

1.2.3

Bulk Properties of Materials

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Introduction

When describing a material, the term “bulk properties” is often used to differentiate, either intentionally or unintentionally, from the term “surface properties.” The importance of surfaces for biomaterials science is highlighted briefly in [Chapter 1.2.2](#) and in great detail in [Chapter 1.2.4](#). While it is important to know that the success or failure of many biomaterials depends on the physical and chemical characteristics of their surface because the surface properties dictate interactions between a material and its environment (thereby determining whether a permanently implanted material will be tolerated or rejected), it does not suggest that bulk properties of biomaterials are any less important. In fact, for almost all biomedical applications, either as short-term degradable applications or long-term structural or load-bearing applications, the bulk properties must meet the physical and/or mechanical demands of these applications over the desired time period, even if the surface properties are deemed to facilitate biocompatible material–tissue interactions. The requirements for biomaterials to exhibit certain *bulk* characteristics are multifaceted, including maintaining physical shapes, carrying mechanical loads, and possessing certain desirable electrochemical behavior, optic index, and/or thermal characteristics, among others.

As discussed in [Chapter 1.2.2](#), the differences in bulk properties of many materials such as metals, polymers, and ceramics are the results of different types of interatomic or intermolecular forces that hold atoms and molecules together. For example, metals and alloys are typically characterized by metallic bonds, ceramics are held together by ionic bonds, and polymers are predominately formed by covalent bonds. The differences in these bonding mechanisms and energies dictate the different bulk properties of these materials. This chapter provides an overview of some of the useful concepts related to the bulk properties of these materials, with an emphasis on mechanical properties. For more in-depth information, the reader is encouraged to consult books on materials science and strength of materials.

Mechanical Variables and Mechanical Properties

The mechanical properties of a material refer to the characteristic values of a material under various mechanical loading conditions. By characteristic values, we refer to quantitative measures of mechanical variables at which certain transitions in the material would occur. For example, stress and strain are two mechanical variables that are commonly referred to for a material, but they are not properties of the material. Only the stresses that cause certain structural changes, such as yielding or breaking, etc. will be regarded as properties. This can be likened to the fact that the temperature of water is a physical variable, but we do consider certain temperatures, at which transitions occur in the state of water, properties such as the freezing point or boiling point of water.

Taking a cylindrical rod with uniform cross-section area (often called prismatic rod) as an example of a material structure, stress (σ) is defined as force per unit area, obtainable by dividing the applied force (F) by the cross-section area (A) of the rod, $\sigma = F/A$.

Stress determined this way is commonly regarded as the *nominal stress* because the actual value of the area (cross-section) to which the load is applied will change as the sample deforms in response to the load. Nominal stress is often synonymously referred to as *engineering stress*.

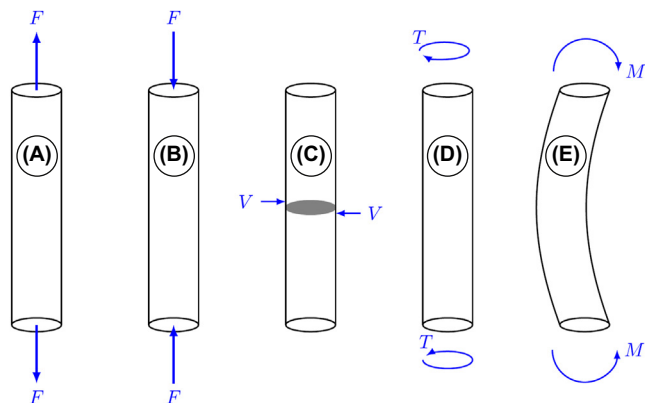
Strain (ϵ) is deformation per unit length, obtainable by dividing the deformation (δ) by the length of the rod (L), $\epsilon = \delta/L$.

By the same argument, strain determined this way is regarded as the *nominal strains* or *engineering strain* because the actual value of the length will change as the sample deforms in response to the load. Stresses and strains determined with reference to the actual cross-section area (A) and length (L) are initially known as true stresses and true strains, respectively. Unfortunately, because A and L change continuously during the course of a mechanical test, true stresses and strains can be difficult to measure in practice. Therefore, nominal stress and strain are commonly used instead of true stress and strain.

Stress (or strain) can be either normal or shear stress (or strain). Normal stress is determined by considering only the normal component of an arbitrary applied force, and shear stress by considering the tangent component of the force. The SI units of stress are newtons per square meter (N/m^2) or pascals (Pa), and strain is dimensionless.

Five Types of Mechanical Loading

A mechanical loading situation in a structure can be represented by a combination of several mechanical loads. In general, there are five basic types of loads as illustrated in Fig. 1.2.3.1 (Beer et al., 2014). When a pair of forces (F – F) with opposite directions is applied to a cylindrical rod along its axial direction, the situation is called axial loading. Under axial loading, we are more concerned with the deformations along the axial direction. If the axial load is making the rod longer, it is regarded as tensile loading, and if the axial load is making the rod shorter, it is compressive loading. When a pair of shear forces (V – V) with opposite directions



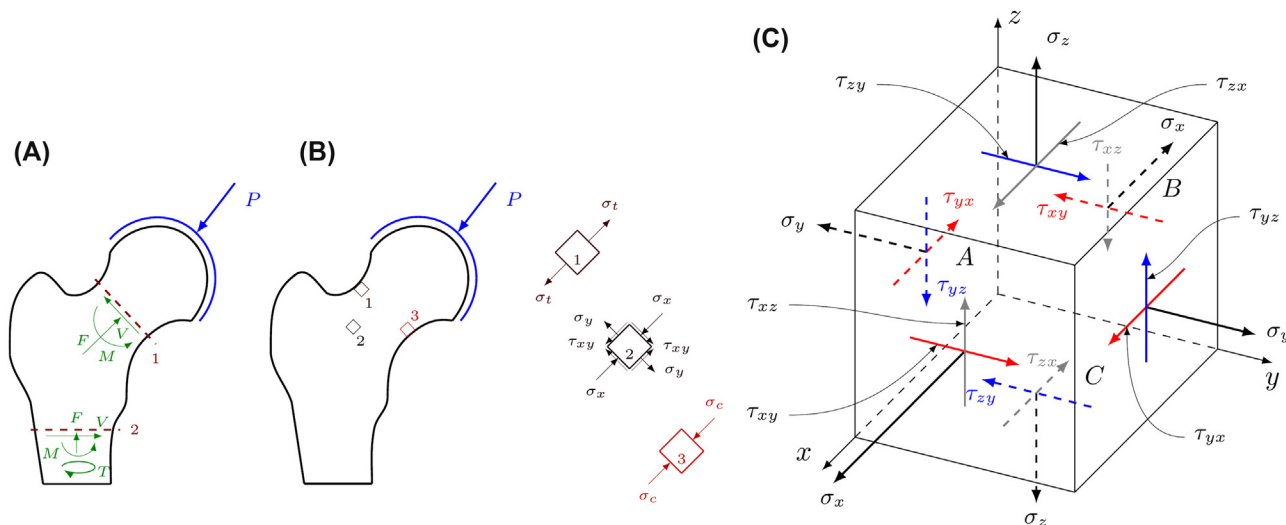
• **Figure 1.2.3.1** Five basic types of mechanical loading situations: (A) tension, (B) compression, (C) shearing, (D) torsion, and (E) bending.

