Biomaterials

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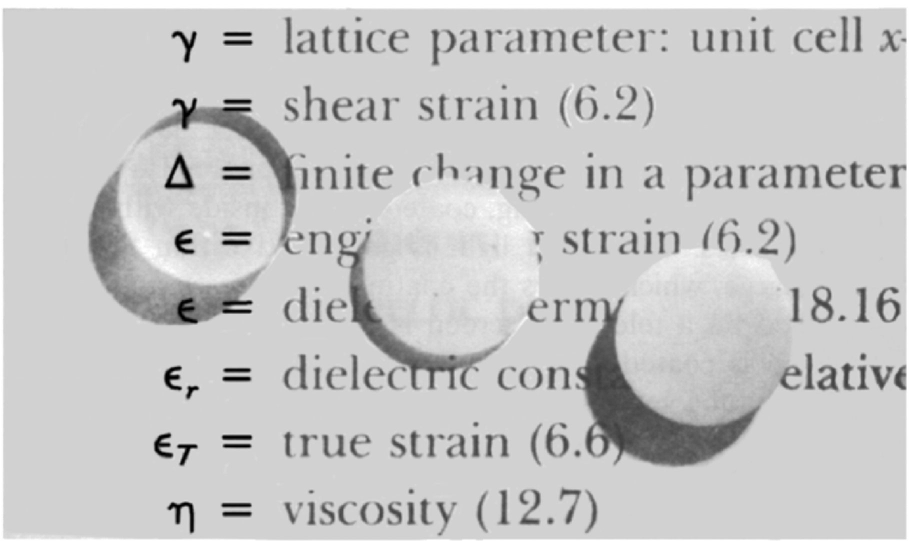
CHARACTERIZATION OF MATERIALS — II: ELECTRICAL, OPTICAL, X-RAY ABSORPTION, ACOUSTIC, ULTRASONIC, ETC.



The light transmittance of three aluminum oxide specimens. From left to right: single-crystal material (sapphire), which is transparent; a polycrystalline and fully dense (nonporous) material, which is translucent; and a polycrystalline material that contains approximately 5% porosity, which is opaque. Specimen preparation, P.A. Lessing; photography by J. Telford. Reprinted with permission from Callister (2000). Copyright © 2000, Wiley.

In addition to the mechanical, thermal, and surface properties of materials, other physical properties could be important in particular applications of biomaterials. Properties considered in this chapter include electrical, optical, absorption of x-rays, acoustic, ultrasonic, density, porosity, and diffusion.

4.1. ELECTRICAL PROPERTIES

The electrical properties of materials are important in such applications as pacemakers and stimulators, as well as in piezoelectric implants to stimulate bone growth.

Electrical resistance, R, is defined as the ratio between the potential difference (voltage) V applied to the object and the current i that flows through:

$$R = \frac{V}{i}. \tag{4-1}$$

If potential difference V is measured in volts (V) and current i in amperes (A), resistance R is in ohms, denoted by a capital Greek omega (Ω). *Ohm's law* states that voltage is proportional to the current in a conductor, so that resistance R is independent of voltage. Metals obey Ohm's law if the temperature does not change much but semiconductors do not. The resistance of an object depends upon both the material of which it is made and the shape. The characteristic of *resistivity*, by contrast, is associated with the material itself. Resistivity ρ_e is defined as the ratio of electric field E to current density J, which is current per cross-sectional area. The electric field is the gradient in electric potential:

$$\rho_e = \frac{E}{J}. \tag{4-2}$$

The unit of resistivity is ohm-meter (Ω-m).

Example 4.1

Consider a pacemaker wire of circular cylindrical shape, $d = 0.1$ mm in diameter and $L = 100$ mm long, made of gold. Determine the electrical resistance.

Answer

In view of the uniform cross-section, the electric field, and the current density will be uniform over the length of the wire (L). The electric field is the gradient in potential, so

$$E = V / L.$$

The current density is the current per unit area, or

$$J = i / A = 4i / \pi d^2.$$

The resistance, therefore, is

$$R = V / i = 4EL / J\pi d^2 = 4L\rho_e / \pi d^2.$$

For gold, $\rho_e = 2.35 \times 10^{-8}$ ohm-m, so $\underline{R = 0.3 \text{ ohm}}$. This is much smaller than the resistance of the surrounding tissue, so it is not likely to be a problem. Single-strand pacemaker wire would be vulnerable to mechanical fatigue due to flexure from a beating heart, so multi-strand wires are used.

