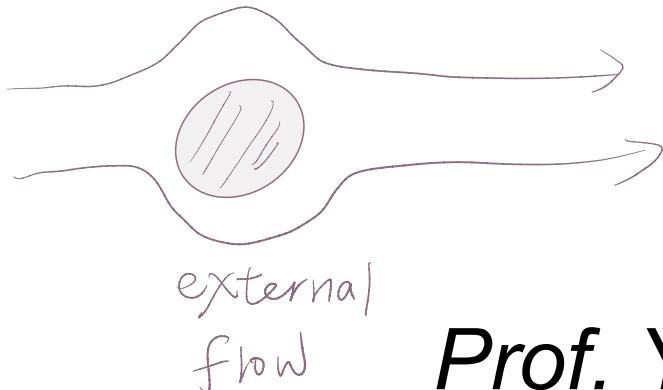


Fluid Mechanics

Internal Flow

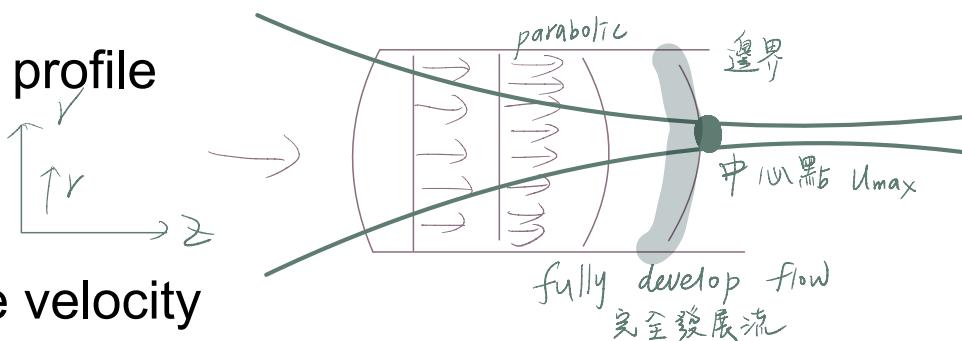


Prof. Yu-Jui Fan (范育睿)

Ray.yj.fan@tmu.edu.tw

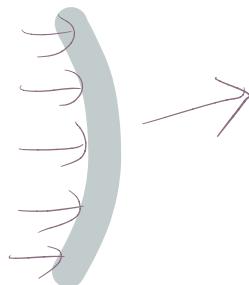
Internal Flow

- Velocity profile



- Average velocity

$$U(r) = U_{max} \left(1 - \left(\frac{r}{R} \right)^2 \right)$$



$$\begin{aligned} U_{avg} &= \frac{L}{A} \int_0^R u dA \\ &= \frac{1}{\pi R} \int_0^R U_{max} \left(1 - \left(\frac{r}{R} \right)^2 \right) 2\pi r dr \\ &= \int_0^R U_{max} \left(1 - \left(\frac{r}{R} \right)^2 \right) 2 \left(\frac{r}{R} \right) d \left(\frac{r}{R} \right) \end{aligned}$$

$$\text{let } \frac{r}{R} = x \Rightarrow \begin{cases} r=0 & x=0 \\ r=R & x=1 \end{cases}$$

$$\Rightarrow \int_0^1 U_{max} (1-x^2) 2x dx$$

$$\begin{aligned} &\Rightarrow \int_0^1 U_{max} (2x - 2x^3) dx = U_{max} \left[x^2 - \frac{1}{4} x^4 \right] \Big|_0^1 \\ &= \frac{U_{avg}}{U_{max}} U_{max} \left[(1-0) - \frac{1}{2} (1-0) \right] = \frac{1}{2} U_{max} \end{aligned}$$

Example

Volume flow rate

- (Zuvio) Water flow steadily through a 8-mm diameter pipe at a rate of 1.151 L/min. The flow is laminar. The velocity in the center of the pipe is

- a) 0.015 m/s
- b) 0.38 m/s
- c) 0.76 m/s
- d) 1.5 m/s
- e) 3.8 m/s

A V

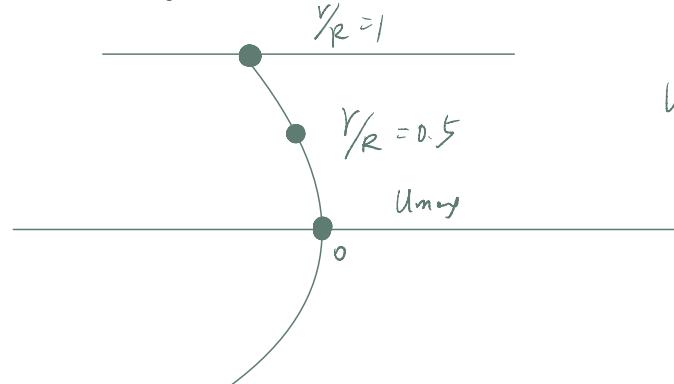
$$= 1.151 \text{ L/min} = \frac{1.151 \times 10^{-3}}{60} \left(\frac{\text{m}^3}{\text{s}} \right)$$

