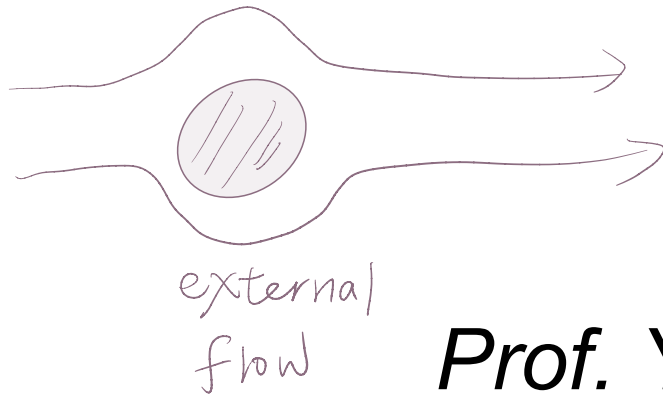


Fluid Mechanics

Internal Flow

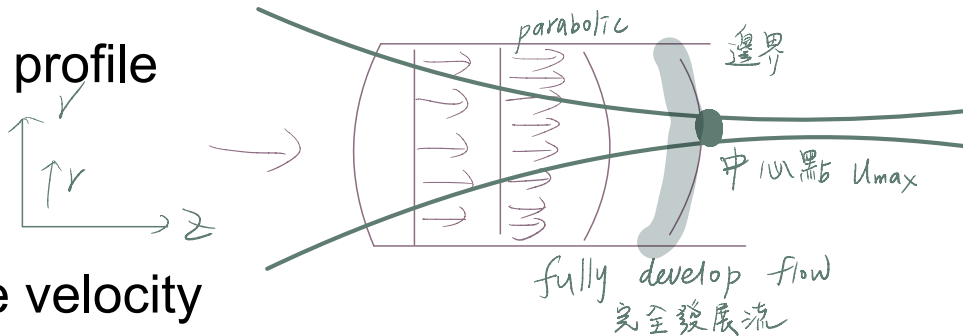


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Internal Flow

- Velocity profile



- Average velocity



$$U(r) = U_{max} \left(1 - \left(\frac{r}{R} \right)^2 \right)$$

$$\begin{aligned} U_{avg} &= \frac{1}{A} \int_0^R u \, dA \\ &= \frac{1}{\pi R^2} \int_0^R U_{max} \left(1 - \left(\frac{r}{R} \right)^2 \right) 2\pi r \, dr \\ &= \int_0^R U_{max} \left(1 - \left(\frac{r}{R} \right)^2 \right) 2 \left(\frac{r}{R} \right) d \left(\frac{r}{R} \right) \end{aligned}$$

$$\text{let } \frac{r}{R} = x \Rightarrow \begin{cases} r=0 & x=0 \\ r=R & x=1 \end{cases}$$

$$\Rightarrow \int_0^1 U_{max} (1 - x^2) 2x \, dx$$

$$\Rightarrow \int_0^1 U_{max} (2x - 2x^3) \, dx = U_{max} \left[x^2 - \frac{2}{4} x^4 \right] \Big|_0^1$$

$$= U_{avg} U_{max} \left[(1-0) - \frac{1}{2} (1-0) \right] = \frac{1}{2} U_{max}$$

Example

volume flow rate

- (Zuvio) Water flow steadily through a 8-mm diameter pipe at a rate of 1.151 L/min. The flow is laminar. The velocity in the center of the pipe is

- a) 0.015 m/s
- b) 0.38 m/s
- c) 0.76 m/s
- d) 1.5 m/s
- e) 3.8 m/s

c

AV

$$= 1.151 \frac{\text{L}}{\text{min}} = \frac{1.151 \times 10^{-3}}{60} \left(\frac{\text{m}^3}{\text{s}} \right)$$

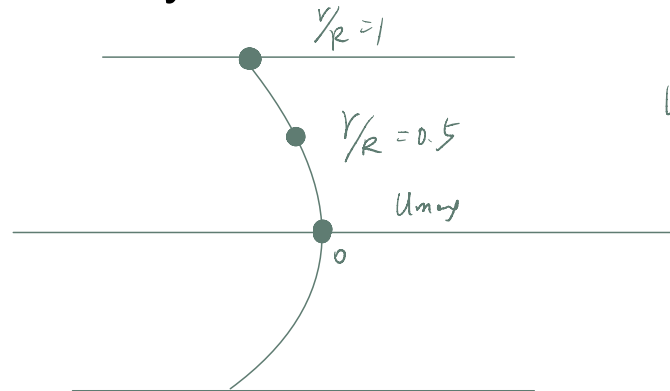
$$U_{\max} = 2 \times 0.38 = 0.76$$

$$V_{\text{avg}} = \frac{AV}{A} = \frac{1.151 \times 10^{-3}}{60} \cdot \frac{1}{\pi \frac{(0.008)^2}{4}} = 0.38 \frac{\text{m}}{\text{s}}$$