is applied to the rod in a transverse direction, the loading condition is regarded as shearing loading with which we are mainly concerned about the transverse shear stress, strain, and deformation. When a pair of torques (T – T) with opposite directions is applied to the rod with respect to its axial orientation, the rod is regarded as under torsion loading in which we are concerned with twisting stress, strain, and angle. Finally, when a pair of moments (M – M) with opposite directions is applied to the rod, it is regarded as under bending. In a bending situation, we are more concerned about bending-induced tensile, compressive, and shear stresses and strains, as well as flexure deformation. Note that the term load is used here to represent all these forces, torques, and moments. Because of that, these loads will have different units, for instance, tensile, compressive, and shearing forces carry the units of newtons (N), and torque and moment have the units of newtons \times meter (Nm).

An actual structure is likely subjected to a combined loading situation consisting of several or all of these five types of basic loading. Very often, we impose certain mechanical constraints to the structure such that we can focus on stresses, strains, and deformations under the mechanical load and ignore other mechanical consequences such as translation and rotation, motion (linear and rotational velocity) and acceleration, among others.

From External Loads to Internal Loads and Stresses

In analyzing the mechanical properties of a material structure, the external loading condition (P) is first considered to determine the equivalent forces, torques, and moments at selected internal section planes. In a simplified two-dimensional (2D) view for example, in section plane 1 (Fig. 1.2.3.2A), the equivalent forces may consist of a compressive force (F), a shear force (V), and a bending moment



• **Figure 1.2.3.2** Two-dimensional illustration of how external mechanical loads are converted to equivalent internal forces, torques, and moments on different section planes (A), and internal stresses in a 2D (B) and 3D sense (C).

(M), but in section plane 2, an additional torque will have to be considered aside from the compressive and shear forces and moment. With these sectional forces, moments, and torques, the resulting internal stresses and strains as well as deformations can be obtained. As depicted by the three 2D stress elements shown in Fig. 1.2.3.2B, stress elements 1 and 3 are taken at the upper and lower surfaces of the neck region, where only tensile stress exists on stress element 1 and compressive stress on stress element 3. On a stress element isolated from an interior location (e.g., stress element 2), however, a much more complex stress state could exist. In a three-dimensional (3D) sense, we will have to deal with stress cubes on which six stress components would exist including σ_x , σ_y , σ_z , τ_{xy} , τ_{yz} , and τ_{xz} .

Linear and Nonlinear Relationship, Elastic and Plastic Behavior

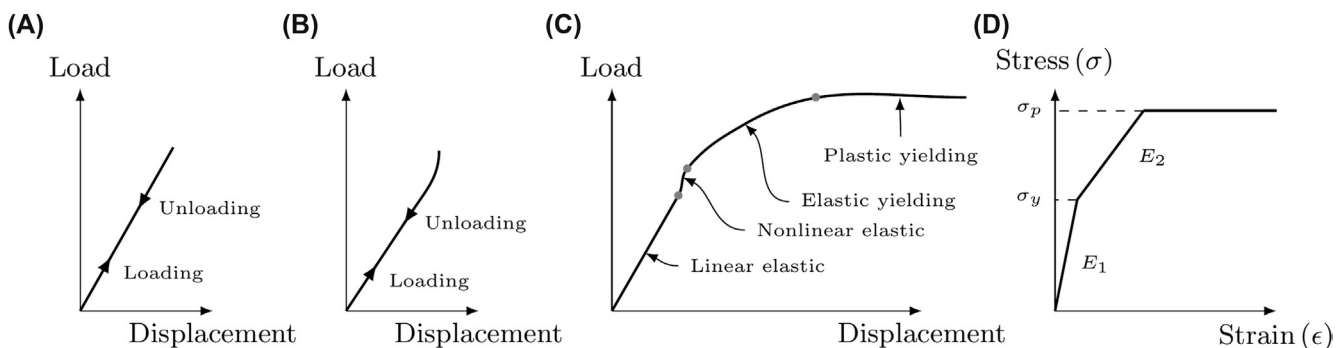
In dealing with materials' mechanical properties, we always encounter terms like linear and nonlinear relationship, and elastic and plastic behavior, and very often we tend to (wrongly) associate a linear relationship with elastic behavior and a nonlinear relationship with plastic behavior. The terms “linear” and “nonlinear” are often used to refer to a relationship between two variables such as in a load–displacement curve or in a stress–strain curve. When these relationships are straight lines, we call them linear, and when they are curved lines, we call them nonlinear. The terms “elastic” and “plastic,” however, refer to a material's deformational behavior, specifically the ability (or inability) to regain the original shape after the removal of loads. When a material structure regains its original shape (all deformations vanish) after the removal of loads, we consider it deforming with elastic behavior, and when it does not regain its original shape, we call it deforming with plastic behavior.

A linear relationship is not necessarily always associated with elastic behavior and a nonlinear relationship plastic behavior. In fact, these terms are often used in combinations to describe a certain mechanical property or behavior (Zhang, 2017). For example, a material structure can exhibit elastic behavior with either linear or nonlinear relationship between load and displacement. As illustrated in

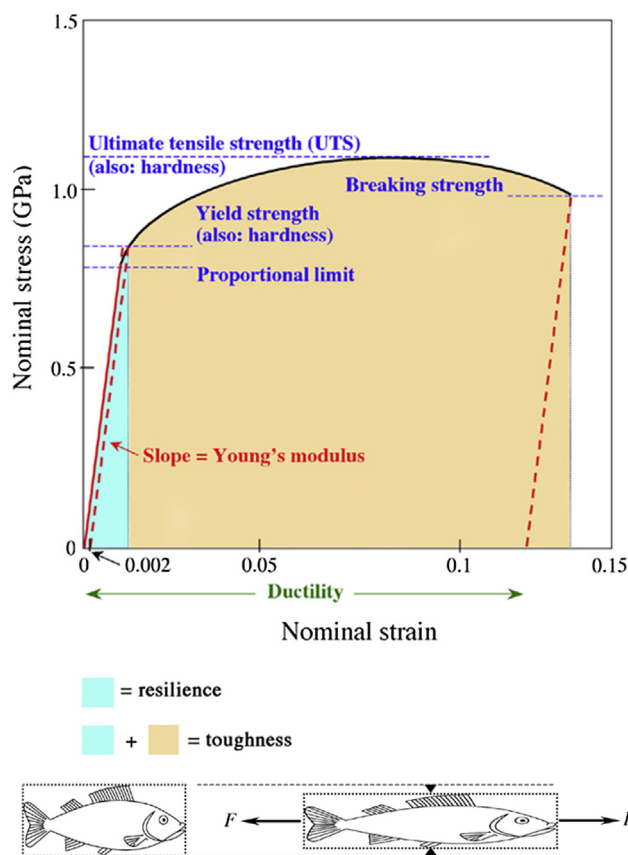
Fig. 1.2.3.3, linear elastic behavior (Fig. 1.2.3.3A) is one in which the load–displacement curve is a straight line (hence linear) and the loading and unloading curves follow the same trace such that the displacement will vanish (hence elastic) when the applied load is removed. Nonlinear elastic behavior (Fig. 1.2.3.3B) describes a curved load–displacement relationship, but the displacement induced by loading will vanish after the removal of the load.