The electrical resistivity of materials varies over many orders of magnitude. Insulators, or materials with very high resistivity, are used to isolate electrical equipment, including implantable devices such as pacemakers and other stimulators, from body tissues. Polymers and ceramics tend to be good insulators. The electrical resistivities of representative materials are given in Table 4-1.

Table 4-1. Electrical Resistivity of Various Materials

Material	Resistivity (Ω-m)
UHMWPE	$>5 \times 10^{14}$
PMMA	10^{14}
Al_2O_3	10^{9}–10^{12}
Zr_2O (3% Y_2O_3)	10^{10}
SiO_2	10^{10}
Bone (wet, longitudinal)	46
Muscle (wet, longitudinal)	2
Physiological saline	0.7
Stainless steel	7.3×10^{-7}
Platinum	10^{-7}
Gold	2.35×10^{-8}
Copper	1.7×10^{-8}
Silver	1.6×10^{-8}

Piezoelectricity is a coupling between mechanical deformation and electrical polarization of a material. Specifically, mechanical stress results in electric polarization, the direct effect; and an applied electric field causes strain, the converse effect. The piezoelectric constitutive equations are as follows:

$$D_i = \sum_{jk} d_{ijk}\sigma_{jk} + \sum_{j} K_{ij}E_j + p_i\Delta T, \tag{4-3}$$

$$\varepsilon_{ij} = \sum_{kl} S_{ijkl}\sigma_{kl} + \sum_{k} d_{kij}E_k + \alpha_{ij}\Delta T, \tag{4-4}$$

in which D is the electric displacement, σ is the stress, d is the piezoelectric sensitivity tensor, K is the dielectric permittivity, E is the electric field, p is the pyroelectric coefficient, T is the temperature, ε is the strain, S is the elastic compliance, and α is the thermal expansion. Only materials with sufficient asymmetry exhibit piezoelectricity or pyroelectricity and consequently have d and p coefficients not equal to zero. The physical origin of piezoelectricity lies in the presence of asymmetric charged groups in the material, as shown in Figure 4-1. As the material is deformed, the charges move with respect to each other so that change in dipole moment occurs. Figure 4-1 also shows the polarization that results from stress via several d coefficients. As for elastic compliance S, the first term in Eq. (4-4) represents Hooke's law, as discussed in the previous chapter. All piezoelectric materials are anisotropic, and the S coefficients represent different compliances for different directions of loading. Young's modulus E is the inverse of compliance S in the direction considered.

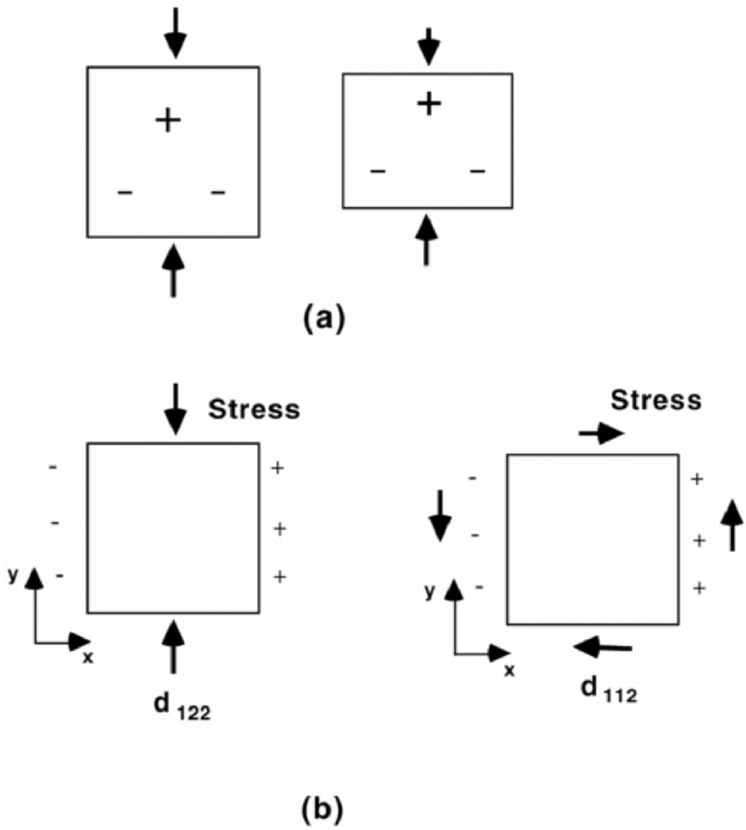


Figure 4-1. Piezoelectric materials. (**a**) Physical origin of piezoelectricity: charge separation in asymmetric unit cell under deformation. (**b**) Piezoelectric *d* coefficients from Eqs. (4-3) and (4-4). **Left**: polarization in the $x(1)$ direction due to compressive stress in the $y(22)$ direction. **Right**: polarization in the x direction due to shear stress in the $xy(12)$ direction.

