%%
%1
```

```
h1 = cross(r01, v01);
h2 = cross(r02, v02);
cross = cross(h1, h2);
l = cross/norm(cross);
lx = linspace(-50000*l(1), 50000*l(1), 1000);
ly = linspace(-50000*l(2), 50000*l(2), 1000);
lz = linspace(-50000*l(3), 50000*l(3), 1000);

%%
%finding total impulse
h1norm = norm(h1);
h2norm = norm(h2);
theta = acos(dot(h1, h2)/(h1norm*h2norm));

%total impulse at apoapsis
speedApo = norm(velocityAtImpulse2);
totalImpulseApo = 2*speedApo*sin(theta/2);

%total impulse at initial LEO
speedLEO = norm(v01);
totalImpulseLEO = 2*speedLEO*sin(theta/2);

%%
%plotting inverse vs f
vcl = sqrt(mu/r1);
f = 0:.01:1;
thetaPlot = f*inc1;
totalImpulseLEOPlot = 2*speedLEO*sin(thetaPlot/2);
impulseNormalized = totalImpulseLEOPlot/vcl;
figure
plot(f, impulseNormalized)
ylabel('total normalized impulse')
xlabel('f')


%%
figure
hold on
plot3(rv1(:,1),rv1(:,2),rv1(:,3))
plot3(rv2(:,1),rv2(:,2),rv2(:,3))
plot3(rt(:,1),rt(:,2),rt(:,3))
quiver3(rt(:,1),rt(:,2),rt(:,3), vt(:,1),vt(:,2),vt(:,3), 2)
plot3(lx, ly, lz, 'b')
```