Mini Project 1

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Project Rules

The rules for this mini-project are as follows:

(1) You are not permitted to communicate with anyone about the contents of this mini-project during the entire duration of this assignment.

(2) Aside from the Professor, you are not permitted to get assistance from anyone in any form whatsoever.

(3) Although you are not permitted to communicate with anyone, you are permitted to use any resources from books, lecture notes, or any online resources you find useful provided you abide by (1) above. If you use any specific external resource (aside from lecture notes or course materials), you *must provide a citation* to the source from which you obtained the information. The citation should also appear as a reference in the bibliography of your submission.

(4) All numerical solutions must be obtained using MATLAB in accordance with the instructions in the questions.

(5) You must type your solutions neatly in either Microsoft Word or LaTeX.

(6) You must be crystal clear with every step of your solution. In other words, every step in a derivation or statement you write must be unambiguous (that is, each step must have one and only one meaning). If any step is ambiguous, it will be assumed to be incorrect.

Anyone suspected of violating the assignment rules will be subject to the rules and regulations of the University of Florida Academic Honesty policy found by clicking here.

Point Distribution

The exam consists of a series of questions with the value of each question clearly indicated. Unless otherwise stated, full credit will be given for a proper application of a relevant concept. Contrariwise, no credit will be given for a concept applied incorrectly, ***even if the final answer is correct***.

University of Florida Honor Code

On your exam you must state and sign the University of Florida honor pledge as follows:

We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the university, the following pledge is either required or implied: On my honor, I have neither given nor received unauthorized aid in completing this assignment.

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Signature: Samantha Harris **Date:** ~~AH~~ 2/7/2020

1 Question 1

Determine a relationship between the change in time of a spacecraft on an orbit, and the spacecraft's true anomaly.

From Homework 1, we proved the relationship,

$$\frac{d\nu}{dt} = \dot{\nu} = \frac{h}{r^2} \tag{1}$$

Rearranging the equation, we get

$$\frac{r^2 d\nu}{h} = dt \tag{2}$$

Our goal is to find the relationship between the change in time of a spacecraft on an orbit, and the spacecraft's true anomaly in terms of the semi-latus rectum, the eccentricity and the gravitational parameter.

Using the orbit equation,

$$r = \frac{p}{(1 + ecos\nu)} \tag{3}$$

we can plug a value for r into equation 2,

$$\frac{p^2}{h(1 + ecos\nu)^2} d\nu = dt \tag{4}$$

Using the fact that,

$$p = \frac{h^2}{\mu} \tag{5}$$

we can rearrange this equation to find a value for h

$$p\mu = h^2 \tag{6}$$

$$\sqrt{p\mu} = h \tag{7}$$

plugging equation 7 into equation 4 we get

$$\frac{p^2}{\sqrt{p\mu}(1 + ecos\nu)^2} d\nu = dt \tag{8}$$

simplifying,

$$\frac{\sqrt{p^3/\mu}}{(1 + ecos\nu)^2} d\nu = dt \tag{9}$$

Therefore,

$$t_2 - t_1 = \int_{\nu_1}^{\nu_2} \frac{\sqrt{p^3/\mu}}{(1 + ecos\nu)^2} d\nu \tag{10}$$

2 Question 2

Script for my time change integrand function:

```
    function f = timeChangeIntegrand(nu, p, e, mu)
% -------------------------------------------------------------%
% Integrand f(nu,p,e,mu) that is used to obtain the time change %
%                           deltat =t2 - t1                     %
% Inputs:                                                       %
%   nu: true anomaly (rad)                                      %
%    p: parameter (semi-latus rectum)                           %
%    e: eccentricity                                            %
%   mu: gravitational paramter                                  %
% Output:                                                       %
%    f: value of function at (nu, p, mu)                        %
% -------------------------------------------------------------%
%function found from part a
f = sqrt(p^3/mu)./((1+e.*cos(nu)).*(1+e.*cos(nu)));
```