Fukada and Yasuda first demonstrated that dry bone is piezoelectric in the classic sense. The piezoelectric properties of bone are of interest in view of their hypothesized role in bone remodeling. Wet collagen, however, does not exhibit a piezoelectric response. Piezoelectric effects occur in the kilohertz range, well above the range of physiologically significant frequencies. Both the dielectric and piezoelectric properties of bone depend strongly upon frequency. The magnitude of the piezoelectric sensitivity coefficients of bone depends on frequency, on direction of load, and on relative humidity. Values up to 0.7 pC/N (= 10–12 C/N) have been observed in bone, to be compared with 0.7 and 2.3 pC/N for different directions in quartz, and 600 pC/N in some piezoelectric ceramics. It is, however, uncertain whether bone is piezoelectric in the classic sense at the relatively low frequencies that dominate in the normal loading of bone. *Streaming potentials* can result in stress-generated potentials at relatively low frequencies even in the presence of dielectric relaxation, but this process is as yet poorly understood. The electrical potentials observed in transient deformation of wet bone in vivo may be mostly due to streaming potentials.

Compact bone also exhibits a permanent electric polarization as well as *pyroelectricity*, which is a change in polarization with temperature. These phenomena are attributed to the polar structure of the collagen molecule; these molecules are oriented in bone. The orientation of permanent polarization has been mapped in various bones and has been correlated with developmental events.

The electrical properties of bone are relevant not only as a hypothesized feedback mechanism for bone remodeling but also in the context of external electrical stimulation of bone to aid its healing and repair.

Example 4.2

Suggest an application of artificial piezoelectric materials in the body.

Answer

In some cases natural growth or repair of bone may be inadequate. Inclusion of an active piezoelectric element in a bone plate or joint replacement will generate electrical signals in vivo that will stimulate the growth of bone. This approach has been used on a research basis.

Example 4.3

A piezoelectric stimulator one square centimeter in cross-sectional area and one millimeter thick is incorporated in a composite bone plate. It experiences 1% of the stress seen in a healthy leg bone during walking (8 MPa). The material is a lead titanate zirconate ceramic for which the relevant piezoelectric coefficient is 100 pC/N and the dielectric constant 1,000. Determine the peak voltage produced by the device. For the purpose of calculation, neglect the leakage of charge through the conductive pathways in bone.

Answer

The charge density may be calculated as follows. The charge density *q/A* is the piezoelectric coefficient times the stress; the stress is 0.01×8 MPa as given above:

$$q/A = 100 \text{ pC/N } [0.01][8 \times 10^6 \text{ N/m}^2] = [10^{-10} \text{ C/N}][8 \times 10^4 \text{ N/m}^2] = 8 \text{ }\mu\text{C/m}^2.$$

Under the assumptions given, the implant behaves as a capacitor of capacitance C, for which the charge q is $q = CV$, in which V is the voltage. $V = q/[k\varepsilon_0 A/d]$, with k as the dielectric constant, ε_0 as the permittivity of space, A as the cross-sectional area, and d the thickness. Using the charge density from above,

$$V = [8 \text{ }\mu\text{C/m}^2][1 \text{ mm}]/[1{,}000 \times 8.85 \times 10^{-12} \text{ F/m}^2] = \underline{0.9 \text{ Volts}}.$$

We have neglected the parallel capacitance and leakage conductance of the surrounding tissue. As for the parallel capacitance, the dielectric constant of muscle, for example, is about 10^5 at 100 Hz and 10^8 at 0.01 Hz. The actual stimulating voltage will therefore be considerably less than determined above and will moreover decay rapidly with time, following each step in walking as a result of the conductivity of the surrounding bone and muscle.

4.2. OPTICAL PROPERTIES

The optical properties of materials are relevant to their performance when used in the eye, as well as to cosmetic aspects of dental materials. A ray of light incident upon a transparent material will be partly reflected and partly transmitted. The transmitted ray is bent or *refracted* by the material. It is observed experimentally (and can be deduced from Maxwell's equations) that the incident ray, normal to the surface, and refracted ray all lie in the same plane, and the angle of incidence equals the angle of reflection. The angle of the refracted ray depends upon a material property known as the *refractive index*, usually denoted by n. The refractive index is defined as the ratio of the speed of light in a vacuum to the speed of light in the medium. The relationship between the angles of incidence and of refraction is given by *Snell's law*:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2, \tag{4-5}$$

in which θ_1 is the angle of the incident ray with respect to the normal to the surface, n_1 is the refractive index of the medium containing the incident ray, and θ_2 is the angle of the refracted ray to the normal to the surface of the material, which has refractive index n_2, as shown in Figure 4-2. Some representative indices of refraction are given in Table 4-2 for yellow-orange light at a wavelength of 589 nm.