As illustrated in Fig. 1.2.3.3C, the linear elastic behavior of a material typically occurs at the beginning of the loading stage when the induced displacement is extremely small. When the displacement increases slightly, the material may exhibit nonlinear but elastic behavior. This behavior is sometimes referred to as geometric nonlinearity. When the displacement increases further, the material will reach an elastic yielding zone where it still exhibits elastic behavior but with reduced rigidity. As the displacement continues to increase the material will enter a plastic yielding zone in which the displacement or deformation will keep increasing without needing any further increase in loads.

Fig. 1.2.3.3D shows the corresponding stress–strain curve derived conceptually from the load–displacement curve shown in Fig. 1.2.3.3C by dividing the load by the cross-section area and the displacement by the length. It presents a different look at the mechanical properties from a pure material's perspective by minimizing geometric-related influences. By convention, the stress is plotted vertically and the strain is plotted horizontally. When the material behaves elastically, in both a linear and nonlinear manner, the stress–strain curve typically exhibits a straight-line relationship with a constant slope E_1 (which is commonly known as the modulus of elasticity or Young's modulus; more discussion in the next section). When the stress in the material reaches its elastic yielding point σ_y it will start to exhibit reduced elastic property with decreased slope E_2 , where $E_2 < E_1$. As the strain in the material further increases, the material will reach its plastic yielding point σ_p after which full plastic deformation (i.e., deformation that keeps increasing without the need of increasing the applied loads) will occur and the stress in the material will remain the same until the material breaks.



• **Figure 1.2.3.3** Sketches of linear elastic (A) and nonlinear elastic (B) load–displacement curves. Sketches of a single load–displacement curve containing linear elastic, nonlinear elastic, elastic yielding, and plastic yielding behavior (C) and its corresponding stress–strain curve (D).



• **Figure 1.2.3.4** Schematic stress versus strain curve for a ductile metal, emphasizing the features that relate to specific mechanical properties and illustration of the Poisson ratio effect $\nu = -\frac{\epsilon_{\text{transverse}}}{\epsilon_{\text{longitudinal}}}$.

Pseudoelastic, Hyperelastic, and Viscoelastic Materials

Some materials, like shape memory alloys (SMAs), will undergo phase transition due to temperature changes, thereby exhibiting recoverable or reversible deformation behavior (hence elastic) as temperature changes back and forth. This type of “elastic” behavior is sometimes called pseudoelastic or superelastic. Superelasticity is an isotherm process in which phase transformation typically occurs from the austenite to martensite phase under an external load. Because the martensite phase is unstable, after removal of the external load the material will transform again to the austenite phase.

Other materials, like many polymeric materials and biological tissues, will undergo polymer-chain or collagen-fiber reorientation under stretching, leading to stiffening of the materials (i.e., an increase in the slope of the load–displacement curve). When the material is in its elastic nature, the observed nonlinear stiffening behavior is called hyperelastic.

Viscoelasticity is a term used to describe a material that possesses time- (or rate-) dependent behavior, meaning that the material may exhibit a different load–displacement or stress–strain curve when the loading rate is different. The main reason for this behavior is that viscoelastic materials are regarded as made of solid components that follow Hooke's law and fluidic components that follow Newton's

law. For viscoelastic materials, a combined Hookean and Newtonian method is often used to evaluate these combined solid/fluid-like characteristics with equivalent models made of springs (representing the elastic part) and dashpots (representing the viscous part). Load–displacement or stress–strain relationships of viscoelastic materials are always nonlinear with loading and unloading curves following two different paths, known as hysteresis. But after the removal of all loads, deformations in the material will vanish (hence elastic), albeit slowly, due to the viscous components. Common viscoelastic materials include paint, rubber, and many biological materials. For viscoelastic materials, the behavior stress relaxation (stress decays under a fixed constant strain condition) and creep (strain increases under a fixed constant stress condition) as well as the rate at which stress relaxes or strain creeps are common properties of interest.

Common Mechanical Properties of Isotropic Materials

Elastic Properties

Fig. 1.2.3.4 shows a typical stress–strain curve obtained from an isotropic ductile metallic slender rod subjected to tensile loading. An isotropic material is one that has properties considered to be independent of the direction one picks to examine the material. The linear segment of the curve reflects the simplified Hooke's law relationship in honor of Robert Hooke. The constant of proportionality (or slope) between stress and strain is known as the *Young's modulus*, E . The equivalent constant of proportionality in the case of a shear stress being applied is known as the *shear modulus*, G . The ratio of the resulting transverse strain (perpendicular to the direction of the applied load) and longitudinal strain (parallel to the direction of the applied load) is another elastic property of the material commonly known as the *Poisson's ratio*, which is defined as:

$$\nu = -\frac{\epsilon_{\text{transverse}}}{\epsilon_{\text{longitudinal}}}$$

The negative sign is used to obtain a convention in which the values of ν are positive for most materials. An illustration of such a Poisson ratio effect is given in Fig. 1.2.3.4, where a stretching force applied to a material causes it to become *narrower* as it gets longer. While most materials possess a positive Poisson ratio, a relatively small number of materials—known as auxetic materials—has a negative Poisson's ratio. Typically, these are low-density, cellular materials, with cell walls that are able to hinge or buckle when loads are applied. Pulling on a piece of auxetic material causes it to become *wider* as well as longer, while attempting to squash it will cause it to become thinner as well as shorter.

It can be shown that of the three elastic constants (E , G , ν) only two are independent, due to the following relationship among them:

$$G = \frac{E}{2(1 + \nu)}.$$

Yield Strength and Ductility

As the stress on the material increases, a point may be reached where the response is no longer linear, generating a permanent (irreversible) deformation, commonly known as *plastic* deformation. This occurs commonly in ductile metals and polymers, and rarely in brittle materials such as ceramics.