Script for my time change integral function:

```
    function deltat = timeChangeIntegral(f, nu1, nu2, p, e, mu, N)
% -------------------------------------------------------------%
% This function employs Legendre-Gauss quadrature to compute an %
% approximation to                                              %
%                               /nu2                            %
%                              /                                %
%                              |                                %
%           deltat = t2 - t1 = | f(nu,p,e,mu)dnu                %
%                              |                                %
%                              /                                %
%                             /nu1                              %
% where f(nu) is a function that used to define the change in   %
% The inputs and outputs of this function are as follows:       %
% Inputs:                                                       %
%   f         = a handle to the function to be integrated       %
%               in the form @(nu,p,mu) <MY-FUNCTION-HERE>
%   nu1       = lower integration limit                         %
%             = initial true anomaly (rad)                      %
%   nu2       = upper integration limit                         %
%             = terminal true anomaly (rad)                     %
%   p         = parameter (semi-latus rectum)                   %
%   e         = eccentricity                                    %
%   mu        = gravitational parameter                         %
%   N         = number of Gauss points & weights                %
%               used to approximate the integral                %
%                                                               %
```

```
% Output:                                                       %
%   deltat    = Gauss quadrature approximation of               %
%               where deltat = t2 - t1 is the                   %
%               time change from nu1 to nu2                     %
% -------------------------------------------------------------%
%get the gauss weight points and x values
 [nu,w] = GaussPointsWeights(nu1,nu2,N);

 %for each value nu, find the value of the function (the values for p, e
 %and mu should be the same for each calculation)
 F = f(nu, p, e, mu);

 %find the area under the curve (solve the integral)
 deltat = w'*F;
```

3 Question 3

```
-------------------------------------------------------------
-------------------------------------------------------------
                        Part (a):
-------------------------------------------------------------
-------------------------------------------------------------
Position of the Spacecraft (rv) [km]:
   [   5634.29739700,  -2522.80786300,  -5037.93088900]
-------------------------------------------------------------
Velocity of the Spacecraft (vv) [km/s]:
   [      8.28617600,      1.81514400,      3.62475900]
-------------------------------------------------------------
Gravitational Parameter of Earth (mu) [km^3/s^2]:
  398600.00000000
-------------------------------------------------------------
Radius of Earth (Re) [km]:
    6378.14500000
-------------------------------------------------------------
Specific Angular Momentum (hv) [km^2/s]:
   [     -0.00048110, -62168.15222054,  31131.49108138]
-------------------------------------------------------------
Magnitude of the Specific Angular Momentum (h) [km^2/s]:
                                       69527.32475413
-------------------------------------------------------------
Semi-Latus Rectum (p) [km]:          12127.56870915
-------------------------------------------------------------
Radius of the Earth (r) [km]:             7968.09979316
-------------------------------------------------------------
Eccentricity vector (ev):
```

```
[     -0.00000014,     -0.33055414,     -0.66010137]
-------------------------------------------------------------
Eccentricity (e):                             0.73824106
-------------------------------------------------------------
Semi-major axis (a) [km]:                   26653.98916786
-------------------------------------------------------------
Orbital period (tau) [seconds]:    43306.64565920
-------------------------------------------------------------
Orbital period (tauhr) [hours]:       12.02962379
-------------------------------------------------------------
-------------------------------------------------------------
 Part (b): Time change from nu1=90 deg to nu2=270 deg
-------------------------------------------------------------
-------------------------------------------------------------
Time elapsed (90,270) deg [hours] (N=10):     11.09570134
Time elapsed (90,270) deg [hours] (N=15):     11.09570134
Time elapsed (90,270) deg [hours] (N=20):     11.09570134
Time elapsed (90,270) deg [hours] (N=25):     11.09570134
-------------------------------------------------------------
-------------------------------------------------------------
 Part (c): Time change from nu2=270 deg to nu1=90 deg
-------------------------------------------------------------
-------------------------------------------------------------
Time elapsed (270,450) deg [hours] (N=10):     0.93392245
Time elapsed (270,450) deg [hours] (N=15):     0.93392245
Time elapsed (270,450) deg [hours] (N=20):     0.93392245
Time elapsed (270,450) deg [hours] (N=25):     0.93392245
-------------------------------------------------------------
-------------------------------------------------------------
 Part (d): See figure 1.
-------------------------------------------------------------
-------------------------------------------------------------
 Part (e): The orbital period is about 12 hours, which
 means that the spacecraft orbits around earth twice a
 day. It also means that the spacecraft faces the same
 side of earth on every second orbit. The time elapsed
 from nu1 = 90 degrees to nu2 = 270 degrees is about 11
 hours. If someone wanted to spy on a certain spot on
 earth, this would be a great orbit to do so. Therefore,
 the Soviet Union is probably spying on the United States
 for about 11 hours every day.
-------------------------------------------------------------
-------------------------------------------------------------
```