$$\begin{aligned} u_{\max} &= 2 \times 0.38 \\ &= 0.76 \text{ m/s} \end{aligned}$$

$$V_{\text{avg}} = \frac{A V}{A} = \frac{1.151 \times 10^{-3}}{60} \cdot \frac{1}{\pi \left(\frac{0.008}{4} \right)^2} = 0.38 \text{ m/s}$$

- The velocity halfway between the wall and center of the pipe is

- a) 0.04 m/s
- b) 0.19 m/s
- c) 0.38 m/s
- d) 0.57 m/s
- e) 0.76 m/s



$$\begin{aligned} V_{\text{avg}} &= u_{\max} (1 - (0.5)^2) \\ &= 0.75 u_{\max} \\ &= 0.75 \times 0.76 \approx 0.57 \end{aligned}$$

Example

- The velocity profile in a pipe of radius R , is given by $u(r) = u_{max}(1 - \frac{r}{R})^{1/6}$. Determine the average velocity in the pipe.

$$\begin{aligned} U_{avg} &= \frac{1}{\pi R^2} \int_0^R u_{max} \left(1 - \frac{r}{R}\right)^{\frac{1}{6}} 2\pi r dr \\ &= \frac{\int_0^R u_{max} (1-x)^{\frac{1}{6}} 2x dx}{\text{laminar}} \quad \text{wolfram Alpha} \\ &= u_{max} (0.291209) \quad \text{turbulent} \end{aligned}$$

Internal Flow

- Laminar or turbulent flow

- Osborne Reynolds

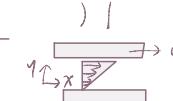
- Reynolds number

$$Re = \frac{\rho V D}{\mu}$$

diameter
inertial force
viscous force

特徵長度
 $\frac{\rho V L}{\mu}$

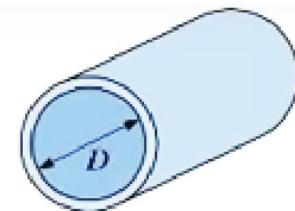
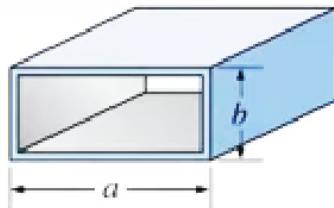
- Critical Re

$$\frac{\cancel{\text{inertial force}} / \cancel{\text{area}}}{\cancel{\text{viscous force}} / \cancel{\text{area}}} = \frac{\text{dynamic pressure}}{\text{viscous stress} (\tau = \mu \frac{du}{dy})} = \frac{\rho V^2}{\mu \frac{du}{dy}}$$
$$Re = \frac{\frac{1}{2} \rho V^2}{\mu \frac{du}{dy}} = \frac{\rho V^2}{\mu \frac{V}{L}} = \frac{\rho V L}{\mu}$$


laminar > 2000 | critical Re | turbulent flow < 4000

Internal Flow

- Non-circular pipe



	Rectangular	Circular
Area	$a \cdot b$	$\frac{\pi D^2}{4}$
Average velocity	$\frac{AV}{A} = \frac{(AV)}{ab}$	$\frac{AV}{A}$
Perimeter	$P = 2(a+b)$	$P = \pi D$
Hydraulic diameter 又称 直径	$\frac{4ab}{2(a+b)} = \frac{2ab}{(a+b)}$	$D_h = \frac{4\pi D^2}{4\pi D} = D$

$$D_h = \frac{4A}{P}$$

Internal Flow

- Entrance region
- Hydrodynamic fully developed
- Entry length
- Example:
Air, 1atm @ 15C ($\rho=1.225 \text{ kg/m}^3$, $\mu=1.802\times 10^{-5} \text{ kg/ms}$) flow steadily through a 5-cm by 8-cm cross section rectangular duct at an average velocity of 4 m/s
 - Determine the Reynolds number.
 - Is it laminar or turbulent

$$D_n = \frac{2ab}{a+b} = \frac{2 \times 5 \times 8 \times 10^{-4}}{13 \times 10^{-2}} = 6.154 \times 10^{-2}$$

$$Re = \frac{\rho V D_n}{\mu} = \frac{1.225 \times 4 \times 6.154 \times 10^{-2}}{1.802 \times 10^{-5}}$$

= 16700 \rightarrow turbulent flow

(大於 4000)