Homework 1
Chapter 1 Problems

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1

Consider the motion of a spacecraft relative the Earth, where the Earth is an inertial reference frame denoted I. Furthermore, let $\{\mathbf{p}_x, \mathbf{p}_y, \mathbf{p}_z\}$ be the perifocal basis which is fixed in I.

(a) A right-handed orthonormal basis $\{\mathbf{u}_r, \mathbf{u}_\nu, \mathbf{u}_z\}$ can be seen in Figure 1.

(b)

$$\mathbf{r} = r\mathbf{u}_r$$

$$^I\mathbf{v} = \frac{^I d}{dt}\mathbf{r} = \frac{^U d}{dt}\mathbf{r} + {}^I\boldsymbol{\omega}^U \times \mathbf{r}$$

$$\frac{^U d}{dt}\mathbf{r} = \dot{r}\mathbf{u}_r$$

$$^I\boldsymbol{\omega}^U \times \mathbf{r} = \dot{\nu}\mathbf{u}_z \times r\mathbf{u}_r = r\dot{\nu}\mathbf{u}_\nu$$

$$\boxed{^I\mathbf{v} = \dot{r}\mathbf{u}_r + r\dot{\nu}\mathbf{u}_\nu}$$

$$^I\mathbf{a} = \frac{^I d}{dt}{}^I\mathbf{v} = \frac{^U d}{dt}{}^I\mathbf{v} + {}^I\omega^U \times {}^I\mathbf{v}$$

$$\frac{^U d}{dt}{}^I\mathbf{v} = \ddot{r}\mathbf{u}_r + (\dot{r}\dot{\nu} + r\ddot{\nu})\mathbf{u}_\nu$$

$$^I\omega^U \times {}^I\mathbf{v} = \dot{\nu}\mathbf{u}_z \times (\dot{r}\mathbf{u}_r + r\dot{\nu}\mathbf{u}_\nu) = \dot{r}\dot{\nu}\mathbf{u}_\nu - r\dot{\nu}^2\mathbf{u}_r$$

$$\boxed{^I\mathbf{a} = (\ddot{r} - r\dot{\nu}^2)\mathbf{u}_r + (2\dot{r}\dot{\nu} + r\ddot{\nu})\mathbf{u}_\nu}$$

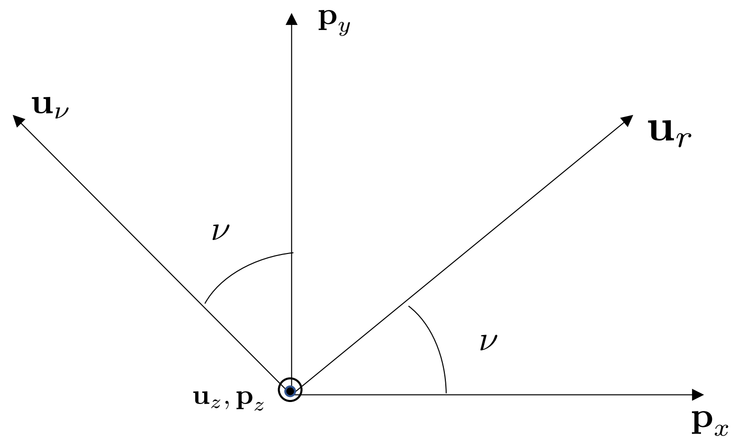

Figure 1: A right-handed orthonormal basis $\{\mathbf{u}_r, \mathbf{u}_\nu, \mathbf{u}_z\}$ fixed in a reference frame U that rotates relative to I with an angular velocity ${}^I\boldsymbol{\omega}^U = \dot{\nu}\mathbf{u}_z$.