Script for Question 3:

```
clear all
```

```
close all
clc
%--------------------------
%Mini Project Question 3
%--------------------------
%part a
%-------
%Starting with the position and velocity at time t=0, perform a series of
%computations such that the following values on the orbit are obtained:
%           semi-latus rectum in km
%           eccentricity
%           the orbital period in hours
%at time t = 0 and in units of km, the position is
rv = [5634.297397; -2522.807863;-5037.930889];

%at time t = 0 and in units of km/s, the velocity measured in the
%inertial reference frame is
vv = [8.286176;1.815144;3.624759];

%the earth gravitational parameter in km^3/s^2
mu = 398600;

%the radius of the earth in km
Re = 6378.145;

%specific angular momentum
hv = cross(rv, vv);

%magnitude of the specific angular momentum
h = norm(hv);

%semi-latus rectum
p = h^2/mu;

%radius of the spacecraft
r = norm(rv);

%eccentricity vector
ev = cross(vv, hv)/mu - rv/r;

%eccentricity
e = norm(ev);

%semi-major axis
a = p/(1-e^2);

%orbital period of the spacecraft in seconds
tau = 2*pi*sqrt(a^3/mu);

%orbital period of the spacecraft in hours
```

```
tauhr = tau/3600;

%--------
%part b
%--------
%Using Gauss Quadrature approximation, estimate the time elapsed on the
%orbit in hours from nu1 = 90 degrees to nu2 = 270 degrees for Gauss points
%N = [10, 15, 20, 25]

%starting and ending true anomaly in degrees
nu1deg = 90; nu2deg = 270;
%nu1 in radians
nu1 = deg2rad(nu1deg);
%nu2 in radians
nu2 = deg2rad(nu2deg);

%Gauss Points
N = [10,15,20,25];

%the time elapsed on the orbit in hours fron nu1 = 90 degrees to nu2 = 270
%degrees for the gauss points N = [10,15,20,25]
% deltat1 = total time elapsed in hours
deltat1 = [];
for i = 1:length(N)
    %deltatsec = total time elapsed in seconds
    deltatsec = timeChangeIntegral(@timeChangeIntegrand, nu1, nu2,p,e,mu, N);
    %converting seconds to hours
    deltat = deltatsec/3600;
    %time elaphsed in hours for each N value
    deltat1 = [deltat1; deltat];
end
%--------
%part c
%--------
%find the time elapsed on the orbit in hours fron nu2 = 270 degrees to nu1 = 90
%degrees for the gauss points N = [10,15,20,25]
%
% delta2 = time elapsed from nu2 to nu1 in hours
deltat2 = [];
for i = 1:length(N)
    %the orbital period of the spacecraft in hours subtracted by the time
    %elapsed on the orbit from nu1 to nu2
    deltat = tauhr - deltat1(i);
    deltat2 = [deltat2; deltat];
end

%--------
%part d
%--------
%make a polar plot of the orbit for this problem alongside a second plot
```

```
%that shows a circle with the radius of the Earth

%nuOrbit = an array that starts at 0, ends at 2pi and has 100 elements evenly
%spaced
nuOrbit = linspace(0,2*pi,100).';
%rOrbit = radius of the spacecraft on its orbit (orbit equation)
rOrbit = p./(1+e*cos(nuOrbit));
%rEarthSurface = create an a matrix of 1's that's the same size as
%nuOrbitm, and multiply every element by the radius of the earth
rEarthSurface = Re*ones(size(nuOrbit));

%polar plot of the orbit of the spacecraft along with an outline of the
%earth
figure
polarplot(nuOrbit, rOrbit,nuOrbit,rEarthSurface)
legend('Spacecraft Orbit', 'Outline of Earth')

fprintf('-------------------------------------------------------------\n');
fprintf('-------------------------------------------------------------\n');
fprintf('                        Part (a):                         \n');
fprintf('-------------------------------------------------------------\n');
fprintf('-------------------------------------------------------------\n');
fprintf('Position of the Spacecraft (rv) [km]:\n   ');
fprintf('[%16.8f,%16.8f,%16.8f]\n',rv);
fprintf('-------------------------------------------------------------\n');
fprintf('Velocity of the Spacecraft (vv) [km/s]:\n   ');
fprintf('[%16.8f,%16.8f,%16.8f]\n',vv);
fprintf('-------------------------------------------------------------\n');
fprintf('Gravitational Parameter of Earth (mu) [km^3/s^2]:\n');
fprintf('\t\t\t\t\t\t %16.8f\n',mu);
fprintf('-------------------------------------------------------------\n');
fprintf('Radius of Earth (Re) [km]:\n');
fprintf('\t\t\t\t\t\t %16.8f\n',Re);
fprintf('-------------------------------------------------------------\n');
fprintf('Specific Angular Momentum (hv) [km^2/s]:\n   ');
fprintf('[%16.8f,%16.8f,%16.8f]\n',hv);
fprintf('-------------------------------------------------------------\n');
fprintf('Magnitude of the Specific Angular Momentum (h) [km^2/s]:\n');
fprintf('                                   \t %16.8f\n',h);
fprintf('-------------------------------------------------------------\n');
fprintf('Semi-Latus Rectum (p) [km]:      \t %16.8f\n',p);
fprintf('-------------------------------------------------------------\n');
fprintf('Radius of the Earth (r) [km]:        \t %16.8f\n',r);
fprintf('-------------------------------------------------------------\n');
fprintf('Eccentricity vector (ev):\n' );
fprintf('[%16.8f,%16.8f,%16.8f]\n',ev);
fprintf('-------------------------------------------------------------\n');
fprintf('Eccentricity (e):                    \t %16.8f\n',e);
fprintf('-------------------------------------------------------------\n');
fprintf('Semi-major axis (a) [km]:              \t %16.8f\n',a);
```

```
fprintf('--------------------------------------------------------------\n');
fprintf('Orbital period (tau) [seconds]:\t\t %16.8f\n'          , tau);
fprintf('--------------------------------------------------------------\n');
fprintf('Orbital period (tauhr) [hours]:\t\t %16.8f\n', tauhr);
fprintf('--------------------------------------------------------------\n');
fprintf('--------------------------------------------------------------\n');
fprintf(' Part (b): Time change from nu1=%i deg to nu2=%i deg \n',nu1deg,nu2deg );
fprintf('--------------------------------------------------------------\n');
fprintf('--------------------------------------------------------------\n');
for i=1:length(N)
    fprintf('Time elapsed (%i,%i) deg [hours] (N=%i):%16.8f\n',nu1deg,nu2deg,N(i), deltat1(i));
end
fprintf('--------------------------------------------------------------\n');
fprintf('--------------------------------------------------------------\n');
fprintf(' Part (c): Time change from nu2=%i deg to nu1=%i deg \n',nu2deg,nu1deg);
fprintf('--------------------------------------------------------------\n');
fprintf('--------------------------------------------------------------\n');
for i=1:length(N)
    fprintf('Time elapsed (%i,%i) deg [hours] (N=%i):%15.8f\n',nu2deg,nu2deg+180,N(i),deltat2(i));
end
fprintf('--------------------------------------------------------------\n');
fprintf('--------------------------------------------------------------\n');
fprintf(' Part (e): The orbital period is about 12 hours, which \n');
fprintf(' means that the spacecraft orbits around earth twice a \n');
fprintf(' day. It also means that the spacecraft faces the same \n');
fprintf(' side of earth on every second orbit. The time elapsed \n');
fprintf(' from nu1 = 90 degrees to nu2 = 270 degrees is about 11\n');
fprintf(' hours. If someone wanted to spy on a certain spot on \n');
fprintf(' earth, this would be a great orbit to do so. Therefore,\n');
fprintf(' the Soviet Union is probably spying on the United States\n');
fprintf(' for about 11 hours every day. \n');
fprintf('--------------------------------------------------------------\n');
fprintf('--------------------------------------------------------------\n');
```