Table 4-2. Refractive Index of Some Materials

Material	Refractive index
Vacuum	1.0
Air	1.0003
Water	1.33
Human aqueous humor	1.336
Human vitreous humor	1.338
Human cornea	1.376
Human lens	1.42
HEMA hydrogel, wet	1.44
PMMA	1.49
Polyethylene(film)	1.5
Crown glass	1.52
Flint glass	1.66
Diamond	2.41

In ophthalmologic biomaterials, transparent materials find application in lenses. Refraction of light by a convex lens is shown in Figure 4-3. The focal length of such a lens is defined as the distance from the lens to the image plane when parallel rays of light (from far away) impinge upon the lens. The focal length f of a simple, thin lens (in air or vacuum) depends on its refractive index n and the surface radii of curvature, r_1 and r_2, as follows:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right). \tag{4-6}$$

This is the “lens maker's equation” and is derivable from Snell's law. The sign convention for the radii of curvature is that they are considered positive if the center of curvature is on the right side of the lens. Consequently, both surfaces of a double convex lens contribute positive power. In biomedical applications, the lens is likely to have one or more interfaces with tissue fluid or the tissues of the eye. The optical power P of lenses is usually expressed in diopters (D) in the ophthalmic setting:

$$P\ (\text{D}) = \frac{1}{f\ (\text{meters})}, \tag{4-7}$$

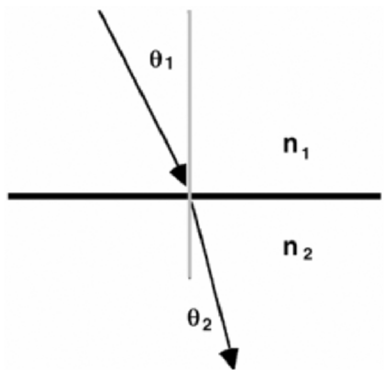


Figure 4-2. Snell's law for refracted light.

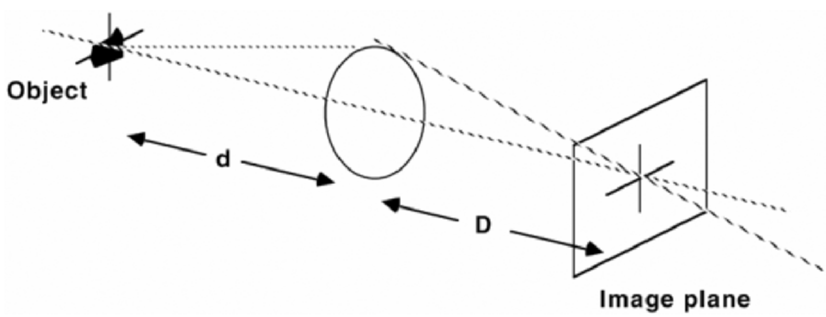


Figure 4-3. Refraction of light by a convex lens.

so that a converging lens with a focal length of 100 mm has a power of +10 D. Diverging lenses have negative focal lengths, hence negative optical power. A typical, normal human eye has an optical power of about 60 D, of which about 43 D is associated with the curvature of the cornea. The eye measures about 22 mm from cornea to retina.

Example 4.4

Consider a polycarbonate intraocular lens of diameter $d = 8.3$ mm, thickness $t = 2.4$ mm, anterior radius of curvature 17.8 mm, and posterior radius of curvature 10.7 mm. Determine the optical power and focal length of this lens in air and in water.

Answer

Use the thin lens Eq. (4-6). In air, assume $n = 1$ for air and take $n = 1.59$ for polycarbonate from Table 4-2.

$$P = (1.59 - 1)[1/0.0178\text{ m} - 1/{-0.0107}\text{ m}] = 88.2\text{ m}^{-1} = \underline{88.2\text{ D}},$$

so

$$f = 1/P = \underline{13.6\text{ mm}} \text{ in air.}$$

Water has an index of refraction $n = 1.33$, and so

$$P = (1.59 - 1.33)[1/0.0178 \text{ m} - 1/{-0.0107} \text{ m}] = 39 \text{ m}^{-1} = \underline{39 \text{ D}},$$

so

$$f = 1/P = \underline{25.6 \text{ mm}} \text{ in water.}$$

We remark that the normal lens of the eye has an optical power of about 19 D.