(c)

$$^I\mathbf{a} + \frac{\mu}{r^3}\mathbf{r} = \mathbf{0}$$

$$^I\mathbf{a} = -\frac{\mu}{r^3}\mathbf{r} = -\frac{\mu}{r^2}\mathbf{u}_r$$

$$-\frac{\mu}{r^2}\mathbf{u}_r = (\ddot{r} - r\dot{\nu}^2)\mathbf{u}_r + (2\dot{r}\dot{\nu} + r\ddot{\nu})\mathbf{u}_\nu$$

To get our first differential equation, we can dot both sides of the above equation by $\mathbf{u}_r$.

$$-\frac{\mu}{r^2} = \ddot{r} - r\dot{\nu}^2$$

$$\boxed{0 = \ddot{r} - r\dot{\nu}^2 + \frac{\mu}{r^2}} \tag{1}$$

To get our second differential equation, dot both sides of the equation by $\mathbf{u}_\nu$.

$$\boxed{0 = 2\dot{r}\dot{\nu} + r\ddot{\nu}}$$

(d)

$$\frac{dE}{dt} = \frac{dT}{dt} + \frac{dU}{dt}$$

$$\frac{dT}{dt} = {}^I\mathbf{a} \cdot {}^I\mathbf{v}$$

$$\frac{dT}{dt} = \dot{r}(\ddot{r} - r\dot{\nu}^2) + r\dot{\nu}(2\dot{r}\dot{\nu} + r\ddot{\nu})$$

$$\frac{dU}{dt} = \frac{\mu}{r^3}\mathbf{r} \cdot {}^I\mathbf{v}$$

$$\frac{dU}{dt} = \frac{\mu}{r^3}r\dot{r}$$

$$\frac{dT}{dt} + \frac{dU}{dt} = \dot{r}(\ddot{r} - r\dot{\nu}^2) + r\dot{\nu}(2\dot{r}\dot{\nu} + r\ddot{\nu}) + \frac{\mu}{r^3}r\dot{r}$$

Since $0 = 2\dot{r}\dot{\nu} + r\ddot{\nu}$

$$\frac{dT}{dt} + \frac{dU}{dt} = \dot{r}(\ddot{r} - r\dot{\nu}^2) + \frac{\mu}{r^3}r\dot{r}$$

$$\frac{dT}{dt} + \frac{dU}{dt} = \dot{r}(\ddot{r} - r\dot{\nu}^2 + \frac{\mu}{r^2})$$

Since $0 = \ddot{r} - r\dot{\nu}^2 + \frac{\mu}{r^2}$

$$\frac{dE}{dt} = \frac{dT}{dt} + \frac{dU}{dt} = 0$$

(e)

$${}^I\mathbf{h} = h\mathbf{p}_z$$

$${}^I\mathbf{h} = \mathbf{r} \times {}^I\mathbf{v} = r\mathbf{u}_r \times (\dot{r}\mathbf{u}_r + r\dot{\nu}\mathbf{u}_\nu) = r^2\dot{\nu}\mathbf{u}_z$$

$$\mathbf{p}_z = \mathbf{u}_z$$

$$h\mathbf{p}_z = r^2\dot{\nu}\mathbf{p}_z \tag{2}$$

$$\frac{{}^I d}{dt}(h\mathbf{p}_z) = \frac{{}^I d}{dt}(r^2\dot{\nu}\mathbf{p}_z)$$

$$0\mathbf{p}_z = (2r\dot{r}\dot{\nu} + r^2\ddot{\nu})\mathbf{p}_z$$

$$0 = 2r\dot{r}\dot{\nu} + r^2\ddot{\nu}$$

$$\boxed{0 = 2\dot{r}\dot{\nu} + r\ddot{\nu}}$$

(f) From equation 2

$$h\mathbf{p}_z = r^2\dot{\nu}\mathbf{p}_z$$

Dot both sides by $\mathbf{p}_z$.