The amount of plastic deformation associated with a given stress can be found by drawing a line from the point of interest on the stress–strain curve, parallel to the initial linear segment of the curve, and marking the point where this constructed line intersects the horizontal (strain) axis. The horizontal distance from the origin to the point of intersection is a measure of the plastic strain. The value of plastic strain required to break the material defines the *ductility* (Fig. 1.2.3.4). The stress at which departure from a linear stress–strain relationship occurs is known as the *proportional limit* (Fig. 1.2.3.4).

It is not always easy to ascertain the proportional limit accurately, and so a more practical determination of the condition for plastic deformation is provided by the *yield strength* (Fig. 1.2.3.4), which is the stress at which *noticeable* plastic strain occurs. In this context, “noticeable” is often taken to be a value of 0.002 (0.2%) for metals—although it may be chosen to be much higher in polymers, where elastic behavior persists to strains that may be 10 or even 100 times higher, and where the transition from elastic to plastic behavior may be difficult to pinpoint.

A marked *yield drop* is often displayed in the stress versus strain plot when the yield stress of a polymer is exceeded. The yield drop occurs because the polymer chains become partially aligned during initial deformation, allowing easier relative motion of chains and reducing the stress needed to further deform the material. The chains then straighten and extend as deformation continues, requiring little increase in stress. Eventually, the opportunities for relative motion of chains are exhausted, along with the capacity for additional chain extension, and the stress versus strain plot then rises sharply.

Strength and Failure

Failure of a loadbearing material occurs when the material ceases to perform its loadbearing function. Different interpretations of “failure” are in common use, and so it is important to be clear about the intended meaning when we speak of materials failure. For materials that must not undergo in situ permanent deformation, failure is synonymous with the yield stress being exceeded, and so the yield stress represents an estimate of the strength of the material in those cases.

For materials in which permanent deformation is acceptable, failure may be deemed to occur when a noticeable “neck” (constriction) develops in the material. The effect of the neck is to concentrate the load on a smaller area; therefore, the load that can be supported by the sample is decreased. On a nominal stress versus nominal strain plot, where stresses are referred to the *original* loadbearing areas

of the sample, the stress required for additional deformation decreases. Therefore, the onset of necking corresponds to a maximum in the nominal stress versus nominal strain plot, defining the *ultimate tensile strength* (UTS) or simply the *tensile strength* of the material (Fig. 1.2.3.4). There is no corresponding maximum on a true stress versus true strain plot, because the stress continues to rise past the onset of necking as values of stress are obtained by dividing the applied load by the actual (small) cross-section of the neck.

On both nominal stress versus nominal strain and true stress versus true strain plots, a *breaking strength* can be defined at the point where the material actually breaks. However, to reach this point practically, at least in the case of most metals, the tensile strength of the material must first be reached, and it is tensile strength that represents the practically measurable maximum stress that a metal can survive. Ceramics, on the other hand, usually reach their breaking stress before they yield.

Thus, the perceived strength of a material depends in part on the definition of failure that is used (onset of plastic deformation, onset of necking, or actual occurrence of breaking). It also depends on the conditions that are used for testing. The *strain rate* used in a tensile test can affect the maximum stress that is reached, depending on the ability of the microstructure to undergo rearrangement to accommodate the imposed deformation.

Over long periods of time, even stresses that are below the conventionally measured yield strength of a material may be sufficient to cause gradual elongation and eventual failure via *creep*. Therefore, creep tests, in which a fixed load is applied while the strain is monitored until the sample breaks, are important for materials that are required to have a long projected service life—as is the case with many implants. Similarly, materials that will be subjected to cyclic loading patterns during service may undergo *fatigue* failure at loads that are smaller than those needed to trigger failure in a conventional stress versus strain test. Testing to determine the relationship between average load, maximum and minimum load, and the number of loading cycles that lead to failure is required in such cases. Finally, the catastrophic failure of a material can be triggered by preexisting flaws that locally concentrate the effect of an applied stress.

Hardness

The hardness of a material is measured by applying a known load to a small indenter of known geometry (typically pyramidal or spherical) in contact with the surface of the material, for a known period of time. The dimensions of the resulting indentation are measured, and this information, together with the experimental conditions, is used to rate the hardness of the material on a relative scale. Indents made in a soft material under a given set of conditions are larger than those made in a hard material. Therefore, hardness provides a measure of how successfully a material resists plastic deformation, which in turn is characterized by both yield strength and tensile strength. It is therefore possible to empirically develop calibration charts that can be used

to convert hardness measurements into both yield strength and tensile strength values. Hardness testing is popular for estimating the yield strength and tensile strength of materials because the equipment is relatively simple and inexpensive, and the tests are nondestructive.

Resilience

Resilience is a measure of the *elastic energy* that can be stored in a unit volume of stressed material. It corresponds to the area underneath a stress versus strain plot, extending from zero strain up to the strain at which the sample yields:

$$U_r = \int_0^{\epsilon_y} \sigma d\epsilon.$$

If the stress versus strain relationship is linear up to the yield strain (typical for ceramics, and approximate for many metals and polymers), the corresponding area under the stress versus strain plot is triangular, and the integral in the previous equation simplifies to:

$$U_r = \frac{1}{2} \sigma_y \epsilon_y = \frac{\sigma_y^2}{2E}.$$

Resilience is an important consideration for materials that need to be “springy,” i.e., that absorb and return significant amounts of energy. Examples include natural and replacement tendons and ligaments, and materials used in foot arch reconstruction or support.

Toughness

Toughness is a measure of the energy required to deform a unit volume of material to its breaking point. Therefore, the definition is similar to that used for resilience, except that we now take into account the entire area under the stress versus strain plot, extending from zero strain up to the strain at which the sample breaks:

$$U_{\text{break}} = \int_0^{\epsilon_{\text{break}}} \sigma d\epsilon.$$

Fracture Toughness and Fatigue Strength

Although fracture toughness and toughness sound superficially similar, they refer to quite different properties. Fracture toughness is a measure of the ability of a material to resist the propagation of a crack. It takes the form of critical stress intensity factor (K_{IC}) of a sharp crack where propagation of the crack suddenly becomes rapid and unlimited.

When a material is subjected to repetitive, or cyclic loading, a process of fracture may result from the initiation and propagation of a crack. The process of initiation and propagation of a crack in response to cyclic stresses is called fatigue. The stresses required to induce a failure by fatigue are typically known as the fatigue strength. Typically, the fatigue strength is well below the yield strength or ultimate strength of the material, but the ultimate fracture of the material would require millions of cycles of loading, during which localized damage accumulates, leading to the crack initiation and propagation.

