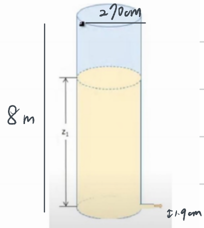
B812109014
江平彤
醫工3

Example

- A cylindrical water tank having 270-cm-diameter and 8-m-height. Water exits through a 1.9-cm-diameter pipe located at the bottom of the tank. The velocity of the water exiting varies according to $(2gz)^{1/2}$ where z is the water level in the tank and g=9.81 m/s^2. The tank starts with an initial volume of 40 m^3 and drains until it has a final volume of 5 m^3 of water. Determine the

1. Initial height of water in the tank
2. Initial velocity of water exiting the tank
3. Final height of water in the tank
4. Final velocity of water exiting the tank
5. Time takes to drain from 40 m^3 to 5 m^3.



1、半徑 $= 270 \div 2 = 135\ cm = 1.35m$

$(1.35)^2 \pi \doteqdot 5.725\ m^2$ (圓柱底面積)

$z_1 = 40m^3 \div 5.725m^2 = 6.986\ m$

2、The velocity of the water exiting varies according to $\sqrt{2gz}$ ($g = 9.81\ m/s^2$)

$\sqrt{2gz} = \sqrt{2 \times 9.81 m/s^2 \times 6.986m}$

$\doteqdot \sqrt{137.06532}\ m/s$

$\doteqdot 11.707\ m/s$

3、$5m^3 \div 5.725m^2 \doteqdot 0.873\ m$

4、代入 velocity $= \sqrt{2gz}$

$= \sqrt{2 \times 9.81 m/s^2 \times 0.873m}$

$\doteqdot \sqrt{17.12826}\ m/s$

$\doteqdot 4.139\ m/s$

5、linear homogeneous differential equation

$$\frac{dV}{dt} = -A \times v \Rightarrow \int dV = \int A v\, dt$$

出水口表面積

水量

velocity of exciting water

$$A = \left(\frac{0.019m}{2}\right)^2 \times \pi = 0.0002835 \doteqdot 2.835 \times 10^{-4}\ m^2$$

$$35m^3 = 2.835 \times 10^{-4} \times \left(\int_{0.873}^{6.986} \sqrt{2gz}\, dz\right) \times t \Rightarrow 35m^3 = 2.835 \times 10^{-4} \times 26.04 \times t$$

$$= \left(\sqrt{2 \times 9.8} \times \frac{1}{3} z^{\frac{3}{2}}\right) \Big|_{0.873}^{6.986}$$

$$= (4.427 \times \frac{1}{3})\left[(6.986)^{\frac{3}{2}} - (0.873)^{\frac{3}{2}}\right]$$

$$= 1.4756 \times (18.4647 - 0.8156)$$

$$= 1.4756 \times 17.6491$$

$$= 26.04$$

$$t = \frac{35}{2.835 \times 10^{-4} \times 26.04}$$

$$= 4741\ s$$

Example

- Water flows in a pipe where at point 1, pressure is 160 kPa, velocity is 1.5 m/s and elevation is 16m.
  At point 2, the pressure is 175kPa, and velocity is 2.3 m/s
  What is the maximum possible elevation at point 2?

According to Bernoulli's principle

$$p_1 + \frac{1}{2} v_1^2 \rho + \rho g h_1 = p_2 + \frac{1}{2} v_2 \rho + \rho g h_2$$

$\rho = 1000\ kg/m^3$

$P_1 = 160\,kPa$ $\quad P_2 = 175\,kPa$

$V_1 = 1.5\ m/s$ $\quad V_2 = 2.3\ m/s$

$h_1 = 16m$ $\quad g = 9.8\ m/s^2$

$$\Rightarrow 160(kPa) + \frac{1}{2}(1.5^2\ m/s) \times 1000(kg/m^3) + 1000(kg/m^3) \times 9.8(m/s^2) \times 16(m)$$
$$= 175(kPa) + \frac{1}{2}(2.3^2\ m/s) \times 1000(kg/m^3) + 1000(kg/m^3) \times 9.8(m/s^2) \times h_2$$

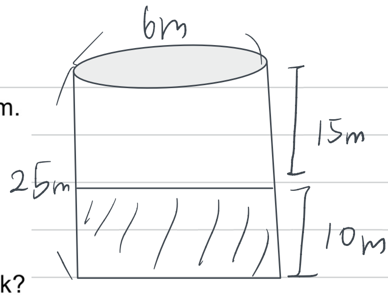
$$\Rightarrow 1000(kg/m^3) \times 9.8(m/s^2) \times (16 - h_2)(m) = \frac{1}{2} \times [(2.3^2\ m/s) - (1.5^2\ m/s)] \times 1000(kg/m^3) + 15(kPa)$$

$$\Rightarrow (16 - h_2)(m) = \frac{1}{2} \cdot \frac{(5.29 - 2.25)}{9.8} + \frac{15(kPa)}{1000 \times 9.8}$$

$$\Rightarrow h_2 \doteq 16 - \left(0.155 + \frac{15(m/s)^2}{9.8}\right)$$
$$\doteq 14.31m$$

- A large cylindrical tank stands 25 m tall and has a diameter of 6m.
  The tank is filled to 10 m with water
  Air above the water is pressurized to 300 kPa
  A valve is located at the bottom of the tank.
  The valve has diameter of 2cm.

  What is the minimum time required to fill a 5L bottle from the tank?