Transparent biomaterials are used to make contact lenses and intraocular lenses. PMMA is the material of choice for intraocular lenses and for hard contact lenses. Its main disadvantage as a contact lens material is that its permeability to oxygen is low, so that the cornea, which receives its oxygen by diffusion from the air, suffers hypoxia. Other materials are used in ocular applications. Representative material properties are given in Table 4-3. Polycarbonate is an amorphous thermoplastic that has been used in the manufacture of contact lenses, particularly those of strong power. The comparatively high refractive index of polycarbonate permits a thinner, lighter lens to be made. Soft contact lenses are commonly made of the water-absorbing hydrogel material poly-HEMA [hydroxyethyl methacrylate], which is highly permeable to oxygen.

Table 4-3. Physical Properties of Some Transparent Materials

Material	Density (g/cm^3)	Refractive index	Young's modulus (MPa)	Tensile strength (MPa)
PMMA	1.19	1.49	2800	55
Silicone rubber	0.99–1.5	1.43	6	2.4–6.9
Silicone rubber, contact lens	1.09	1.43	6	1.4
Polycarbonate	1.2	1.59	2200	60

4.3. X-RAY ABSORPTION

The ability of materials to absorb x-rays is of interest in the context of the visibility of an implanted object in diagnostic radiographs. X-rays are electromagnetic waves similar to light except that their wavelength is much shorter and their energy much higher. The index of refraction for x-rays in matter of any kind is very nearly unity. Consequently, x-rays are neither bent nor reflected to any appreciable extent as they interact with matter. The contrast that gives rise to an image in an x-ray film comes about from differences in materials' ability to absorb x-rays. The absorption follows *Beer's law*:

$$I = I_0 e^{-\alpha x}, \tag{4-8}$$

in which I is the intensity at a depth x, and α is the absorption coefficient. Absorption of x-rays in matter is governed by the *photoelectric effect*, in which the incident x-ray photon is absorbed by one of the bound electrons in an atom of material, and the electron is ejected; and by the *Compton effect*, in which the x-ray photon is scattered from a free or weakly bound electron in the material. The absorption due to the photoelectric effect process is proportional to the fifth power of the atomic number N, and increases with the wavelength (λ) (hence decreases with energy) in the x-ray energy range 100 to 350 keV:

$$\alpha = N^5\lambda^{7/2}. \tag{4-9}$$

At energies below 20 keV, a resonant effect occurs in which absorption becomes very strong when the x-ray energy is equal to the binding energy of the electrons in the inner "K" shell of the atoms of the absorber material. Clinical x-ray diagnostic equipment operates at tube voltages of from 20 to 200 kV. The emitted x-rays are at energies (in electron volts) equal to or less than the tube voltage. Most radiological techniques involve tube voltages between 60 and 100 kV, for which absorption by the photoelectric effect and the Compton effect are comparably important. Since the x-ray energy is

$$E = h\nu = \frac{hc}{\lambda}, \tag{4-10}$$

in which h is Planck's constant and c is the speed of light, the x-rays have wavelengths from 100 pm (0.1 nm) at 10 keV to 5 pm at 200 keV. These wavelengths are much smaller than those of visible light — 400 to 700 nm.

Table 4-4. Mass Absorption Coefficient for X-Rays in Various Materials

Material	Atomic no.	Density ρ (g/cm^3)	Specific absorption coefficient μ/ρ (cm^2/g)
Al	13	2.70	48.7
P	15	1.82	73
Ca	20	1.55	172
Cr	24	7.19	259
Fe	26	7.87	324
Co	27	8.9	354
Pb	82	11.34	241

For CuKα x-rays, wavelength λ =1.54 Å or 0.154 nm.

It is clear that the heavier elements absorb x-rays strongly, as shown in Table 4-4. Heavy metals such as lead are commonly used to shield x-ray equipment. Human soft tissue contains a great deal of the lighter elements (hydrogen, carbon, and oxygen) and is consequently relatively transparent to x-rays. Bone, by virtue of its calcium and phosphorus content, absorbs more strongly and therefore shows up well in x-ray images. Metallic implants absorb strongly and also are highly visible in x-ray images. Polymers, by contrast, are relatively transparent to x-rays. Barium sulfate is incorporated in bone cement to make it visible in diagnostic x-ray images.

4.4. ACOUSTIC AND ULTRASONIC PROPERTIES

The acoustic and ultrasonic properties of biomaterials are relevant in the context of their importance in diagnostic ultrasound images. Important properties are the acoustic velocity v, the acoustic attenuation α, and the material density ρ. The relation for the attenuation of ultrasound is identical to that for x-rays — Eq. (4-8).