Fatigue strength is often assessed by subjecting a sample to a cyclic loading test, counting the number of cycles applied until failure and plotting the stress (S) versus the log of the number (N) of cycles to failure in so-called S – N curves. Detailed discussions on fracture toughness and fatigue strength of metallic materials can be found in [Chapter 1.3.3](#).

Generalized Hooke's Law and Anisotropy of Materials

As discussed in “[Common Mechanical Properties of Isotropic Materials](#),” in the simplest form, the stress and strain relationship of an isotropic elastic material can be expressed as $\sigma = E\epsilon$. While this equation captures well, conceptually, the stress–strain relationship governed by Hooke's Law, it is valid only for situations where uniaxial stress and strain in a one-dimensional structure are of concern. In a complex situation, one must deal with all six stress components (see “[From External Loads to Internal Loads and Stresses](#)”), in which case the generalized Hooke's law takes the following form:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\ c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\ c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix}$$

in which the 6×6 square $[c]$ matrix is sometimes called the modulus matrix. Since the $[c]$ matrix is symmetric square matrix, the highest number of independent constants for an anisotropic material is 21. However, the actual number of independent constants can be much less due to certain symmetry in materials of different isotropy.

For instance, as shown in [Fig. 1.2.3.5A](#), for isotropic materials the $[c]$ matrix has only two independent nonzero constants, c_{11} and c_{12} . These two constants are related to the three E , G , ν constants discussed earlier through the following relationships:

$$c_{11} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \text{ and } c_{12} = \frac{E\nu}{(1+\nu)(1-2\nu)}$$

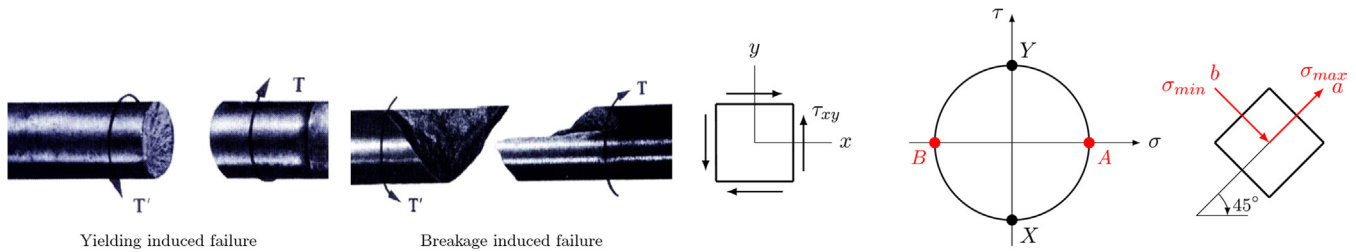
$$\frac{c_{11} - c_{12}}{2} = \frac{E}{2(1+\nu)} = G$$

Note that $\frac{c_{11} - c_{12}}{2} = \frac{E}{2(1+\nu)}$, confirming that of the three constants (E , G , ν), only two are independent.

Moreover, as also illustrated in [Fig. 1.2.3.5](#) ([Zhang, 2020](#)), materials with cubic (crystal) symmetry have three independent nonzero constants, c_{11} , c_{12} , and c_{44} ([Fig. 1.2.3.5B](#)); materials with hexagonal symmetry have five independent constants, c_{11} , c_{12} , c_{13} , c_{33} , and c_{44} ([Fig. 1.2.3.5C](#)); materials with tetragonal symmetry have seven independent constants, c_{11} , c_{12} , c_{13} , c_{16} , c_{33} , c_{44} , and c_{66} ([Fig. 1.2.3.5D](#)); materials with orthorhombic symmetry have nine independent constants, c_{11} , c_{12} , c_{13} , c_{22} , c_{23} , c_{33} , c_{44} , c_{55} , and c_{66} ([Fig. 1.2.3.5E](#)); and materials with monoclinic symmetry have 13 independent constants, c_{11} , c_{12} , c_{13} , c_{15} , c_{22} , c_{23} , c_{25} , c_{33} , c_{35} , c_{44} , c_{46} , c_{55} , and c_{66} ([Fig. 1.2.3.5F](#)). These

$$\begin{aligned}
 \text{(A)} \quad [c] &= \begin{bmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{c_{11}-c_{12}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{c_{11}-c_{12}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{c_{11}-c_{12}}{2} \end{bmatrix} & \text{(B)} \quad [c] &= \begin{bmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{44} \end{bmatrix} & \text{(C)} \quad [c] &= \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{c_{11}-c_{12}}{2} \end{bmatrix} \\
 \text{(D)} \quad [c] &= \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & c_{16} \\ c_{12} & c_{11} & c_{13} & 0 & 0 & -c_{16} \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ c_{16} & -c_{16} & 0 & 0 & 0 & c_{66} \end{bmatrix} & \text{(E)} \quad [c] &= \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} & \text{(F)} \quad [c] &= \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & c_{15} & 0 \\ c_{12} & c_{22} & c_{23} & 0 & c_{25} & 0 \\ c_{13} & c_{23} & c_{33} & 0 & c_{35} & 0 \\ 0 & 0 & 0 & c_{44} & 0 & c_{46} \\ c_{15} & c_{25} & c_{35} & 0 & c_{55} & 0 \\ 0 & 0 & 0 & c_{46} & 0 & c_{66} \end{bmatrix}
 \end{aligned}$$

• **Figure 1.2.3.5** Independent, nonzero constants in the $[c]$ matrix for isotropic materials (A), and materials with cubic symmetry (B), hexagonal symmetry (C), tetragonal (D), orthorhombic symmetry (E), and monoclinic symmetry (F).



• **Figure 1.2.3.6** Two fractured rods after torsional tests. Mohr's circle for torsional rods and two important stress elements. (Reproduced with permission from McGraw Hill.)

types of crystal symmetry are commonly found in metals and ceramics.

Loading Modes, Stress States, and Mohr's Circle

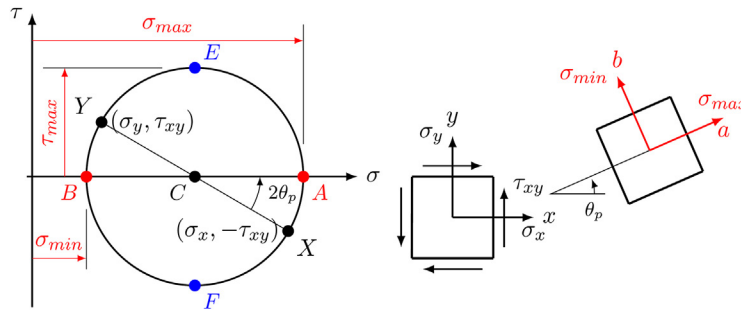
As we discussed in “From External Loads to Internal Loads and Stresses,” external loads dictate the resulting internal stresses. However, the external loading mode does not always reflect directly the failure mode of the material tested. For example, let us look at the two images in Fig. 1.2.3.6, in which two cylindrical rods failed in two different modes under the same type of torsional loading. The rod on the left has a fracture surface that resembles a perpendicular cross-section cut and the rod on the right has a fracture surface of a helical shape. The fact that these two rods, having the same geometric shape and subjected to the same type of loading, fail in two different modes confirms that the failure mode of a material does not always reflect directly its loading mode. To better understand the failure modes, it is necessary to consider the actual stress state induced in the material and its causal relationship with the breakage of the atomic bonds within the materials.