$$h = r^2\dot{\nu}$$

$$\boxed{\dot{\nu} = \frac{h}{r^2}}$$

(g) Deriving an expression for $\frac{d\rho}{d\nu}$

$$\dot{r} = \frac{dr}{dt} = \frac{dr}{d\nu}\frac{d\nu}{dt}$$

$$r = \frac{1}{\rho}$$

$$\dot{r} = \frac{d}{d\nu}\frac{1}{\rho}\frac{d\nu}{dt}$$

$$\frac{d}{d\nu} = \frac{d}{d\rho}\frac{d\rho}{d\nu}$$

$$\dot{r} = \frac{d}{d\rho}\frac{1}{\rho}\frac{d\rho}{d\nu}\frac{d\nu}{dt}$$

$$\frac{d}{d\rho}\frac{1}{\rho} = \frac{-1}{\rho^2}$$

$$\frac{d\nu}{dt} = \dot{\nu} = \frac{h}{r^2} = h\rho^2$$

$$\dot{r} = \frac{-1}{\rho^2}\frac{d\rho}{d\nu}h\rho^2$$

$$\dot{r} = -h\frac{d\rho}{d\nu}$$

$$\boxed{\frac{d\rho}{d\nu} = -\frac{\dot{r}}{h}}$$

Deriving an expression for $\frac{d^2\rho}{d\nu^2}$

$$\ddot{r} = \frac{d\dot{r}}{dt} = \frac{d\dot{r}}{d\nu}\frac{d\nu}{dt}$$

$$\ddot{r} = \frac{d}{d\nu}(-h\frac{d\rho}{d\nu})\frac{d\nu}{dt}$$

$$\ddot{r} = \frac{d}{d\nu}(-h\frac{d\rho}{d\nu})h\rho^2$$

$$\ddot{r} = -h^2\rho^2\frac{d^2\rho}{d\nu^2}$$

$$\boxed{\frac{d^2\rho}{d\nu^2} = \frac{-\ddot{r}}{h^2\rho^2}}$$

(h) Starting with equation 1,

$$0 = \ddot{r} - r\dot{\nu}^2 + \frac{\mu}{r^2}$$

and plugging in the value for $\dot{\nu}$ from part (f),

$$0 = \ddot{r} - r\frac{h^2}{r^4} + \frac{\mu}{r^2}$$

next plug in the value for $\ddot{r}$ from part (g)

$$0 = -h^2\rho^2\frac{d^2\rho}{d\nu^2} - r\frac{h^2}{r^4} + \frac{\mu}{r^2}$$

lastly, plut in $r = 1/\rho$

$$0 = -h^2\rho^2\frac{d^2\rho}{d\nu^2} - h^2\rho^3 + \mu\rho^2$$

To simplify, take out $-h^2\rho^2$

$$\boxed{0 = \frac{d^2\rho}{d\nu^2} + \rho - \frac{\mu}{h^2}}$$

(i) To find the general solution, we will first rewrite the differential equation as a homogeneous differential equation.

Let

$$\widetilde{\rho} = \rho - \frac{\mu}{h^2}$$

$$\rho = \widetilde{\rho} + \frac{\mu}{h^2}$$

We can see that

$$\frac{d^2\widetilde{\rho}}{d\nu^2} = \frac{d^2\rho}{d\nu^2}$$

substituting these values into our differential equation

$$0 = \frac{d^2\widetilde{\rho}}{d\nu^2} + \widetilde{\rho} + \frac{\mu}{h^2} - \frac{\mu}{h^2}$$

$$0 = \frac{d^2\widetilde{\rho}}{d\nu^2} + \widetilde{\rho}$$

This is in the same form as the differential equation seen in Question 1 of Homework #0. The general solution to this differential equation is:

$$\widetilde{\rho} = cos(\nu) + sin(\nu)$$

pluggin in our value for $\widetilde{\rho}$

$$\rho - \frac{\mu}{h^2} = cos(\nu) + sin(\nu)$$

$$\boxed{\rho = cos(\nu) + sin(\nu) + \frac{\mu}{h^2}}$$

(j)

$$\rho = cos(\nu) + sin(\nu) + \frac{\mu}{h^2}$$

$$\frac{1}{r} = cos(\nu) + sin(\nu) + \frac{\mu}{h^2}$$

$$r = \frac{1}{cos(\nu) + sin(\nu) + \frac{\mu}{h^2}}$$

$$r = \frac{\frac{h^2}{\mu}}{1 + \frac{h^2}{\mu}cos(\nu) + \frac{h^2}{\mu}sin(\nu)}$$

$$r = \frac{\frac{h^2}{\mu}}{1 + \frac{h^2}{\mu}(cos(\nu) + sin(\nu))}$$

2

Consider the following two Earth orbits:

- Orbit 1: Periapsis Radius = $r_{p1} = r_p$; Semi-Major Axis = a_1
- Orbit 2: Periapsis Radius = $r_{p2} = r_p$; Semi-Major Axis = $a_2 > a_1$

1. Which orbit has the larger speed at periapsis?

   From equation (1.94) in the class notes, we can determine the speed of the spacecraft at any point on the solution of the two-body differential equation.

$$v = \sqrt{\mu}\sqrt{\frac{2}{r} - \frac{1}{a}}$$

   From this equation, we can see that if a gets larger, the term $\frac{1}{a}$ will get smaller. A smaller amount being subtracted from the term $\frac{2}{r}$ will result in an overall larger value for v. Therefore, orbit 2 will have the larger speed at periapsis.

2. Which orbit has the larger speed at apoapsis?

   From equation (1.48) in the notes, we can see how the semi-major axis is related to the apoapsis radius.

$$a = \frac{r_p + r_a}{2}$$

   From this equation, we can see that if a gets larger, r_a will get larger. Therefore, orbit 2 will have a smaller value for the term $\frac{2}{r}$ and a smaller value for the term $\frac{1}{a}$. The value $\frac{2}{r}$ will have bigger overall effect on the speed and since that value is smaller for orbit 2, we know that orbit 1 has the larger speed at apoapsis.

3

The answer for to question 3 is below:

```
    >> HARRIS_HW1_Q3
Semi-major axis   a =    7028.14500000
Eccentricity    e =      -0.02134276
Semi-latus rectum   p =    7024.94358626
Magnitude of Specific angular momentum   h =   52916.37282998
Periapsis Speed   vp =       7.37187293
Apoapsis Speed   va =       7.69340757
>>
```

The script for question 3 is below:

```
    % Solution to Question 3.
%given
mu      = 398600; %graviational parameter for Earth in km^3/s
Re      = 6378.145; %radius of the earth in km
hp      = 800; %perapsis altitude in km
ha      = 500; %apoapsis altitude in km

%find
ra      = ha + Re; %apoapsis radius
rp      = hp + Re; %periapsis radius
a       = (rp +ra)/2; %semi-major axis
e       = (ra - rp)/(ra + rp); %eccentricity
p       = a*(1- e^2); %semi latus rectum
h       = sqrt(p*mu); %magnitude of angular momentum
vp      = sqrt(mu)*sqrt(2/rp - 1/a); %periapsis speed
va      = sqrt(mu)*sqrt(2/ra - 1/a); %apoapsis speed
```

```
% Print results using fprintf statements
fprintf('Semi-major axis \t\t\t a = %16.8f\n' ,a);
fprintf('Eccentricity \t\t\t\t  e = %16.8f \n',e);
fprintf('Semi-latus rectum \t\t\t p = %16.8f\n' ,p);
fprintf('Magnitude of Specific angular momentum \t h = %16.8f\n' ,h);
fprintf('Periapsis Speed \t\t\t vp = %16.8f\n' ,vp);
fprintf('Apoapsis Speed \t\t\t\t va = %16.8f\n' ,va);
```

4

In this problem we know,

$$\begin{aligned} r_{pA} &= r_{pB} \\ r_{aA} &< r_{aB} \\ b_A &= b_B \end{aligned} \tag{3}$$

and the spacecraft should be at r_p in order to view point Q that lies in the direction from the planet to the apoapsis.

Since $r_{aA} < r_{aB}$ we know that $a_A < a_B$ from equation (1.48). From equation (1.94), we can see that orbit B will have a smaller value for the term $\frac{2}{r}$ and a smaller value for the term $\frac{1}{a}$. The value $\frac{2}{r}$ will have bigger overall effect on the speed and since that value is smaller for orbit B, we know that orbit B will enable the spacecraft the longer visualization time of point Q.

