Homework

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- Calculate the gradient of divergence: $\mathrm{P} = 2xyz^2$

$$\nabla \cdot P = \frac{\partial}{\partial x} + \frac{\partial P}{\partial y} + \frac{\partial P}{\partial z} = 2yz^2 + 2xz^2 + 4xyz$$

令 $\nabla \cdot P = V$

$$\nabla(\nabla \cdot V) = \nabla V = \frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}$$

$$= (2z^2 + 4yz)\hat{i} + (2z^2 + 4xz)\hat{j} + (4yz + 4xz + 4xy)$$

- Calculate the Laplacian: $\mathrm{P} = 2xyz^2$

$$\nabla^2 P = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} = 4xy$$

- Calculate the divergence of

$$\vec{V} = (z(b - cx)^2)\,\widehat{e_x} + (2cy + 2xy)\widehat{e_y} + (3axz + bxy)\,\hat{e}_z$$

$$\nabla \cdot \vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

$$= -2cz(b - cx) + 2c + 2x + 3ax$$

- Calculate the gradient of P in cylindrical coordinates: $\mathrm{P} = 2rz + 3z^2$

$$\nabla P = \frac{\partial P}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial P}{\partial \theta}\hat{\theta} + \frac{\partial P}{\partial z}\hat{z}$$

$$= (2z)\hat{r} + 0\,\hat{\theta} + (2r + 6z)\hat{z}$$

- Calculate the divergence of $\vec{V}$ in cylindrical coordinates

$$\vec{V} = 5rz\widehat{e_r} + 2r\widehat{e_\theta} + 3rz^2\widehat{e_z}$$

$$\nabla \vec{V} = \frac{1}{r}\frac{\partial r V_r}{2r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z}$$

$$= \frac{1}{r}\frac{\partial 5rz}{2r} + \frac{1}{r}\frac{\partial 2r}{\partial \theta} + \frac{\partial 3rz^2}{\partial z} = 10z + 6rz$$