If we look closely at the fracture surfaces, we will note that the textures of the two fracture surfaces are also different. The one on the left shows the sign of material yielding and the one on the right sign of tensile breakage. So then, why do yielding and breakage failure modes cause the two rods to fracture in two different fracture surfaces? To answer

this question, we will need to know the stress states in these rods with the help of Mohr's circle.

Mohr's circle is a 2D graphical representation of the stress state of a given point in a material using a circle (for a 2D stress state; three circles will be needed for a 3D stress state) in a σ - τ coordinate system with a pair of opposite points on the circle representing the stress state of a stress element in a given direction. To visualize this, let us look at Fig. 1.2.3.7, where a Mohr's circle and two stress elements in a plane stress situation (see “Plane-Stress and Plane-Strain Simplification” for further discussion) are sketched. For the stress element shown in the middle, representing a viewing angle through the x - y orientation, we consider a normal stress σ_x and a shear stress τ_{xy} in the x -cut plane, and a normal stress σ_y and a shear stress τ_{xy} in the y -cut plane. By using these paired stress values as coordinates, we mark two points $X(\sigma_x, \tau_{xy})$ and $Y(\sigma_y, \tau_{xy})$ in the σ - τ coordinate system shown on the left with σ as the abscissa and τ as the ordinate. A sign convention is followed in this process in which tensile normal stresses (σ_x and σ_y) are considered positive and compressive normal stresses negative, and clockwise shear stresses (τ_{xy}) are positive and counterclockwise shear stresses negative. By linking the two points, X and Y , with a line we find an intersect with the abscissa at point C . Then using point C as the center point, we construct a circle passing through both points X and Y . This circle is the so-called Mohr's circle, honoring its creator Christian Otto Mohr.

On a Mohr's circle, any paired opposite points (with respect to the center point C), like the X - Y pair, represent



• **Figure 1.2.3.7** Mohr's circle and stress elements for depicting the stress state of a given point.

the stress state of the given point in a certain viewing orientation. This means that the stress state of the given point can be represented by countless different combinations of stress components depending on our viewing orientations. For the A–B pair, while still representing the same stress state of the point, because τ_{xy} is zero, it describes an orientation in which a stress element only has normal stresses acting on it. This orientation is called the principal directions and the corresponding stresses are called principal stresses.

From the Mohr's circle, we can see that a counterclockwise rotation by an angle of 2θ will transform the X–Y pair to the A–B pair. For the stress element, a counterclockwise rotation by θ (half of the rotation in the Mohr's circle) will transform the element in the x–y orientation to the one in the principal a–b orientation, as shown on the right in Fig. 1.2.3.7. With a further 90° counterclockwise rotation from the A–B orientation we will reach the E–F orientation where the shear stress reaches its maximum, τ_{\max} . This means that a 45° counterclockwise rotation from the a–b stress element will obtain a stress element having maximum shear stresses.

Let us now return to the two torsional rods discussed earlier and analyze the cause for fracture by examining the stress states in them. Fig. 1.2.3.6 also shows the Mohr's circle for a surface point on a cylindrical rod undergoing pure torsional loading. From this Mohr's circle, we can see that in the x–y orientation (corresponding to the X,Y points on the circle) the stress element has only shear stresses acting on it with no normal stresses. This means that the x–y stress element experiences the maximum shear stresses. On the other hand, the principal stress element, the a–b stress element (corresponding to the A,B points on the circle), is a 45° counterclockwise rotation away, because the X–Y and A–B lines are 90° apart in the Mohr's circle. In the principal stress element, the maximum principal is maximum tension and the minimum principal stress is maximum compression. Taken together, when the two rods undergo a torsion loading test, the x-cut plane (as well as the y-cut plane) endures maximum shearing, the 45° -cut plane experiences the maximum tension and the 135° -cut plane the maximum compression.

Keep in mind that a ductile material tends to fail due to shear-induced material yielding and a brittle material due to tension-induced breakage. It becomes clear that the rod on the left failed in a shear mode due to its ductile material nature and the highest shear stress existing in the x-cut plane (incidentally this cut plane has the smallest cross-section

area). For the rod on the right, it failed in a tension mode due to its brittle nature and the highest tensile stress existing along a plane oriented at 45° . It is thus not surprising that the fracture surface caused by tension-induced breakage is oriented at a 135° angle from the x-cut plane, perpendicular to the direction of the maximum tension.

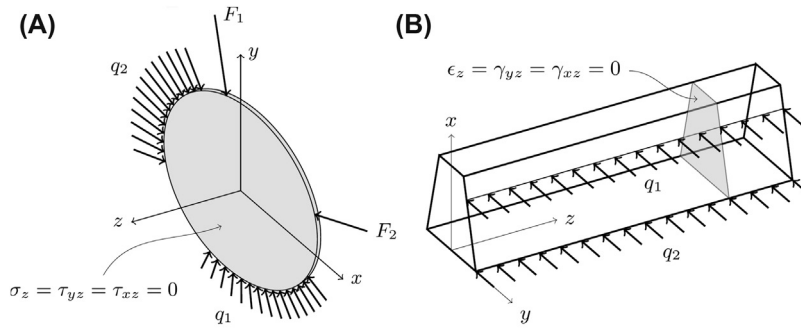
In short, the failure mode of a material is not determined by the loading mode but by the induced stress state (i.e., the maximum stresses and their orientations) and the type of materials. If the material is of the ductile type, shearing-induced yielding failure should be of concern and if the material is of the brittle type, tension-induced breakage should be avoided. To put this knowledge to use, the reader is encouraged to refer to Chapter 3.1.9 for case studies of some failure modes of various implant devices based on the observations made from implant retrieval analyses.