5

In this problem we know,

$$a_1 = a_2$$

$$a_1 = r_1$$

therefore,

$$a_2 = r_1$$

from equation (1.94),

$$v_{p2} = \sqrt{\mu}\sqrt{\frac{2}{r_p} - \frac{1}{a_2}} \tag{4}$$

from equation (1.54),

$$r_p = a(1-e)$$
$$r_p = r_1(1-e)$$

plugging r_p and a_2 into Equation 4

$$v_{p2} = \sqrt{\mu}\sqrt{\frac{2}{r_1(1-e)} - \frac{1}{r_1}}$$

Rearranging to get e by itself,

$$\frac{v_{p2}^2}{\mu} = \frac{2}{r_1(1-e)} - \frac{1}{r_1}$$
$$\frac{v_{p2}^2 r_1}{\mu} = \frac{2}{(1-e)} - 1$$
$$\frac{v_{p2}^2 r_1}{\mu} + 1 = \frac{2}{(1-e)}$$
$$1 - e = \frac{2}{\frac{v_{p2}^2 r_1}{\mu} + 1}$$
$$-e = \frac{2}{\frac{v_{p2}^2 r_1}{\mu} + 1} - 1$$
$$\boxed{e = -\frac{2}{\frac{v_{p2}^2 r_1}{\mu} + 1} + 1}$$

6

The answer to Question 6 is below:

```
    >> HARRIS_HW1_Q6
Inertial Acceleration,   av = [      0.22508768,      0.37514614,     -0.28135960]
>>
```

The script for Questions 6 is below:

```
    % Solution to Question 6.
%given
mu      = 1;
rv      = [-12; -20; 15]/20;
vv      = [16; -9; 9]/20;

%find
r       = norm(rv); %magnitude of r
av      = (-mu/r^3) * rv; %inertial acceleration of spacecraft

% Print results using fprintf statements
fprintf('Inertial Acceleration, \t\t av = [%16.8f,%16.8f,%16.8f]\n',av);
```

7

(a) Because the specific angular momentum lies along the direction $\mathbf{u}_z$ where $\mathbf{u}_z = \mathbf{p}_z$ and is orthogonal to the orbit plane,

$$^I\mathbf{h} = \mathbf{r} \times {}^I\mathbf{v} = rv\mathbf{u}_z sin\phi \tag{5}$$

Where ϕ is the zenith angle. Taking the magnitude of equation 5

$$h = ||^I\mathbf{h}|| = ||\mathbf{r} \times {}^I\mathbf{v}|| = rvsin\phi \tag{6}$$

The flight path angle is

$$\gamma = \frac{\pi}{2} - \phi \tag{7}$$

Using Equation 7 together with the fact that $sin\phi = sin(\pi/2 - \gamma) = cos\gamma$, equation 6 becomes

$$h = rvcos\gamma \tag{8}$$

The scalar product of $\mathbf{r}$ with $^I\mathbf{v}$ gives

$$\mathbf{r} \cdot {}^I\mathbf{v} = ||\mathbf{r}||||^I\mathbf{v}||cos\phi = rvcos\phi \tag{9}$$

Using the identity $cos\phi = cos(\pi/2 - \gamma) = sin\gamma$ gives

$$\mathbf{r} \cdot {}^I\mathbf{v} = rvsin\gamma \tag{10}$$

Combining the results in Equation 8 and 10, the tangent of the flight path angle is given as

$$tan\gamma = \frac{\mathbf{r} \cdot {}^{I}\mathbf{v}}{h} \tag{11}$$

Furthermore,

$$\mathbf{r} \cdot {}^{I}\mathbf{v} = \frac{1}{2}\frac{d}{dt}(\mathbf{r} \cdot \mathbf{r}) = \frac{1}{2}\frac{d}{dt}(r^2) = r\dot{r} \tag{12}$$