According to Bernoulli's principle

$$P_1 + \cancel{\tfrac{1}{2}\rho V_1^2} + \rho g h_1 = P_2 + \tfrac{1}{2}\rho V_2^2 + \rho g h_2$$

$$300\ kPa + 1 \times 9.8 h_1 = 300\ kPa + 1000\ kg/m^3 \times 9.8\ m/s^2 \times 15\ m$$
$$+ \rho \frac{V_2^2}{2} + 1 \times 9.8 \times h_2$$

$$\Rightarrow 300\ kPa + 9.8\ m/s^2\ h_1 = 300\ kPa + 147000\ kg/ms^2 + \tfrac{1}{2}\rho V_2^2 + 9.8\ m/s^2 \times h_2$$

$$9.8\ m/s^2 \cdot h_1 = 147000\ kg/ms^2 + \tfrac{1}{2}\rho V_2^2 + 9.8 h_2$$

$$\tfrac{1}{2}\rho V_2^2 = -147\ kg/ms^2 + 147\ m^2/s^2 \cdot kg/m^3$$

$$P_2 + \tfrac{1}{2}\rho V_2^2 = P_1$$

$$P_{Air} = 300\ kPa$$

$$147\ kPa + \tfrac{1}{2} V_2^2 = 300\ kPa$$

$$\tfrac{1}{2} V_2^2 = 153\ kPa$$

$$V_2 = 17.49\ m/s$$

$$A = 17.49 \times 3.14 \times 10^{-4}$$
$$= 0.0054\ m^3$$

$$0.005 \div 0.0054 = 0.925\ s$$

Example

- A pump increases the pressure of water from 100 kPa to 900 kPa.

  The mechanical energy increase of the water, in kJ/kg, is
  (A) 0.8
  (B) 8
  (C) 80
  (D) 800
  (E) 800,000

$900\ kPa - 100\ kPa = 800\ kPa$

$\Delta h = (P_2 - P_1)/\rho$

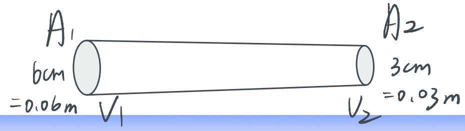
$= 800\ kPa / \rho$

$= 8 \times 10^5 \left(\frac{kg}{ms^2}\right) / 1000 \left(\frac{kg}{m^3}\right)$

$= 800\ \frac{m^2}{s^2}$

$= \boxed{}\ (kJ) \times (kg \times m^2 / kg \cdot s^2)$

$\Rightarrow \boxed{} = 0.8$ 選 A



Venturi Meter: example problem

- Air is flowing through a venturi flow meter. The inlet diameter is 6 cm and the throat is 3cm. The pressure measured at the inlet is 210 kPa and 160 kPa at the throat. Assume the air density is 1.2 kg/m³ and estimate the flow rate of air.

According to Bernoulli's Principle: ① $P_1 + \frac{1}{2}V_1^2\rho + \underbrace{\cancel{\rho g h_1}}_{\text{無}} = P_2 + \frac{1}{2}V_2^2\rho + \underbrace{\cancel{\rho g h_2}}_{\text{無}}$

$A_1 V_1 = A_2 V_2$

$\Rightarrow$ ② $V_2 = \frac{A_1}{A_2} V_1$

① + ②:

air density 1.2 kg/m^3

$P_1 + \frac{1}{2}V_1^2\rho = P_2 + \frac{1}{2}\left(\frac{A_1}{A_2}V_1\right)^2\rho$

$210\,(kPa) + \frac{1}{2}V_1^2 \times 1.2\,(kg/m^3) = 160\,kPa + \frac{1}{2}\left(\frac{0.06(m)}{0.03(m)}V_1\right)^2 \times 1.2\,(kg/m^3)$

$50\,(kPa) = \frac{1}{2}(4V_1^2)\times 1.2 - \frac{1}{2}V_1^2 \times 1.2$

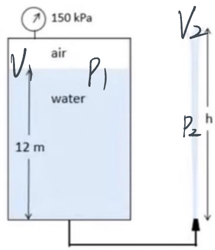
$50\times10^3\,(kg/ms^2) = \frac{1}{2}(3V_1^2)\times 1.2 = 1.8\,V_1^2$

$V_1^2 = 27770\,(m^2/s^2)$

$\boxed{V_1 \fallingdotseq 166.643\ m/s}$

- A sealed water tank is filled with 12m of water and air at 150 kPa gage pressure. The tank is connected to a hose with a nozzle that shoots the water straight up

What is the maximum height that the water could reach?



According to Bernoulli's Principle : $P_1 + \frac{1}{2}\cancel{V_1^2}\rho + \rho g h_1 = P_2 + \frac{1}{2}\cancel{V_2^2}\rho + \rho g h_2$

$P_1 + \rho g h_1 = P_2 + \rho g h_2$

$\Rightarrow 150(kPa) + 1000\,(kg/m^3) \times 9.8\,(m/s^2) \times 12(m) = 101.325\,(kPa) + 1000\,(kg/m^3) \times 9.8\,(m/s^2) \times h_2$

$\Rightarrow 150 - 101.325\,(kPa) = (h_2 - 12)(m) \times 1000\,(kg/m^3) \times 9.8\,(m/s^2)$

$\Rightarrow 12m + \dfrac{48.675\,kPa}{1000\,(kg/m^3) \times 9.8\,(m/s^2)} = h_2$

$\Rightarrow h_2 = 16.96\,m$