Plane-Stress and Plane-Strain Simplification

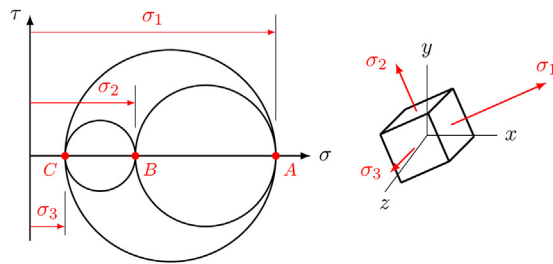
In real life we always encounter three-dimensional (3D) objects, yet we often use the terms two-dimensional (2D) and one-dimensional (1D) problems. Technically speaking, all problems are 3D in a geometry sense, but in certain circumstances they can be simplified through approximation into 2D and 1D problems. Due to the approximation nature, the conditions for making the simplification should be considered carefully. For example, when a material is thin in its third dimension (say, along the z axis) compared with the other two dimensions, and if the loading and deformations in the material occur only within the thin plane, the stresses in the third dimension, as illustrated in Fig. 1.2.3.8A, σ_z , τ_{yz} , and τ_{xz} are negligible. In this case, we can simplify the 3D problem into a 2D plane-stress problem. Another situation is when the material is very long in its longitudinal z direction as compared with the x, y dimensions and the loading, deformations, and cross-section area remain unchanged over the entire longitudinal length, as depicted in Fig. 1.2.3.8B. In this case, the strains in the third dimension, ϵ_z , γ_{yz} , and γ_{xz} are negligible, and we can simplify the 3D problem into a 2D plane-strain problem.

Trajectories of Tensile and Compressive Stress Lines

At any given point in a structure under mechanical loading we can identify the principal stresses and their directions by



• **Figure 1.2.3.8** 2D simplification of 3D solid mechanical problems: (A) plane-stress situation, and (B) plane-strain situation.



• **Figure 1.2.3.9** 3D Mohr's circles and the first, second, and third principal stresses.

constructing Mohr's circles and determining the orientation of the stress element that corresponds to the pairs of extrema with zero shear stresses, i.e., the A–B, B–C, or A–C pairs on the Mohr's circles in Fig. 1.2.3.9.

Let us now consider a plane stress situation. Referring to Fig. 1.2.3.10, we assume that the first principal stress (σ_1) is positive (tensile), the second one (σ_2) zero, and the third one (σ_3) negative (compressive). After identifying the directions of the first (tension) and third (compression) principal stresses at randomly selected points across the plane structure, we mark the two directions using perpendicular crosses at these points (note that these two principal stresses are always perpendicular to each other). To distinguish the two stresses, we use red to represent the tensile direction and blue the compressive direction. By connecting the red and blue lines together, respectively, we obtain two sets of lines representing the directions of maximum tensile stress (red lines) and maximum compressive stress (blue lines), which sometimes are also referred to as the stress trajectories. Fig. 1.2.3.10 shows the trajectories of tension and compression lines in a rectangular beam structure subjected to three-point bending. With a close inspection, we note that these two sets of lines are indeed orthogonal, indicating that, at each intersect, a red line is perpendicular to a blue line, as expected.

Assuming the beam structure is made of concrete, we should expect tension-induced breakage as the failure mode. Referring to the principal stress element shown in the figure, a fracture surface caused by tensile stresses should be perpendicular to the tensile stress, meaning that the breaking surface in the concrete should coincide with the compression lines. By examining the images of the actual fracture lines near the corner and at the bottom of the beam shown

in Fig. 1.2.3.10, one can see that these fracture lines follow exactly the compression lines in these regions.

Knowing the stress trajectories provides us with crucial insight into how to strengthen certain structural design. The structure of the human femoral head is an example that our body knows how to strengthen bone structure according to the stress trajectories as a result of constant bone remodeling. The architecture of the spongy bone is likely the result of the bone remodeling process with the purpose of meeting the strengthening needs of human growth and daily activities, guided by the stress trajectories inside the femoral head.

Other Bulk Properties

The bulk properties of a material are of course not limited to performance in mechanical tests. Other types of bulk properties that may have to be taken into account during the selection of a biomaterial include: thermal properties; optical properties; electrical properties; and magnetic properties. Even cost—financial and environmental—can be considered to be a bulk property, given the dependence of cost on: (1) the type and volume of material that is used; and (2) the complexity of microstructural rearrangement during processing.

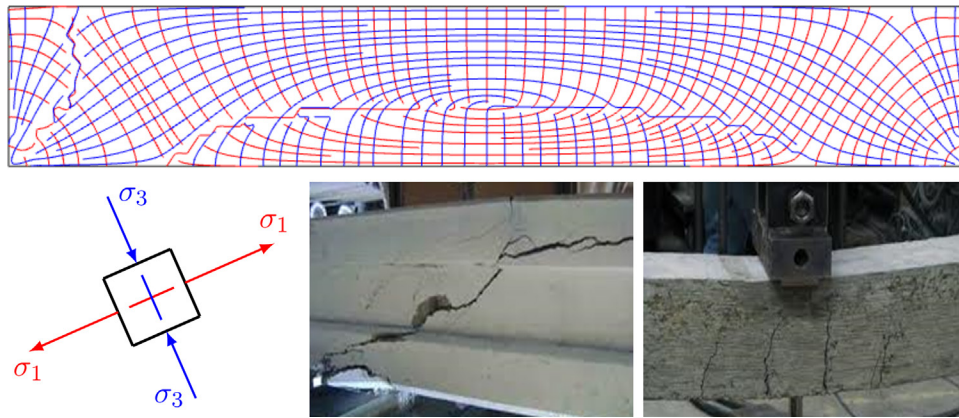
Thermal Properties

Thermal conductivity becomes a significant consideration if an implanted material contributes to an unnatural flow of heat through the surrounding tissue. For example, metal rods selected for their combination of stiffness, strength, fracture toughness, and biocompatibility can promote heat loss and cause the patient to feel colder than normal. Heat conduction through a metal dental filling can also be a source of discomfort and may therefore play a role in the choice between a metal or nonmetal filling.

Thermal expansion is a (nominal) strain that occurs when the temperature of a material is changed:

$$\epsilon_{\text{thermal}} = \alpha (T_{\text{final}} - T_{\text{initial}})$$

where α is the thermal expansion coefficient; its units are (degrees)⁻¹, with “degrees” measured on whatever standard scale is used to measure temperature T .



• **Figure 1.2.3.10** Trajectories of tension (red) and compression (blue) lines in a beam structure subjected to three-point bending along with a principal stress element and two images of fracture lines in concrete beams under the same type of loading.

Attempts to match the properties of a dental filling to those of the surrounding tissue must therefore extend beyond achieving compatible elastic constants to include consideration of thermal expansion, to avoid premature failure of the interface.

Optical Properties

In the context of biomaterials, the most significant bulk optical properties are color, refractive index, and transparency; all three are important in the selection of materials for intraocular lenses or fluids. The color of a transparent material is controlled by composition, and therefore demands a high degree of quality control to avoid impurities that could adversely affect color. Long-term stability of material composition is also important: components should not selectively diffuse into the surrounding tissue, and no chemical changes should occur—either by reactions between the components or in response to light.