$$tan\gamma = \frac{r\dot{r}}{h} \tag{13}$$

We know that,

$$\dot{r} = \frac{hesin\nu}{p} \tag{14}$$

Plugging equation 14 into equation 13

$$tan\gamma = \frac{r\frac{hesin\nu}{p}}{h} = \frac{resin\nu}{p} \tag{15}$$

we know that

$$r = \frac{p}{1 + .ecos\nu} \tag{16}$$

Plugging equation 16 into equation 15 we get,

$$\boxed{tan\gamma = \frac{esin\nu}{1 + ecos\nu}} \tag{17}$$

(b) The maximum value of the flight path angle will be when the denominator of equation 17 is small, this is when ν is close to π. The minimum value would be when $\nu = 0$.

8

The solution to question 8 is below:

```
>> HARRIS_HW1_Q8
Position,   rv = [      0.00000000,      2.00000000,      0.00000000]
Inertial Velocity, vv = [     -0.57735027,      0.81649658,      0.00000000]
Specific Angular Momentum, hv = [      0.00000000,     -0.00000000,      1.15470054]
Eccentricity Vector, ev =   [      0.94280904,     -0.33333333,      0.00000000]
Eccentricity , e =         1.00000000
hv dot ev =         0.00000000
Semi-Latus Rectum, p =         1.33333333
Semi-Major Axis, a =   -3002399751580331.50000000
True Anomaly [deg], nu =       109.47122063
>>
```

The script for question 8 is below:

```
% Solution to Question 8.  Note that quantities such as \rv", \vv", etc, are vecto
% whereas quantities such as \r", \v", etc are scalars.
mu      = 1;
rv      = [0; 2; 0];
vv      = [-1; sqrt(2); 0]/sqrt(3);
hv      = cross(rv,vv);
h       = norm(hv,2);
r       = norm(rv,2);
v       = norm(vv,2);
ev      = cross(vv, hv)/mu - rv/r;
e       = norm(ev,2);
hvdotev = dot(hv, vv);
p       = h^2/mu;
a       = p/(1-e^2);
cosnu   = (p/r - 1)/e;
nu      = acos(cosnu);
nudeg   = nu*180/pi;
% Print results using fprintf statements
fprintf('Position, \t\t rv = [%16.8f,%16.8f,%16.8f]\n',rv);
fprintf('Inertial Velocity, vv = [%16.8f,%16.8f,%16.8f]\n',vv);
fprintf('Specific Angular Momentum, hv = [%16.8f,%16.8f,%16.8f]\n',hv);
fprintf('Eccentricity Vector, ev = \t [%16.8f,%16.8f,%16.8f]\n',ev);
fprintf('Eccentricity , e = \t\t %16.8f \n',e);
fprintf('hv dot ev = \t\t\t %16.8f \n',hvdotev);
fprintf('Semi-Latus Rectum, p = \t\t\t %16.8f \n',p);
fprintf('Semi-Major Axis, a = \t\t\t %16.8f \n',a);
fprintf('True Anomaly [deg], nu = \t\t\t %16.8f \n',nudeg);
```

9

(a)

$$r = \frac{p}{1 + ecos\nu}$$

$$1 + ecos\nu = \frac{p}{r}$$

$$ecos\nu = \frac{p}{r} - 1$$

$$cos\nu = \frac{p}{re} - \frac{1}{e}$$

$$\nu = cos^{-1}[\frac{p}{re} - \frac{1}{e}]$$

Since $p = a(1 - e^2)$ and $r = a$

$$\nu = cos^{-1}(\frac{a(1 - e^2)}{ae} - \frac{1}{e})$$

$$\nu = cos^{-1}(\frac{1}{e}(1 - e^2 - 1))$$

$$\nu = cos^{-1}(\frac{-e^2}{e})$$

$$\boxed{\nu = cos^{-1}(-e)}$$

(b)

$$v = \sqrt{\mu}\sqrt{\frac{2}{r} - \frac{1}{a}}$$

At $r = a$

$$v = \sqrt{\mu}\sqrt{\frac{2}{a} - \frac{1}{a}}$$

$$v = \sqrt{\mu}\sqrt{\frac{1}{a}}$$

$$\boxed{v = \sqrt{\frac{\mu}{a}}}$$

10

The soluton to question 10 is below:

```
>> HARRIS_HW1_Q10
Magnitude of specific angular momentum h = 1190386.17270834
semi-latus rectum   p = 3554990.56742400
periapsis radius   rp = 2962492.13952000
apoapsis radius   ra = 4443738.20928000
>>
```