Signals for ultrasonic imaging devices are generated by *reflection* of the waves from interfaces in the body, by contrast to diagnostic x-rays, in which differences in attenuation are exploited to create an image. The amplitude reflection coefficient associated with an interface between material 1, containing the incident wave, and material 2, is given by

$$R_A = \frac{Z_2 - Z_1}{Z_2 + Z_1}. \qquad (4\text{-}11)$$

The acoustic impedance Z is defined as

$$Z = \rho \text{v}, \qquad (4\text{-}12)$$

in which the acoustic velocity v is proportional to the square root of the stiffness (the modulus C_{1111} for longitudinal waves, or the shear modulus for shear waves) divided by the material mass density. By contrast to x-rays, the atomic number has no bearing upon ultrasound. The reflection of electromagnetic waves such as light is similarly determined from the index of refraction, which is also defined in terms of wave velocity.

Table 4-5. Acoustic Properties of Some Materials

Material	Velocity v(m/s)	Impedance Z (kRayl)	Attenuation coeff. α (dB/cm)
Air (at STP)	330	0.04	12
Water	1480	148	0.002
Fat	1450	138	0.63
Blood	1570	161	0.18
Kidney	1560	162	1.0
Soft tissues(avg)	1540	163	0.7
Liver	1550	165	0.94
Muscle	1580	170	1.3–3.3
Bone	4080	780	15
PMMA	2670	320	–
UHMWPE	2000	194	–
Ti6Al4V	4955	2225	–
Stainless steel	5800	4576	–
Barium titanate	4460	2408	–

kRayl = 10^4 kg/m^2/s

Comparative acoustic properties of some relevant materials are given in Table 4-5. Attenuations are given for a frequency of 1 MHz, at the lower end of the range used clinically. The ultrasonic attenuation α (per unit length) is directly related to the viscoelastic loss tangent (tan δ) discussed in Chapter 3; the attenuation therefore depends on frequency. The relation is

$$\alpha = \left(\frac{2\pi \nu}{v}\right) \tan\frac{\delta}{2}, \qquad (4\text{-}13)$$

in which ν is the frequency of the wave and v is its velocity. As for biomaterials, we observe that polymeric implants, which do not show up well in x-ray images, will usually be highly visible in a diagnostic ultrasound scan as a result of the differences in acoustic properties in comparison with natural tissues.

4.5. DENSITY AND POROSITY

The density ρ of a material is defined as the ratio of mass to volume for a sample of the material:

$$\rho = \frac{m}{v}. \tag{4-14}$$

A biomaterial that replaces an equivalent volume of tissue may have a different weight, as a result of the differences in density. In some applications this can be problematic. Densities of representative materials are given in Table 4-6.

Table 4-6. Density of Some Materials

Material	Density(g/cm^3)
Air (at STP)	0.0013
Fat	0.94
Polyethylene (UHMWPE)	0.94
Water	1.0
Soft tissue	1.01–1.06
Rubber	1.1–1.2
Silicone rubber	0.99–1.50
PMMA	1.19
Compact bone	1.8–2.1
Glass	2.4-2.8
Aluminum	2.8
Diamond	3.5
Titanium	4.5
Stainless steel	7.93
Wrought CoCr	9.2
Gold	19.3

Porous materials are used in a variety of biomedical applications, including implants and filters for extracorporeal devices such as heart-lung machines. In other applications such as bone plates, porosity may be an undesirable characteristic since pores concentrate stress and decrease mechanical strength. Perhaps the most important physical quantity associated with porous materials is the *solid volume fraction* (V_s). The *porosity*, often expressed as a percent figure, is given by

$$\text{Porosity} = 1 - V_s. \tag{4-15}$$

It is also noted that there are three measurements of volume, i.e., true, apparent, and total(bulk)volume:

$$\text{True volume} = \text{ total (bulk) volume} - \text{total pore volume}, \tag{4-16}$$

$$\text{Apparent volume} = \text{total (bulk) volume} - \text{open pore volume}, \tag{4-17}$$

$$\text{Total pore volume} = \text{open pore volume} + \text{closed pore volume}. \tag{4-18}$$

We can also have three types of densities corresponding to the three definitions of the volume.