The *refractive index* n of a material is defined as:

$$n = \frac{c_{\text{vacuum}}}{c_{\text{material}}}$$

where c denotes the speed of light in the subscripted medium. When light crosses the interface between two media (materials), it is deviated from its original path by an angle that is an increasing function of the difference between the refractive indices of the media. Therefore, the effectiveness of a material as a lens is directly related to its refractive index. Refractive index increases along with the content of electron-rich atoms, which is why lead “crystal” is so useful in both lenses and decorative glassware.

Transparency is a qualitative term that describes the ability of a material to transmit light without attenuating (absorbing or scattering) it. To minimize absorption, the primary bonds in the material must be strongly covalent or ionic (and definitely not metallic). Scattering is minimized if the material is free of internal interfaces (which could

reflect light) and compositional differences (which would be associated with refractive index differences that could deviate light from an uninterrupted path through the material). Thus, an optimally transparent material will either be a homogeneous single crystal or it will be completely and homogeneously amorphous.

There is no commonly accepted quantitative definition of transparency. Instead, it is usual to consider the complementary property, *opacity*. This is itself a colloquial term, but it lends itself to a more formal description by way of the *mass attenuation coefficient*. If light of a given frequency (color) and initial intensity I_0 travels a distance x through a material, then the intensity is decreased to:

$$I(x) = I_0 \exp(-\mu \rho x)$$

where μ is the mass attenuation coefficient of the material, and ρ is the density.

Piezoelectric Properties

Piezoelectricity deals with the coupled mechanical and electrical properties of a material that becomes electrically polarized when it is mechanically strained. There are two forms of piezoelectric effect. Direct piezoelectric effect describes the generation of electric dipoles and polarization as a result of displaced atoms and the associated redistribution of electrons inside a solid when it is mechanically strained. This polarization in turn generates an electric field. By contrast, reverse piezoelectric effect is a phenomenon in which a solid becomes strained when placed in an electric field. Piezoelectric effect was first discovered and demonstrated in its direct form by the brothers Pierre and Jacques Curie in 1880. Since then, the science of piezoelectricity has slowly moved from research laboratories to real-world applications such as sonars, devices for the production and detection of sound and for mechanical actuation, sensors for thickness and temperature measurements, even biosensors, among others. The theory of piezoelectricity allows us to deal with relationships between the generated electrical field and applied strains, or vice versa, within the material's elastic deformation limit.

Weibull Modulus and Nonbrittle Materials

The Weibull distribution of fracture probability for materials provides a statistical measure of brittleness, via the specification of a Weibull modulus m (Derby et al., 1992). The value of m can in theory range from zero (totally random fracture behavior, where the failure probability is the same at all stresses, equivalent to an ideally brittle material) to infinity (representing a precisely unique, reproducible fracture stress, equivalent to an ideally nonbrittle material).

This quantitative approach to defining nonbrittleness is more robust than a qualitative description, since there is no single antonym of “brittle.” Both “tough” and “ductile” are in particular senses the opposite of “brittle,” but we have seen that “tough” and “ductile” are not interchangeable, and their corresponding nouns are associated with very different features on a stress–strain curve (Fig. 1.2.3.4).

Weibull statistics were initially adopted to quantify failure in classically brittle materials (Kelly and Macmillan, 1986), where tensile strength is dominated by the size distribution of preexisting flaws, and the use of this method is rarely extended to materials that are regarded as nonbrittle. However, there is no a priori reason to restrict Weibull statistics to the analysis of failure in classically brittle materials. For example, although natural silks exhibit large extensions to failure, and many of them behave like an elastomer at least during the initial stages of deformation,

Weibull analysis has proved to be useful for characterizing and contextualizing the failure strength variability of these materials (Pérez-Rigueiro et al., 1998, 2001).

Why is a Weibull analysis appropriate for nonbrittle materials? A justification can be formulated simply on phenomenological grounds: after significant deformation, silk *develops* the statistical failure *characteristics* of a brittle material, even though it initially deserves classification as an elastomer. While the existence of strength-limiting defects can be inferred from this description, their nature has yet to be confirmed; it is not clear whether they are present in as-spun material, or whether they begin to develop during the earlier stages of deformation.

The appropriateness of Weibull analysis for a nonbrittle material can also be justified fundamentally (Viney, 2002), given the equation that is used as the basis for measuring m (see “Statistical Aspects of Failure”):

$$1 - F = \exp\left(-\frac{\sigma}{\sigma_0}\right)^m$$

If F is to be independent of σ , which is the hallmark of an ideally brittle material, this equation requires m to be equal to zero. However, practical Weibull analysis necessarily admits to nonzero values of m , so it is implicit that ductile contributions to failure can also be accommodated.

Under the conditions of constant stress and constant strain, the constitutive relation governing piezoelectricity can be expressed as

$$D = \epsilon E^e + dT \text{ and } D = \epsilon E^e + eS$$

respectively, where D is electric displacement vector, E^e electric field, T stress vector, S strain vector, ϵ permittivity constants, d piezoelectric strain constants, and e piezoelectric stress constants (Zhang, 2020). Of these seven terms, D , E^e , T , and S are physical variables, and d , e , and ϵ are piezoelectric properties. For an anisotropic piezoelectric material in a 3D space, D and E^e are 3×1 vectors, T and S are 6×1 vectors, d and e are 3×6 matrices, and ϵ is a 3×3 matrix. Very often, due to structural symmetry in the material, the actual numbers of constants in these matrices is much lower. For example, a cubic crystal with class 23 symmetry has only one independent nonzero constant in d , e , and ϵ matrices, respectively.

Electrochemical Properties

The electrochemical properties of a biomaterial (typically of metals and alloys) are the material's characteristics in an electrochemically corrosive environment, including electrochemical potentials, reaction constants, etc. These properties dictate the responses of such metallic biomaterials governed by corrosive oxidation and reduction reactions, where the oxidation processes take metal (zero valence) and increase its valence to make ions (cations) or oxides (or other solid oxidation products). On the reduction side, processes may include water and oxygen reduction to form hydroxide ions and/or reactive oxygen intermediates, aside from many other biologically based molecules that are susceptible to redox processes. The oxidation and reduction reactions are electrically connected through the metal and complete the circuit through the solution (commonly known as the

two half-cell reactions), resulting in currents (electronic and ionic) through both phases. All metallic biomaterials exhibit some form of corrosion process, whether passive dissolution of ions through the passive film, to more aggressive process caused by wear or crevice geometries. Fortunately, the common implantable metals and alloys (with the exception of noble metals such as gold or platinum) have protective oxide films to control corrosion of the material to acceptable levels. These oxides are commonly referred to as passive films, typically of only a few nanometers in thickness, and will limit transport of metallic ions to the implant surface. More detailed discussions on the electrochemical properties of many metallic biomaterials are given in Chapters 1.3.3 and 2.4.3.

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