The script for question 10 is below:

```
% Solution to Question 10.
mu      = 398600; %graviational parameter for Earth
E       = (-200000000)*.0003048*.0003048; %conversion from ft^2/s^2 ro km^2/s^2
e       = .2; %eccentricity
a       = (-mu)/2*E;
p       = a*(1-e^2);
h       = sqrt(p*mu);
rp      = p/(1+e);
ra      = p/(1-e);
% Print results using fprintf statements
fprintf('Magnitude of specific angular momentum h = %16.8f\n' ,h);
fprintf('semi-latus rectum \t\t\t p = %16.8f\n' ,p);
fprintf('periapsis radius \t\t\t rp = %16.8f\n' ,rp);
fprintf('apoapsis radius \t\t\t ra = %16.8f\n' ,ra);
```

11

The solution to question 11 is below:

```
>> HARRIS_HW1_Q11
Apoapsis altitude   ha =    1869.58777778
Specific mechanical engergy   E = -1494339220.55555582
magnitude of the specific angular momentum h =   54394.77600561
semi-latus rectum   p =    7422.95950000
>>
```

The script for question 11 is below:

```
    % Solution to Question 11.
%given
```

```
mu      = 398600; %graviational parameter for Earth
Re      = 6378.145; %radius of the earth
rp      = 370 + Re; %periapsis radius
e       = .1; %eccentricity

%find
p       = rp*(1+e); %semi-latus rectum
ra      = p/(1-e); %apoapsis radius
ha      = ra - Re; %apoapsis altitude
a       = p/(1-e^2); %sami-major axis
E       = -mu/2*a; %specific mechanical energy
h       = sqrt(p*mu); %magnitude of the specific angular momentum

% Print results using fprintf statements
fprintf('Apoapsis altitude \t\t\t ha = %16.8f\n' ,ha);
fprintf('Specific mechanical engergy \t\t E = %16.8f\n' ,E);
fprintf('magnitude of the specific angular momentum h = %16.8f\n' ,h);
fprintf('semi-latus rectum \t\t\t p = %16.8f\n' ,p);
```

12

The answer to question 12 is below:

```
    >> HARRIS_HW1_Q12
Specific mechanical engergy   E = -1042871013.55099690
magnitude of the specific angular momentum h =    8302.51600000
semi-latus rectum   p =     172.93470128
periapsis radius   ha =      87.19382138
Apoapsis radius   ha =   10378.14500000
>>
```

The script for question 12 is below:

```
    % Solution to Question 12.
%given
mu      = 398600; %graviational parameter for Earth
Re      = 6378.145; %radius of the earth
ho      = 4000; %altitude of a space object
v       = 800/1000; %speed of space object in km/s
fpa     = 0; %flight path angle

%find
r       = ho +Re; %radius of space object
```

```
a       = 1/(-(v^2)/mu + 2/r); %semi-major axis
E       = -mu/2*a; %specific mechanical engergy
h       = r*v*sin(pi/2 - fpa); %magnitude of angular momentum
p       = h^2/mu; %semi latus rectum
e       = sqrt(-p/a + 1); %eccentricity
ra      = p/(1-e); %apoapsis radius
rp      = p/(1+e); %periapsis radius

% Print results using fprintf statements
fprintf('Specific mechanical engergy \t\t E = %16.8f\n' ,E);
fprintf('magnitude of the specific angular momentum h = %16.8f\n' ,h);
fprintf('semi-latus rectum \t\t\t p = %16.8f\n' ,p);
fprintf('periapsis radius \t\t\t ha = %16.8f\n' ,rp);
fprintf('Apoapsis radius \t\t\t ha = %16.8f\n' ,ra);
```