- The velocity halfway between the wall and center of the pipe is

- a) 0.04 m/s
- b) 0.19 m/s
- c) 0.38 m/s
- d) 0.57 m/s
- e) 0.76 m/s

d



$$U_{\text{avg}} = U_{\max} (1 - (0.5)^2) = 0.75 U_{\max} = 0.75 \times 0.76 = 0.57$$

Example

- The velocity profile in a pipe of radius R , is given by $u(r) = u_{max}(1 - \frac{r}{R})^{1/6}$. Determine the average velocity in the pipe.

$$\begin{aligned} U_{avg} &= \frac{1}{\pi R^2} \int_0^R u_{max} \left(1 - \frac{r}{R}\right)^{1/6} 2\pi r dr \\ &= \frac{\int_0^R u_{max} (1-x)^{1/6} 2x dx}{\text{wolfram Alpha}} \\ &= u_{max} (0.791209) \quad \begin{array}{l} \text{laminar} \\ \text{turbulent} \end{array} \end{aligned}$$

Internal Flow

- Laminar or turbulent flow

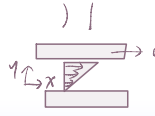
- Osborne Reynolds

- Reynolds number

$$Re = \frac{\rho V D}{\mu} \quad \begin{array}{l} \text{inertial force} \\ \text{viscous force} \end{array} \quad \begin{array}{l} \text{diameter} \\ \text{特徴長度} \end{array}$$

- Critical Re

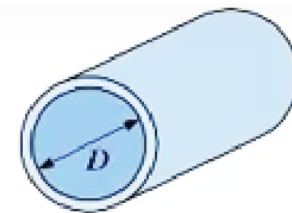
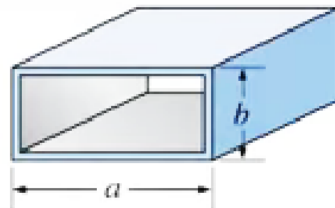
$$\frac{\text{inertial force} / \text{area}}{\text{viscous force} / \text{area}} = \frac{\text{dynamic pressure}}{\text{viscous stress}} \quad \left(\tau = \mu \frac{du}{dy} \right)$$

$$Re = \frac{\frac{1}{2} \rho V^2}{\mu \frac{du}{dy}} = \frac{\rho V^2}{\mu \frac{V}{L}} = \frac{\rho V L}{\mu}$$


laminar > 2000 critical Re 4000 turbulent flow

Internal Flow

- Non-circular pipe



| | Rectangular | Circular |
|----------------------------|--|-------------------------------------|
| Area | $a \cdot b$ | $\frac{\pi D^2}{4}$ |
| Average velocity | $\frac{AV}{A} = \frac{(AV)}{ab}$ | $\frac{AV}{A}$ |
| Perimeter | $p = 2(a+b)$ | $p = \pi D$ |
| Hydraulic diameter 水 直径 | $\frac{4ab}{2(a+b)} = \frac{2ab}{(a+b)}$ | $D_h = \frac{4\pi D^2}{4\pi D} = D$ |

$$D_h = \frac{4A}{p}$$

Internal Flow

- Entrance region
- Hydrodynamic fully developed
- Entry length
- Example:
Air, 1atm @ 15C ($\rho=1.225 \text{ kg/m}^3$, $\mu=1.802 \times 10^{-5} \text{ kg/ms}$) flow steadily through a 5-cm by 8-cm cross section rectangular duct at an average velocity of 4 m/s
 - Determine the Reynolds number.
 - Is it laminar or turbulent

$$D_h = \frac{2ab}{a+b} = \frac{2 \times 5 \times 8 \times 10^{-4}}{13 \times 10^{-2}} = 6.154 \times 10^{-2}$$

$$Re = \frac{\rho V D_h}{\mu} = \frac{1.225 \times 4 \times 6.154 \times 10^{-2}}{1.802 \times 10^{-5}}$$

$$= 16700 \rightarrow \text{turbulent flow}$$

(大於 4000)