4 Conclusions

In this mini project, the goal was to determine the time elapsed for a spacecraft to move from one position to another. The rate of change of a spacecraft's position with respect to time cannot be solved, but the rate of change between a spacecraft's true anomaly with respect to time can be solved. Using the result obtained from homework 1, $\dot{\nu} = h/r^2$, along with the orbit equation, we were able to find an equation that relates the change in the true anomaly to the change in time. The solution was solved numerically using Gauss quadrature, which showed the time elapsed from an initial true anomaly to a final true anomaly.

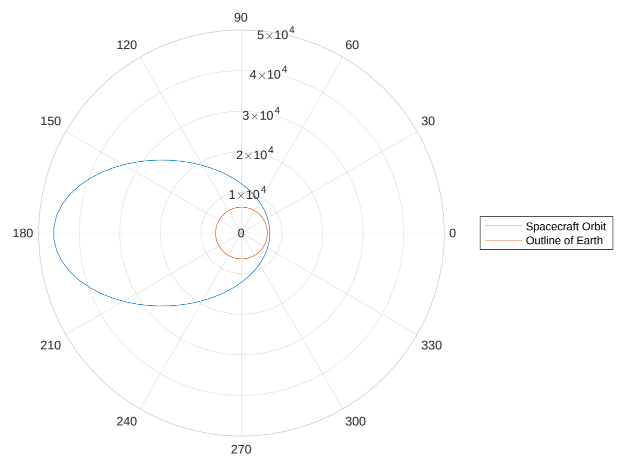

Figure 1: The orbit of the spacecraft is in blue, and a circle with the radius of the earth is in red.