13

The answer to Question 13 is below:

```
    >> HARRIS_HW1_Q13
Position,   rv = [     -0.60000000,     -1.00000000,      0.75000000]
Inertial Velocity,   vv = [      0.80000000,     -0.45000000,      0.45000000]
Specific Angular Momentum, hv = [     -0.11250000,      0.87000000,      1.07000000]
Eccentricity Vector,   ev = [     -0.44026893,     -0.18540655,      0.10446117]
Eccentricity ,  e =       0.48900354
 hv dot ev =       0.00000000
Semi-Latus Rectum,    p =       1.91445625
Semi-Major Axis,   a =       2.51612274
True Anomaly [deg],   nu =       38.86686744
>>
```

The script for question 13 is below:

```
    % Solution to Question 13.
mu      = 1;
rv      = [-.6; -1; .75];
vv      = [.8; -.45; .45];
hv      = cross(rv,vv);
h       = norm(hv,2);
r       = norm(rv,2);
v       = norm(vv,2);
```

```
ev      = cross(vv, hv)/mu - rv/r;
e       = norm(ev,2);
hvdotev = dot(hv, vv);
p       = h^2/mu;
a       = p/(1-e^2);
cosnu   = (p/r - 1)/e;
nu      = acos(cosnu);
nudeg   = nu*180/pi;
% Print results using fprintf statements
fprintf('Position, \t\t\t rv = [%16.8f,%16.8f,%16.8f]\n',rv);
fprintf('Inertial Velocity, \t\t vv = [%16.8f,%16.8f,%16.8f]\n',vv);
fprintf('Specific Angular Momentum,\t hv = [%16.8f,%16.8f,%16.8f]\n',hv);
fprintf('Eccentricity Vector, \t\t ev = [%16.8f,%16.8f,%16.8f]\n',ev);
fprintf('Eccentricity ,\t\t\t  e = %16.8f \n',e);
fprintf('\t\t\t hv dot ev = %16.8f \n',hvdotev);
fprintf('Semi-Latus Rectum,  \t\t p = %16.8f \n',p);
fprintf('Semi-Major Axis, \t\t a =  %16.8f \n',a);
fprintf('True Anomaly [deg], \t\t nu =  %16.8f \n',nudeg);
```

14

The solution to question 14 is below:

```
    >> HARRIS_HW1_Q14
Eccentricity    e =       2.17979310
Periapsis Altitude   hp =  146090.81224769
Periapsis Speed   vp =       2.88321722
>>
```

The script for question 14 is below:

```
    % Solution to Question 14.
%given
mu      = 398600; %graviational parameter for Earth in km^3/s
Re      = 6378.145; %radius of the earth
r       = 403000; %geocentric radius of terrestrial object in km
nu      = deg2rad(151); %true anomaly in radians
v       = 2.25; %earth relative speed in km/s

%find
a       = 1/(-(v^2)/mu + 2/r); %semi-major axis
h       = r*v*sin(nu); %magnitude of angular momentum
p       = h^2/mu; %semi latus rectum
```

```
e       = sqrt(-p/a + 1); %eccentricity
rp      = p/(1+e); %periapsis radius\
hp      = rp - Re; %periapsis altitude
vp      = sqrt(mu)*sqrt(2/rp - 1/a); %periapsis speed

% Print results using fprintf statements
fprintf('Eccentricity \t\t\t\t  e = %16.8f \n',e);
fprintf('Periapsis Altitude \t\t\t hp = %16.8f\n' ,hp);
fprintf('Periapsis Speed \t\t\t vp = %16.8f\n' ,vp);
```

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(a) True, from equatio (1.34) we can see that $\mathbf{e} \cdot {}^{I}\mathbf{h} = \mathbf{0}$.

(b) False, it is normal to the orbit plane.

(c) True, the eccentricity vector is along the direction from the planet to the periapsis.

(d) False, it is largest when it lies in the direction opposite of the eccentricity vector.

(e) True, this can be seen from equations (1.90) and (1.50).