Pore size is important in situations in which tissue ingrowth is to be encouraged, or if the permeability of the porous material is of interest. Porous materials may be characterized by a single pore size, or may exhibit a distribution of pore sizes. Porosity and pore size can be measured in a variety of ways. If the density of the parent solid is known, a measurement of the apparent density of a block of material suffices to determine the porosity. Mercury intrusion porosimetry is a more precise method that delivers a measurement of the porosity and the pore size distribution. In this method, mercury is forced into the pores under a known pressure and the relationship between pressure and mercury volume is determined. Since the mercury has a high surface tension and does not wet most materials, higher pressures are required to force the mercury into progressively smaller pores. If a single pore size predominates, it can be measured by optical or electron microscopy.

Example 4.5

A polyurethane open cell foam is to be used for a lining for an artificial leg. The solid volume fraction is 4%, the pores are 0.5 mm in diameter and the solid polyurethane has a density of 0.9 g/cm^3. Determine the porosity and apparent density of the material, and the weight of a sheet 200 mm by 200 mm by 1 cm thick.

Answer

$$\text{Porosity } P = 1 - V_s, \ \text{ so } \ P = 1 - 0.04 = 0.96, \ \text{ or } \underline{96\% \text{ porosity}}.$$

$$\text{Apparent density} = \ \rho_{\text{app}} = V_s \rho_{\text{true}} = 0.04[0.9 \text{ g/cm}^3] = \ \underline{0.036 \text{ g/cm}^3}.$$

$$\text{Mass} = V\rho_{\text{app}} = 20 \times 20 \times 1 \ (\text{cm}^3)\ 0.036 \text{ g/cm}^3 = \underline{14.4 \text{ g}}, \text{ so weight} = \underline{0.032 \text{ lb}}.$$

The weight of the foam lining is much less than that of an artificial leg. In some cases, the effect of implant density is important. For example, intraocular lenses made of PMMA are denser than the soft tissues of the eye, as shown in Table 4-6. During rapid eye movement, the lens is accelerated by the structures to which it is attached, e.g., the iris. By contrast, the natural eye lens is essentially neutrally buoyant. Damage to these eye structures by intraocular lenses has been observed clinically. Lenses of a silicone rubber composition of lower density are currently under investigation; these offer the added benefit of being softer, and hence easier to insert through a small incision. Another example of the density of implants is augmentation mammoplasty, in which the added weight of the implant can be problematical.

4.6. DIFFUSION PROPERTIES

The diffusion properties of materials are important in applications in which transport of biologically significant constituents is required. Examples include transport of oxygen and carbon dioxide from the atmosphere to the blood in the artificial lung component of the heart-lung machine and transport of oxygen to the cornea through contact lenses. The diffusion equation that governs the motion of dissolved materials under a gradient of concentration C is given by

$$\frac{\partial C}{\partial t} = D\nabla^2 C, \tag{4-19}$$

in which D is the diffusion coefficient, and ∇^2 is the Laplacian, which in one dimension reduces to $\partial^2 C/\partial x^2$. The driving force for material transport may be a pressure gradient rather than a concentration gradient. Moreover, the geometry in many biomedical applications may be approximated by a thin film. In that case, the volumetric flux F (in units of volume per unit time) across a layer of area A is given by

$$F = KA\ DP, \tag{4-20}$$

in which ΔP is the pressure difference across the layer, and K is the permeability coefficient. Representative permeabilities for oxygen transport are given in Table 4-7. Permeabilities for other gases are in general different. Carbon dioxide, for example, diffuses through these materials from two to five times more rapidly than oxygen.

Table 4-7. Gas Permeability of Various Materials

Material	Permeability to O_2	Applications
Silicone rubber	50	Contact lens, lung
Polyalkylsulfone	6	Lung
Polyethylenecellulose-perfluorobutyrate	5	Lung
Teflon film	1.1	Lung
Poly(HEMA)	0.69	Contact lens
PMMA	0.0077	Contact lens

Units: $[\mathrm{cm^3/sec}][(\mathrm{cm\ thick/cm^2})(\mathrm{cm\ Hg} \times 10^{-9})]$

When high oxygen transport is desired, a material with a large permeability coefficient should be chosen, if all other aspects of the materials under question are comparable. In the case of contact lenses, poly-HEMA (poly hydroxyethyl methacrylate) lenses are commonly used for soft lenses, even though the permeability is lower than that of silicone rubber. The permeability of poly-HEMA is adequate for the oxygenation of the cornea, and it is chosen for other reasons, such as ease of manufacture. As for membrane materials for oxygenators in heart-lung machines, oxygen transport depends on membrane thickness as well as on permeability. Minimum thickness is dictated by membrane strength, so a strong material with a lower permeability may result in the highest oxygen flux.

PROBLEMS

4-1. The crystalline lens of a normal human eye has radii of curvature 10.2 and 6 mm, is convex, and has a thickness of 2.4 mm. The refractive index varies with position but may be taken as 1.386. Determine the optical power and focal length, recognizing that in vivo the lens is immersed in ocular fluid. We remark that "crystalline" in this context is an anatomical term that means transparent; it does not mean the lens has a regular atomic arrangement. The lens is actually fibrous.

4-2. Barium compounds and iodine compounds are used to enhance contrast in diagnostic radiology. Why?

4-3. Show that a contact lens which fits the cornea exactly will provide the same optical correction whether the lens is in fact on the cornea or whether it is separated by an infinitesimal air gap. *Hint*: Consider two lenses of different indices of refraction and one common curvature. Determine the focal length for two cases: lenses in contact and lenses separated by a layer of air.

4-4. Consider a hemispherical silicone augmentation mammoplasty 130 mm in diameter. Determine the weight of two such mammoplasties. Compare with a corresponding volume of natural tissue. Discuss the implications.

4-5. Determine the reflection coefficient for ultrasonic waves crossing a muscle-to-bone interface. Some of the needed material properties may be found elsewhere in the book. Discuss the implications for ultrasonic imaging through bone. Discuss the use of other kinds of waves for imaging through bone.

4-6. Calculate the voltage generated in bone by mechanical deformation associated with walking. Use the given piezoelectric coefficient for bone, and find any other needed material properties elsewhere in the book.

4-7. It is often difficult to remove the bone cement from leg bones from which a hip joint prosthesis must be removed for revision arthroplasty. An experimental technique for this purpose is extracorporeal shock wave lithotripsy (ESWL), a method that was originally developed to shatter kidney stones by intense sonic shock pulses without surgery. Calculate the relative sound intensity of a wave after passing through 5 cm of muscle and 2 cm of bone, as a percent of the initial intensity.

4-8. Discuss the advantages and disadvantages of the extracorporeal shock wave lithotripsy method described in Problem 4-7.

SYMBOLS/DEFINITIONS

Greek Letters

α: Attenuation coefficient for ultrasound or for x-rays.

λ: Wavelength of light, x-rays, or ultrasound.

ν: Frequency of a wave.

ρ: Density.

ρ_e: Electrical resistivity.

∇^2: Laplacian operator, which in one dimension reduces to $\partial^2/\partial x^2$.

Latin Letters

c: Speed of light.

C: Concentration.

dB: Decibel — a logarithmic unit of ratios, often applied to sound pressure levels.

D: Diopter, see below.

E: Young's modulus, the ratio of stress to strain for simple tension or compression.

f: Focal length of a lens.

F: Volumetric flux: volume per unit time.

h: Planck's constant, associated with the quantum nature of light.

K: Permeability coefficient.

m: Mass.

n: Refractive index.

N: Atomic number.

R: Electrical resistance. Also, amplitude reflection coefficient.

S: Elastic compliance. For anisotropic materials the compliance tensor describes the strain-to-stress ratio for different directions. For example, $S_{1111} = 1/E_1$.

v: Ultrasonic wave speed.

V: Voltage.

$\boldsymbol{V_s}$: Solid volume fraction of porous material.

Z: Acoustic impedance, equal to density times sound velocity.

Words

Attenuation: Absorption of sound or ultrasound waves.

Compton effect: Mechanism for scattering of x-rays by free or weakly bound electrons in matter.

Diopter: Optical power of lens, or degree of ray divergence of a bundle of light rays. The power in diopters is the inverse of the focal length measured in meters.

Piezoelectricity: Electrical polarization of a material in response to mechanical stress.

Permeability: Ability of a material to pass a gaseous or ionic species, under a pressure gradient or a concentration gradient.

Photoelectric effect: Mechanism for the extinction of x-rays by the x-ray photon knocking bound electrons out of atoms.

Pyroelectricity: Electrical polarization of a material in response to temperature change.

Streaming potential: Electric potential developed when charged particles such as ions flow through a tube or porous medium such as bone; potential may be attributed to the imbalance created by washing away the electric double layer .

Maxwell's equations: Set of four field equations that govern electricity, magnetism, radio wave propagation, and the behavior of light.

Refractive index: Ratio of the speed of light in a vacuum to the speed of light in a material. It is a measure of the ability of a material to refract [bend] a beam of light.

Snell's law: Governing equation for the angle of incident and refracted light rays.

Tensor: A mathematical expression with well-defined transformation properties under changes in coordinates. A vector is a tensor of rank one; a scalar is a tensor of rank zero. Stress and strain are examples of tensors of rank two.

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